Fuzzy Sets, Logic and Systems and Applications
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Lecture - 48
Fuzzy Rules and Fuzzy Reasoning

Welcome to lecture number 48th of Fuzzy Sets, Logic and Systems and Applications. In this lecture we are going to discuss Fuzzy Rules and Fuzzy Reasoning.

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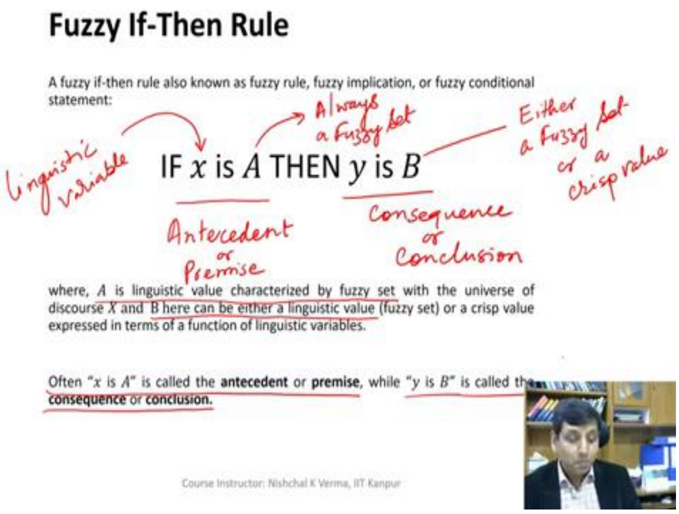

So, in this lecture we are going to first take up the fuzzy if-then rule. So, let me before going ahead tell you that fuzzy if-then rule is an essential component of a fuzzy system. So, without fuzzy if and then rules there is no fuzzy system.

So, this means that in any fuzzy system we must have a set of fuzzy $if - then$ rules. A fuzzy $if - then$ rule is also known as fuzzy rule, fuzzy implication or fuzzy conditional statement. So, fuzzy if and $then$ rule has a form of $IF\ x\ is\ A, THEN\ y\ is\ B$. So, this is the syntax of syntax of any fuzzy if and $then$ rules. So, here we have two components first component of this fuzzy $if - then$ rule is the antecedent or premise.

So, this part is called the antecedent or premise and then the second part which is just after then is called the consequence or conclusion. So, in any fuzzy $if - then$ rule will have antecedent part or premise part and consequent part or conclusion part? And it is very important here to note that antecedent part in fuzzy if and $then$ rule is always fuzzy, this means that A is always fuzzy, A is a linguistic value and which is always a fuzzy quantity, a fuzzy set.

Whereas we have the consequent part or conclusion part and this consequent and or conclusion part has B and this B can be either can be either a fuzzy set or a crisp value. And this crisp value can be expressed in terms of some function which is actually the function of the generic variable. So, we can say that the A that has been taken here in this fuzzy rule fuzzy $if - then$ rule.

A is a linguistic value A is a linguistic value characterized by a fuzzy set with the universe of discourse X and B here B here can be either B here can be either a linguistic value and of course we all know that linguistic value can always be represented by a fuzzy set. So, A is a linguistic value A is a linguistic value characterized by a fuzzy set, whereas B can be either a linguistic value that means the fuzzy set or a crisp value expressed in terms of a function of linguistic variables used in this for example, here the linguistic variable is x, so this is the linguistic variable all right.

So, now, often x is A is called antecedent or premise as I have already mentioned here, while y is B is called the consequence or the conclusion. So, this must be understood very clearly and as I have already mentioned the fuzzy if and then rule is very important component of any fuzzy system and without a set of fuzzy if and then rule the fuzzy system cannot be cannot exist.

