Fuzzy Sets, Logic and Systems and Applications
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Lecture - 44
Dilation and Composite Linguistic Term and Some Examples

So, welcome to lecture number 44 of Fuzzy Sets Logic and Systems and Applications. In this lecture we will discuss a Dilation of fuzzy set and Composite Linguistic Term and also we will discuss some of the examples.

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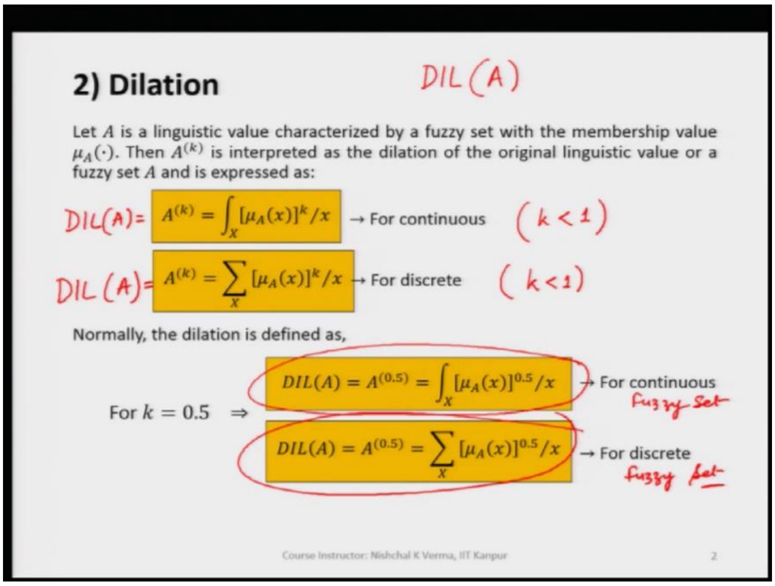

So, let us first take the dilation of any fuzzy set, so as we already know that any linguistic value is always represented by a suitable fuzzy set. So, let us say if we have linguistic value which is represented by a fuzzy set A and we are interested in the dilation of this fuzzy set.

So, the dilation basically here is expressed by or I would say the represented by DIL. So, dilation in short symbolically represented by DIL and if the fuzzy set A which is being dilated is represented by $DIL(A)$. So, dilation of any fuzzy set here is

$$DIL(A) = A^{(k)} = \int_X [\mu_A(x)]^k/x$$

So, what we are doing here in dilation of any fuzzy set is we are basically taking the we are raising the power of the membership function, but this here this raise of k is the opposite and in other way; in other words we can say we are decreasing the power. So, here $[\mu_A(x)]^k/x$ and here please understand that this k is always less than 1.

So, the value of k here is for dilation is always less than 1 and similarly here if we have any discrete fuzzy set if we have any discrete fuzzy set A. So, on the same lines we go for dilation and here also we write

$$DIL(A) = A^k = \sum_X [\mu_A(x)]^k/x$$

And here this k is again is the value of k is less than 1, but normally the value of k here is 0.5.

So, if we do not mention any value of k the value of k here is understood as 0.5 for dilation, so then if we are let us say dilating a fuzzy set A continuous fuzzy set A. So, we can represent the dilation of A, as

$$A^{(0.5)} = \int_X [\mu_A(x)]^{0.5}/x$$

Similarly, we have the discrete fuzzy set, then we can use this expression for dilating fuzzy set A, dilating a discrete fuzzy set A. And, this fuzzy set will be $A^{(0.5)}$

$$A^{(0.5)} = \sum_X [\mu_A(x)]^{0.5}/x$$

So, this is for the discrete fuzzy set the first one is for the discrete fuzzy set and then second one is sorry, first one is for the continuous fuzzy set and the second one is for the discrete fuzzy set here.

So, this way we have understood the formulation remain the same, formulations remain the same for the dilation same as the concentration except the value of k. So, in concentration the value of k remained always more than 1, whereas for dilation the value of k is always less than 1.

