**Fuzzy Sets, Logic and Systems and Applications**
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# Lecture – 43
# Concentration and Dilation & Composite Linguistic Term and Some Examples

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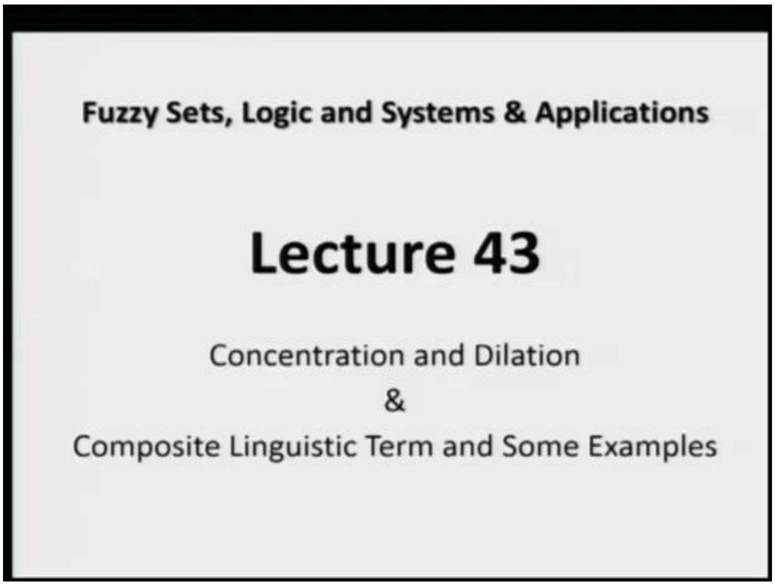


So, welcome to the Lecture Number 43 of Fuzzy Sets, Logic and Systems and Applications. In this lecture, we will discuss the Concentration and Dilation of fuzzy sets and also we will discuss a composite linguistic terms and the related examples.

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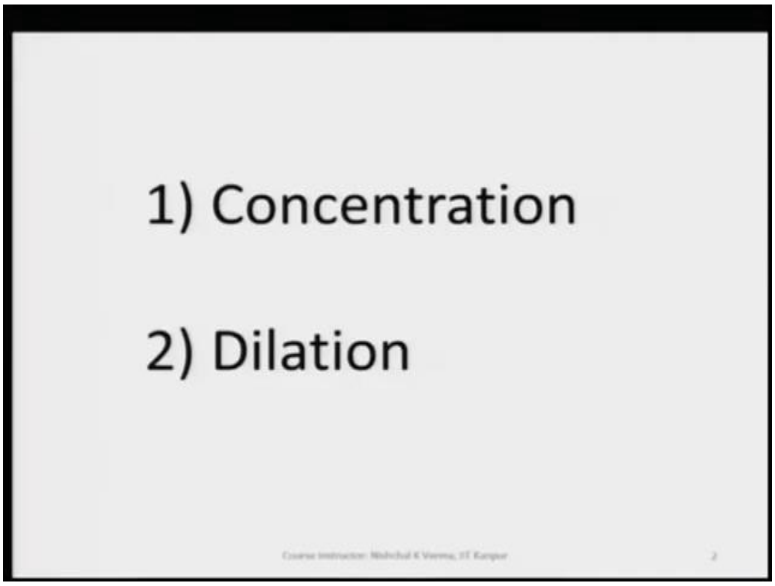


So, here let us first have the concentration and then the dilation.

