Fuzzy Sets, Logic and Systems & Applications
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Lecture - 05
Membership Functions

Welcome to lecture number 5 of Fuzzy Sets Logic and Systems and Applications. So, today in this lecture we will discuss various types of Fuzzy Membership Functions.

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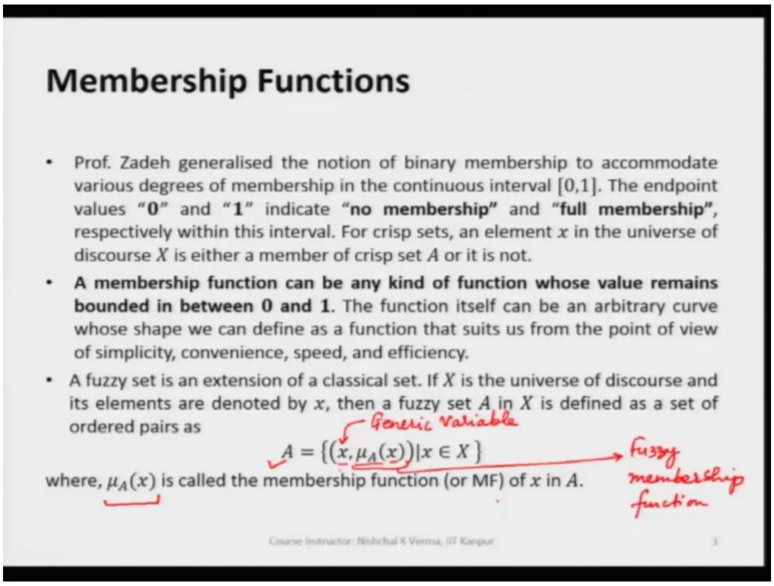

As we know we have seen in the previous lectures that we can define a fuzzy $A=\{¿\vee x\in X$. So, you see here x is the generic variable this is generic variable and this is along with $\mu_A(x)$. So, this $\mu_A(x)$ is nothing but the fuzzy membership function.

And this fuzzy membership function is responsible for giving the membership values to the corresponding value of the generic variable. So, we see here this $\mu_A(x)$ is very important in the sense that this provides us the membership values. So, fuzzy membership function is very important in the sense that this function, this membership function assigns the corresponding membership values which are more than 0 up to 1.

So, since fuzzy membership value can be more than 0 up to 1, any value in between that and that is why the membership value, fuzzy members function can be in between 0 to 1 or I would say the fuzzy membership function can go up to at the most up to 1.

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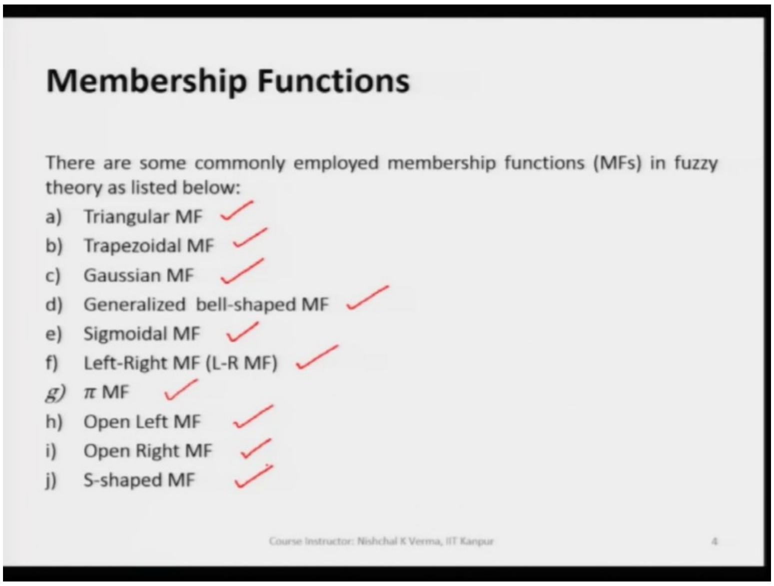

So, there are some commonly implied membership functions in fuzzy systems theory and these membership functions are also called a standard membership functions which are listed as the triangular membership function, trapezoidal membership function, Gaussian membership function, generalized bell-shaped membership functions, sigmoidal membership function, left-right membership function, pi membership function, open left membership function, open right membership function and S-shaped membership functions. So, the details of these membership functions will be described given in the coming slides.

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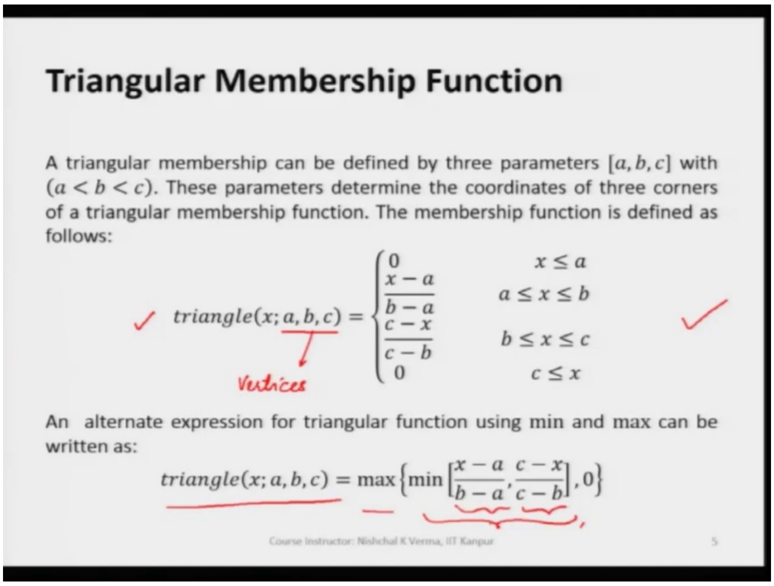

So, if we take up first the triangular membership function, so this fuzzy membership function we defined by this syntax.

$$triangle(x; a, b, c) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & c \leq x \end{cases}$$

So, this is how we define triangular membership function. We write a triangle membership function. And here we have an alternate expression for triangular membership function

$$triangle(x; a, b, c) = max\left\{min\left[\frac{x-a}{b-a}, \frac{c-x}{c-b}\right], 0\right\}$$

So, this called min-max function for triangle triangular membership fuzzy function.