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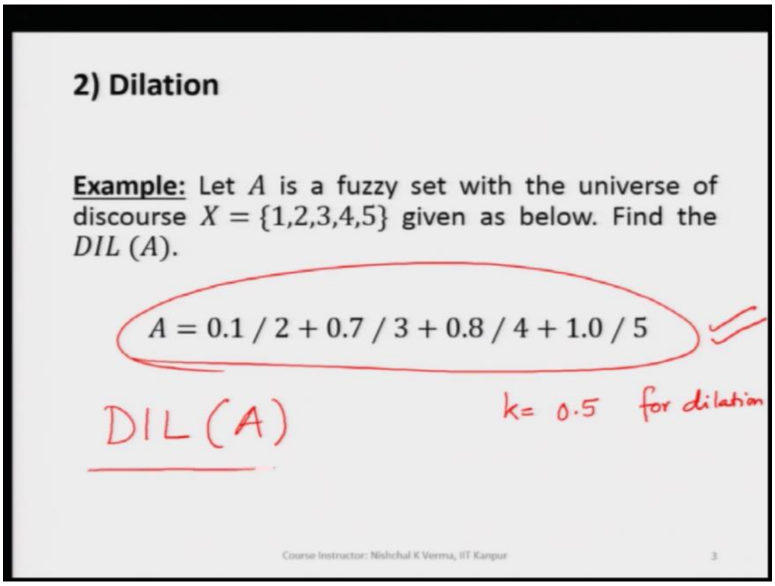

Now, let us take an example here to understand the dilation of any discrete fuzzy set here in our case it is A. So, A is given to us and it is defined by the elements here the $0.1/2 + 0.7/3 + 0.8/4 + \ 1/5$ see here. So, this is a discrete fuzzy set within the universe of discourse X that is 1 to 5 and you see now we have to find DIL means dilation of A. So, we see here that for dilation of any fuzzy set we need the value of k as well, but here since the value of k is not mentioned in this example.

So, we can take we will take value of k as default here 0.5, so the value of k is automatically coming as 0.5, for dilation for dilation if nothing is mentioned, similarly if it was a concentration it was for concentration we could have taken $k = 2$.

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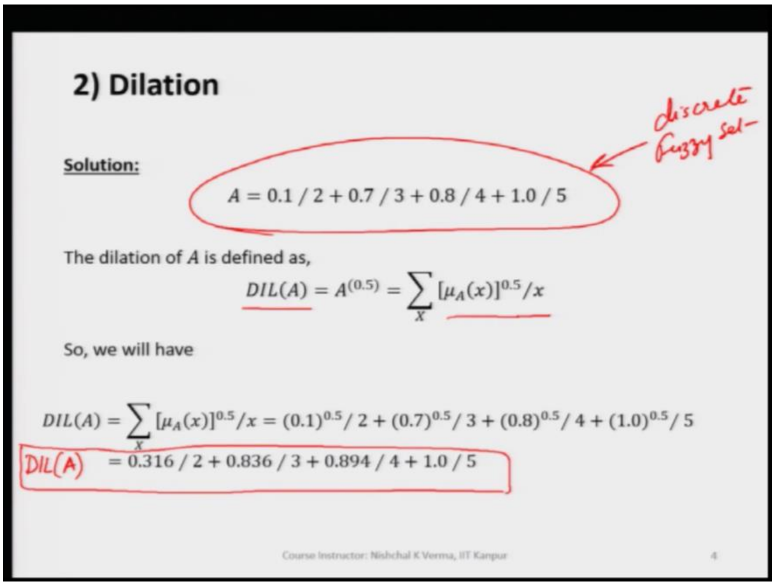

So, now, let us take the dilation of A let us do the dilation of A. So, we have A as a discrete fuzzy set here discrete fuzzy set. Now, when we do the dilation we need to increase the power we need to raise the power of the respective membership values to 0.5 which is here corresponding its generic variable values. So, when we do that what we are getting here is this see $DIL(A)$ is coming out to be $0.316/2 + 0.836/3 + 0.894/4 + \ 1/5$.

So, we get a new fuzzy set which is a dilated fuzzy set, which is the outcome of the dilation of a fuzzy set A. And as we know that we have got this by taking the square roots of the respective membership values corresponding to all generic variable values. So, this way we have found the dilation of a fuzzy set, discrete fuzzy set A here and similarly other fuzzy sets can be taken and the same on the same lines that dilation of the fuzzy set A can be found out.

