Fuzzy Sets, Logic and Systems and Applications
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Lecture – 47
Orthogonality of Fuzzy Sets

Welcome to lecture number 47 of Fuzzy Sets, Logic and Systems and Applications. In this lecture, we will be discussing the Orthogonality of Fuzzy Sets.

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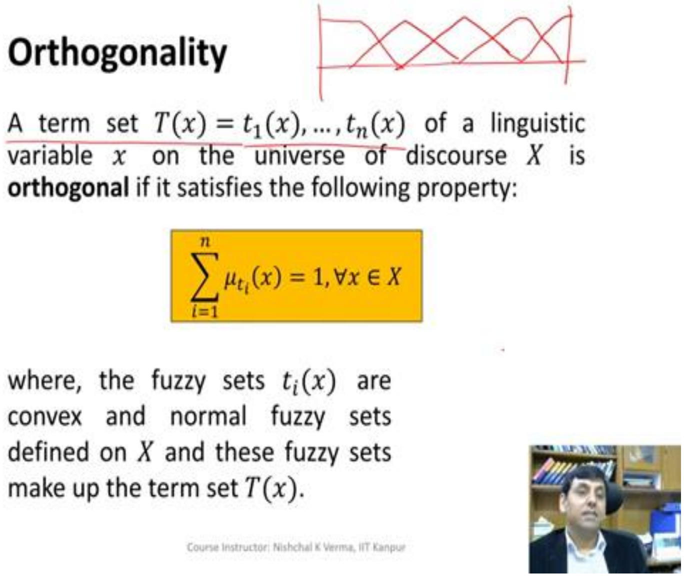

Now, let us discuss another term, let us understand the orthogonality of fuzzy sets. So, if we have a fuzzy if we have a term set $T(x)$. So, we have already seen that the $T(x)$ includes many linguistic values and these linguistic values if these are represented by fuzzy sets.

So let us say the term sets $T(x)$ has multiple linguistic values and which are represented by fuzzy sets $t_1(x), t_2(x), t_3(x), \ldots\ldots, t_n(x)$. This means that term set has n number of fuzzy sets. So this means what this means that if we have any variable any linguistic variable and if these linguistic variable if this linguistic variable is within the universe of discourse, let us say this is the universe of discourse, within this, we have multiple fuzzy sets. Means this the reason within the universe of discourse is covered by multiple fuzzy sets and let us say we have here n fuzzy sets, small n number of fuzzy sets. So here what is the orthogonality? Is that the if we have a term set $T(x)$ and this term set has

$t_1(x), t_2(x), t_3(x), \dots\dots, t_n(x)$ of a linguistic variable x on the universe of discourse X. So this is called the orthogonal if it is satisfies the following property.

So this means that the we take summation of all these fuzzy summation of all these membership values which are there for a particular generic variable value x. So here, this

$$\sum_{i=1}^{n} \mu_{t_i}(x) = 1, \forall x \in X$$

So this is very simple to compute by taking an example, I will be able to understand you better I hope.

And so the $t_i(x)$ we can say, the i^{th} fuzzy set in the term set in the complete term set are the convex and normal fuzzy sets that is very interesting to know that all these fuzzy set which are present in the term set, must be a convex and normal fuzzy sets.

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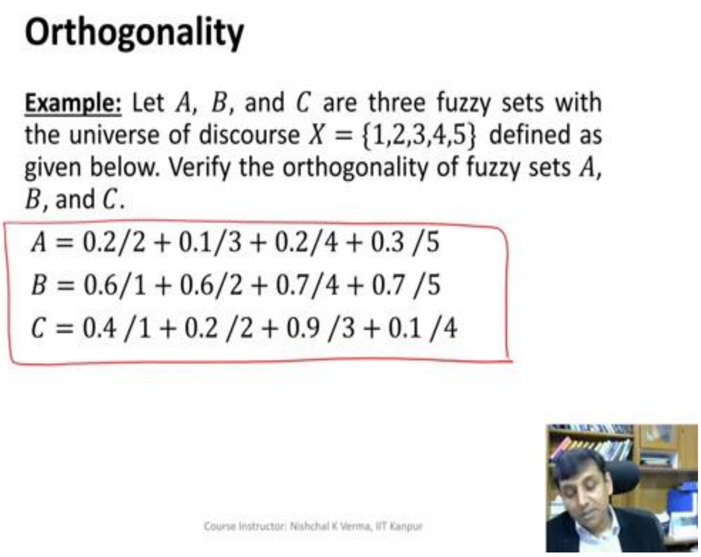

