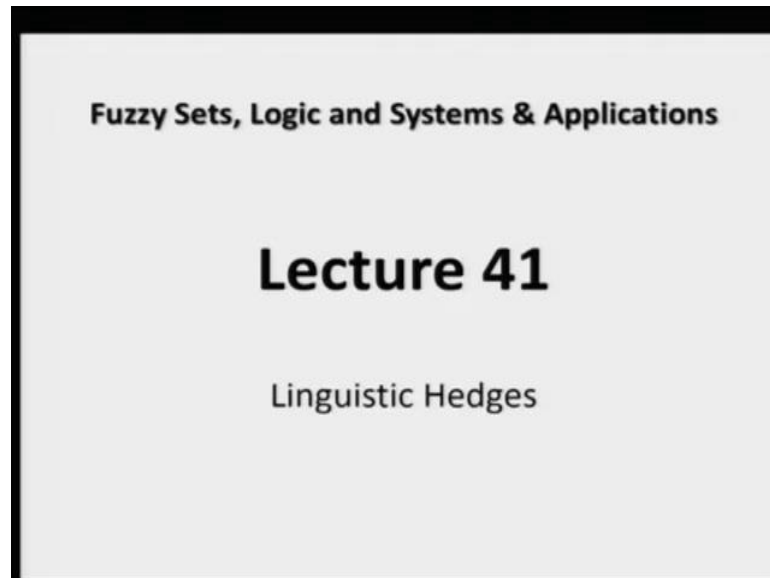


Fuzzy Sets, Logic and Systems and Applications
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Lecture - 41
Linguistic Hedges

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Welcome to lecture number 41 of Fuzzy Sets Logic and Systems and Applications. In this lecture, we will discuss Linguistic Hedges. So, before we finally discuss linguistic hedges let us go through the term linguistic variables.

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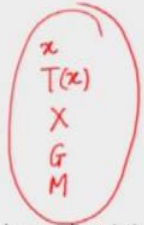
Linguistic Variables

A linguistic variable is characterized by a quintuple as,

$$(x, T(x), X, G, M)$$

Here,

- x is the name of the linguistic variable.
- $T(x)$ is the term set of x i.e., the set of its linguistic values or linguistic terms.
- X is the universe of discourse.
- G is a syntactic rule which generates the term in $T(x)$.
- M is a semantic rule which associates with each linguistic value x i.e. $M(x)$, where $M(x)$ denotes a fuzzy set with the universe of discourse X .



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So, a linguistic variable is characterised by a quintuple and this quintuple basically has five variables and here these five variables are $x, T(x)$. So, x is generic variable and $T(x)$ here is the term set, X is the universe of discourse G here is this syntactic rule which generates the term set $T(x)$. And we have M which is nothing but, the semantic rule which associates with each linguistic value.

So, that is how we have these five variables and these five variables are put together to be called as quintuple. So, if here let us say we take an example of a linguistic variable.

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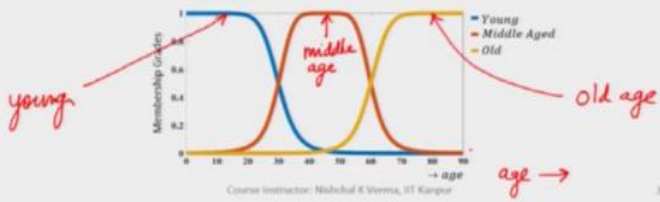
Linguistic Variables

For a linguistic variable "age",

- **Linguistic variable (x):** "age" is a linguistic variable.
- **Term set ($T(x)$):** If "age" is interpreted as a linguistic variable, then its term set $T(\text{age})$ could be as follows:

$$T(\text{age}) = \{\text{young, middle aged, old}\}$$

The "age is young" denotes the assignment of the linguistic value "young" to the linguistic variable "age".



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Let say a linguistic variable here is age. So, if we have age as the linguistic variable. So, let's see how these five variables look like. So, we see that if we have *age*, *a g e*, *age*. So, what is the generic variable in this case? So, here the linguistic variable or the generic variable basically, the linguistic variable is the *age*, but we have x as the generic variable, generic variable. And then here we have the term set $T(x)$.

So, for *age*, we can have term set and this term set can be a collection of multiple linguistic values. When we say linguistic values it means fuzzy sets. So, every linguistic value is represented by a particular fuzzy set and here the term set $T(x)$ and since here *age* is a linguistic variable. So our $T(\text{age})$ or I would say the term set for *age* can be here in this case *young, middle aged, old*. So, this is just for example, and we can have multiples segregations multiple fuzzy sets for *age*. So, term set can be basically a collection of the linguistic values for a particular linguistic variable.