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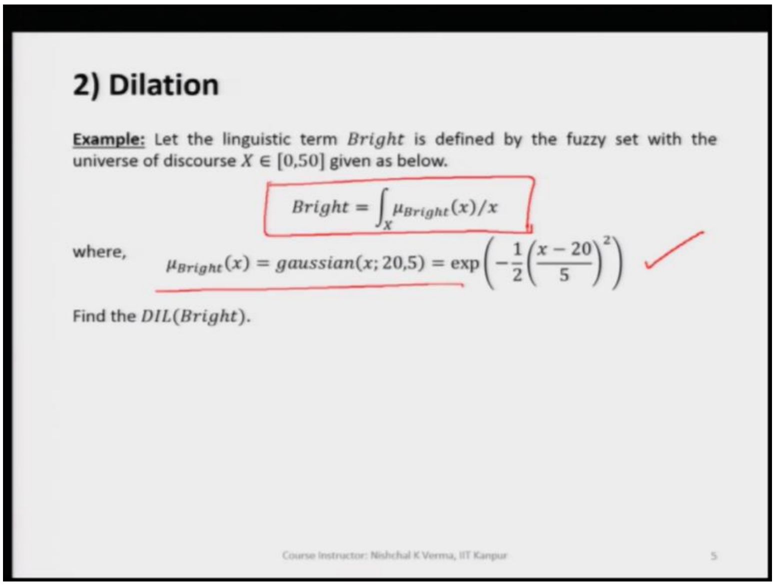

Now, if we take another example which is on the continuous fuzzy set. So, here we have a continuous fuzzy set, let us say this fuzzy set is a $bright$ this fuzzy set is used this fuzzy set is to represent the linguistic term $bright$. So, we have this example where we are taking a fuzzy set continuous fuzzy set here to represent the linguistic term bright. Now, if we are interested in finding the dilation of this fuzzy set again we have to do the same which we have done in the previous example.

But here we have since we have the continuous fuzzy set, since $bright$ is a continuous fuzzy set. It means the membership function is a continuous fuzzy set to give this continuous fuzzy set. So, in our case in this example $\mu_{bright}(x)$ is a Gaussian membership function which is

$$\mu_{bright}(x) = gaussian(x; 20,5) = exp\left(-\frac{1}{2}\left(\frac{x-20}{5}\right)^2\right)/x$$

So, this is the continuous membership function and this continuous membership function is used to represent the continuous fuzzy set.

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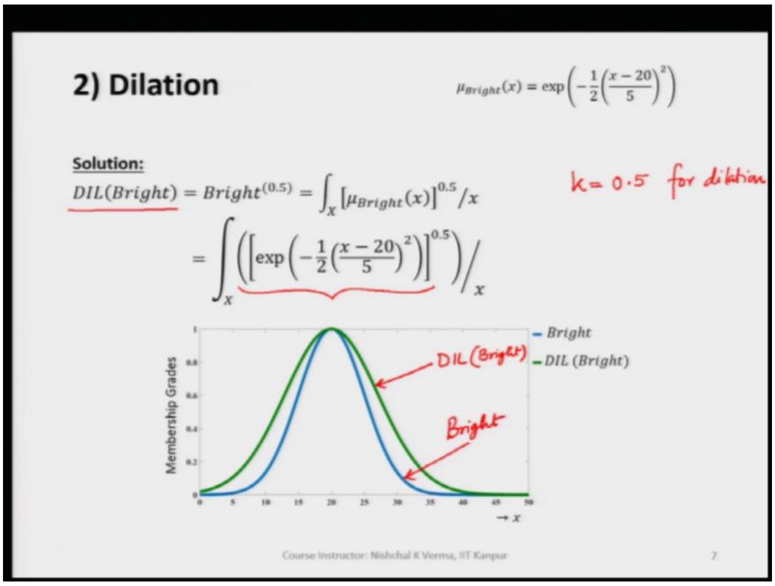

Now, if we are interested in finding the dilation of this fuzzy set, the $bright$ fuzzy set we can write the dilation of $bright$. And then in the in in this way the this can be written by the $bright^{(0.5)}$. So, since we are going for dilation so we have to have the value of k less than 1 and since the value of k has not been given in this problem as well.

So, we have to take the standard value of k that is 0.5 for dilation for dilation. And when we do that we simply raise the power by raise the power of the membership function by k and here k is 0.5. So, we simply write here $exp\left(-\frac{1}{2}\left(\frac{x-20}{5}\right)^{0.5}\right)/x$

So, this way when we plot this when we represent this here so this blue coloured fuzzy set basically is for this is for $bright$ and then here the green coloured fuzzy set is nothing but the dilation of $bright$. So, we can clearly see here that what is happening here is the original fuzzy set original continuous fuzzy set which after taking the dilation of it has wider spread.