So let us take an example here, and understand. So, first we take a discrete fuzzy set and first we take discrete fuzzy sets A, B and C to understand the orthogonality betters. So very simple, so by taking the example by going through these examples 2, 3 examples, we are taking will be able to understand the orthogonality of fuzzy sets better.

So here, we are taking 3 fuzzy sets, 3 discrete fuzzy sets and this these 3 fuzzy sets are within the universe of discourse $1, 2, 3, 4, 5$. So means from 1 to 5. So now, we see we have a fuzzy set discrete fuzzy set A $0.2/2 + 0.1/3 + 0.2/4 + \ 0.3/5$.

We have another fuzzy set B which is $0.6/1 + 0.6/2 + 0.7/4 + \ 0.7/5$. The C fuzzy set that we have is $0.4/1 + 0.2/2 + 0.9/3 + \ 0.1/4$. So this way we have 4. So, we have the 3 fuzzy sets and now let us check whether the fuzzy sets, the orthogonality condition is satisfied for these fuzzy sets or not, for the term set or not.

So, basically the orthogonality when we talk of orthogonality, it’s for the whole term set. So, we simply check for a particular generic variable value if we sum all the membership values if it is equal to 1, we say the, and it should be there for all the generic variable values. Means, throughout all the generic variable values at any generic variable value we should have the summation of the membership values is equal to 1. So this is the condition.

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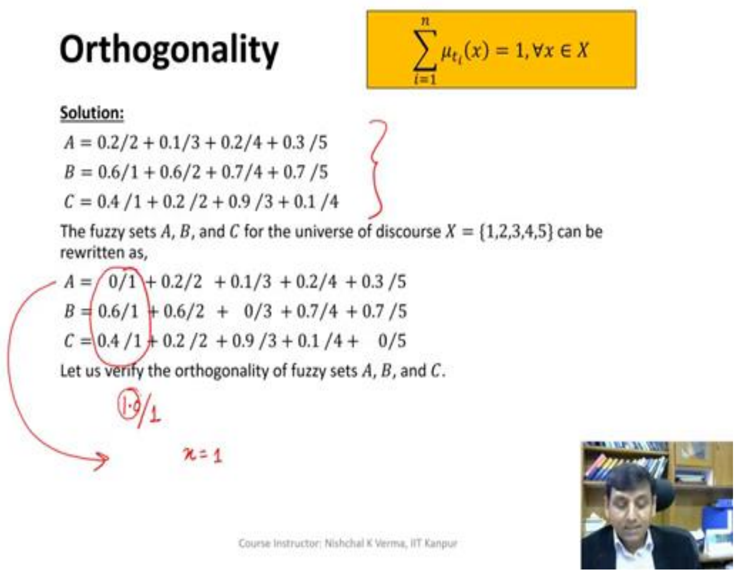

So, let us now quickly check. So since we have the 3 discrete fuzzy sets, and if we take a particular a generic variable let us say x is equal to 1, so see here that if we take the generic variable 1 the summation here is going to give us the 1, the membership value 1. So we can say that at generic variable 1 generic variable value 1, the summation the total of the membership values of all the coming from all the fuzzy sets is equal to 1.

So this way we can say that this is 1, I mean for corresponding the generic variable value 1, we are getting the total of the membership values is equal to 1.