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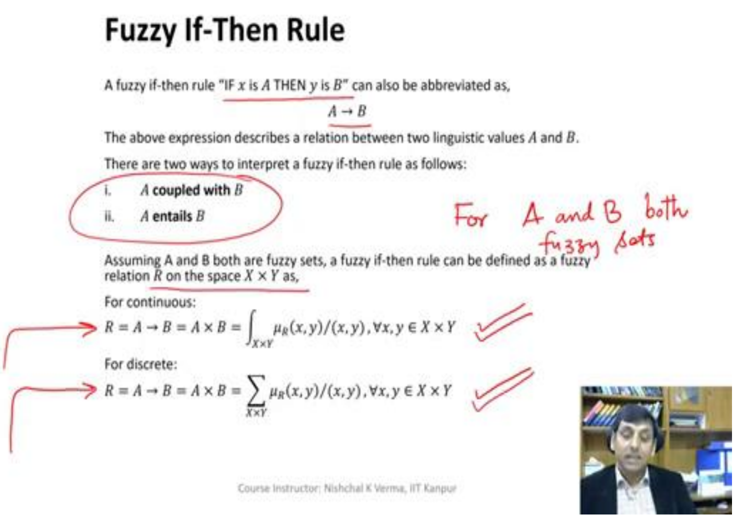

So, let us now move ahead and we discuss a fuzzy $if - then$ rule where we have let us say the fuzzy if and $then$ rule like this like if $x\ is\ A\ then\ y\ is\ b$. So, let us first understand that this can also be represented by $A\ entails\ B\ or\ A\ coupled\ with\ B$. So, please understand here as it is written here that there are two ways to interpret a fuzzy $if - then$ rule. First way is $A\ coupled\ with\ B$ or the other way I mean the second way is $A\ entails\ B$.

So, fuzzy $if - then$ rule when it is written in that syntax that I have already mentioned like if A x A is A, x is A then y is B. So, this can also be interpreted like this like A coupled with B or A entails with B. So, assuming A and B both are fuzzy sets. So, let us please understand let us assume here that A and B both are the fuzzy sets. So, when we assume A and B means for A and B for A and B both fuzzy sets all right.

So, please understand why are we saying this that for A and B both fuzzy set because B can be here in fuzzy if and $then$ rule B can be either crisp or fuzzy, but here we are assuming for this discussion that we have both A and B fuzzy sets. So, assuming A and B both fuzzy sets if and then rule can be defined as a fuzzy relation. So, this is very important to note when we have A and B both fuzzy sets. So, in this case this $A\ entails\ B$ $A\ couples\ coupled\ with\ B$ can be regarded as a relation.

And this is represented by R on a space $X \times Y$ where this X is the universe of discourse for the generic variable x. And Y is the universe of discourse for generic variable small y if B is the fuzzy set. So, this needs to be noted. So, when we have a relation when we have a fuzzy relation in between A and B we can always write this as $A \times B$ we have already done this. So, $R = A\ entails\ B = A \times B$ you can see here.

So, this way we can write that for continuous fuzzy set this is for the continuous fuzzy set that we have the

$$R = A \rightarrow B = A \times B = \int_{X \times Y} \mu_R\ (x, y)/(x, y), \forall x, y \in X \times Y$$

And similarly for discrete for discrete here we have the representation

$$R = A \rightarrow B = A \times B = \sum_{X \times Y} \mu_R\ (x, y)/(x, y), \forall x, y \in X \times Y$$

So, what we have seen here is that if we have in fuzzy if and then rule both A and B. A is coming from the antecedent part or premise part and Y and B is coming from the consequence or conclusion part. So, if we have A and B both fuzzy sets that are coming from fuzzy rule if and then fuzzy rule then we can write the R relation fuzzy set in terms of A and B; that means, $A \times B$.

And A cross B for continuous we have seen here and for discrete we have seen here as to how we can write.

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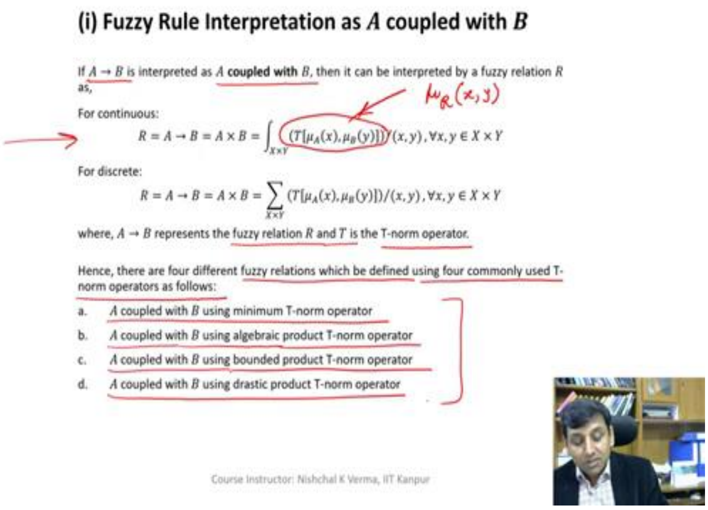