So, we see that the spread of the fuzzy set is increased, so dilation of any fuzzy set always increases its spread. So, in this case in this example we have a Gaussian membership function for the fuzzy set which is used to represent the linguistic term $bright$ and when we have dilated the $bright$ linguistic term. So, the dilation the normal dilation is giving us a new fuzzy set which has more spread and it is shown by a green colour here this picture.

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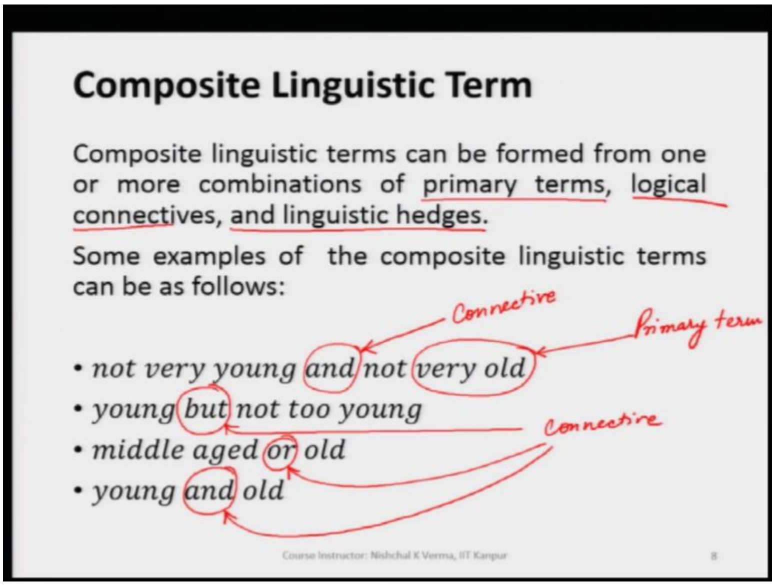

So, this way we have understood the dilation here and now let us go further to talk about the composite linguistic terms. So, composite linguistic terms can be formed from one or more combinations of primary terms here logical connectives and linguistic hedges. So, let us use all of these words all of these combinations together to get or to build the better linguistic variables or in other words I would say that is use all of these to modify to get the modified linguistic variables.

So, if we can use primary terms logical connectives and linguistic hedges, so we see that when we use all of these one of the composite linguistic terms can be like this like we have $not\ very\ young$ and $not\ very\ old$. So, here we have and as the connective and we see that what is the primary term primary term is basically $young$ and $very\ old$. So, primary term is this primary term, so like that when we use all of these together we can form a new linguistic variable this is also called composite linguistic term.

And, similarly we can have another composite linguistic term which is here as $young\ but\ not\ too\ young$. So obviously, here we see that we have but is the another connective which is used for and so here but is connective. And then in other case in the third one we have $middle\ aged$ or old so we have or as the connective in this case.

Similarly, in the fourth case we have and as the connective so this way in all the cases we are getting a new linguistic term and we would call these as a composite linguistic terms.

And this we are getting out of the out of all the primary terms logical connectives and in some cases the linguistic hedges.

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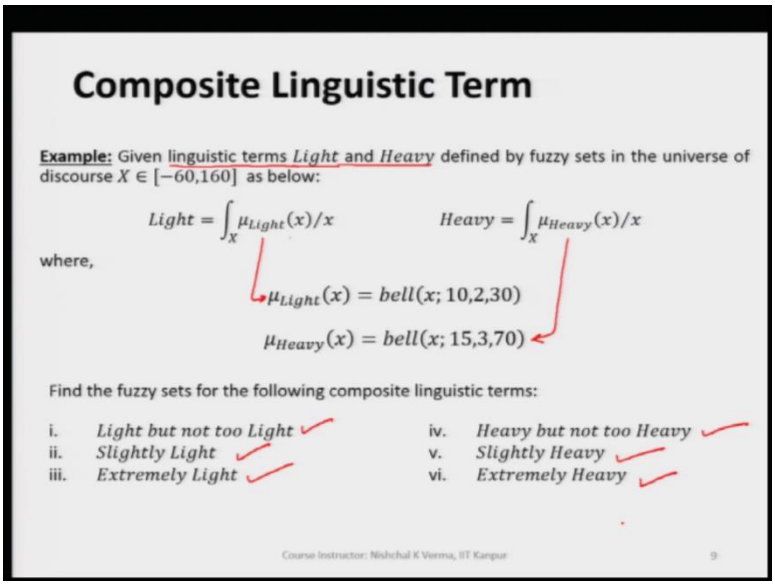