So, if we have a linguistic term like *age is young*, this will denote the assignment of the linguistic value young to the linguistic variable *age*. So, the linguistic variable as I have mentioned here if we have *age* as the linguistic variable. So, linguistic variable is normally represented in terms of the generic variable x .

So, as I mentioned that this linguistic variable can be segregated can be divided into multiple linguistic values. Here in this case, we have divided the whole region of *age* in 3. So, these three regions these three regions basically are *young, middle aged, old* like that, but we can have multiple such regions which could be represented by the linguistic values and these linguistic values are normally represented by fuzzy sets.

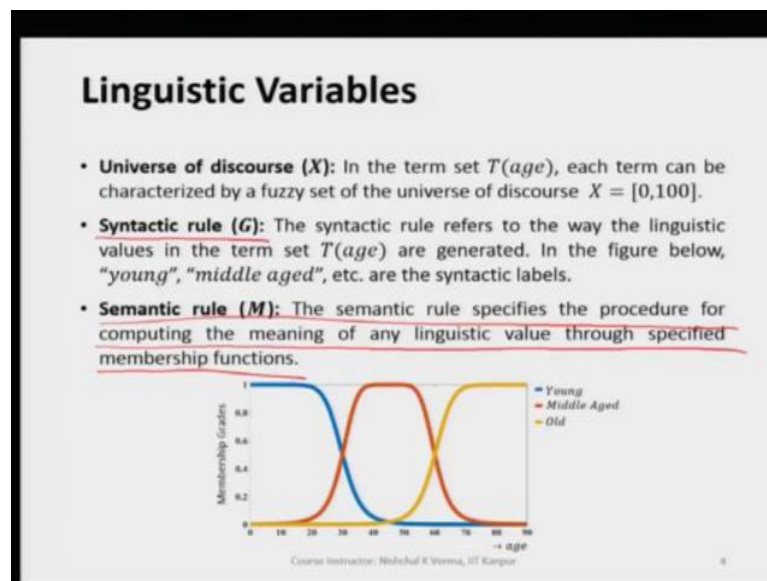
So, here we have this fuzzy set, this linguistic value *young* and this *young* is nothing but, the lower side of the *age* representation. So, *young* is a fuzzy set here is a left open fuzzy set and we have this the middle one is *middle age* and here this *middle age* is represented by another fuzzy set. Similarly, we have the *old age* here which is represented by a fuzzy set which is right open.

So, as I mentioned that when we are dealing with the term set we can suitably divide a particular linguistic variable into multiple fuzzy regions and every fuzzy region is represented by a particular linguistic value which is represented by a fuzzy set. So this way a linguistic variable is normally represented and as in this case we have seen that *age* if

we have taken age as the linguistic variable *age* has been divided into various fuzzy regions. And all these regions are nothing but, is the collection of all these regions is term set.

So, here we have a generic variable x which is the measure of age, for example we have 1 year, 2 years, 3 year like that and it can further go up to here in this case we are we have 90. So, these values are actually the values of the generic variable x and x here is the *age*.

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Now, what is the universe of discourse? So, the universe of discourse can be all you know the possible limit of *age* where the all possible values of the generic variable age, linguistic variable age could be settle. So, within the total a space basically is here termed as universe of discourse X . So, in the term set $T(\text{age})$ each term can be characterized by a fuzzy set of a universe of discourse.

So, the X here is the universe of discourse and this universe of discourse basically says that whatever value that x can take or *age* can take will be belonging into the limit of the universe of discourse. Then comes the syntactic rule here, so, the syntactic rule is G and it is symbolically defined by G . So, the syntactic rule refers to the way the linguistic values in the term set T are generated.

So, in the figure that we have just discussed for *age* where we have created multiple fuzzy regions for *age*; *young*, *middle age* etcetera are the syntactic levels. So, based on this

syntactic rule, we create *young, middle aged, old* etcetera. Similarly, semantic rule M . So, M basically helps us in specifying the procedure for computing the meaning of any linguistic value through specified membership function. So, we will have couple of examples and with this we will be able to understand all these parameters.

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Composite Linguistic Term

For linguistic variables, we use words as values of linguistic variables. In case of linguistics, we often use more than one word to describe a variable.

