Fuzzy Sets, Logic and Systems and Applications
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Lecture - 46
Contrast Intensification and Orthogonality

Welcome to lecture number 46 of Fuzzy Sets, Logic and Systems and Applications. In this lecture, we are going to discuss a Contrast Intensification and Orthogonality. And this lecture is in continuation to our previous lecture which was on linguistic hedges where we have learnt the concentration and dilation of fuzzy sets.

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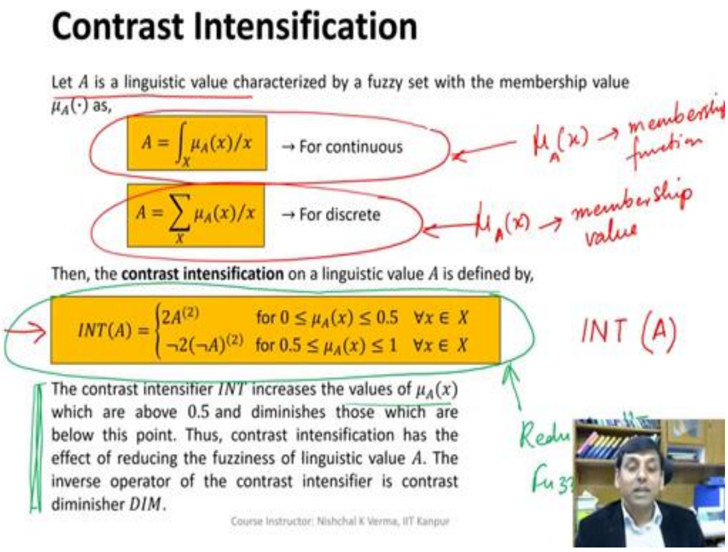

So, since we are going to discuss the contrast intensification in this lecture and we have already the idea of the concentration of a fuzzy set. So here, in contrast intensification, we use the concentration of fuzzy sets.

So let us assume that we have a fuzzy set A and this is nothing but this fuzzy set A is linguistic, this fuzzy set is for a linguistic value. So in other words, we can say we have a linguistic value and which is characterized by a fuzzy set and if it is a continuous fuzzy set, then we have $\mu(x)$ here as the membership function $\mu(x)$ as the membership function.

And if we have a discrete fuzzy set, which is just the representation of linguistic value then the $\mu_A(x)$ is the membership value. And of course, this membership value is basically the corresponding to the generic variable x. So the contrast intensification is basically represented by INT which is you can see here $I-N-T$ is the symbol for the intensification, the contrast intensification.

So we have A if we have A as a fuzzy set which whose intensification is needed or whose contrast intensification is needed to be done, then we write this as the $INT(A)$. So we normally write this INT of the normal bracket the small bracket, here. And this is represented by, this is expressed by the $INT(A) = 2A^{(2)}$.

This means that we are doing the concentration and this concentration is a normal concentration because for normal concentration, we have k is equal to 2. So here we have the intensification of A, this will be equal to $2A^{(2)}$. It means we are concentrating the fuzzy set A or in other words I can say the A is being normally concentrated so that you know we have k is equal to 2. So $A^{(2)}$ and then whatever comes here as we multiply it by 2.

So this is for the membership value, this computation is for the membership value. So let us now understand this, this is true for $0 \leq \mu_A(x) \leq 0.5$. And this of course, is for all x for every x belonging into the universe of discourse X. And then we have another expression, this is valid for $0.5 \leq \mu_A \leq 1, \forall x \in X$.

And if this is the case, then we have another expression. What is this expression? We have the $\neg 2(\neg A)^{(2)}$. So this needs to be completely understood that we apply this intensification in 2 zones.

So, first zone starts with it starts below 0.5 and the second zone is starts above 0.5. So above 0.5 as the membership value, we have $2A^{(2)}$ and below 0.5 as the membership value of A holds the value of the intensified membership $\neg 2(\neg A)^{(2)}$.

which is clearly written here you can understand by this expression. So the intensification is very important concept and this is used for you know, the reducing the for reducing the, I am writing here for reducing for reducing the fuzziness. So whenever, we intensify we take the we do contrast intensification of any linguistic value which is characterized by a fuzzy set, we essentially do the reduction of fuzziness which is present in the fuzzy set.