So, let us first understand that the how the linguistic edges we have we can obtain we have already discussed these linguistic hedges and these linguistic hedges basically we obtain by either dilation dilating the original fuzzy set or concentrating the fuzzy sets. So, let us take an example here to understand the composite linguistic term out of the primary term and the connectives and hedges.

So, let us take this example, so in this example here we have the linguistic term $light$ and $heavy$ and of course, these $light$ and $heavy$ are since these are the linguistic terms, linguistic values. So, these will be defined by some fuzzy set and here $light$ is defined by this fuzzy set heavy is defined by this fuzzy set and these two are the continuous fuzzy sets. And these membership values membership functions μ_{light} is defined by a bell shaped fuzzy set.

Here this mu light is μ_{light} is this and then μ_{heavy} is this, so this way both the fuzzy sets are continuous fuzzy sets and both the fuzzy sets are different fuzzy sets. Now our job is to find the composite fuzzy sets out of the given fuzzy sets as primary terms. So, we see that we can have the composite linguistic terms like this like we have we can have $light$ $but\ not\ too\ light$ and then we can have $slightly\ light$.

We can have $extremely\ light$, we can have $heavy\ but\ not\ too\ heavy$ we can have $slightly\ heavy$, but slightly heavy and then finally we can have $extremely\ heavy$. So, like that all these composite linguistic terms we can manage to get by using the primary terms, primary linguistic terms for $light$ and $heavy$ and then its hedges and connectives.

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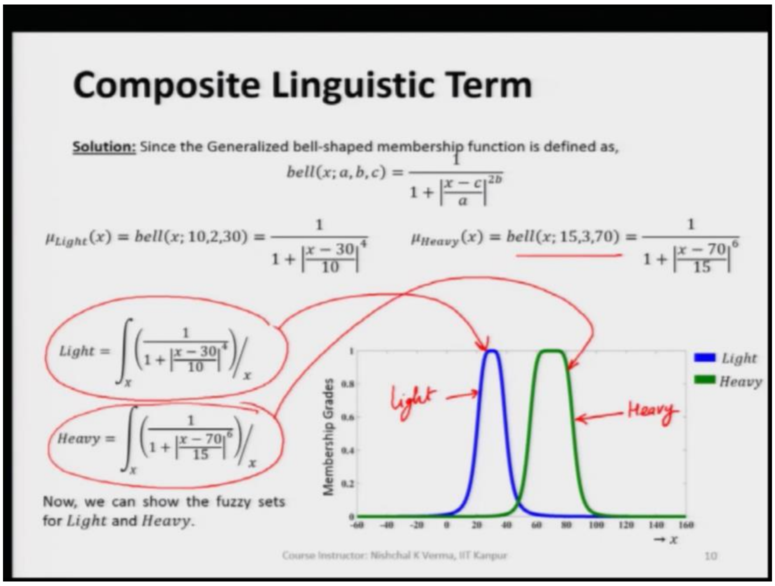

So, let us go 1 by 1 so here since the primary terms, primary linguistic terms are defined by or characterized by suitable continuous fuzzy sets. And here we have these fuzzy sets its membership functions as well bell functions for $light$ and for $heavy$ as well. So, it means that we have $\mu_{light}(x) = bell(x; 10,2,30)$ and then we have $\mu_{heavy}(x) = bell(x; 15,3,70)$. So, this means that we have two bell functions which are the membership functions for the $light$ fuzzy set, the fuzzy set for $light$ and the fuzzy set for $heavy$.

So, let us first define the fuzzy set for $light$ so fuzzy set for $light$ is this here you can see. And since the $light$ has its $\mu(x)$ its membership function and this membership function is coming from here, this membership function is coming from here. So, the light fuzzy set can be defined by

$$Light = \int_X \left(\frac{w}{1+\left|\frac{x-30}{10}\right|^4}\right)\Big/ x$$

So, we see that we have a bell shaped membership function for $light$ and for $heavy$ both and we all know that the parameters that are given here for the bell shaped are 10 to 30 in place of a, b, c respectively. So, we have basically a is equal to 10 and b is equal to 2, c is equal to 30 for membership function for light.

