Fuzzy Sets, Logic and Systems and Applications
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Lecture – 51
Mamdani Fuzzy Model

Hi, welcome to lecture number 51 of Fuzzy Sets, Logic and Systems and Applications. In this lecture I will discuss Mamdani Fuzzy Model.

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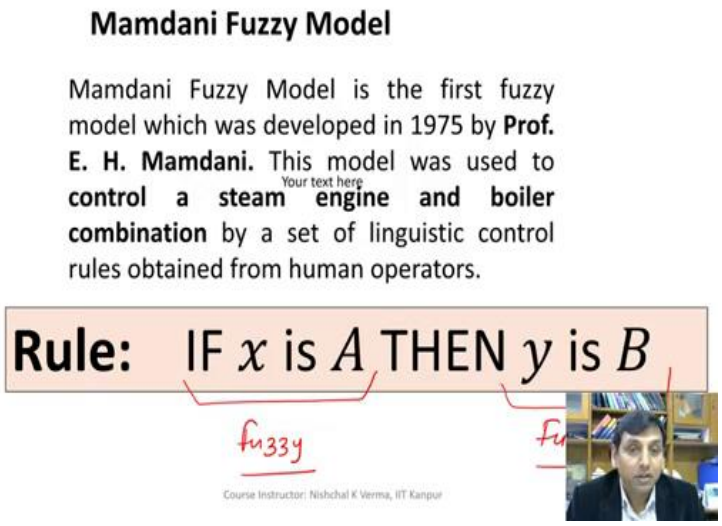

So, the Mamdani Fuzzy Model basically is the first fuzzy model which was developed in 1975, by Professor E. H. Mamdani. At this model was used to control a steam engine and boiler combination by a set of linguistic control rules obtained from human operators, that means, the experience, human experience.

So, Mamdani model is a very interesting model, in the sense that we use the set of fuzzy rules of a specific type. So, Mamdani model we use when we have fuzzy rules, fuzzy if-then rules of the type here, where in the fuzzy if-then rules we have see here the premised part and here we have the consequent part. So, when both the parts of the rule, the premise as well as the consequent both the parts are fuzzy, then we use Mamdani model.

So, in other words we can say that, Mamdani model is used when we have a set of rules, set of fuzzy rules available is like this where the premise part of the rule is fuzzy and the

consequent part of the rule is also fuzzy. So, this means that both the parts the premise and consequent are fuzzy. Please also note that the Larsen also, Larsen model which is also one of the fuzzy models is also using the same kind of if-then rules.

So, since we are discussing here, the Mamdani fuzzy model. So, we will be dealing all the rules of this type, all the fuzzy rules of this type.

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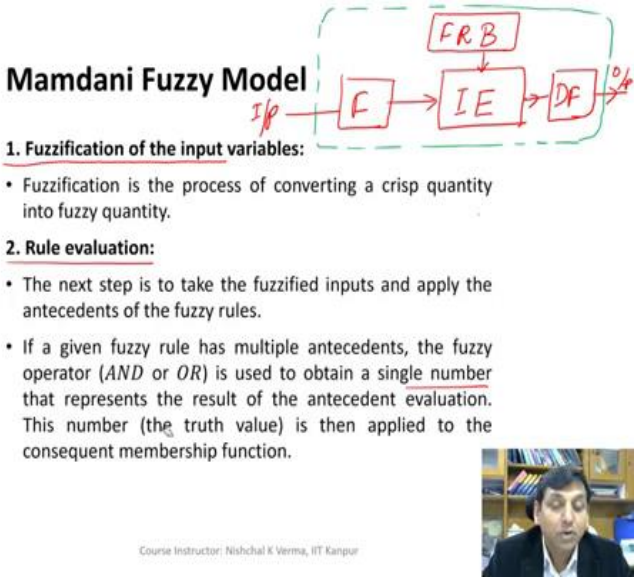

So, we already know that we have a fuzzy inference system and fuzzy inference system has multiple components. How many components? 4 components. The first component is the fuzzifier which I am denoting by F. And the second component here is the inference engine, I am denoting this by IE. The third component here is the fuzzy rule base, I am writing here as FRB and the fourth component here is the defuzzifier. I am denoting this by DF.

So, whenever any input comes to the fuzzy inference system here, I will just mark the fuzzy inference system like this here by here. So, this is what is the fuzzy inference system, a typical fuzzy inference system. And inside it we have 4 blocks, 4 components which are acting and to give the suitable output corresponding to the input data fed, input data fed. So, here we have the output.