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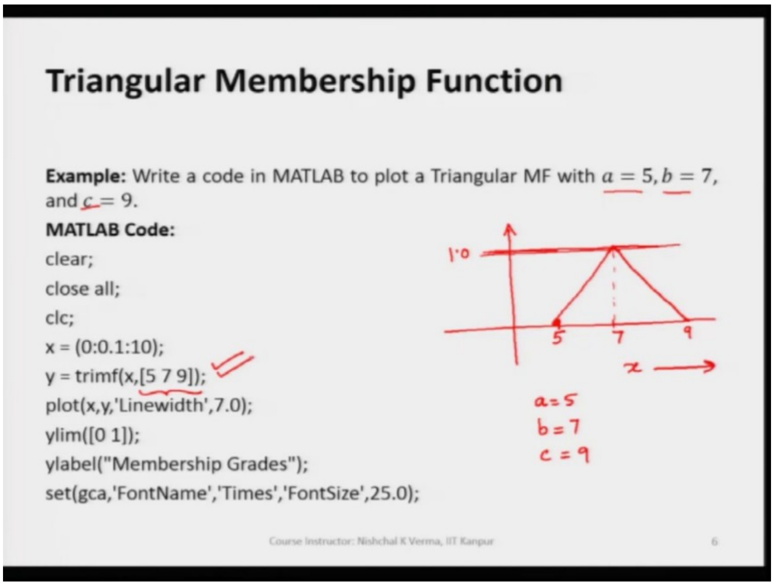

So, either of these we can use to generate a triangular fuzzy membership function. So, here is here you see we have a MATLAB code for generating triangular fuzzy membership function. So, you we see here the code, MATLAB code which generates a triangular membership fuzzy function whose the it generates the triangular shaped fuzzy membership function whose vertices are a, b, c. So, a here is equal to 5, b is 7 and c is equal to 9.

So, we have basically if we make a triangular members fuzzy function it will be like this. If we have let us say a here, so a, b, c and here the highest membership value that it can attain is 1. So, you see here that if this is a let us say and this is c, so it starts from a goes up to c and this is the generic variable x's and here is the membership value. So, the highest membership value here that it attains is 1, ok.

So, the vertex the touches at 1, ok. So, a is equal to 5, b is equal to 7, c is equal to 9. So, if we use this MATLAB program we will be getting this kind of shape whose vertices will be at 5, 7 and 9 like this. So, this is 5, this is 7, this is 9. So, if we use this program MATLAB program we will be able to generate a shape, a triangular membership function like this.

We see here the values of the vertices that are given. So, 5, 7, 9 are given, if we change these values we will get accordingly the triangular membership function whose vertices we assign here. So, this needs to be noted.

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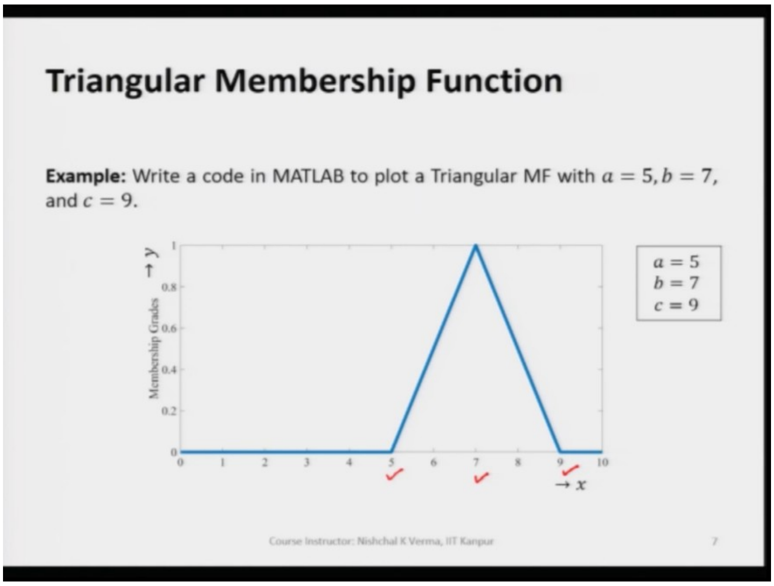

Now, we see here the plotted shape that we used in MATLAB. So, if we run the, this program in the MATLAB we will be able to get this shape plotted. So, this is the fuzzy membership function which is triangular membership function whose vertex whose vertices are, whose vertices are at 5 and then 7 and then at 9. So, as I mentioned before that if we change if we assign the values of vertices a different values of vertices we can different get the different kinds of triangular membership fuzzy functions.

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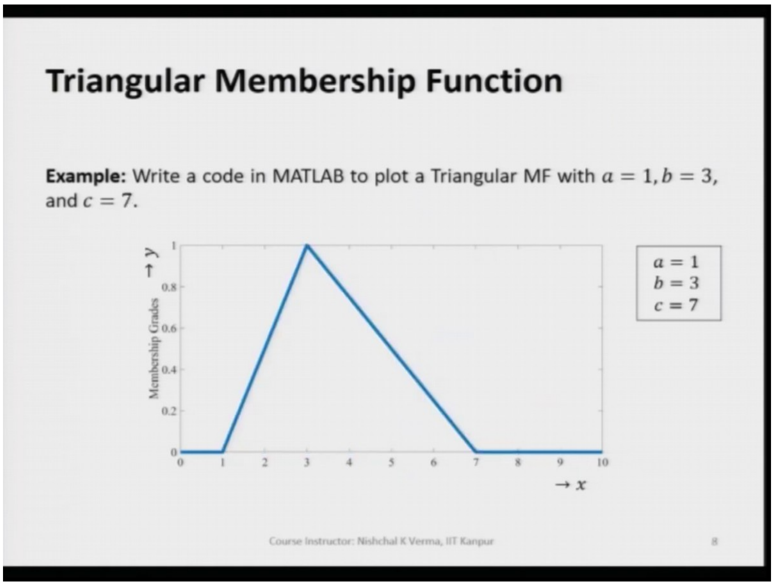

So, as we see here if we assign a is equal to 1, b is equal to 3 and c is equal to 7 it means one of the vertex is, one of the vertices is 1 and then the other one is at 3 the and then the last vertex is 7 at 7. So, we see this kind of shape gets generated. So, this is how a triangular membership function whose vertices are 1 ,3 ,7 gets generated.

So, by using this program, this MATLAB program you can generate any kind of triangular membership functions. So, the input that you need to give to this function is only the vertex, the, and the vertices and accordingly you can get the ship membership functions generated, ok.