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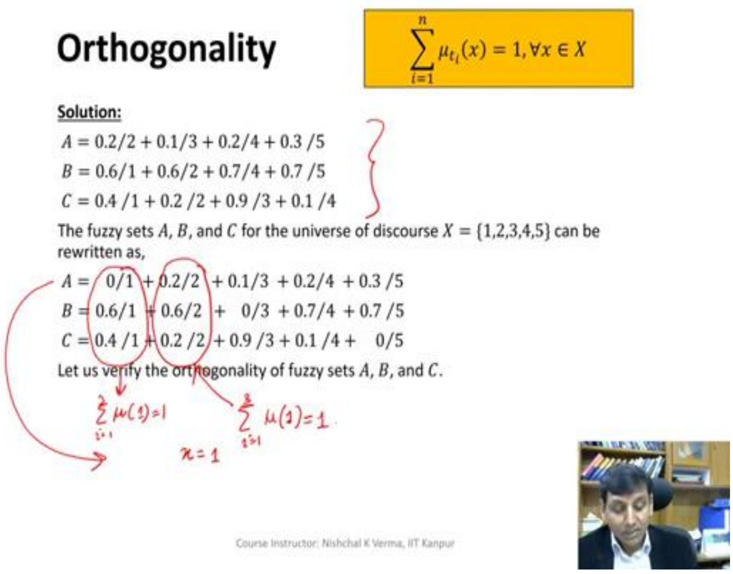

Similarly now, we check for the other. So here, we can write it like this and similarly here also we see that summation of membership values summation of membership values for all the 3 membership for all the 3 fuzzy sets. So, this is also equal to 1.

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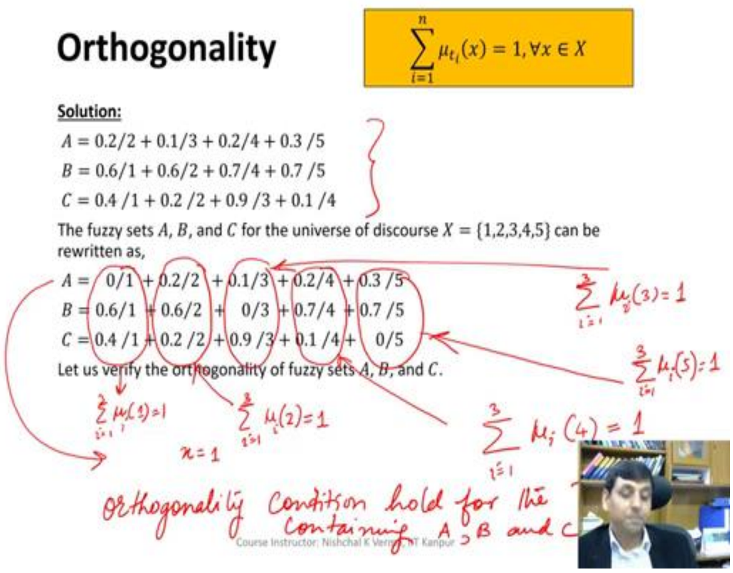

Similarly, we see that this is for 2. This is for the generic variable value 2. Here we see for generic variable value 3. If we sum all the membership values. So, this is for 3. So, i is equal to 1 to 3, all the 3 membership values corresponding to the 3 fuzzy sets if we are adding we are getting here one.

Similarly, here for this also for the generic variable value 4. When we sum this, we see that i is equal to 1 to 3, μ this I can write here i. So μ_i and now if I write the $\mu_i(4)$ of 4, this again, you see 0.2, 0.7 and 0.1 both are going to give us 1. Similarly, here also the summation of the i is equal to i 1 to 3 and then $\mu_i(5)$ this is again is going to give us 1.

So we see that for all the generic variable values in the universe of discourse, we are getting the summation of all the membership values in the corresponding fuzzy sets in the term set is equal to 1. So this way, we can say for these 3 fuzzy sets these 3 discrete fuzzy sets, the orthogonality the orthogonality condition holds for the term set containing A, B and C.

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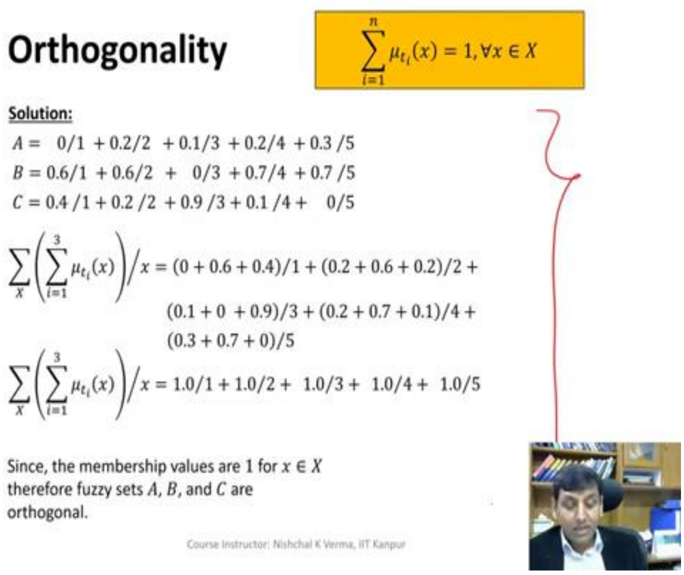