We have already done we have already discussed in detail a fuzzy relation. So, we can accordingly manage to write the relation fuzzy relation matrix. So, if $A\, entails\, B$ is interpreted as or $A\, coupled\, with\, B$. So, both are same so, $A\, entails\, B$, $A \rightarrow B$ is interpreted as $A\, coupled\, with\, B$ then it can be interpreted by a fuzzy relation R as we have just discussed.

So, for continuous so, for continuous the relation can be written as R can be written as here $A\, coupled\, with\, B$ is equal to $A \times B$ means the Cartesian product, simple Cartesian product as we have already discussed in previous lectures. So, here the $\mu(x, y)$ how will we find this $\mu(x, y)$ is here. So, this $\mu(x, y)$ you see here $\mu(x, y)$ this $\mu(x, y)$ we can get by suitably taking the Cartesian product means we can we can get by taking by using the T-norm.

So, here this is nothing, but this gives us $\mu_R(x, y)$. So, this we can get by taking the T-norm of A and B means we take the T-norm of $\mu_A(x)$ and $\mu_B(y)$. So, as we have already done this T-norm in previous lectures. So, the fuzzy relation R and T let us say we have two fuzzy relations sets right.

And then accordingly we can have the T-norms, T-norm operators we can apply. And then we can say that there are four different fuzzy relations which are defined using four commonly used T-norm operators what does this mean this means that we have we have in T-norm we have multiple we have multiple T-norm operators. So, we have four commonly used T-norm operators first is $A\, coupled\, B$ using minimum T-norm operator and then the second is the $A\, coupled\, B$ is using algebraic product T-norm operator then we have the third one is $A\, coupled\, B$ using bounded product T-norm operator.

And then we have the $A\, coupled\, B$ using drastic some product operator. So, this way we can use any of these T-norm operators suitably to get the $\mu_R(x, y)$ and this was for the continuous on the same lines we can get the R for discrete.

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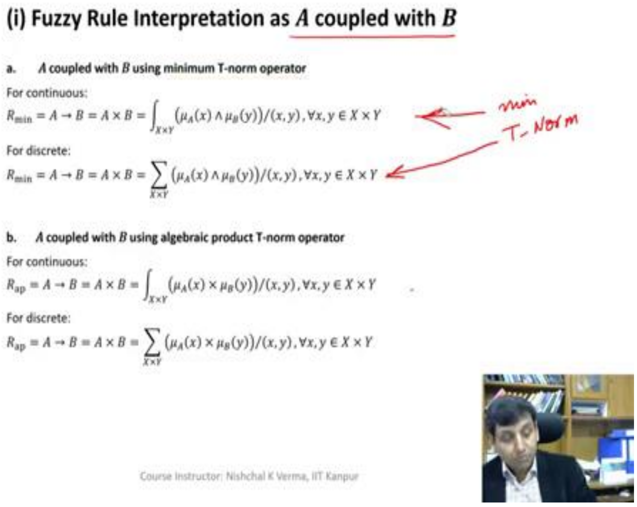

Relation matrix, the relation in between A and B for discrete fuzzy sets.

So, let us now go one by one. So, fuzzy rule interpretation as $A\ coupled\ B$ here and this A and B are coming from the if and then fuzzy rule. So, let us take the first case where we have the minimum T-norm operator. So, we have let us first type of T-norm operator. So, first type of T-norm operator is min.

So, for continuous fuzzy set fuzzy sets A and B where this R is the relation fuzzy set which is coming out by coming out from $A \times B$. So, the Cartesian product of A and B. So, we write here the relation fuzzy relation set as R_{min} because min is here denoting that the minimum T-norm operator that is being used for getting the fuzzy relation in between A and B.

So, this can be represented by this and similarly when A and B are is discrete fuzzy sets then there R_{min} can be written as this. So, here we have the fuzzy relation set when we use min. So, we can write min T-norm and then for discrete here we have when we have A and B both are discrete. So, this way we can represent the fuzzy relation matrix sorry fuzzy relation set and when we have instead of min as the T-norm the algebraic product T-norm.