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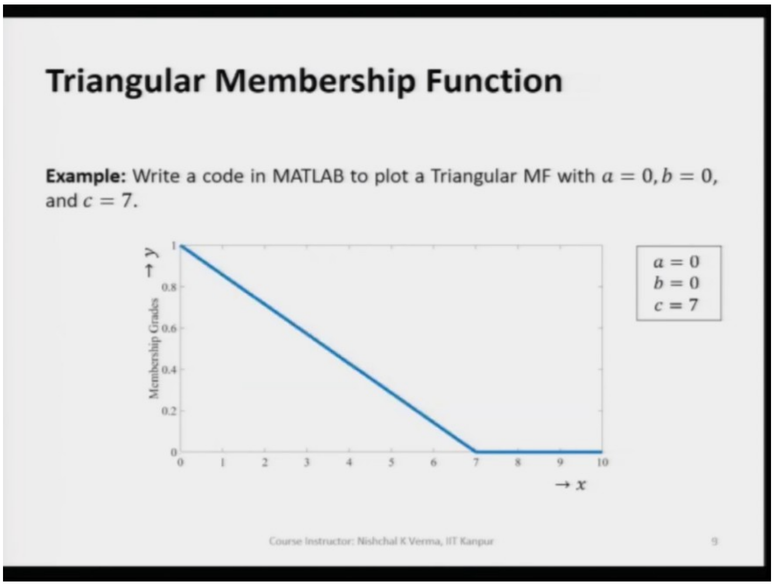

So, let us know using say MATLAB program let us give two vertices at 0 ,0, and the other the third one 7. So, let's see what is happening. So, if we give two vertices at 0 ,0, means two vertices are at 0, so we see here these two vertices of the triangle are membership function they are at 0 here and then the third one is at 7.

So, we see this kind of shape gets generated. So, this is also a triangular membership function whose two vertices are at 0 and the third vertex is at 7. So, this kind of membership is also called left side open, left open triangular membership function, all right.

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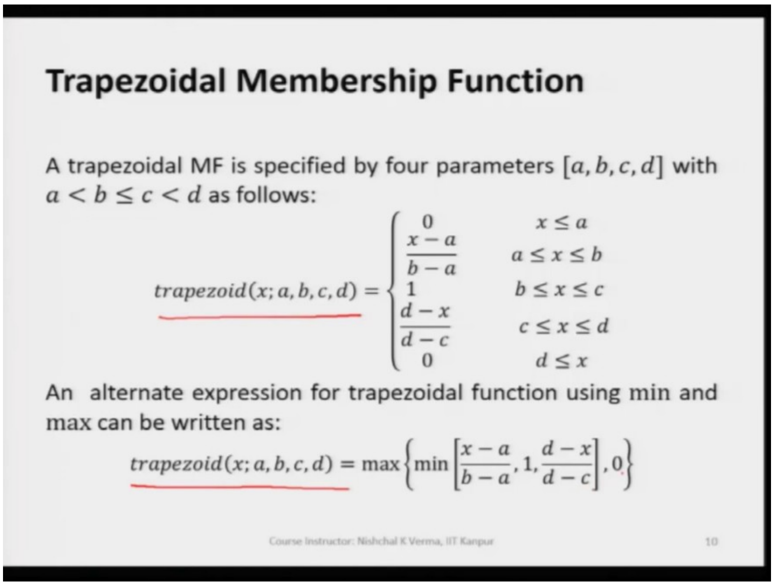

So, then we have another kind of a membership, fuzzy membership function. So, this is a trapezoidal membership function and this can be defined by this function, you see here we can write simply

$$trapezoid(x; a, b, c, d) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c \leq x \leq d \\ 0 & d \leq x \end{cases}$$

And same as the triangular membership function here we this also has an alternate expression for generating trapezoidal membership function.

$$trapezoid(x; a, b, c, d) = max\left\{min\left[\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right], 0\right\}$$

So, this is how, so, either of these can be used for generating a trapezoidal membership function.

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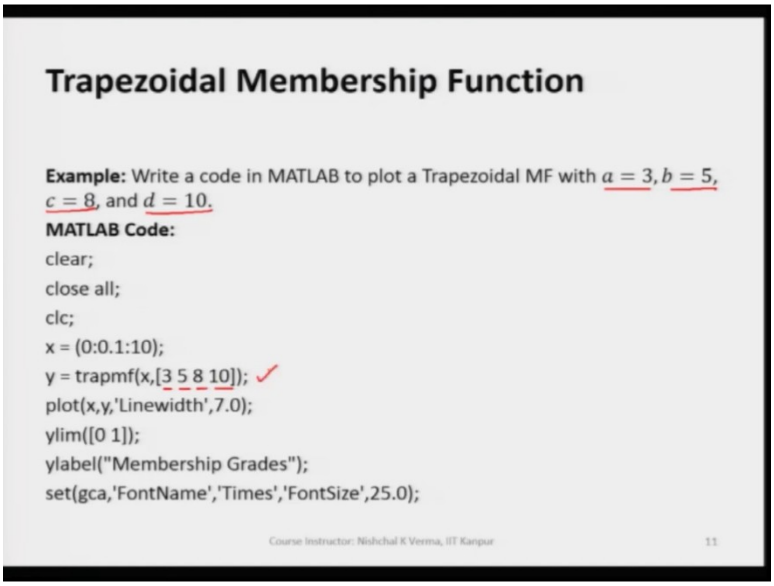

Now, here we have a member a MATLAB code for generating or plotting the trapezoidal membership function. So, in this example MATLAB code we have a trapezoidal membership function with a is equal to 3, b is equal to 4 and c is equal to $8, d$ is equal to 10, means we have assigned the values a, b, c, d as $3, 5, 8, 10$ respectively. And if we run this code we generate this kind of function.

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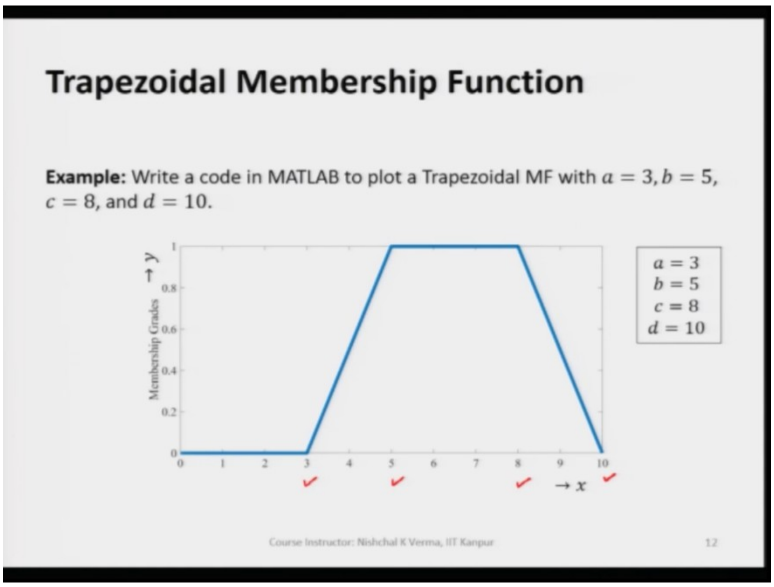

So, you see we have 4 vertices and these 4 vertices are like this, that first vertex is at 3 and then the second vertex is at 5 and the third vertex is at 8 and a 4th vertex is at 10. So, if we change the values of a, b, c means we can change we can assign accordingly the place of these vertices and similarly, we can change the here if we assign the values of these vertex these vertices we get different shapes of the trapezoidal membership function.