And similarly, the membership function for heavy set $heavy$ fuzzy set we have a is equal to 15, b is equal to 3, c is equal to 70 you can see here. And on the same lines we can have a fuzzy set for fuzzy set representation for heavy.

$$Heavy = \int_X \left(\frac{w}{1 + \left| \frac{x - 70}{15} \right|^6} \right) \Big/ x$$

So, these both the fuzzy sets light and heavy are different because of its membership functions. So, membership functions are different the nature of membership function remains the same both are bell shaped membership functions, but the parameters are different and we can represent these fuzzy sets here.

So, here we have the fuzzy set for $light$ which is represented by blue colour and then we have the fuzzy set for heavy which is represented by the green colour here. So, this way we have two fuzzy sets as the primary terms or in other word we can say the primary linguistic terms $light$ and $heavy$, so we can say this is as $light$ and this is as say $heavy$. So, these both the fuzzy sets are there so these are already given. So, now, we are interested in the composite linguistic term versus here which is $light\ but\ not\ too\ light$.

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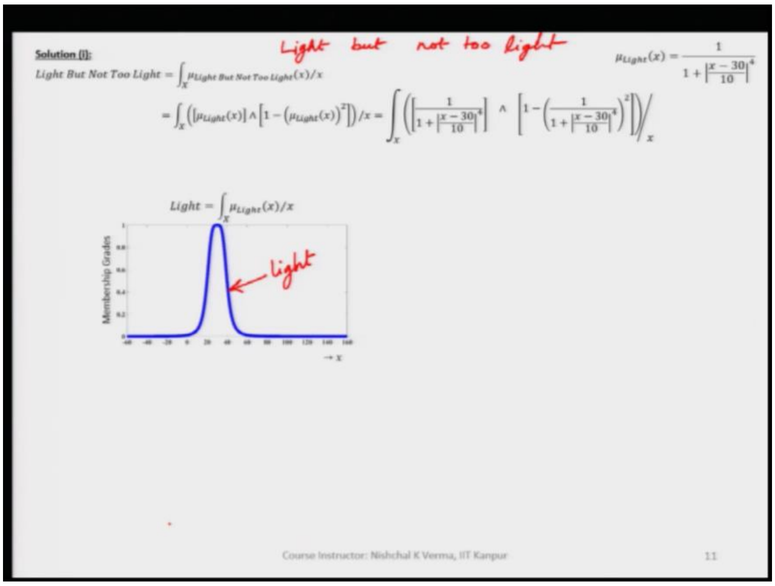

So, let us try to get $light\ but\ not\ too\ light$ I am writing this here $light\ but\ not\ too\ light$. So, please understand that the fuzzy set for $light$ is already there so I am writing here $light$ this is fuzzy set for $light$. Now, we have to make the composite fuzzy set means we need to find first the $too\ light$. So, when we say $too\ light$ it means you know for getting $too\ light$ we need to concentrate the fuzzy set $light$.

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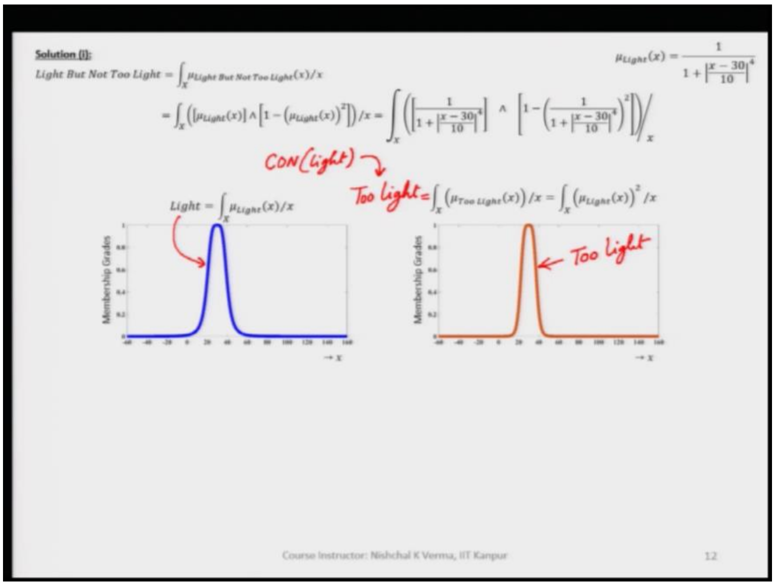