So you can just see here this paragraph mentions the same. That means, that the contrast intensification INT increases the value of $\mu_A(x)$ which are above 0.5 and diminishes means reduces those which are below this point means the 0.5 as the membership value of fuzzy set A or the linguistic value A. Thus, the contrast intensification has the effect of reducing the fuzziness of the linguistic value A, the inverse operator of the contrast intensifier is also available is also used and this is called the contrast diminisher, which is represented by DIM, $D-I-M$ which is as already mentioned that it is the opposite of the contrast intensification.

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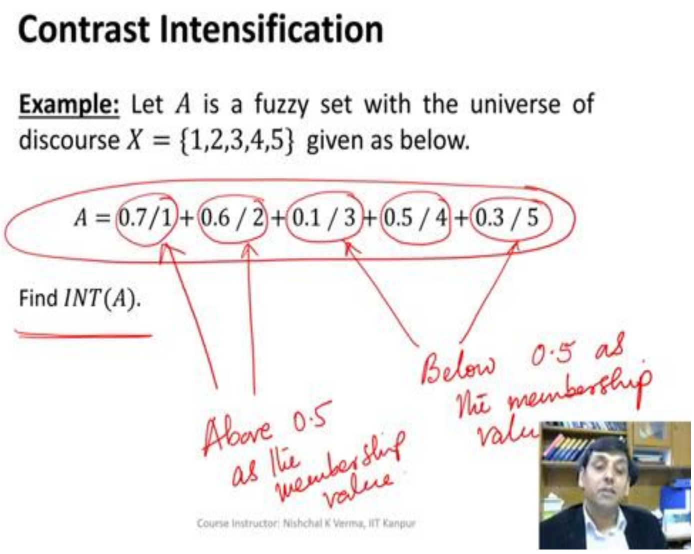

So let us have 1 example of a discrete fuzzy set to understand the contrast intensification better. So here we are taking this example where we have a fuzzy set A which is of course, a linguistic value and this is with the universe of discourse which is defined here and the universe of discourse here we have $1, 2, 3, 4$ and 5.

So if we see here that we have a fuzzy set which is a discrete fuzzy set which is available with us and this represents the fuzzy linguistic value and if we are interested in finding the contrast intensification or the $INT(A)$, then let us see how we can manage to get. So as I have already mentioned that we first look for the point where we have to apply the first expression that I mentioned that above 0.5 as the membership value and below 0.5 as the membership value.

So if we look at this fuzzy set where we have these 2 you see here, these 2 terms so $0.7 / 1$, $0.6 / 2$ both are above 0.5. And so here the expression for contrast for contrast intensification that we will be using is nothing but the $2(A)^{(2)}$. Means we are going to concentrating the fuzzy set and then we will the then the membership value that we have for this fuzzy set we will multiply this membership value by 2.

Similarly, we have 1 term here which is with 0.5. So this 0.5 now the question comes which form which expression we should apply because both the expression are applicable. So whatever what whatever expression that we apply both are going to give the same result because both are applicable for the 0.5 as the membership value. So now, here we see the here we see that here we see that we have these two terms which are below 0.5. So, these are below 0.5, these are below 0.5 as the membership value and here these are above 0.5 as the membership value.

So this way, we see that we can clearly apply the expression which are valid for these membership for computing these membership value for intensification of A.