So this way we can say that the both the all the 3 discrete fuzzy sets are basically helping the term set to be the orthogonal. Now you can see the solution in more detail here, on the same lines it has been explained and we can have one more example here.

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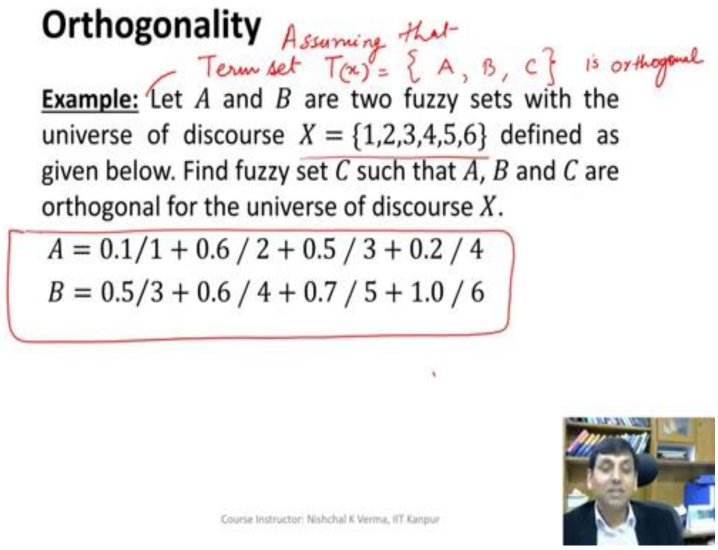

So we are assuming that we have a term set we have a term set $T(x)$ which is nothing but which has A, B and C.

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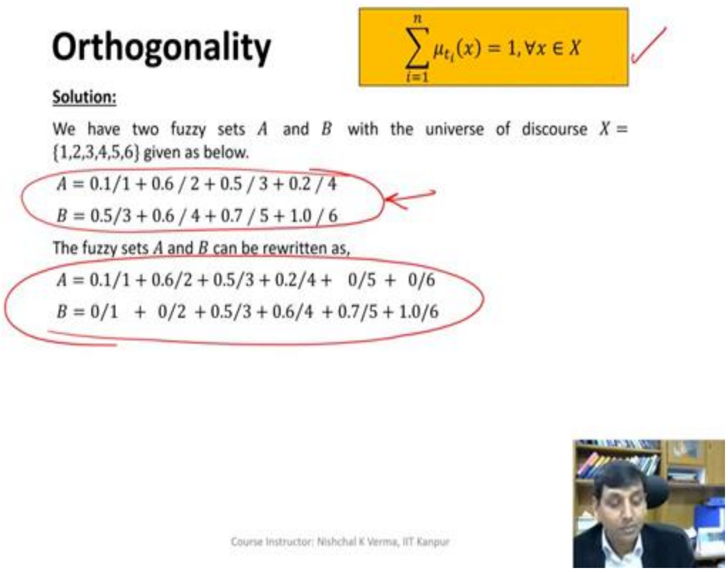

So, we assume that these term set $T(x)$ is orthogonal. So here, we assume that this term set. So here we are assuming. So assuming that term set $T(x)$ which has the linguistic values A, B, C and A, B, C are nothing but the fuzzy sets.

So out of these A, B, C, 2 fuzzy sets A and B are which is again in the universe of discourse here as mentioned 1 to 6 and they are these A and B are known to us. Now the question is if the term set $T(x)$ is orthogonal then what is C? What is the fuzzy set C? So its very simple, we apply the same condition here for orthogonality and since we already know that we have 2 fuzzy sets A and B and all of the membership values corresponding the generic variable values which are known.