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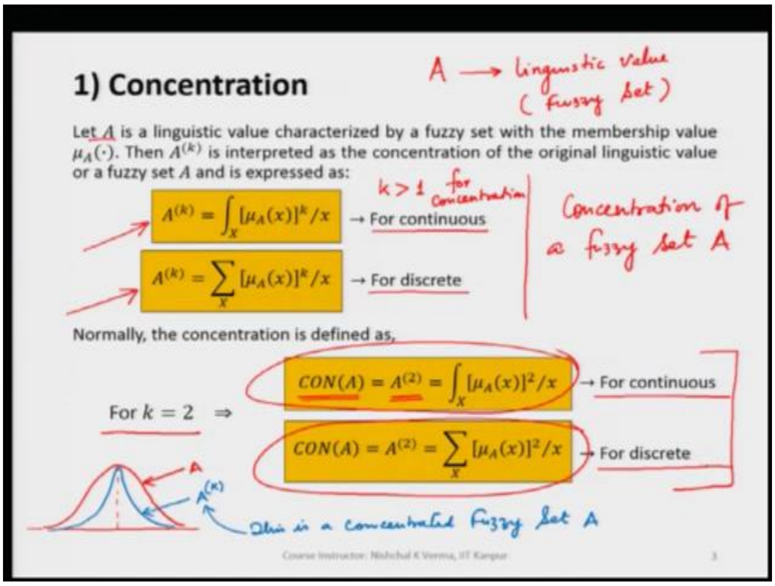


Concentration basically is nothing but its a actually the concentration of a fuzzy set. And when we say fuzzy set it means that we have some linguistic value and which is characterized by a fuzzy set or which is represented by a fuzzy set.

Let us say $A$, then of course, we will have its membership value or membership function in case of continuous linguistic value. So, we'll have $\mu_A(x)$. In case of $x$, we have the generic variable. So if let us say we have $A$ which is a linguistic value, and when we say linguistic value it means in fuzzy system, we represents a linguistic value by a suitable fuzzy set.

So we can say that it is nothing but a suitable fuzzy set. So, if we have a linguistic value which is represented by a suitable fuzzy set $A$ and if we are interested in having the concentration of a fuzzy set $A$, so this concentration of fuzzy set is nothing but $A^{(k)}$. So concentration of a fuzzy set $A$ is normally represented by here, $A^{(k)}$.

And if it is a continuous fuzzy set which is being concentrated, then of course, this will be represented by the integral sign and then we have the universe of discourse $X$ and then we have $\mu_A(x)$ and since we have the $A^{(k)}$ so this $k$ will be here as well. So, this way the concentration of $A$ is represented by the expression $A^{(k)}$.

And similarly, if we have a discrete fuzzy set which is being concentrated, we will use sigma and then we use again the suitable universe of discourse. So, in this case we have the universe of discourse as $X$. So, for continuous and discrete fuzzy sets, we have the concentrated versions of these, the concentrated fuzzy sets. So in summary, a concentration of any fuzzy set basically is nothing, but we get another set which has its membership values or its membership functions raised to the power $k$.

So, rest other things remains the same. Which we can see here that we simply we raise the power of membership function in case of continuous fuzzy set and we raise the power of membership values in case of discrete fuzzy sets. And as I mentioned, rest other things will remain the same.

So here as I mentioned, the power of membership function is raised by $k$. So, $k$ is here some number. In general, we write $k$ for concentration. But for normal concentration are mentioned here that when we do not mention any value of k then normally, the for simpler concentration the value of $k$ can be 2 and the value of $k$ can be any value more than 1.

So if nothing has been mentioned, then we can simply write the value we can take the value of $k$ as 2. So when we take a normal concentration, we have here we have taken $k = 2$ and when we substitute the $k = 2$, what we are getting is here for continuous membership concentration.

So, if we have a fuzzy set $A$ which is a continuous fuzzy set and if we are interested in finding the concentration of fuzzy set $A$, then we simply write here $A^{(2)}$. Why within bracket? Because we are not exactly squaring the fuzzy set $A$, we are only squaring here the membership values or membership functions of the fuzzy set $A$.

So here, this way this $A^{(2)} = \int_X [\mu_A(x)]^2/x$. So, this is going to be our concentrated fuzzy set and when we have a discrete fuzzy set. So for discrete fuzzy set, everything remain the same except the summation in place of the integration sign. So, this way we have understood that what is the concentration of a fuzzy set.

So once again, I would like to tell you that if we are interested in finding the concentration of any fuzzy set, we will simply write the fuzzy set and we will try to find the value of $k$ if the value of $k$ is given here, the when we are interested in concentration, the value of $k$ will always be greater than 1. This has to be noted, for concentration.