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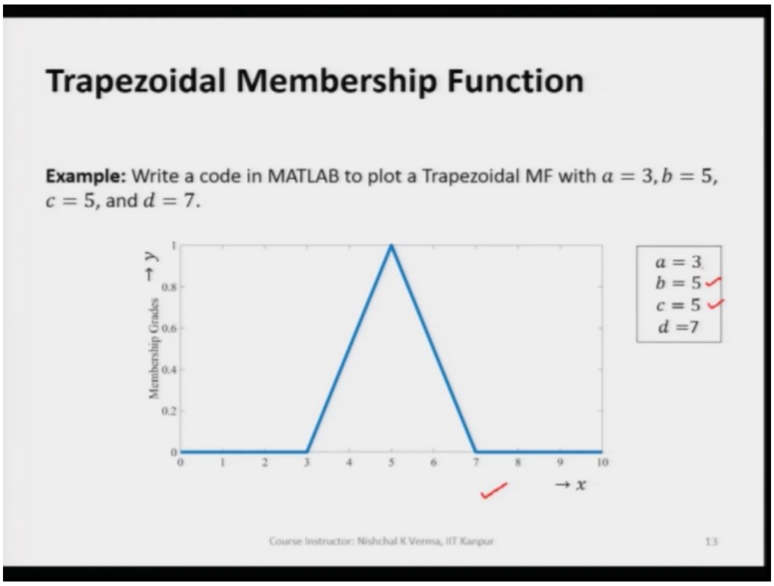

Very interesting thing here is to note is that if we make the two vertices common, so means v and c here they are at $5, 5$ if we assign a, b, c value; a, b, c, d values like that we get at triangular membership function. So, we can use a trapezoidal membership function formula to you know generate a triangular membership function also. So, what we do here is that we simply have b and c common point, means here in this case b is at 5 and c is also 5 at 5.

So, with this we are generating a triangular membership function instead of trapezoidal membership function. So, by using this MATLAB program for trapezoidal membership function we can generate a different kinds of trapezoidal membership functions by changing vertices suitably.

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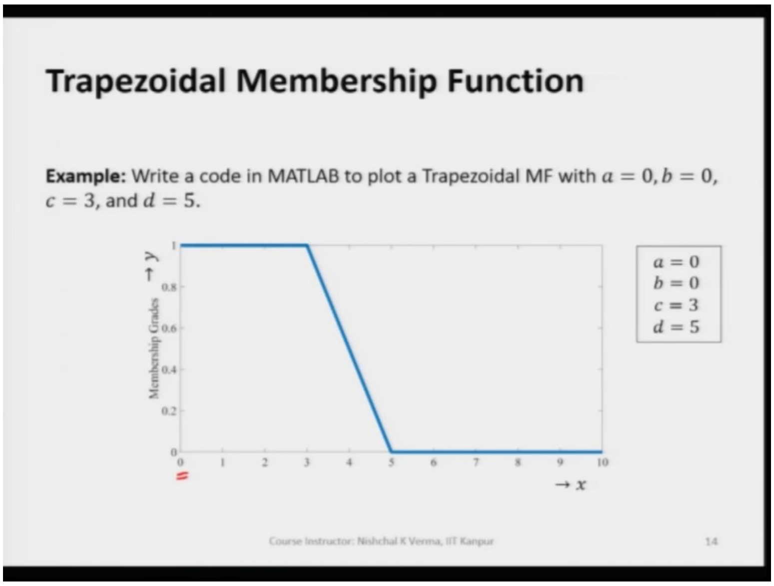

So, then here we have another kind of plot generated by the trapezoidal membership function. So, if we keep a is equal to 0 ,b is equal to 0, we see that a and b come at the same point, 0. So, here the two vertices a and b are here at this point and then c and c is equal to 3, d is equal to 5, so we have two more vertices at different points. So, here we can make a left open membership function also using this formula, this trapezoidal membership functions formula.

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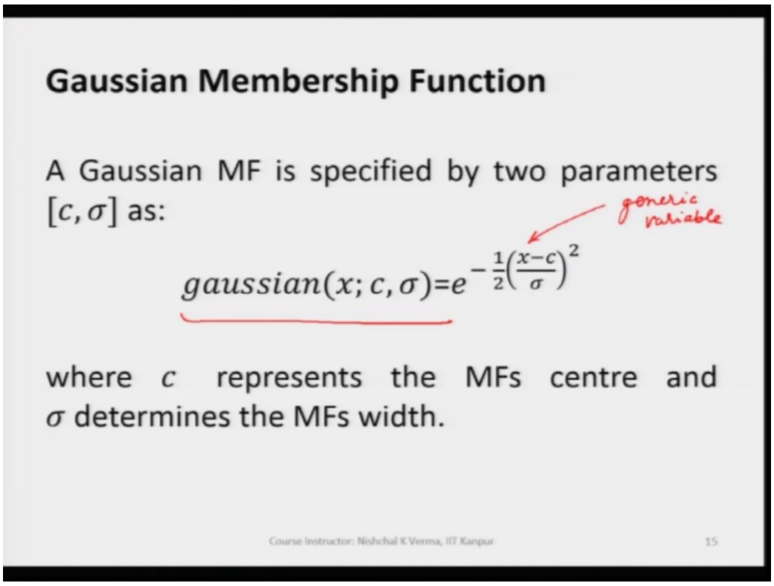

Now, comes the Gaussian membership functions. So, this is a very interesting membership function, fuzzy membership function. So, this can be defined by a Gaussian and then x, so this is actually the syntax so, this is the way we write a Gaussian membership function. So,

$$gaussian(x;c,\sigma)=e^{\frac{-1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

So, this way Gaussian membership function gets generated.