Now we can have a fuzzy set C and since it is already said that the term set is orthogonal, so we can quickly manage to find the third fuzzy set applying this condition. The $\mu_{t_i}(x)$ and the summation i is equal to 1 to n. So, here we have 3 fuzzy sets. So, n is equal to 3 here A, A, B and C.

So, let us now quickly go through we have this fuzzy sets this 3 fuzzy sets, I can again write the x is nothing, but this A, B, C.

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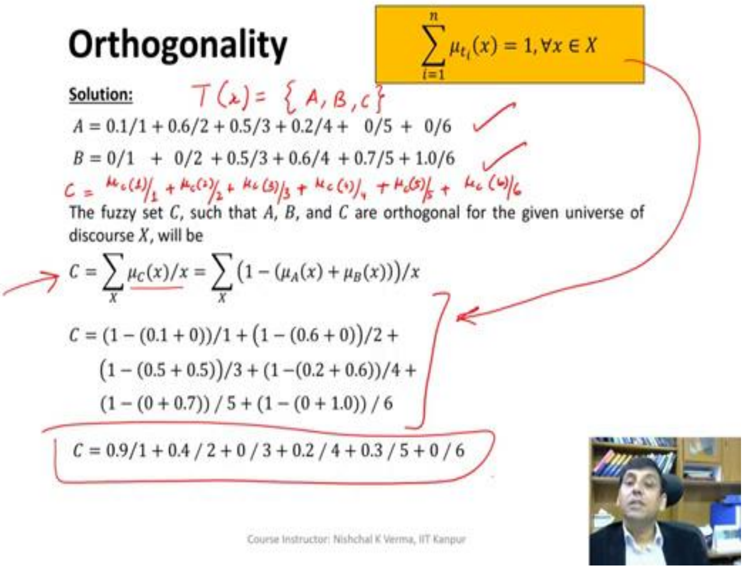

So these 3 fuzzy sets we have and A is known, B is known. So C can be known by subtract by having 1 minus these membership values for the corresponding membership for the corresponding generic variable value.

So, we can assume here the C is going to be the $\mu_C(x)/1$ and then we can have similarly the μ_C, we can write like this. There is let us say C. So C can be $\mu_C(1)$ and then we can have $\mu_C(2)/2$ then we can have $\mu_C(3)/3$ then we can have $\mu_C(4)/4$ then we can have $\mu_C(5)/5$ then we can have $\mu_C(6)/6$.

So now here, we are we can compute all these respective membership values $\mu_C(1)$, $\mu_C(2)$, $\mu_C(3)$, $\mu_C(4)$, $\mu_C(5)$, $\mu_C(6)$ and this way we can represent it like C is equal to

$$C = \sum_X \mu_C(x)/x$$

So this can be written like this here and when we use the condition, this condition here we can find the C is equal to $0.9/1 + 0.4/2 + 0/3 + 0.2/4 + 0.3/5 + 0/6$. So the condition here is that the sum of all the values of memberships corresponding to the generic variable 1 here.

So, for 1, for 2, for 3, for 4, for 5, for 6 all the generic variable values the corresponding membership values are known now and when you sum these values. So at any point at any point of the generic variable value, the summation of the membership values are 1. So this way, given the condition that a $T(x)$ which is comprising of A, B and C fuzzy sets of the linguistic values and if A and B are given C can be computed if $T(x)$ is orthogonal.

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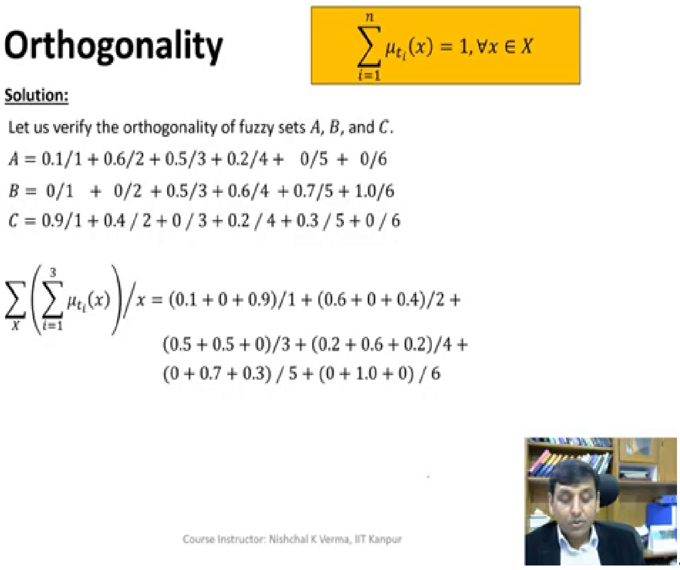