So, we will first look for the value of $k$ and if the value of $k$ has been given, then we will use the value of $k$ which will be more than 1. So we will use that, but if the value of $k$ is not mentioned, then we will go for the normal concentration. Means, we will take the value of $k$ as 2. So, as we have done here.

So, concentration is very simple and we simply use the value of $k$ which is more than 1 and this value of $k$ basically increases the power of the membership function for continuous fuzzy set or the power of membership value for discrete fuzzy sets.

And here, the notion of this concentration is nothing but, when we concentrate any fuzzy set whatever fuzzy set that we have here let us say we have a fuzzy set $A$, let us say this is a fuzzy set $A$, and if we are interested in concentrating this fuzzy set so this the concentrated fuzzy set will be something like this, depending upon the value of $k$. So, I can say this is my concentrated discrete fuzzy set.

So, I can write here that this is a concentrated fuzzy set $A$. So what do we see here? What basically do we see here is that, when we go for the resulting fuzzy set that is a concentrated fuzzy set gets a squeezed. Means, they spread gets reduced. So that needs to be understood here, that whenever we concentrate any fuzzy set, its spread is going to get reduced.

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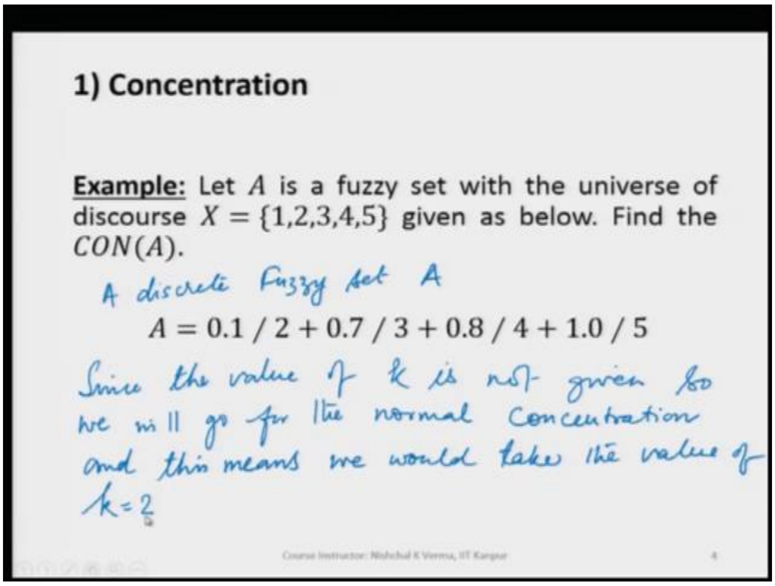


Let us take couple examples on the concentration of fuzzy sets to understand the concept better. So here we have a simple discrete fuzzy set $A$, a discrete fuzzy set $A$ with a universe of discourse 1 to 5 and here we are interested in the concentration of $A$ and $k$ has not been given to us. So, obviously, we will have to go for the value of $k = 2$. So, I am writing here since the value of $k$ is not given, so we will go for the normal concentration and this means we would take the value of $k = 2$.

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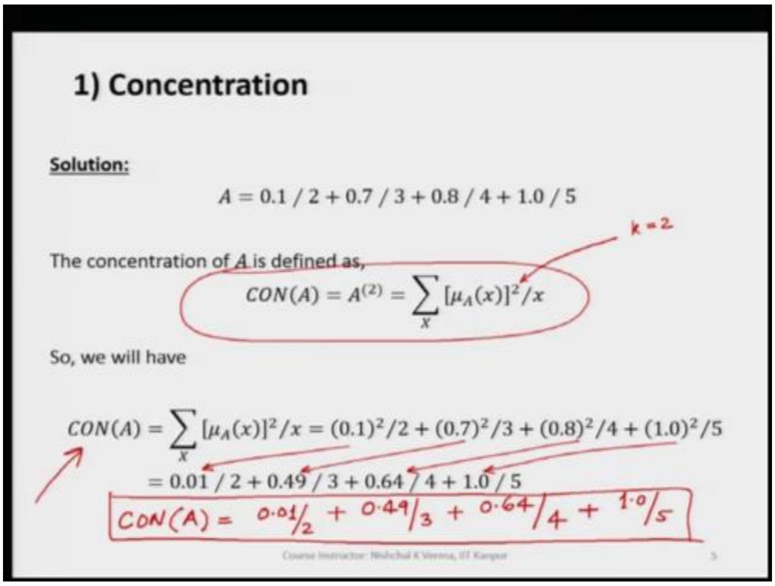