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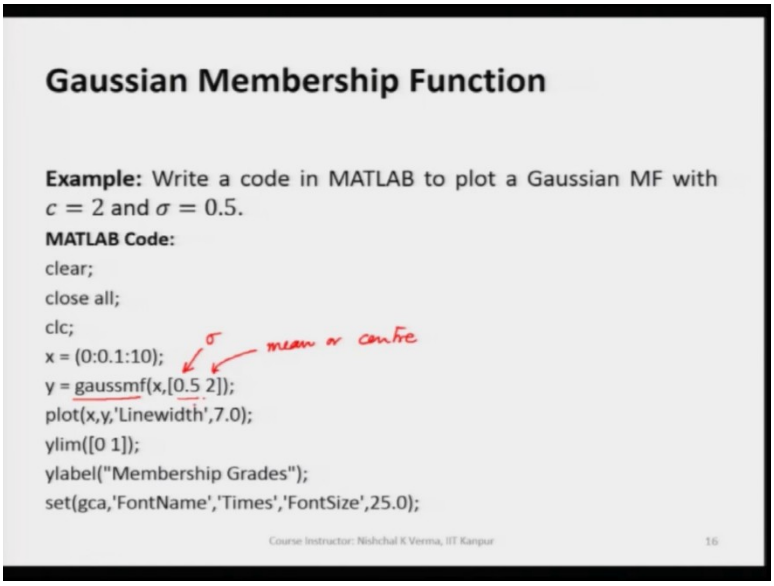

So, here is an example by using a MATLAB code the Gaussian membership function has been generated and if you like to use this MATLAB code you can use for generating a membership Gaussian membership function. So, gaussmf is the function which is being used here. It is already in built MATLAB function, so here this is sigma and here this value is the mean or the center. So, if we substitute these values in this syntax the gaussmf MATLAB function, so we see this plot generated.

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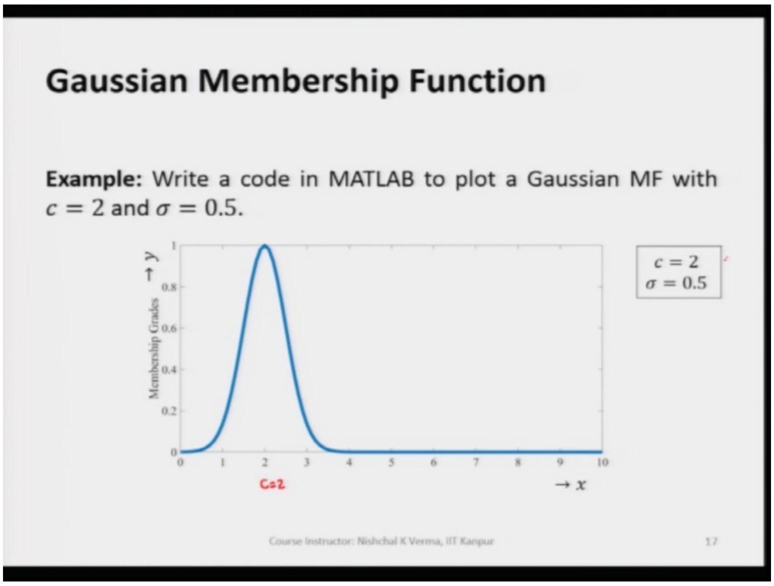

So, you see here the center of the Gaussian and the mean of the Gaussian comes at c, so c is equal to 2 and then we have the standard deviation 0.5 which is used to generate this plot. So, this is called a fuzzy Gaussian membership function.

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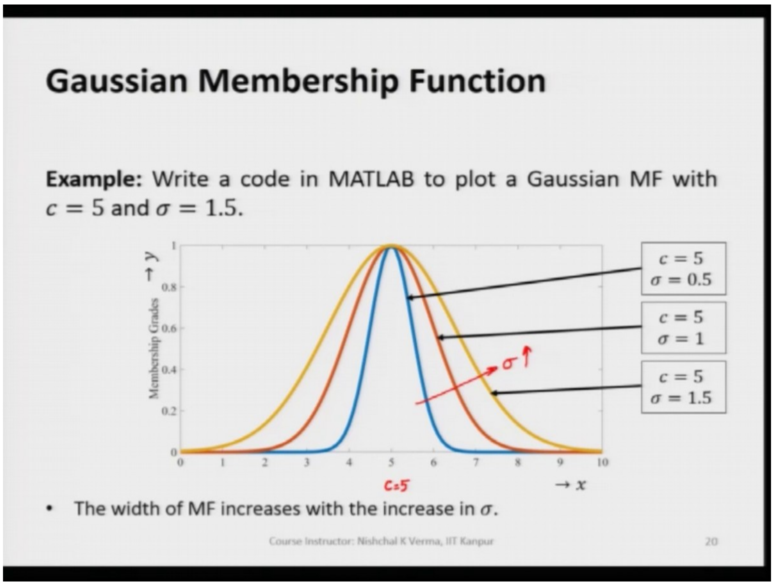

Now, if we change these values we will get different types of or different you know Gaussian function membership functions, so if we change here see here if we take c is equal to 5 and σ is equal to 0.5, so we get this Gaussian membership function generated. See the center is changed here. So, now as the center is at c is equal to 5 and the standard division remains 0.5.

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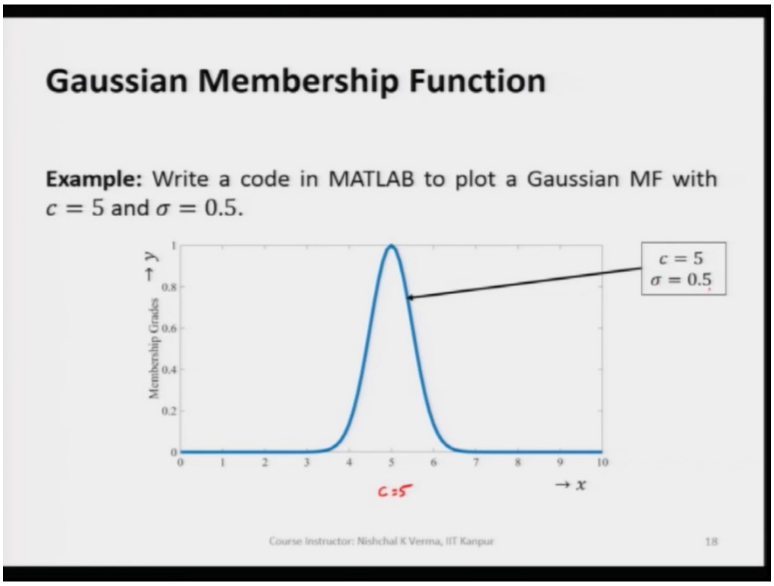

Now, if we change the standard deviation, we see the shape of this Gaussian membership function changing. So, this gets this the later one has the more spread. So, by substituting sigma is equal to 1, we are spreading the Gaussian membership function. Similarly, if we increase the sigma we will get more spread like this.