So, exactly the same are mentioned here, you see here the fuzzification the first the fuzzification and then the rule evaluation means we should have a set of fuzzy rules, fuzzy

if-then rules which basically helps in helps the inference engine to generate the suitable output, suitable fuzzy output. So, it's written over here that, if a given fuzzy rule has multiple antecedent. So, a fuzzy rule can have can be a various kind which I will be discussing. And the fuzzy operators AND or OR is used to obtained a single number that represents the result of the antecedent evaluation this number is then applied to the consequent membership function.

So, in nutshell what is happening here is that, we have a fuzzy rule base which helps the inference engine to give the output to generate the output corresponding to the input the fuzzy input that is fed to IE.

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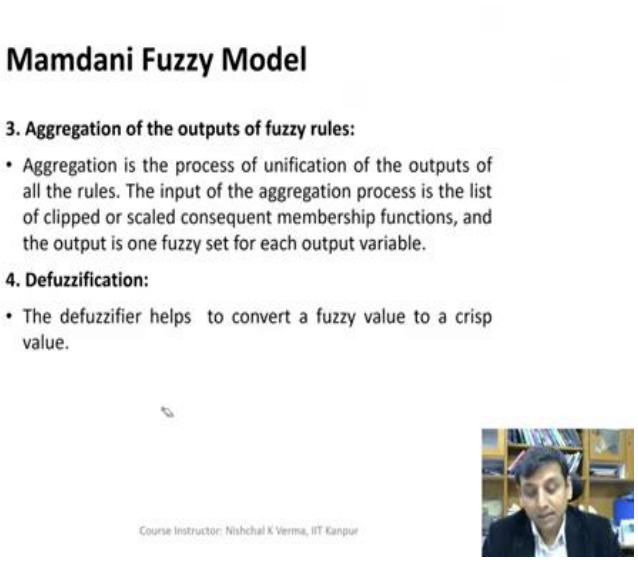

And when it is done, the output is basically the every rule is generating certain output. So, we aggregate all the outputs corresponding to the fuzzy rules and then the aggregated output is in Mamdani fuzzy model we have the aggregated output which is fuzzy output, which is a fuzzy quantity. So, then we need to defuzzify this output to generate a crisp output, crisp value.

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So, let us understand how a Mamdani fuzzy model works for the following cases. So, I am going to talk about 3 different cases, where we have the Mamdani model and Mamdani model can use the max-min composition or max-product composition which is mentioned here. And then again the input that is fed to the fuzzy model can be either the fuzzy or crisp.

So, then we have 3 cases, first case here is that the single rule when we have a model with single rule, with single antecedent. So, this means that if I have let us say a fuzzy model, which is having only a single rule and that rule is with the single antecedent. And then we have another case where the model can be with single rule with multiple antecedents.

Then the third case could be the multiple rules with multiple antecedents. So, all these 3 scenarios will be discussed in coming slides, in this lecture. And again these 3 scenarios we will be discussed when we will be using max-min composition, max-product compositions and again for the fuzzy inputs and crisp inputs. So, let us go one by one.

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So, now let us start with the first scenario, where we have a Mamdani fuzzy model. And this Mamdani fuzzy model has only single rule with single antecedent.

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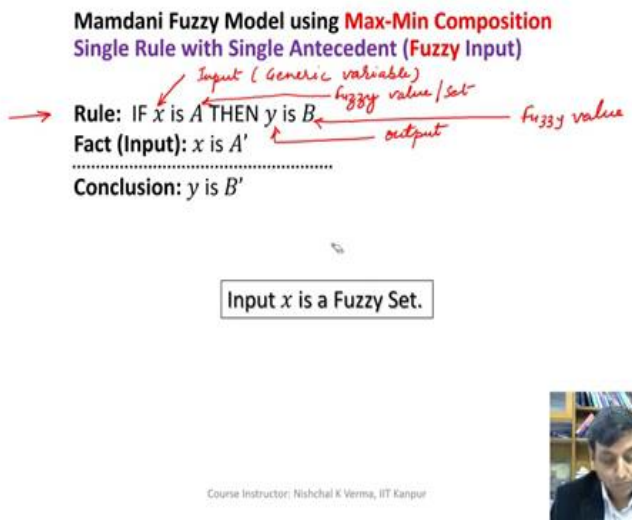

You can see the rule here in the Mamdani model, Mamdani fuzzy model. So, the rule basically says $IF\ x\ is\ A\ THEN\ y\ is\ B$, so we already know what is x. So, x is nothing but the generic variable, the input and this input is nothing, but the generic variable.