So, when the when we have algebraic product T-norm operator then we simply instead of taking min we write we take the product in between μ_A and μ_B you can see here.

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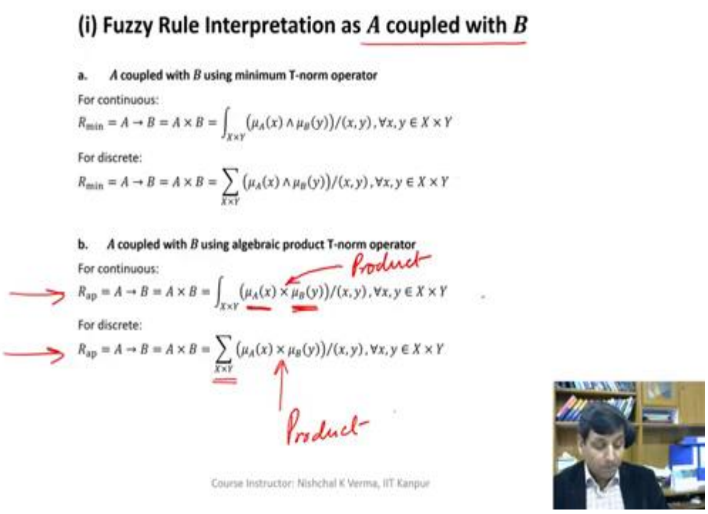

So, we take the product here we take the product here. So, $\mu_A(x) \times \mu_B(y)$. So, we take the product in case of algebraic product T-norm operator. So, and we write this as R_{ap} you can see here and similarly for discrete R_{ap}. So, for discrete A and B for a discrete fuzzy sets A and B we write the relation fuzzy set by R_{ap} which is nothing, but $A \times B$ summation $X \times Y$ as the universe of discourse

And then here again we have the product here again we have the product in between $\mu_A(x)$ and $\mu_B(y)$ and then rest of the things remain the same.

So, only difference here is that the, this operator here this operator gets change. So, when we have min we use we simply take the min when we use min T-norm we take the min sign here and inverted when we take min it means we take the minimum of $\mu_A(x)$ and $\mu_B(y)$ and when we use product then simply we multiply.

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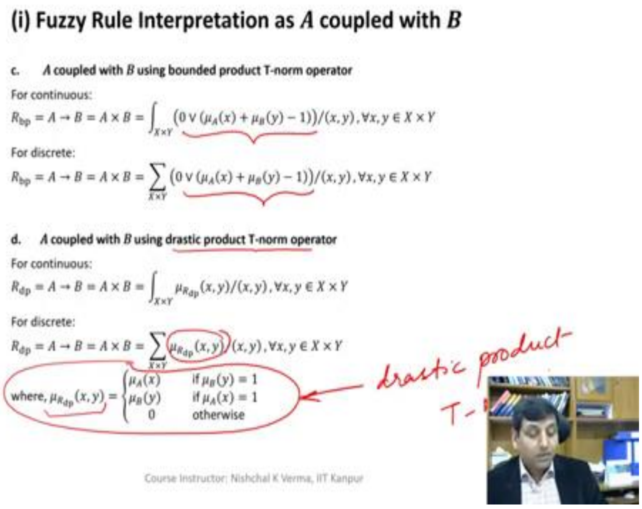

Similarly, when we take the bounded product T-norm so, we use this formula we have already done this when we have discussion discussed the all T-norm operators. So, here we have bounded product. So, for bounded product in between bounded product of $\mu_A(x)$ and $\mu_B(y)$ we write 0. And then the max sign of means the $0 \vee (\mu_A(x) + \mu_B(y) - 1)$, similarly when we have discrete fuzzy set we write it this way.

So, let us understand now that as to how the various T-norm operators change the computations. So, you here you see here the fourth one is $A\ coupled\ B$ using drastic T-norm operators. So, here when we have drastic operators we use drastic computation for drastic product computation this function. So, when we use this for $\mu_A(x)$ and $\mu_B(y)$ we can write it like this $\mu_{R_{dp}}(x, y)$ and this $\mu_{R_{dp}}(x, y)$ can be computed by the drastic product T-norm.