For example:

If "the intensity of light" is a linguistic variable, then its linguistic values might be "very bright," "slightly dim," and "more or less bright," etc.

So, the values of a linguistic variable may be a composite term and can be classified into three groups:

- 1) Primary Terms → Low, medium, high
- 2) Linguistic Hedges → Very low, more or less medium
- 3) Negation / Complement and Connectives → Not low, not medium
Low but not very low
low
Connective

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Now, comes the composite linguistic term. So, here for linguistic variables we use word as values of linguistic variables in cases of linguistics we often use more than one word to describe a variable. So, for example, here the intensity of light, if we choose the intensity of light as a linguistic variable. So, let us now see how these five parameters as we have seen those parameters which were as a quintuple. How actually are these coming up for this linguistic variable.

So, intensity of the light basically here, if we are taking this as the linguistic variable. So, what is the generic variable? How we are going to measure the intensity of light? How we are going to represent the intensity of light, let us say it is x , so, x is the generic variable and then what is the universe of discourse X . So, that the limit the range within which we are supposed to take the generic variable values and then the term set.

So, the term set here for this case could be if it is the intensity of light, it could be simply either *low* or *high* or *medium* or like that or may be *dim, bright*. So, these kinds of fuzzy regions or the linguistic values can be included in the term set. And similarly, the syntactic and semantic rules could be created for these the linguistic variable the intensity

of light. Here we have written *very bright, slightly dim, more or less bright*, all these are little further if we discussed the hedges. So, these comes under that.

So, we will understand these terms when we move little ahead. So, the linguistic variables we see here the term set which we have for linguistic variables. So, this can further be generated and let us now understand first that the linguistic variable may be a composite term and can be classified into three groups. So, as a whole linguistic variable can have primary terms and then linguistic hedges and then we have the negation and complement or and connectives. For example, if we take intensity of light so, what are the primary terms for the intensity of light?

So, primary terms could be for intensity of light, it could be as I already mentioned that could be low intensity, medium intensity or may be high intensity. Similarly, the linguistic hedges for the intensity of light could be maybe we if we add an adjective here, so we could simply write *very low* intensity or may be *more or less, medium* or whatever.

So, similar linguistic values could be included. So, the linguistic hedges basically are with the adjectives in along with the primary terms. And then comes the negation class here we have you know the linguistic fuzzy regions where the primary terms are taken as either the complement or negation or with some other connectives like and or, or. So, for example, here we can write we can use a term *not low* or may be *not medium*. Similarly, we can have another fuzzy value which could be like this *low, but not very low*.

So, here we see that, but is the, but is a connective. So, this way we see that we have some composite terms and these are basically divided into three groups; first group is primary terms, second group is the linguistic hedges and the third group is the complement and connectives. So, let us further understand this by taking some examples and these primary terms and then the linguistic hedges negation complements and connectives can be understood further.

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1) Primary Terms

- For instance, if “age” is a linguistic variable then its primary terms could be *young*, *middle aged*, *old*, etc.
- Each primary term can be defined by a fuzzy set.
- For example, *young* can be expressed by a fuzzy set and *middle aged* and *old* can also be expressed by other fuzzy sets.

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So, if we talk of the primary terms let us say the *age* which we have already taken in the beginning as the linguistic variable, then its primary term could be the *young* as I have already mentioned and in this primary terms, the *middle aged* can also be one of the linguistic value, similarly *old*. So, we have already seen that we have three fuzzy regions. This could be like this.

So, we have let's say this is the universe of discourse here and within this we have the generic variable x which is nothing but the *age* the measure of *age* let's say years. So, let us say we have created three primary terms and these three primary terms here are *middle age* or *middle aged* and then we have the *old* and here we have let's say *young*. So, these three are the primary terms which have been created here and further these primary terms can be used with some adjectives or connectives to get the linguistic hedges.

So, we will discuss this in the coming slides. So, here the primary terms basically are the fuzzy regions, primary fuzzy regions which are being created just to represent the basic building regions basic regions. So, for example, here the age as the generic variable, *age* as the linguistic variable here is divided into three region, three basic regions and hence these *young*, *middle aged*, *old* are called as basically the primary terms.

Now, these primary terms as I have already mentioned can be used as hedges by using the adjectives along with the primary terms.