So, it’s very simple and the solution is here, you can further extend it. So now we have an other example here. We have.

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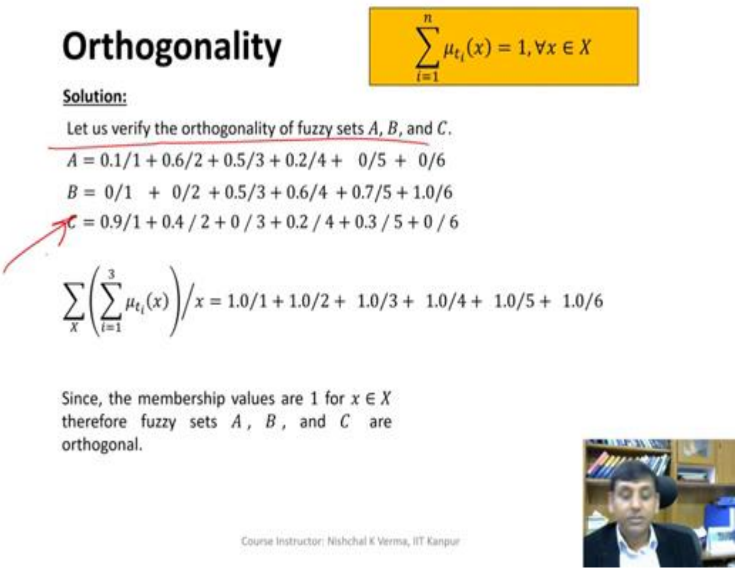

So before this, let me just quickly go through this and see that the with the orthogonality we are getting the C very easily.

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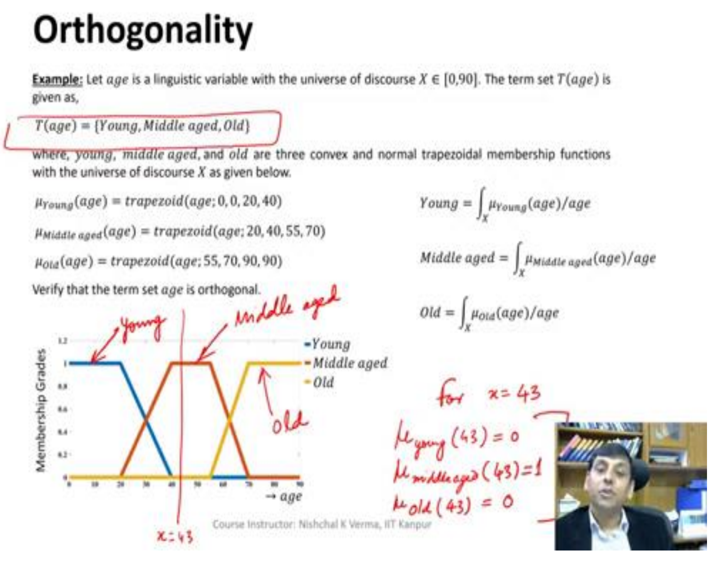

Okay so let us now take the continuous fuzzy sets or means a term set which has multiple continuous fuzzy sets. So here the same formula applies means at any point of time or any point of generic variable value, the summation of membership values given the universe of discourse are within the universe of discourse this going to be 1.

So we can clearly see that if we have a term set let us say $T(age)$ and this term set has 3 linguistic values which is which are $young, middle - aged$ and old. And these 3 linguistic values are represented by or characterized by fuzzy sets. So if we have this as $young$ and this as $middle\ ages, middle\ aged$ and this as the old.