So, this way we have seen that as to how we can manage to get the fuzzy relations set by using all the four types of T-norm operators.

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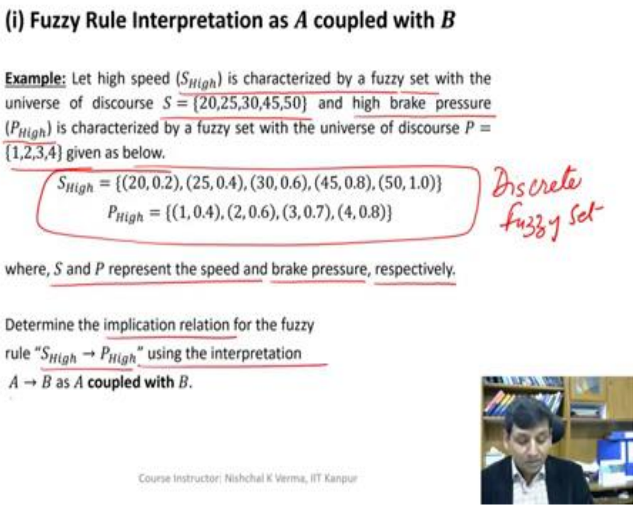

And if we have here an we take an example here where we have a high speed S which is characterized by a fuzzy set. So, S_{High} is the set with the universe of discourse $S = (20, 25, 30, 45, 50)$. And we have another fuzzy set here P_{High} and with the universe of discourse $1, 2, 3, 4$. So, we have two we have two discrete fuzzy sets. So, so both I can write here both the fuzzy sets are the discrete fuzzy sets.

So, S_{High} and P_{High} both are discrete fuzzy sets where S, P both represent the speed and brake pressure respectively. So, determined the implication relation, that means that relation fuzzy set for this. So, S_{High} and then here $S_{High}\ coupled\ with\ P_{High}$ using the interpretation the same interpretation that we have just discussed.

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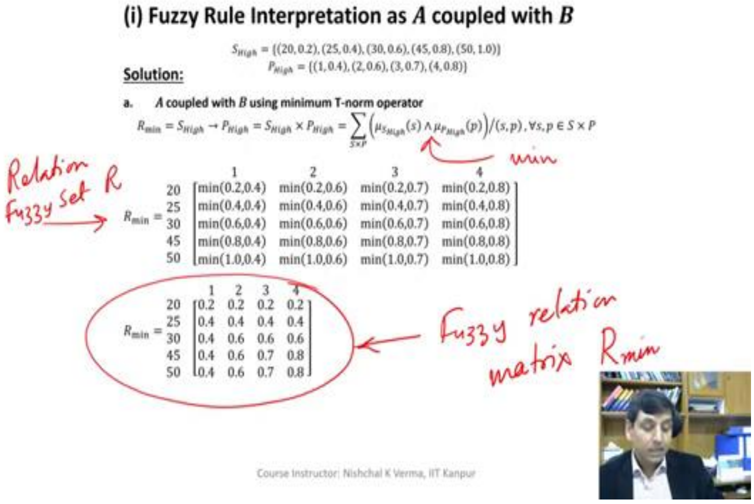

So, let us quickly use all four operators four T-norm operators.

And let us see how we are getting various relation fuzzy sets. So, we have four T-norm operators. So, for the first T-norm operator that is the min T-norm operator we can quickly find the R_{min} the relation fuzzy set the relation fuzzy set R and since we are using min T-norm operators. So, we use the min here see the here we use min operator as the T-norm. So, we have already done this exercise in previous lecture. So, when we do that we are going to get this as the fuzzy relation matrix. So, this is fuzzy relation matrix R_{min}.