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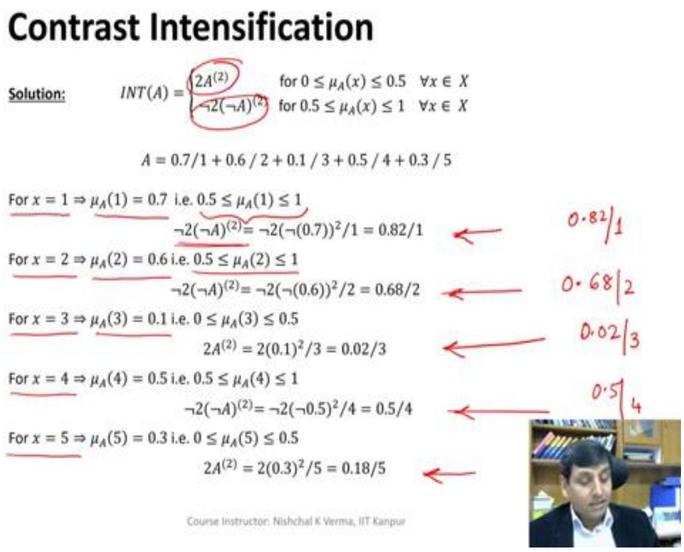

So let us quickly go ahead and see how we proceed. So for x is equal to 1, we have $\mu(A)$ of 1 which is 0.7 and this is within this range. So this is above 0.7. So this expression applies. What is this expression? Expression here that applies is the $\neg 2(\neg A)^{(2)}$. So when we compute, we are going to get the new membership value which is point which is 0.82 for the generic variable 1. Similarly, for x is equal to 2 that is $\mu_A(2)$ which was given as 0.6.

So here and now since this is 0.6, so this is very clear that the 0.6 is lying within this range where μ_A is more than or equal to 0.5 or less than or equal to 1. So this way, when we apply the expression of INT contrast intensification, we get $0.68\,/\,2$. We here 2 is the generic variable value that is, x is equal to 2. Similarly, for x is equal to 2, the same can be computed and this please remember that this membership value for the generic variable 3 for the generic variable value 3 which is 0.1.

So it is clearly understood that this membership values since it is below 0.5. So the contrast intensification expression will change and the contrast intensification basically here will be $2(A)^{(2)}$ is equal to $2 \times (0.1)^2$. Means, the new value that comes out to be here is 0.02 for 3.

So similarly for x is equal to 3, we can compute and we see that we are getting for generic variable value 0.5; 4 0.5 and similarly for generic variable value 5, we are getting the membership value of the contrast intensified fuzzy set as 0.185. So, this is how we get a new fuzzy set.

So please remember that this is how we get a new fuzzy set which is the intensified fuzzy set, this is called contrast intensification of fuzzy set. And when we apply these expressions as I just discussed and for you know these 2 conditions and when we when we compute these new membership values, new membership grades for respective membership for respective generic variable values.

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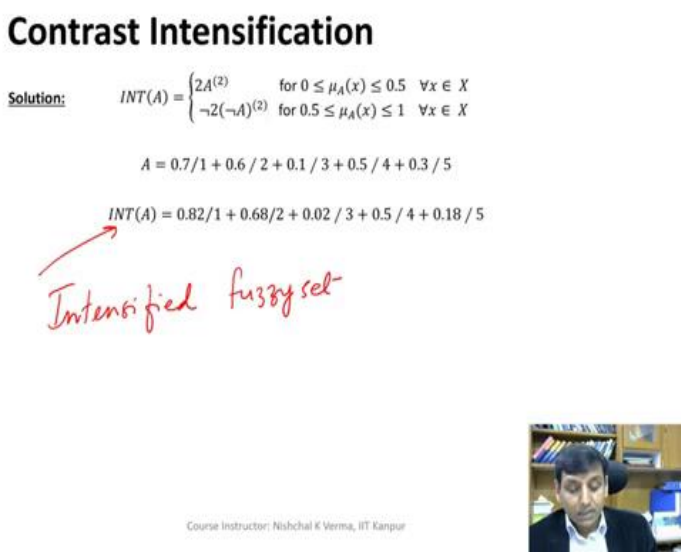

So this way we get the all these computed and then when we write the new fuzzy set which is nothing but the intensified fuzzy set which is the contrast intensification of A and this is nothing but when we since we have already computed all these new membership values of respective fuzzy respective generic variable values 1, 2, 3 and 4. So we get this new fuzzy set. So this is the intensified fuzzy set. INT or in other words, we can say the intensified or we can say the intensification of a fuzzy set A. So that is how we managed to get the contrast intensification of a fuzzy set that was given to us.