So, let us concentrate the fuzzy set here so when we concentrate the fuzzy set here this is the fuzzy set for $too\ light$ this is fuzzy set for $too\ light$. So, the fuzzy set for $light$ is already there and here what we are doing is we are concentrating the *light* fuzzy set to get the *too light*.

So, when we concentrate the fuzzy set *light* we are getting the fuzzy set as *too light* which you can now very easily understand. So, this way we have got the fuzzy set for *too light* I can write it like this the *too light*. Now, since we have obtained the fuzzy set for *too light* and we need *not too light* also *not too light* the fuzzy set for *not too light*, so now let us take the complement of *too light* to get the *not too light* fuzzy set.

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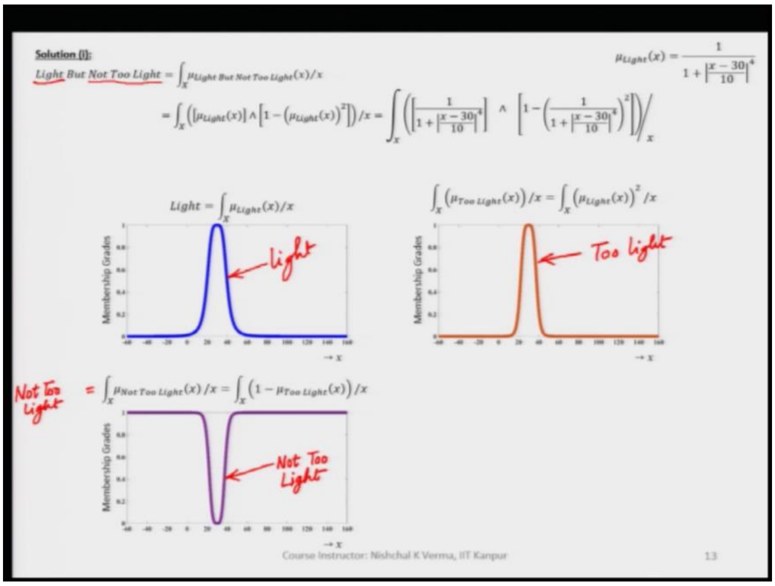

So, here we are taking the complement of *too light* and this is going to give us the fuzzy set representation for *not too light*, *too light* and this way we have getting here the fuzzy set for *not too light*. Now we are interested in *light*, but *not too light* so first we have here the fuzzy set for *light* then we have the fuzzy set for *too light* and then here we have the fuzzy set for *not too light*.

Now, we have to find a composite fuzzy set which is out of the *light*, but not here a connective but, and but is nothing, but and it is equivalent to and so we will basically compose these fuzzy set. So, we have a composition of *light* fuzzy set here because we are interested in *light but not too light*.

So, we should take $light$ and then $not\ too\ light$ and then we connect this by but, so but is as I already said it is and so let us now find the composite fuzzy set here, the modified fuzzy set here out of $light$ and $not\ too\ light$ and both the fuzzy sets are composed by composed with and connective.

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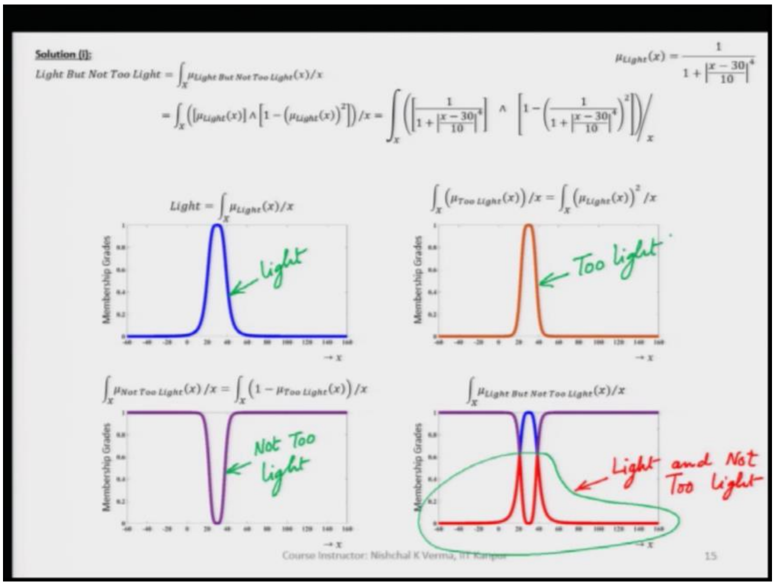