So, if we see that the center remains the same, center is at 5, but we see 3 Gaussian membership functions with different σ's. So, sigma is increasing. So, with the increase of sigma the spread is increasing. So, if we increased the sigma the spread is increasing. So, that is how if you would like to plot a fuzzy Gaussian membership function we can very easily plot Gaussian membership function by using this MATLAB code, ok.

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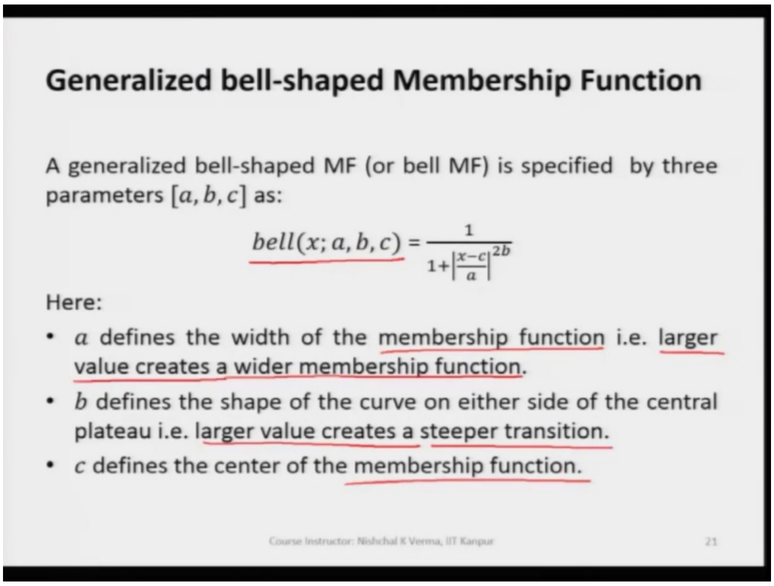

Then comes the generalized bell-shaped membership function. So, the formula for generalized bell-shaped membership function is here. So, this the syntax of this bell bell-shaped membership function is written as

$$bell(x; a, b, c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}.$$

So, the a here defines the width of the membership function, so I would say a controls the width of the membership function, that means, the larger value creates a wider membership function. So, larger value of a creates a wider membership function. And b defines the shape of the curve on either side of the central plateau; that means, the larger value of b creates a steeper transition; c defines the center of the membership function means the either we say center or we say mean.

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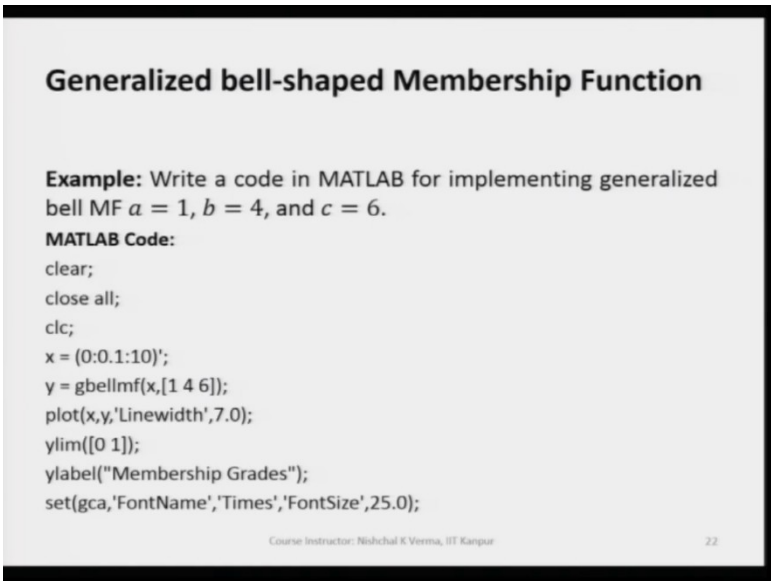

So, here also we have a MATLAB code for generating generalized bell-shaped membership function. So, as we saw the formula for generalized bell-shaped membership function, if we substitute the values of a, b and c we can generate the bell-shaped membership function like this.

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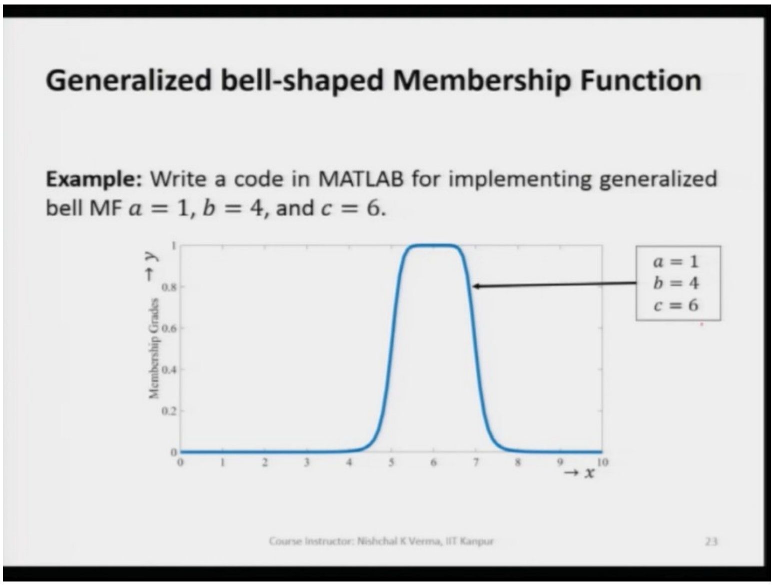

So, we can use the same MATLAB code for generating this generalized bell-shaped membership function. We have to substitute the values of a, b and c and then we will get this bell-shaped membership function generated.