Similarly, A is a fuzzy region, A is some fuzzy, some fuzzy value; that means, the fuzzy set. And what is y here? y is the output. And B here is basically, B here is again the fuzzy value. So, when we have a single rule and the input is coming to the model, let us see what happens. So, as I have already mentioned in FIS, when we discussed FIS we saw that we have 5 block, 4 blocks the first block takes the input, and it converts the input into a fuzzy value.

So, this means that this the input is first fuzzified, then it passed on to the inference engine where various compositions and aggregations are done with the help of fuzzy rule set that is available. And then this output that is generated out of the inference engine is normally a fuzzy output and which is converted into crisp value by the defuzzification, defuzzifier or defuzzification.

So, let us now give an input here to this fuzzy model and let us see what happens. So, we have a single rule only, only one rule with one antecedent, means with a single antecedent. So, when we say single antecedent means we have the only one input, this means x is the single antecedent, $x\ is\ A$ is the single antecedent.

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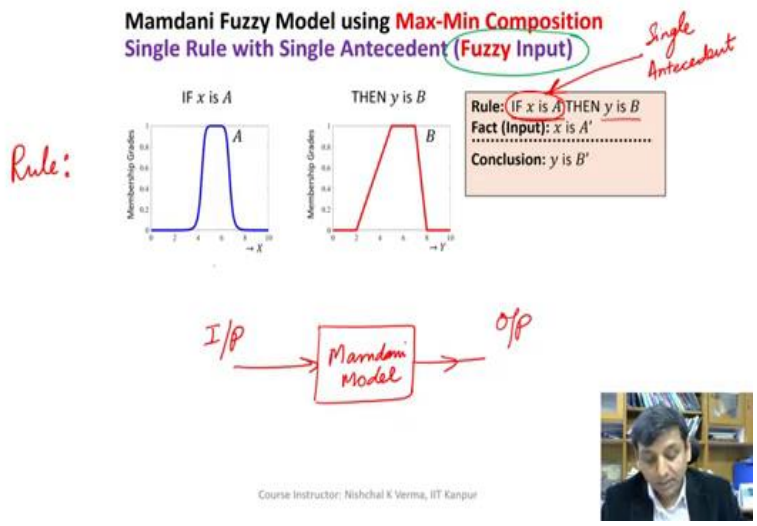

So, here since the rule is already with us, here we have the rule and this rule is in this form, $if\ x\ is\ A\ then\ y\ is\ B$. And rule is single as we can already see and also here the antecedent also is single. So, I can write here a single antecedent, antecedent. So, single antecedent

and single rule, all right. So, now, when we have this kind of scenario in Mamdani model or this kind of scenario of Mamdani model.

If we substitute or if we feed an unknown input, what is this fuzzy model is going to give us? So, this means what? This means that if we have let us say model here, a fuzzy model and this form fuzzy model is the Mamdani model, Mamdani type of model Mamdani model. And if I am giving an input here some input x is equal to something and this model is going to give us some output here which normally is the crisp output. So, let us see what we are going to get.

And as I have already mentioned that this model here in this case has this Mamdani model has a single fuzzy rule. So, this model has a single fuzzy rule, and also this rule has only a single antecedent, which I have already mentioned. Now, if a new input comes, if a new unknown input is comes unknown means this that input which this model has never seen. So, if some input is coming to this model as input and then this model is going to generate some output corresponding to that input.

Now, what is this output is going to be if my input is a fuzzy input or maybe a crisp input. So, when we say fuzzy input it means we provide, we feed a fuzzy value as the input. So, there can be 2 kinds of input; one is here is the fuzzy input, directly fed to the model or maybe the crisp input fed to the model. So, when we say fuzzy input. So, fuzzy input means a fuzzy set or fuzzy value is directly given to the model. And when we say crisp input then we feed some value of x. So, let us take the first case where we are giving the fuzzy input. So, when we say fuzzy input it means at x is a fuzzy value.

So, here we see that we have the fuzzy input. So, fuzzy input means x is, x as the input is fuzzy. So, the model is ready, model is known when we say model is known means this rule is known here, they and this model has a single rule and the premise part of this rule is $x\ is\ A$, which you can see here, $x\ is\ A\ and\ y\ is\ B$. So, this means that for any x which is falling in A fuzzy region, corresponding to that the output is going to fall in y in B fuzzy region.