So, here when both the fuzzy sets are superimposed on each other so the $light\ but\ not\ too\ light$ is going to give us the intersection of $light\ and\ not\ too\ light$. So, this way we are getting the this portion when we take the intersection this I will again mention as $not\ too\ light$ and this is for $light$ this is for $too\ light$.

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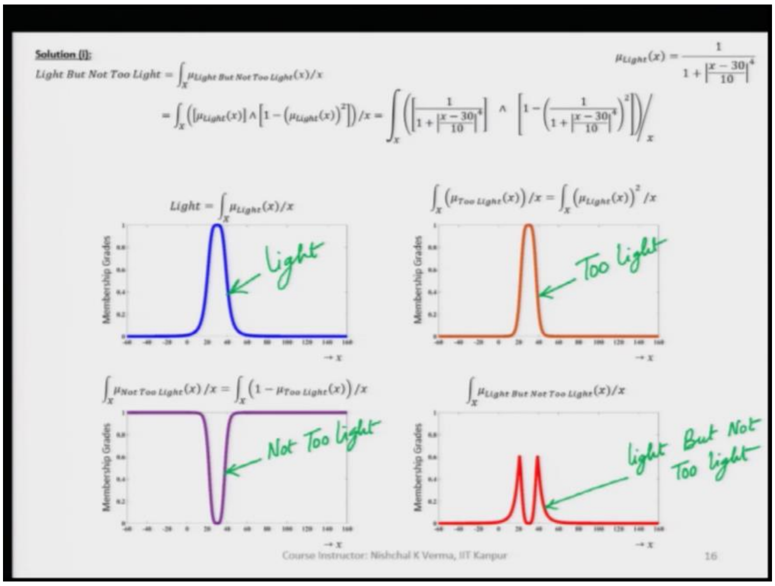

So, this way we have you see composite fuzzy set which is the outcome of $light\ but\ not\ too\ light$ and of course, here this fuzzy set is for $not\ too\ light$ and this fuzzy set is for too light, this fuzzy set is for $light$. So, this way we have obtained the composite linguistic term which is represented by the fuzzy set $light\ but\ not\ too\ light$ which is here.

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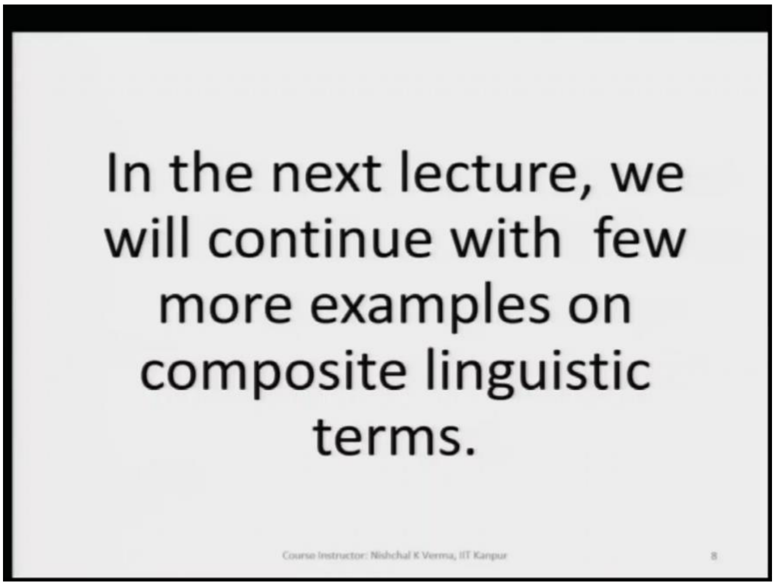

So, with this I would like to stop here and in the next lecture we will continue with few more examples on composite linguistic terms.

Thank you.