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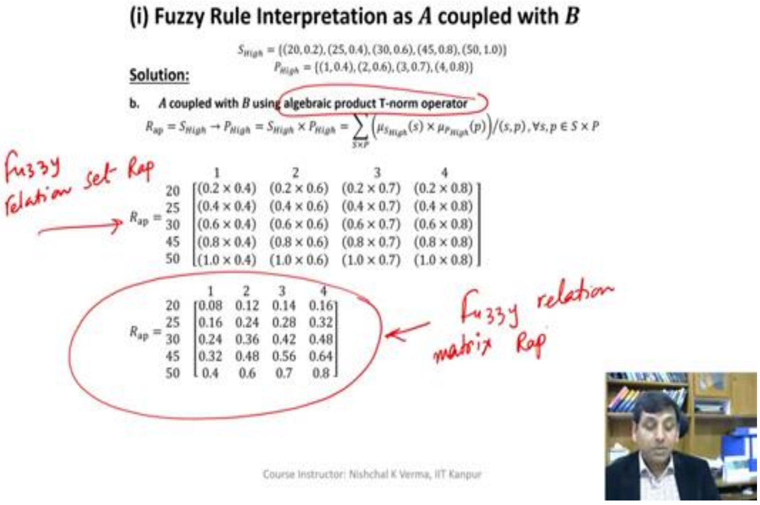

Similarly, when we use the algebraic T-norm operator here, so, then now the computation becomes little bit different, so instead of taking min of the membership values we multiply the respective membership value you can see here, so like 0.2 multiplied by 0.4. So, this is the case when we have the algebraic product. So, we simply take the product of the membership values.

So, this fuzzy relation matrix R_{ap}. So, please understand that this R_{ap} is a fuzzy relation set which is represented in the matrix form. So, R_{ap} is a fuzzy relation set. So, fuzzy relation set R_{ap} which is represented in the form of a matrix. So, when we take the multiplication when we do the multiplication in between the respective membership values. So, now, what we are getting is this. So, this is nothing but the fuzzy relation matrix, fuzzy relation matrix R_{ap} when we have used the algebraic product T-norm.

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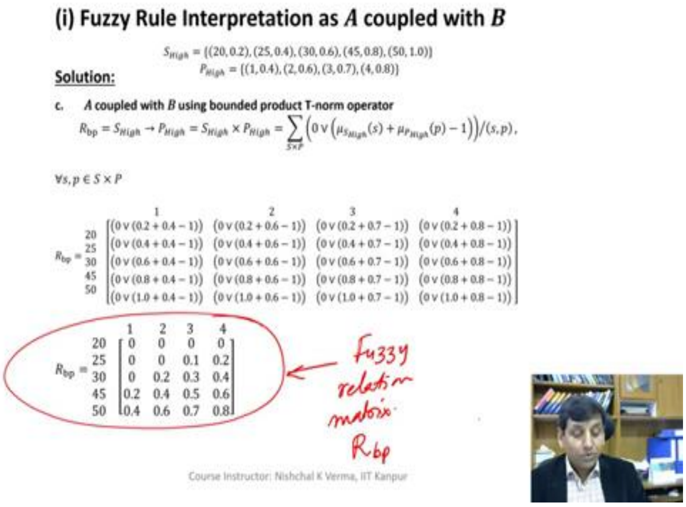

Similarly, let us now take the third type of T-norm operator. So, when we take the bounded product T-norm let us see what happens. So, when we take bonded product T-norm so, we use this formula this relation this expression for finding the bonded product T-norm and this is nothing, but we take the max in between $0 \vee (\mu_{S_{High}}(s) + \mu_{P_{High}}(p) - 1)$. So, this way when we apply this to all the pair of membership values here you can see all these values.

And please note that when we are taking the when we are finding the relation in between A and B discrete fuzzy set here and since we are taking the $A \times B$ for getting the relation fuzzy set R_{bp} maybe whatever type of T-norm we apply, but A and B must be multipliable. So, this means the order of A and B should be in such a way that they can be multiplied. So, these two matrices can be multiplied.

So, whatever we are getting after this is here. So, when we use the bounded product T-norm the relation the fuzzy relation fuzzy relation matrix is if it if I write it by R_{bp}. So, this is fuzzy relation matrix and this comes out when we you in between when we use A and B discrete fuzzy sets and we use the bounded product T-norm operator.