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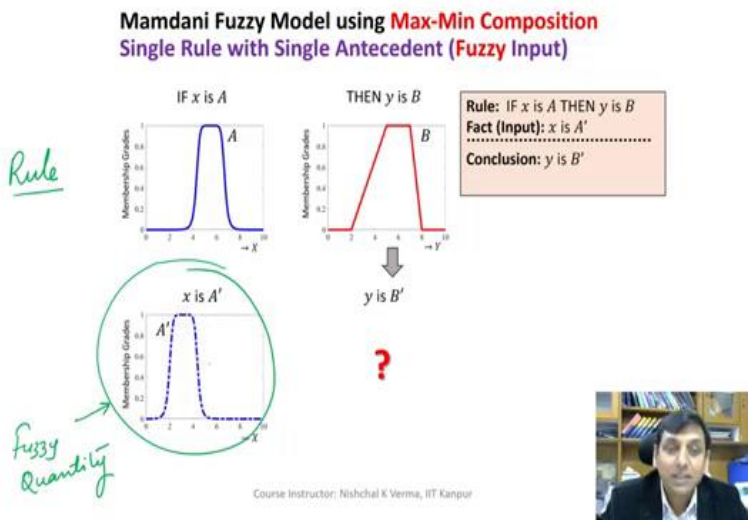

So, here let us now, let us now apply the input to this fuzzy model, where we have a single rule with single antecedent.

So, let us do this quickly and see what we are going to get. So, our input is A', so our input is x which is A'. A' is here this is a fuzzy quantity, this is a fuzzy quantity. So, fuzzy quantity is always in the form of a fuzzy set. So, this is a fuzzy quantity or fuzzy value or it's a fuzzy set basically. So, we see that here we have a fuzzy set, which is fed to the model as input. So, now, let us apply this and see what is the corresponding output that we are going to get.

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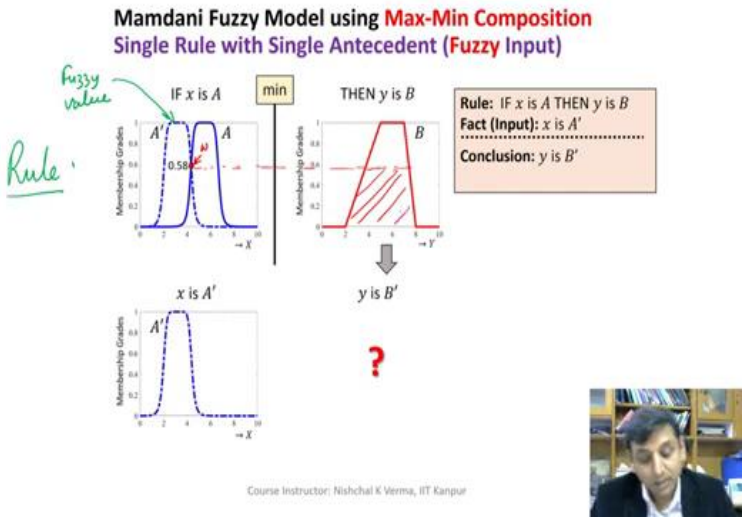

So, what we have done here is that we have the rule that is with us single rule and this is the rule $x\ is\ A\ then\ y\ is\ B$.

Now, the A', the fuzzy see here the fuzzy value as the input that is fed to the model, fuzzy value here. So, when this is fed to the model, now let us superimpose this A' with A and when we superimpose these 2 fuzzy sets. So, A' is the fuzzy value which is given as the input to the model and A was already known, A is already existing here in the fuzzy rule.

Now, when we superimpose these 2 fuzzy sets, A' and A, we see here that these 2 are intersecting to some point and this is the point of intersection. So, here when we see we find that corresponding to this intersection point we its membership value is 0.58. So, we note this point 0.58, the intersection point and please note that there may be multiple points which point of intersection you might get.

So, when we get multiple points, then we take the maximum of these two and the maximum will apply. So, we take the maximum of the intersection points and then the with the maximum we will proceed.

So, the next step here is two, in this case we have 0.58, 0.58 as the value of the point of intersection, this we call as the weight. So, we call this as the w, we call this as the w. So, this is the point of intersection. Now with this value, with this weight we truncate the

output. So, output you see here and with this value of w, see here this is my w and with this value if we truncate the output fuzzy set B.