So, when we do this, here the concentration of a discrete fuzzy set $A$, this can be written as this equation. So, $A^{(2)} = \sum_X [\mu_A(x)]^2/x$. So the $k$ comes here. Here, $k = 2$. So, the membership values of this discrete fuzzy set will get squared.

So when we go ahead, the we write the symbol of the concentration as $CON$, so $CON(A) = \sum_X [\mu_A(x)]^2/x =$ (0.1)^2/2 + (0.7)^2/3 + (0.8)^2/4 + $(1.0)^2/5$.

So we are doing nothing except we are squaring the membership values of the corresponding generic values of $x$. So, when we are squaring this what we are getting we see here that, we are getting 0.01 here. When we square 0.1 and we are getting 0.49 when we square 0.7, we are getting 0.64 when we square 0.8 and we are getting 1 when we are square 1. So this way, we are getting a new expression of the discrete fuzzy set which is here, corresponding to the generic variable values, we are getting the modified values of the membership.

So, we get the concentration of the fuzzy set which we have taken as a discrete fuzzy set. So, we can write here the concentration of $A$ is nothing but $0.01/2$, then $0.49/3$, then $0.64/4$ then $1/5$. So this is what is the new fuzzy set as a result of the concentration of the discrete fuzzy set that was given to us. All right, so this was the example for the discrete fuzzy set or I would say the concentration of discrete fuzzy set.

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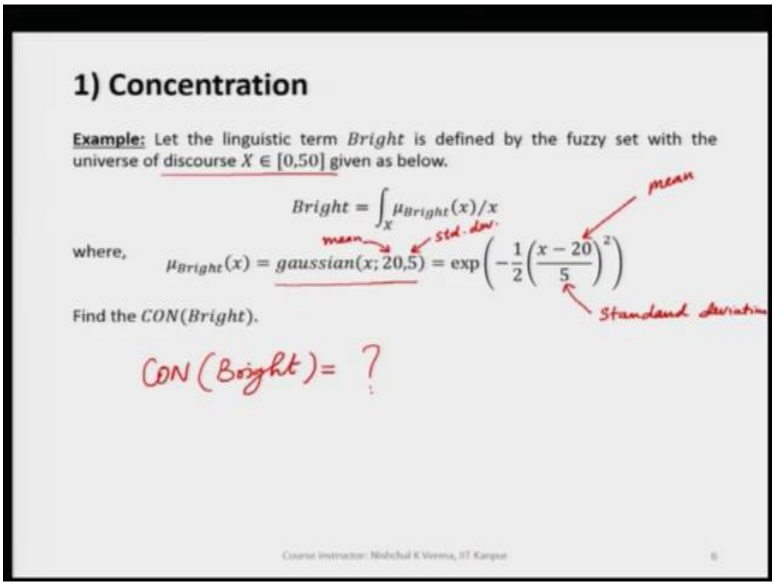

Now, let us take an another example for the concentration of a continuous fuzzy set. So, if we have a linguistic term which is defined by a suitable fuzzy set let us say it is named as a $Bright$. So, fuzzy set for $Bright$. So, fuzzy so, $Bright$ is a fuzzy set, $Bright$ is a represented by a suitable continuous fuzzy set and the universe of discourse here is from 0 to 50. So, I can write here linguistic term $Bright$ is represented by a suitable fuzzy set here.

Where, the membership of this $Bright$ fuzzy set is represented by $\mu_{Bright}(x)$ here and this is nothing, but a gaussian function. So, this is represented by $gaussian(\,x;\ 20{,}5) = exp\left(-\frac{1}{2}\left(\frac{x-20}{5}\right)^2\right)$. Where, this 5 is nothing but the standard deviation. And this 20 is nothing, but the mean.