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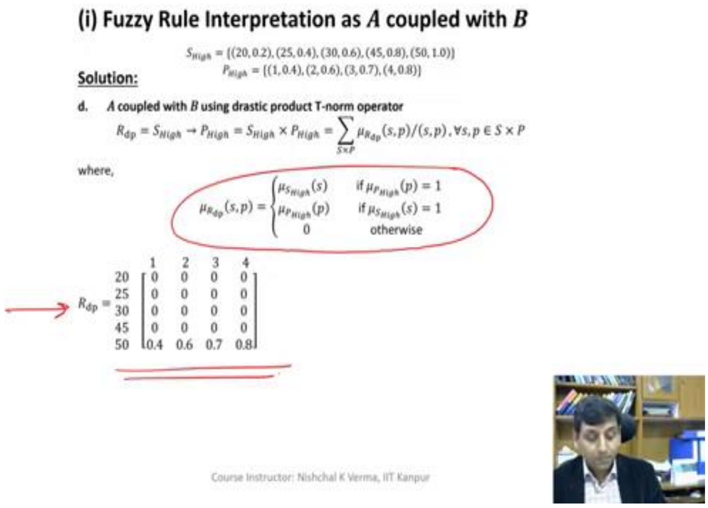

So, now let us take the fourth kind of T-norm operator that is drastic product T-norm operator and we all know that when we use drastic product in our operator this expression applies this means that if $\mu_{P_{High}}(p)$ is 1 then $R_{dp}(s,p) = \mu_{S_{High}}(s)$. And then if we have $\mu_{S_{High}}(s)$ is equal to 1 then $R_{dp}(s,p) = \mu_{P_{High}}(p)$, otherwise the $R_{dp}(s,p) = 0$. So, when we do that when we apply this condition then the R_{dp} which is here which is the fuzzy relation matrix based on the drastic product T-norm operator they get here R_{dp}.

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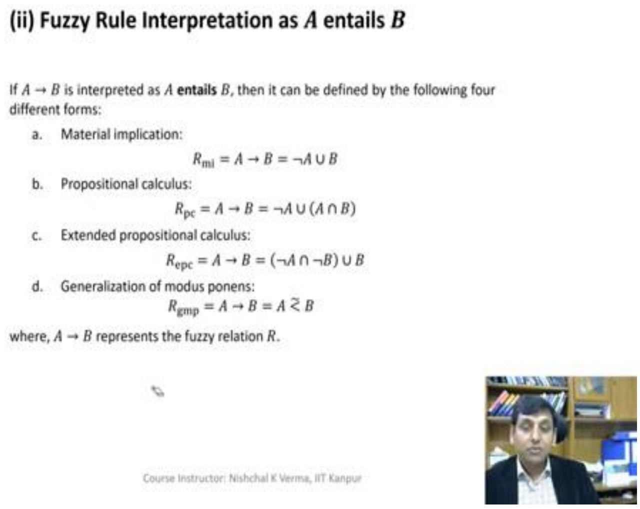

So, this way we have seen that as to how we can get the fuzzy relation matrix by using various kinds of T-norms all four kinds of T-norms that we have used in previous lectures. So, the first was the minimum T-norm operator and then the second one was the algebraic product T-norm operator the third one was the bounded product T-norm operator and the fourth one was the drastic product T-norm operator.

So, all these four types of T-norm operators we have already studied and we are applying this here to get the relation built by using all these four kinds of operators T-norm operators.

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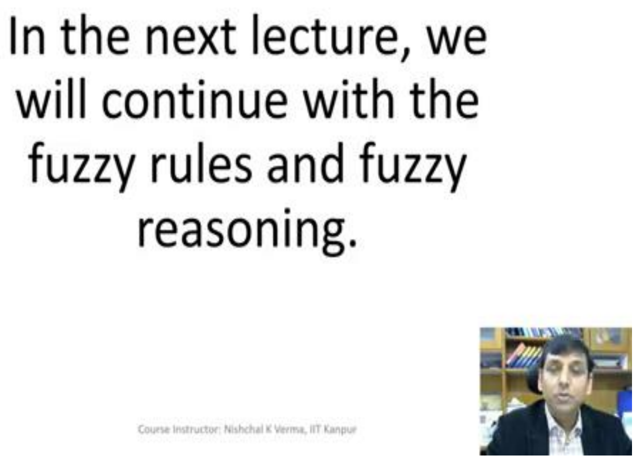

So, with this I would like to stop here in the next lecture we will continue with the fuzzy rules and fuzzy reasoning.

Thank you.