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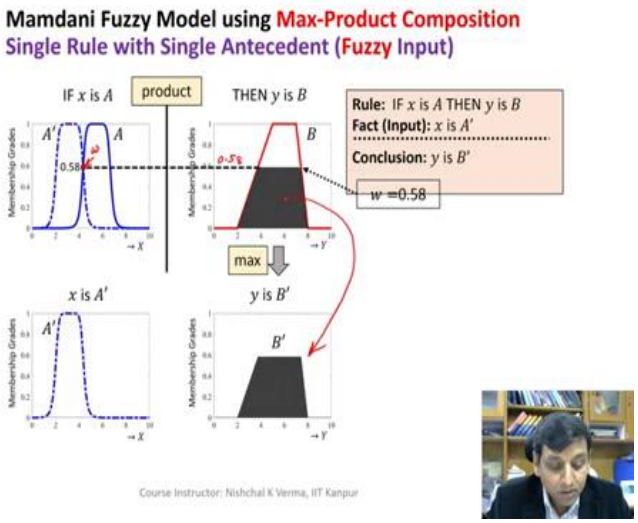

So, what is that we are getting here is the this solid area the truncated area which is blackened. So, this area is the output corresponding to the fuzzy input A', we may call this fuzzy area which is the blackened one, the shaded one is B'. So, here the corresponding to the fuzzy value, is fuzzy value is fuzzy input we are getting the output and again this output is the fuzzy output.

So, this means that when we have a fuzzy when we have a Mamdani fuzzy model with single rule with single antecedent we get the output like this corresponding to fuzzy input. So, now here in this case the output has been fuzzy has been corresponding to the fuzzy input. Now, what if we have the crisp input for the same model?

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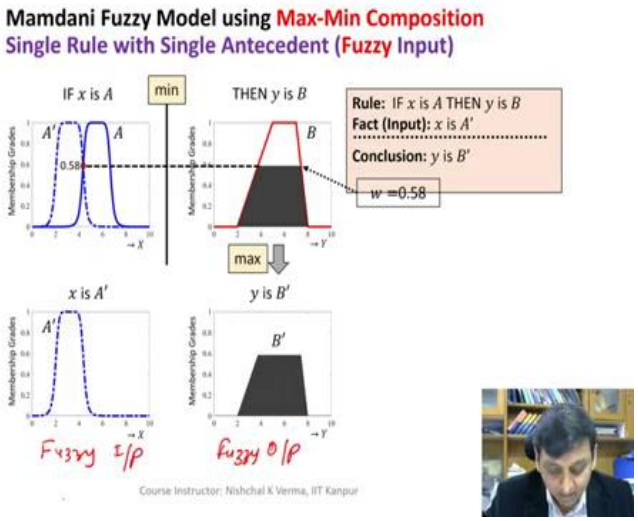

So, let us and before that here I would like to mention one more thing that since we are using the max-min composition. So, here we have the *w* which is here the *w*, if we would have multiple antecedents this min would have been applicable. Because here we have a single point of intersection, so we are getting only one *w*.

So, single antecedent I would say single antecedent. So, for single antecedent we have single *w*, even if we would have multiple point of intersections in this case we would have avoided the conflict by taking the max and we for single antecedent we will have single w.

But if we would have multiple antecedents, so for multiple antecedents we would have either used max-min composition or max-product composition. So, even if we take the min of *w*, we are going to have the same value that is point 0.58. So, with this value we truncate. Now, we can have the max-product composition. So, since we have here the w which is again this single value.

So, for the single value whether you take the min or product both will remain the same. So, here also will have the same value of 0.58. So, here in this case the both the max-min or max-product both are going to give the same truncated fuzzy value. Now, comes here after this the output of this single rule, now we take the max of it. So, since we have only one rule even if we take max the same value is going to come.

So, that is why whatever is here is coming here directly. So, max of this same value is going to give us the same fuzzy value.

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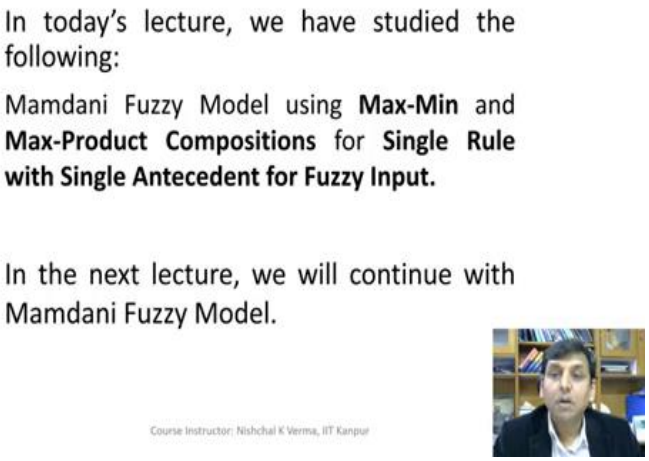

So, in today's lecture we have discuss the Mamdani fuzzy model using max-min and max composition, max-product compositions for single rule with single antecedent for fuzzy input. In the next lecture we will continue our discussion with Mamdani fuzzy model.

Thank you.