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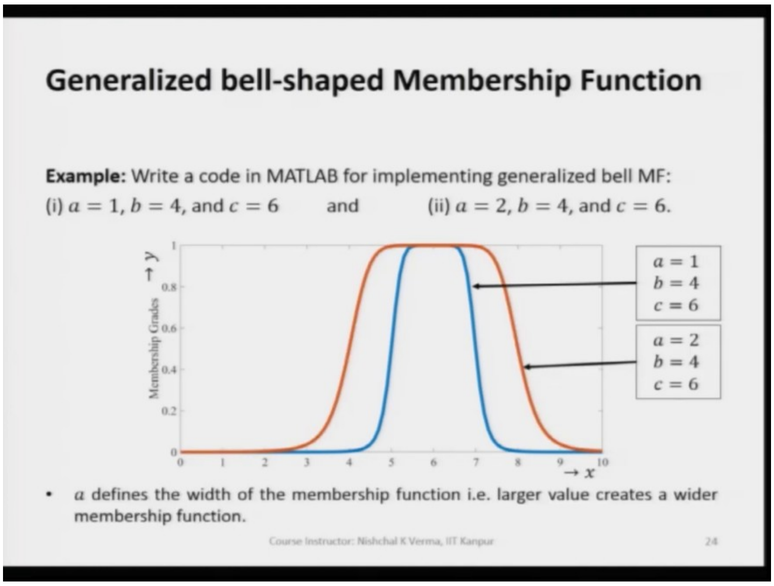

So, here if we change the values of a, b, c accordingly we get the different generalized bell-shaped membership functions generated.

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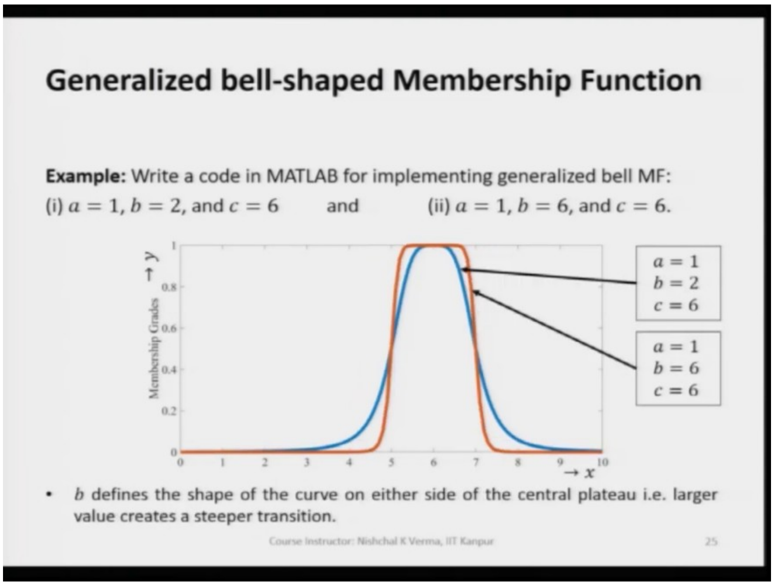

Here also we see that as we change these values we get different kinds of membership, generalized bell-shaped membership functions generated.

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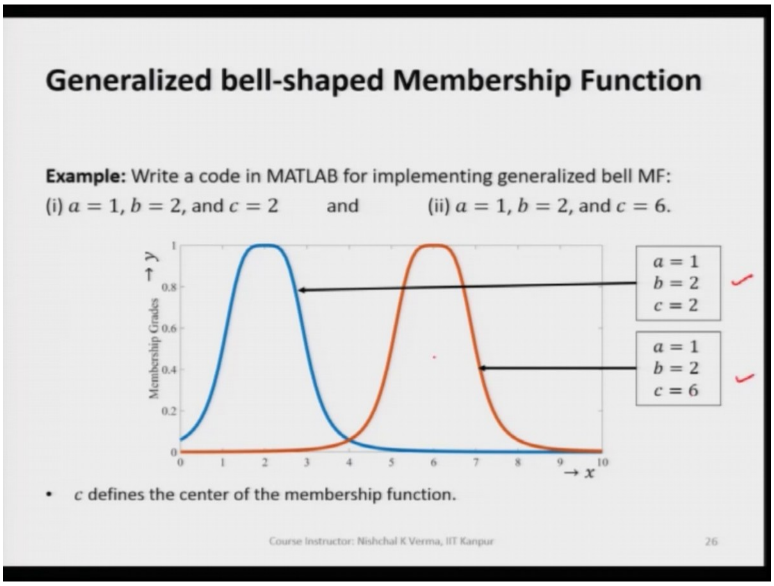

Here also we see by changing the, these values of $a, b, c; a, b, c$ here also we get these values they generalized membership functions generated.

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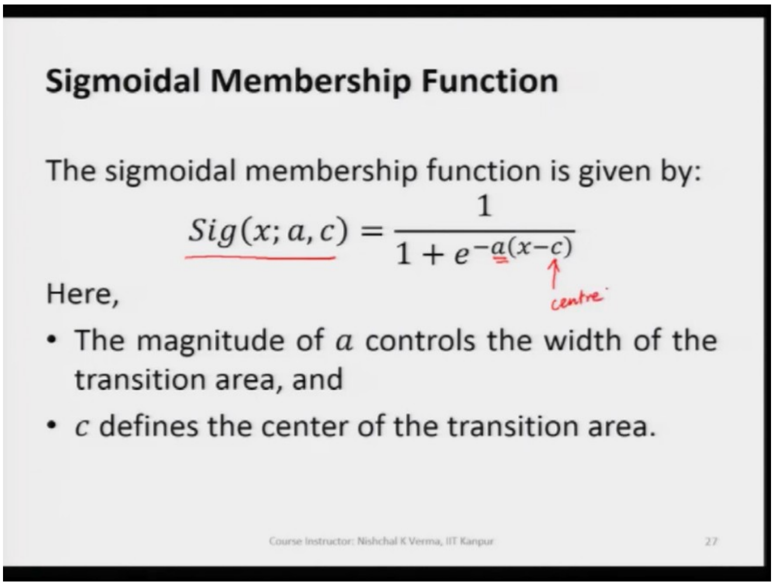

Now, we come to another kind of a membership function which is sigmoidal membership function. So, sigmoidal membership function is given by

$$Sig(x; a, c) = \frac{1}{1 + e^{-a(x-c)}}$$

So, here also a controls the width of the transition area and this a here is control the, a controls the width of the transition area and c is nothing but again the center of the plot, the center of the transition area, center of the transition area, this is center of the transition area.