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2) Linguistic Hedges

- In linguistic, fundamental atomic terms are often modified with adjectives (nouns) or adverbs (verbs) such as very, low, slight, more or less, fairly, slightly, almost, barely, mostly, roughly, approximately, etc.
- These modifiers are known as linguistic hedges i.e. the singular meaning of an atomic term is modified or hedged from its original interpretation.

Let us understand the mathematical expressions of linguistic hedges.

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So, here as I mentioned linguistic hedges. So, in linguistic, fundamental atomic terms. Basically, atomic terms are often modified with adjectives. So, here with adjectives and adverbs such as *very, slight, more or less, fairly, slightly, almost, barely, mostly, roughly, approximately* etcetera. So, we see that everywhere we have some adjectives or adverbs and when this is being used along with the primary terms or atomic terms then this becomes linguistic hedges.

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2) Linguistic Hedges

Let us consider a fuzzy set A with the universe of discourse X defined as,

For continuous $\Rightarrow A = \int_{x \in X} \mu_A(x)/x$ ← Continuous fuzzy set

For discrete $\Rightarrow A = \sum_{x \in X} \mu_A(x)/x$ ← Discrete fuzzy set

$\mu(x) \rightarrow$ membership function of Primary fuzzy set

i. The membership values for the linguistic term "Very or Too" is defined as,

$$\mu_{\text{Very } A}(x) = \mu_{\text{Too } A}(x) = [\mu_A(x)]^2$$

$A_{\text{young}} = \int_X \mu_{\text{young}}(x)/x \rightarrow \text{very young} \rightarrow A_{\text{very young}} = \int_X [\mu_{\text{young}}(x)]^2/x$

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So, let us understand this further with some mathematical expressions of linguistic hedges. So, if we have here a fuzzy set let say A and if it is continuous fuzzy set then we represent a fuzzy set, continuous fuzzy set like this. And similarly, we represent discrete fuzzy set like this. So, the membership values which we use here for representing any fuzzy set let say A whether it is a continuous fuzzy set or discrete fuzzy set.

So, if we have a membership function $\mu(x)$ which has been used in the basic fuzzy set in the primary term or the atomic term that the symbol that has been used. So, if we have $\mu(x)$, then how can we represent a linguistic hedge using the membership function of the primary term. So, here if we have the $\mu(x)$ which is here. The $\mu(x)$ is nothing but the membership function of primary fuzzy value.

So, if we have $\mu(x)$ as the primary fuzzy value when we say fuzzy set, primary fuzzy set it means it is the primary term like in case of *age* we have seen the *young*. So, *young* is represented by a primary set, this is a primary fuzzy set these also sometimes termed as a fuzzy value or the linguistic value because all these the linguistic value, fuzzy value etcetera are represented by a suitable fuzzy set

So, now let us make use of $\mu(x)$ which is used in the primary fuzzy set and let us make linguistic hedge out of it. So, if we have $\mu(x)$ for a primary fuzzy set then if we have to let say use very for example, if we have let say this $\mu(x)$, I will write $\mu(x)$ here and this $\mu(x)$ has been used for let say *young*. This $\mu(x)$ has been used for *young*. It means, we have a fuzzy region or fuzzy value which is represented by a fuzzy set A and this A is nothing but, this is for *young*. So, this $\mu(x)$ is for $\mu(x)$ here is a membership function and this is for the primary fuzzy set primary region primary term.

So, if let say we would like to say it like this. We would like to modify the fuzzy set like this like *very young*. So, how can we make use of this $\mu(x)$ and we convert it into very *young*. So, very young we have used very before *young*, *young* we already know. So, how can we get the a fuzzy set let say which is A and this is for *very young*. It is very easy and we simply make use of the $\mu(x)$ that was given to us we have to square the $\mu(x)$. So, the $\mu(x)$ that was given to us means simply take it and then we will square it and that is it.

So, μ_A can be converted into $\mu_{very A}$ very easily by just taking square of the membership functions. So, this way we are able to convert this into a linguistic hedge.