So we have 3 fuzzy sets. So now we see within the universe of discourse 0 to 90 which is given. Within this, at any point of the generic variable we see that when we sum the membership value corresponding to a particular membership value we get its summation always 1. So the young fuzzy set has the trapezoidal membership function and $middle - aged$ also has the membership function trapezoidal and for old also, we have trapezoidal as the membership function.

But $young$ and old are open trapezoidal means left open, $young$ has left open trapezoidal, old is right open trapezoidal. So this way, we when we see that we at any point of time. So if we let us say if we draw a line here and let us say here for x is equal to let us say 43. So, for 43 we see that we have the membership value. So, let us say $\mu(43)$, I am just writing here.

For x is equal to 43 the $\mu_{young}(43)$ is 0. $\mu_{middle} - aged(43)$ is 1. You can see here the membership value corresponding to x is equal to 43.

And then we have μ_{old} this is $\mu_{old}(43)$ is equal to 0. So now, when we sum this for x is equal to point for x is equal to 43, we see that we are getting all μ's for 43, let us say this is 1 to 3. So this is here and then we are getting 1. So likewise, if we move this line throughout within the universe of discourse, we are getting at each and every place the summation of all the membership values unity.

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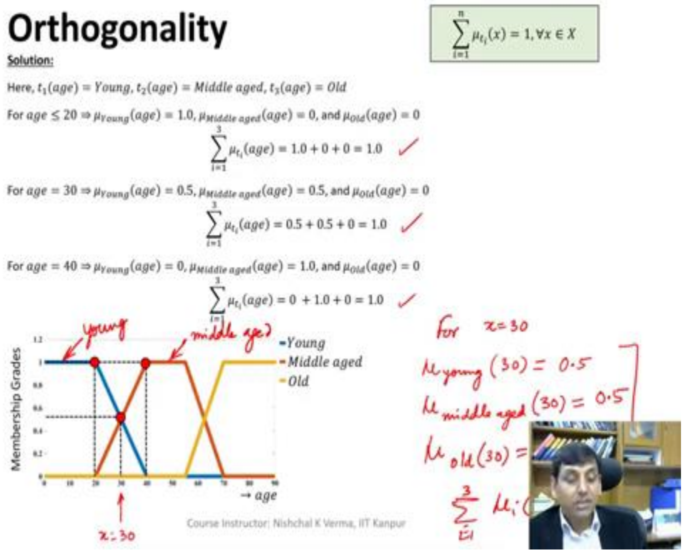

So this way here you can see every place we are getting the summation of its membership values 1. Similarly, if we take this value which is x is equal to 30, the generic variable value is equal to 30. So we see that for generic variable value that is x is equal to 30. What is happening? So what is happening here is that $\mu_{young}(30)$ is.

You see here, just look at the membership function characterizing the fuzzy set young which is represented by the red colour. So no which is represented by the yellow colour. This is sorry, blue colour. So this is characterized by this is represented by this mu this is the fuzzy set young. This represented by blue colour. So if we look at this at x is equal to 30 we are getting its membership value 0.5.

Similarly now so at x is equal to 30 as the membership as the generic variable value, we see that we are coming across 2 membership functions. So first is the young and the second one is middle aged. So we see that which is represented by red colour and then when we see that $\mu_{middle\ aged}(30)$, we are getting this also as 0.5.

So no other membership is available here for x is equal to 30. So we can write here μ_{old}, so this is for 30 or at 30 this is at 30, 0. So when we sum these 3, when we take this summation of these mu all μ's at 30, i equal to 1 to i is equal to 1 to 3, we see that this is 1.

So similarly, for this these terms for this term set having the fuzzy sets $young, middle - aged, old$, we see that within the universe of discourse no matter what value you take for x as the generic variable value for each and every value of x, we are getting the summation of its corresponding membership values 1. And this way we can say the term set here given is the orthogonal term set. So orthogonality condition for this term set is satisfied.