So, this is the standard deviation and this 20 is the mean. Now, here we have been asked to concentrate this fuzzy set $Bright$ and $Bright$ is a continuous fuzzy set. So, let us do that. So, we'll write the concentration of $Bright$ fuzzy set by the $CON$s of $Bright$, the concentration write concentration and then we will write $Bright$ like this.

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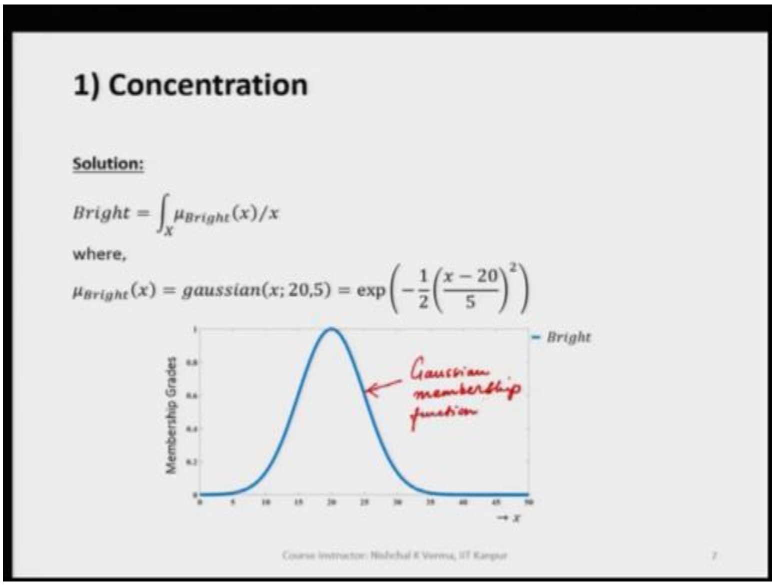


So, when we do this, let us first take this fuzzy set which has been given to us, let us first understand the continuous fuzzy set that is for the linguistic term $Bright$. So, when we plot the fuzzy set here, the $Bright$ fuzzy set, this looks like this and this is nothing, but the gaussian membership function.

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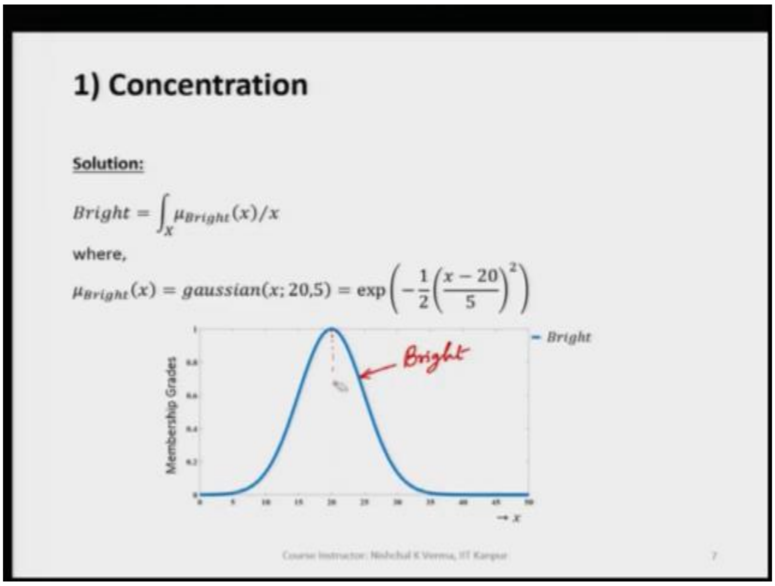


So, we have the fuzzy set which is for $Bright$. And this $Bright$ has the membership function as the gaussian with its mean 20 and its standard deviation 5. We can see here, is 20. So, we can represent the $Bright$ fuzzy set like this.