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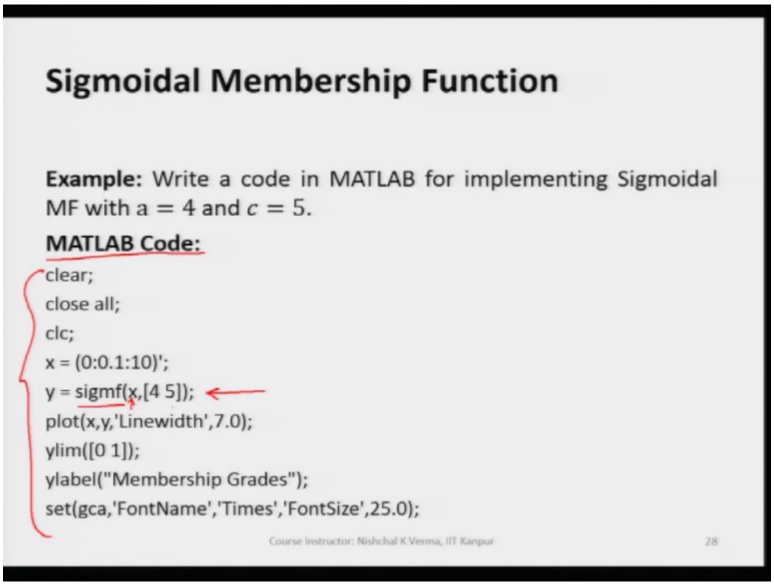

So, like other membership functions we have the MATLAB code for sigmoidal membership function also. So, this MATLAB code is given for the, for generating sigmoidal membership function. If we use this MATLAB code and we feed the value of a and c, suitably we can get the sigmoidal membership function generated.

So, like in previous MATLAB codes here also we have a sigmf function which is nothing but MATLAB function, MATLAB function for generating sigmoidal membership function and x is nothing but the generic variable, and this 4 and 5 are the values of a and c. So, this way if we suitably input these values we can get you know suitable sigmoidal membership function generated.

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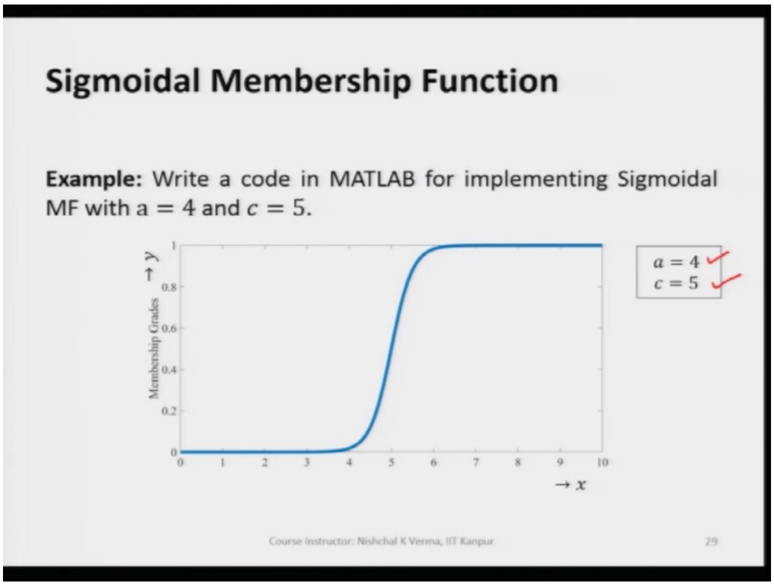

So, for $a=4$ and $c=5$ you see this kind of sigmoidal function will be generated. This is for $a=4$ and $c=5$.

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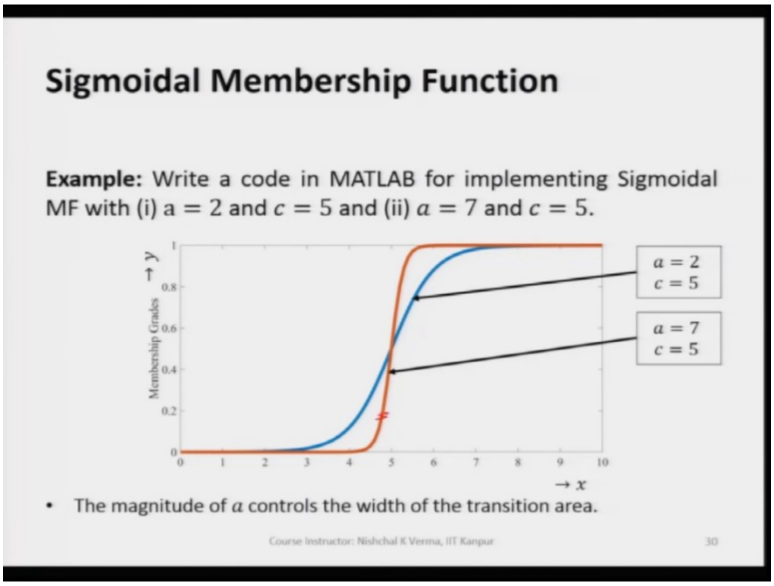

Now, if we choose another set of a and c we see that we get different type of a different sigmoidal curve here, different sigmoidal plot here. So, this plot is for $a=2$ and $c=5$ and this plot is for $a=7$, $c=5$.

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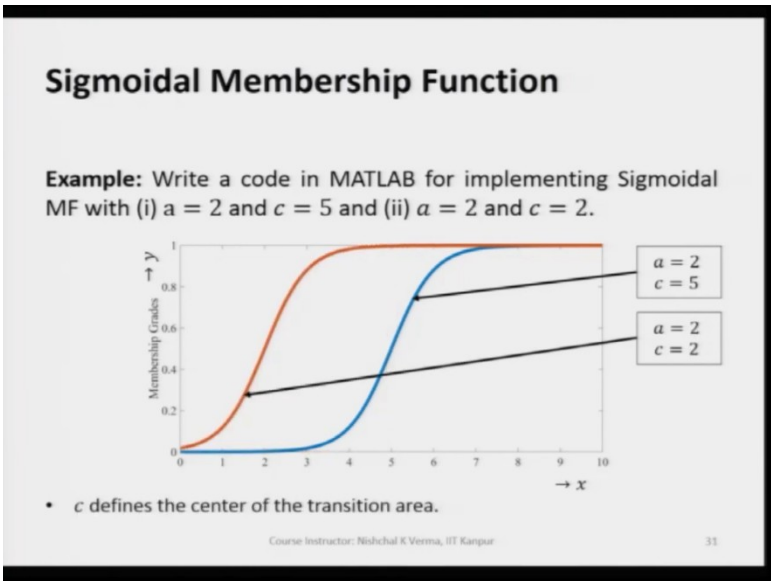

Similarly, we can have other values like this and we can get other sigmoidal membership functions created.

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So, far we have discussed the following membership functions in this lecture. So, these membership functions are triangular membership function, trapezoidal membership function, Gaussian membership functions, generalized bell-shaped membership function, sigmoidal membership functions. And remaining membership functions, remaining 5 membership functions out of 10 commonly used membership functions we will be discussing in the next lecture.

Thank you.