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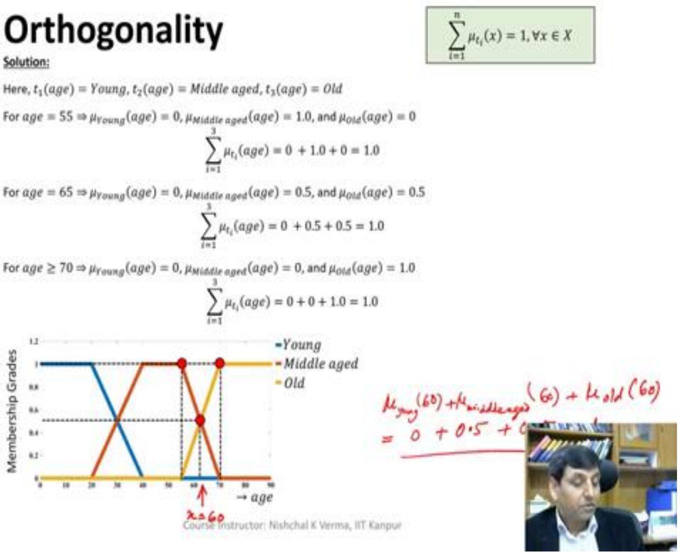

Similarly, here if we take x is equal to 60. So for this also we see that we are coming across 2 membership functions. For 2 membership functions of $middle - aged$ and old, respectively. So we see that both of these we have 0.5 as the membership values and when we see that for x is equal to 60 when we sum these sum all the membership values.

We find 0.5 for $middle - aged$ and 0.5 for old and the summation is 1. And please understand that at x is equal to, at x is equal to 60, μ_{young} is 0. I can write here $\mu_{young}(60)$, $\mu_{middle\ aged}(60)$ and then $\mu_{old}(60)$. So this is going to give us the summation 1.

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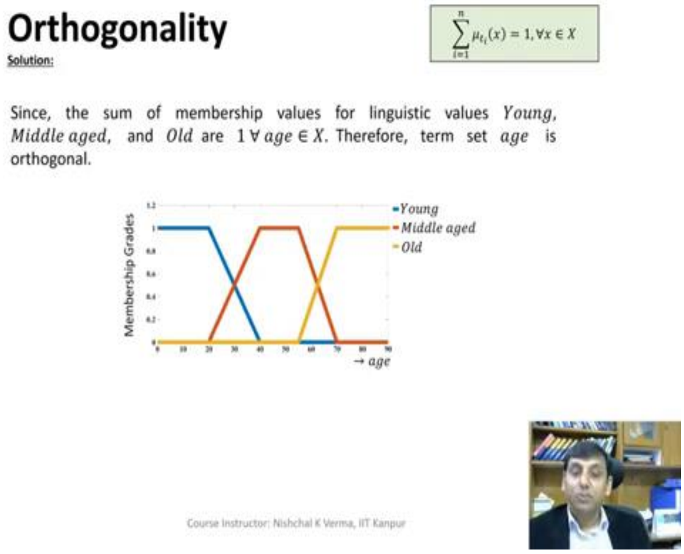

So this way, we can say that the sum of the membership values for linguistic values $young, middle - aged$ and old are 1 for every generic variable value belonging into the universe of discourse X. And therefore, the term set which is for age is orthogonal.

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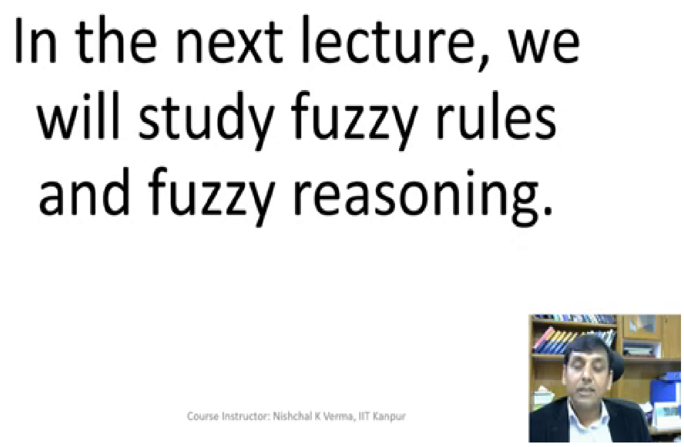

So that is how we test the orthogonality of the term set, now at this point we will stop and in the next lecture, we will discuss the fuzzy rules and fuzzy reasoning.

Thank you very much.