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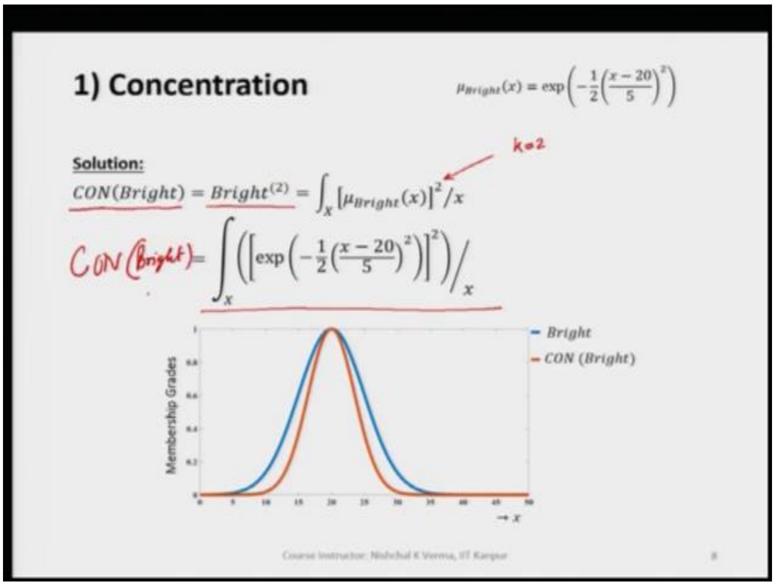


Now, let us concentrate this fuzzy set $Bright$ and as I already mentioned that we can write the $CON(Bright)$ means, the concentration of $Bright$ like this. And this can also be written as $Bright^{(2)}$ and then this is again going to be equal to since this is continuous fuzzy set so, we will use the integral sign to represent this fuzzy set.

So, integral sign over $X$ which is the universe of discourse, and then we will write the $\mu_{Bright}(x)^2/x$. So initially, for $Bright$ we had simply the $\mu_{Bright}(x)$. But when we are concentrating the fuzzy set $Bright$, then we will have to raise the power of the membership function by $k$. And $k$ since is not given in this example so, we can take $k = 2$.

If $k$ has been given, then we will use the same value of $k$ and this $k$ since we are concentrating the fuzzy set so the value of $k$ is going to be always more than 1. So here we are taking $k = 2$, we are squaring the membership function. So, when we do that the whole fuzzy set can be written

$$CON(Bright) = \int_X \left(\left[exp\left(-\frac{1}{2}\left(\frac{x-20}{5}\right)^2\right)\right]^2\right) \Big/ x.$$

And then again whatever is here, as the membership function is squared, means raised to the power 2 and then we have the oblique $x$. So, this is $CON(Bright)$, again.

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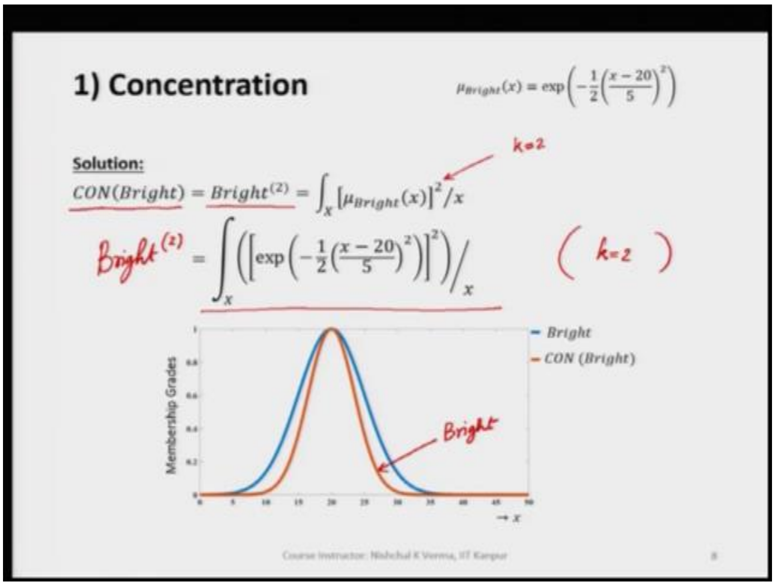


Or in other words, we can write here the $bright$ and then within bracket 2. And here please note that we have used $k = 2$ only. We have used the $k = 2$. So, let us not get confused with the value of $k$, as I have already mentioned if no value of $k$ has been given, then we will simply use the value of $k$ as 2 for concentration. So when we plot the concentration of $Bright$, means the fuzzy set which has come out of the concentration of $Bright$, we get this fuzzy set.