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2) Linguistic Hedges

Let us consider a fuzzy set A with the universe of discourse X defined as,

For continuous $\Rightarrow A = \int_{x \in X} \mu_A(x)/x$

For discrete $\Rightarrow A = \sum_{x \in X} \mu_A(x)/x$ $\mu_A(x)$

ii. The membership values for the linguistic term "More or Less" is defined as,

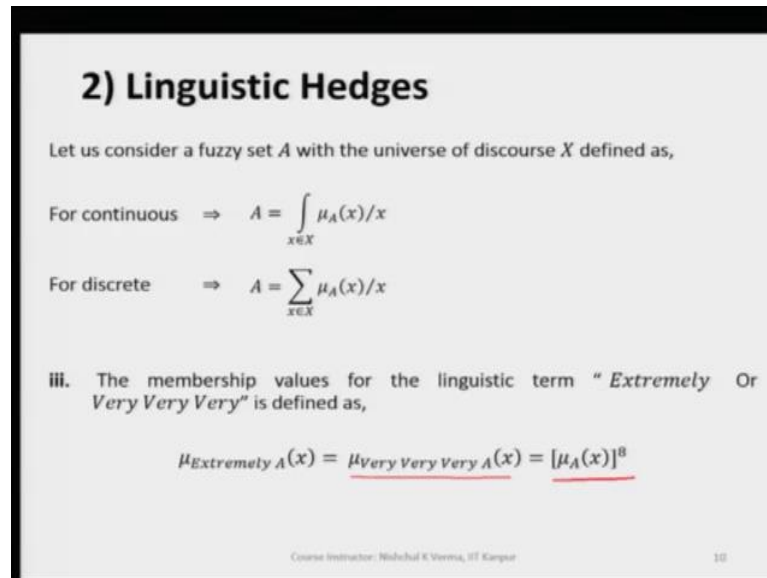
$$\mu_{More\ or\ Less\ A}(x) = [\mu_A(x)]^{\frac{1}{2}}$$

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Similarly, a membership value if it is given let's say again this is the basic membership value μ_A and we are interested in more or less as the linguistic hedge, we have to simply dilate it. When we say dilate means we are reducing the power, means we are taking these square root of it here.

So, more or less can be represented by μ , the membership function for more or less can be simply the $\mu_{more\ or\ less\ A}(x) = [\mu_A(x)]^{1/2}$.

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2) Linguistic Hedges

Let us consider a fuzzy set A with the universe of discourse X defined as,

For continuous $\Rightarrow A = \int_{x \in X} \mu_A(x)/x$

For discrete $\Rightarrow A = \sum_{x \in X} \mu_A(x)/x$

iii. The membership values for the linguistic term "Extremely Or Very Very Very" is defined as,

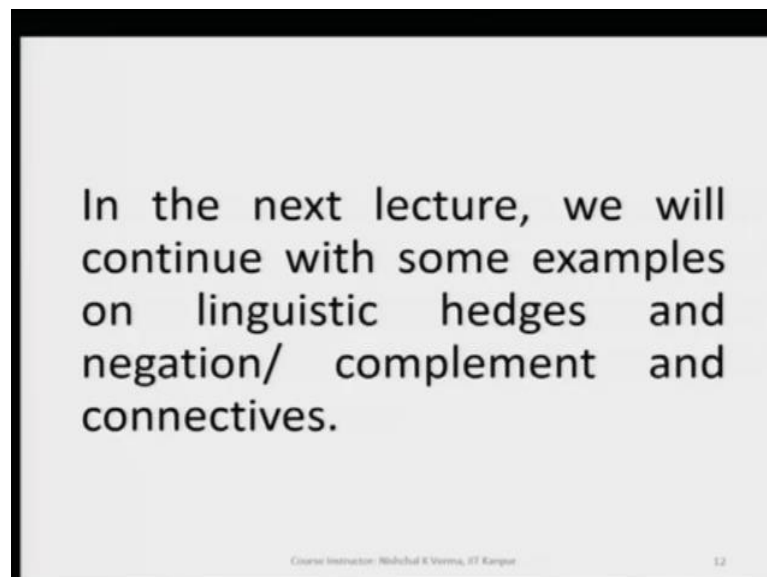
$$\mu_{\text{Extremely } A}(x) = \mu_{\text{Very Very Very } A}(x) = [\mu_A(x)]^8$$

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Similarly, when we want to find the membership function of the extremely something extremely if we want to use the hedge is *extremely*, $\mu_{\text{extremely } A}(x) = \mu_{\text{very very very } A}(x) = [\mu_A(x)]^8$. This means we have applied very 3 times. If we take very one time it is squares the membership function means we write $\mu_A(x)$ raise to the power 2.

So, this way we are able to make use of the hedge and we can applied the adjective over the primary term to make the linguistic hedge. So, with this I would like to stop here.

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In the next lecture, we will continue with some examples on linguistic hedges and negation/ complement and connectives.

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And in the next lecture we will continue with some examples on linguistic hedges and negations complements and connectives.

Thank you.