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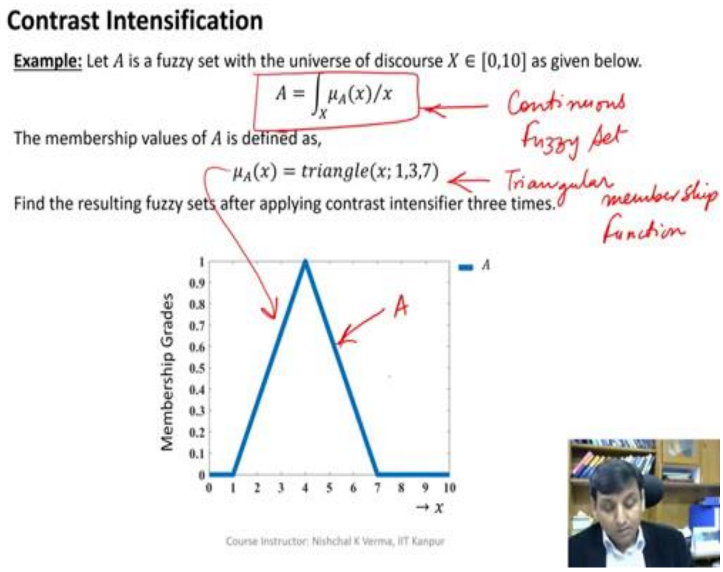

Now let us take another example and this example is for the continuous fuzzy set. So if we have a fuzzy set A which is a continuous fuzzy set. If we have a fuzzy set A which is a continuous fuzzy set you see here, we have a fuzzy set which is a continuous fuzzy set continuous fuzzy set. So and if we look at this fuzzy set, we this fuzzy set is characterized by membership function and this membership function is $\mu_A(x)$ and $\mu_A(x)$ here is triangular membership function.

So I can write here a Triangular Membership Function. This is represented by the $triangle(x; 1,3,7)$ are the vertices of the triangle that we are intending to know, intending to use for to represent the membership function.

So here we have a triangular membership function which is a triangle triangular membership function and the vertices are $1, 3, 7$. We can see here the same membership function and this membership function is for fuzzy set A. So this is for this is a membership function which is characterizing a fuzzy set A.

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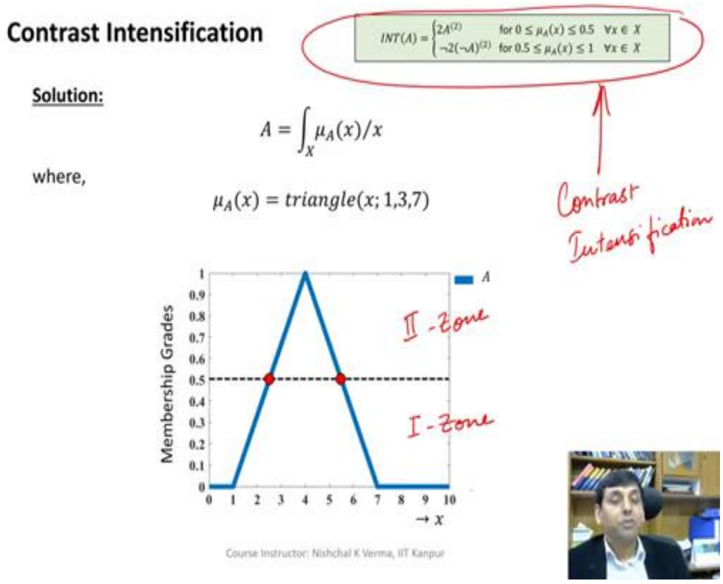

Now let us use the contrast intensification function INT for this fuzzy set and we can use this formula this expression for contrast, intensification and this is going to give us a new fuzzy set which is the intensified fuzzy set which is contrast intensified fuzzy set and we will look as to how this fuzzy set is, we will see as to how this fuzzy set looks like.