So, which is represented by the red color here. So, this fuzzy set is the concentration of $Bright$.

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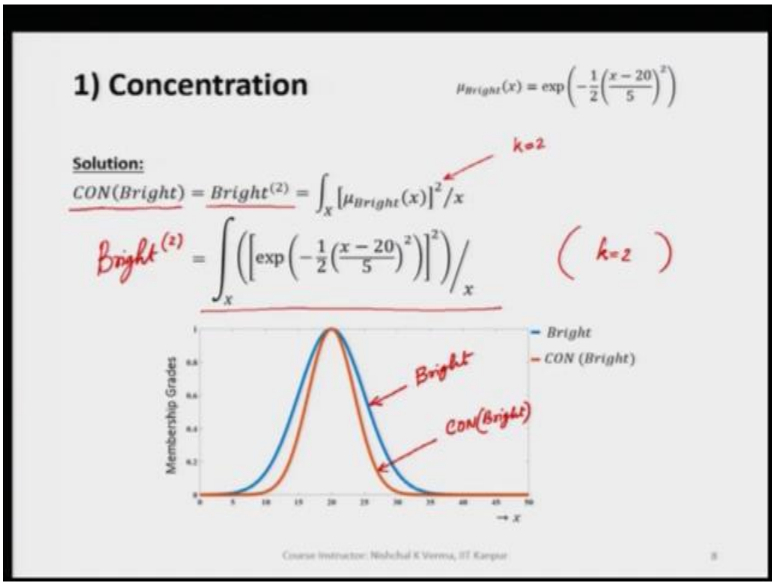


I can write it either this way or the $CON(Bright)$. And this blue plot is for the $Bright$ fuzzy set. So, we can clearly see that when we concentrate any fuzzy set, we get its spread reduced or squeezed. So, here if we once again go for further concentration of the concentrated $Bright$ fuzzy set, we will further get this spread reduced. So, here the concentration basically helps us in reducing the fuzziness the uncertainties that is involved in the fuzzy representation.

So, concentration of any fuzzy set basically gives us reduces spread than that of the original spread of the fuzzy set that was taken for concentration. So, with this the discussion on the concentration of a fuzzy set is over.

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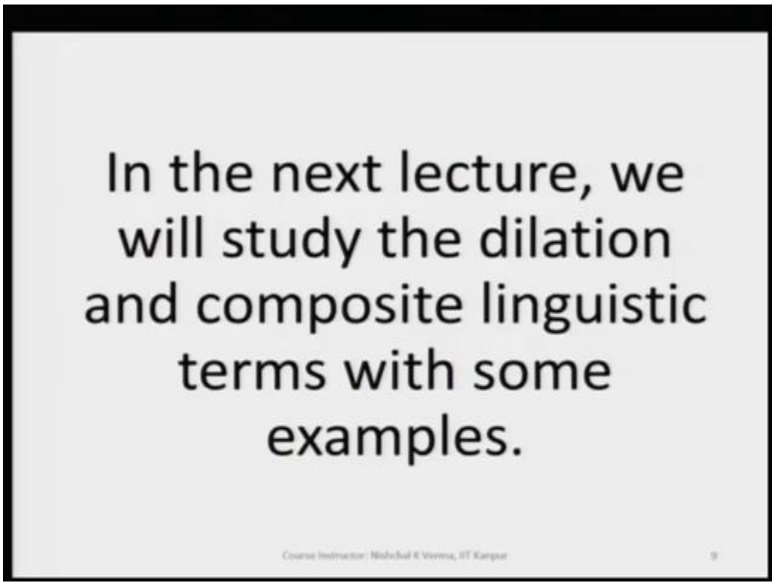


And in the next lecture we will discuss the dilation and the composite linguistic terms with some suitable examples.

Thank you.