So as this intensification function the contrast intensification function is applicable and these are the, this intensification function has 2 expressions and the first expression is applicable in the first zone and the second intensification function expression is applicable for the second zone. So first zone is basically the zone which is below the 0.5 membership value.

So this is the first zone, I write here the first zone and then we have the second zone which is more than which is applicable to more than 0.5 as the membership value. So now, and please understand that any membership value which is exactly equal to 0.5, so please understand that both of them means either of these expressions can be applied so and of course, these any of these if you apply both of these are going to give the same value of the intensified value of the membership function of the intensified fuzzy set.

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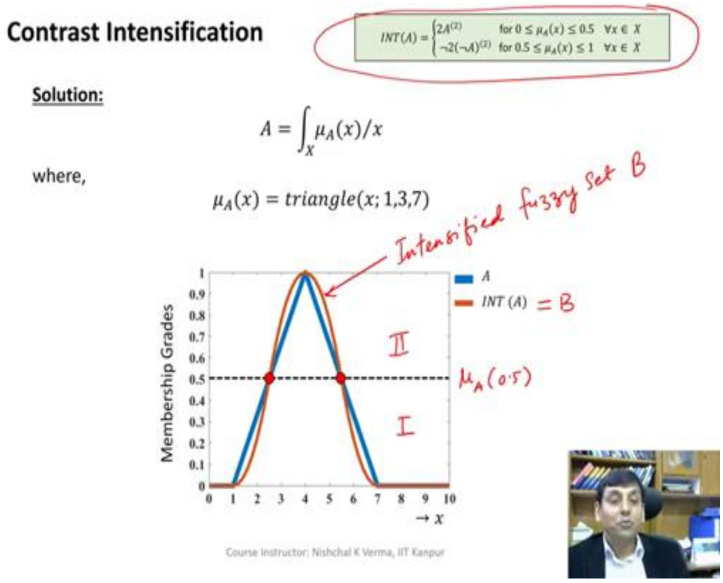

So let us now see what is happening. So when we apply this for the first time so for the first time means we apply the intensification function here, the contrast intensification function. So when we do that we see that below 0.5 line this is below 0.5 line is μ is equal to I can write here is μ_A is equal to 0.5. So below this this in Zone 1, the fuzziness is getting reduced because you see the membership values of the corresponding generic variables are getting reduced and when we talk of the Zone 2, here the Zone 2. So in Zone 2, we see that the membership values of the intensified fuzzy set A is increased as compared to the original membership value of the fuzzy set A for corresponding generic variable values.

So this way we have when we are when we apply this contrast intensification for continuous fuzzy set, we and when we plot the intensified the contrast intensified I can write here intensified fuzzy set. So the fuzzy set that was given to us was A and when we intensify this will look like this. So this is shown by a red colour fuzzy set. So now, when we have found another fuzzy set which is the intensified fuzzy set, I can write here intensified fuzzy set let us say B.

If it is B, if it is represented by let us say B fuzzy set. So this is B and now we can further intensify this fuzzy set and let us see what happens.

So now, when we further go for the contrast intensification of fuzzy set B, let us see what it how it looks like.

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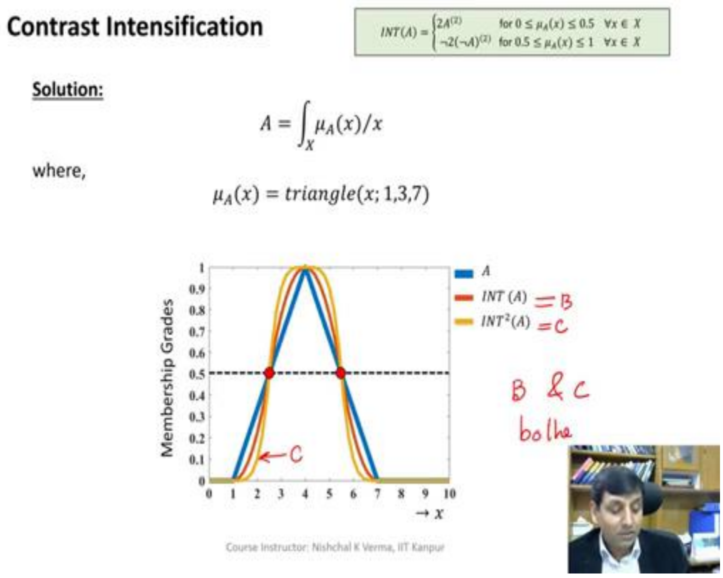

So intensification once again is going to give us if this is B then we let us represent this by C the new fuzzy set. So, intensification is going to give us, you see here the new fuzzy set is represented by fuzzy set C and which is shown by a yellow colour see here. So this was the, so this new fuzzy set C is compared to the fuzzy set B, this is B fuzzy set the red colour fuzzy set B as I have already mentioned. So this is B.

So I can write here that B and C both are both B and C both have reduced fuzziness as compared to the fuzzy set A. So fuzziness can be you know can by taking the intensification by taking the contrast intensification the fuzzy set gets reduced.

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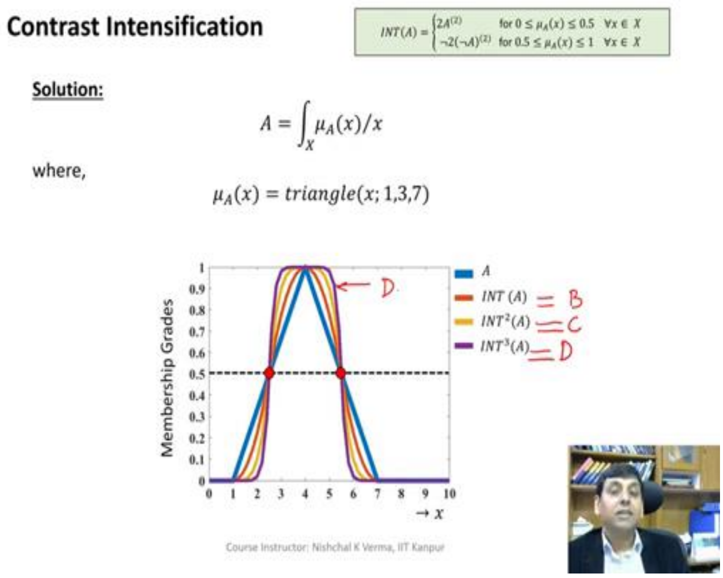

When we go further, let us say this fuzzy set is D fuzzy set intensification once again. So if we represent this fuzzy set by a violet colour which is D. So this is further intensified and we can clearly see that this the you know the 0.5 line is the separator. Below this, the behaviour is different and the above this line the behaviour is different. Means, below this line whatever membership values are there, it is getting reduced above 0.5 line, the membership values are getting increased.

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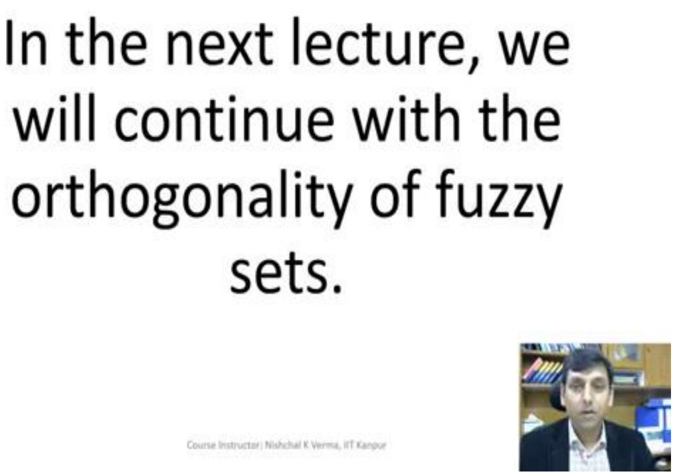

So this way, we are able to intensify the membership value. With this, I would like to stop here and in the next lecture, we will continue with the orthogonality of fuzzy sets.

Thank you.