Fuzzy Sets, Logic and Systems and Applications
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Lecture – 40
Fuzzy Tolerance and Equivalence Relations –III

So, welcome to lecture number 40 of Fuzzy sets, Logic and Systems and Applications, and this lecture is in continuation to our previous lectures and here we will discuss a Fuzzy Tolerance and Equivalence Relations.

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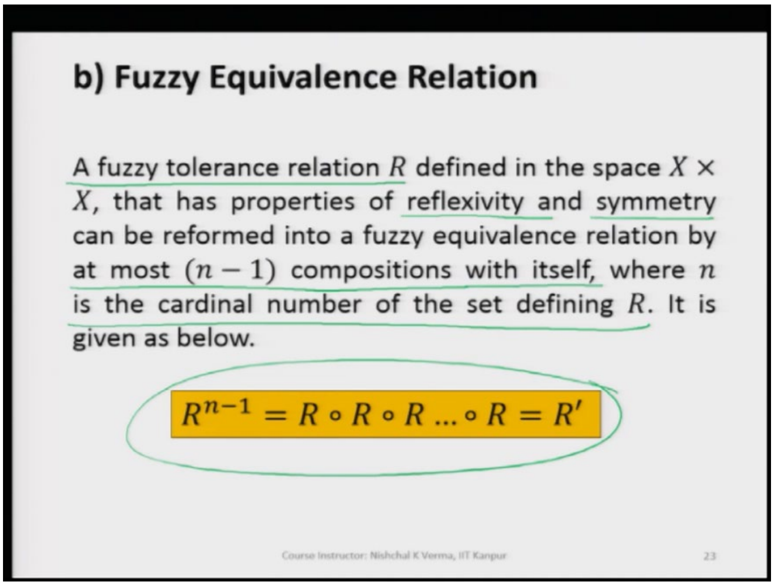

So, this way we have understood as to, when we can say that particular fuzzy relation set is a fuzzy equivalence relation are not.

One thing we have understood that a fuzzy equivalence relation is always a fuzzy tolerance relation that is an important outcome of this discussion. And there are fuzzy relation sets which many a times they are not fuzzy equivalence relations, but we can make them fuzzy equivalence relation.

So, here is a very important concept that I am going to mention is that a fuzzy relation set which is a fuzzy tolerance relation let us say fuzzy tolerance relation R, if it is defined in this space $X \times X$ so; that means, these R is already a reflexive and symmetric, means this fuzzy relation set R is satisfying the reflectivity and symmetry property. So, if we have a fuzzy tolerance relation and this fuzzy tolerance relation if it is not fuzzy equivalence relation it can be converted into fuzzy equivalence relation by at most n minus 1 compositions.

So, this is very important concept that we need to know here. So, if we can make a composition, at most $n-1$ composition with itself. So, within this any fuzzy tolerance relation R can be converted into a fuzzy is equivalence relation. So, as I mentioned a fuzzy tolerance relation R which is defined in the space $X \times X$ that has already satisfied the properties of reflexivity and symmetry can be reformed into a fuzzy equivalence relation by at most $n-1$ composition with itself. Where n is the cardinal number of the set defining R. So, this can be written as this expression.

So, R^{n-1} this shows that we are making $n-1$ compositions. So, this is the maximum number of composition that it can go through and within this the fuzzy relation set which is not a fuzzy equivalence relation can be transformed into fuzzy equivalence relation.

So, I would like to just amend here that a fuzzy tolerance relation R only can get transformed into fuzzy equivalence relation in this process. And the process is at most n minus 1 compositions with itself, where n is the cardinal number of the set defining R.

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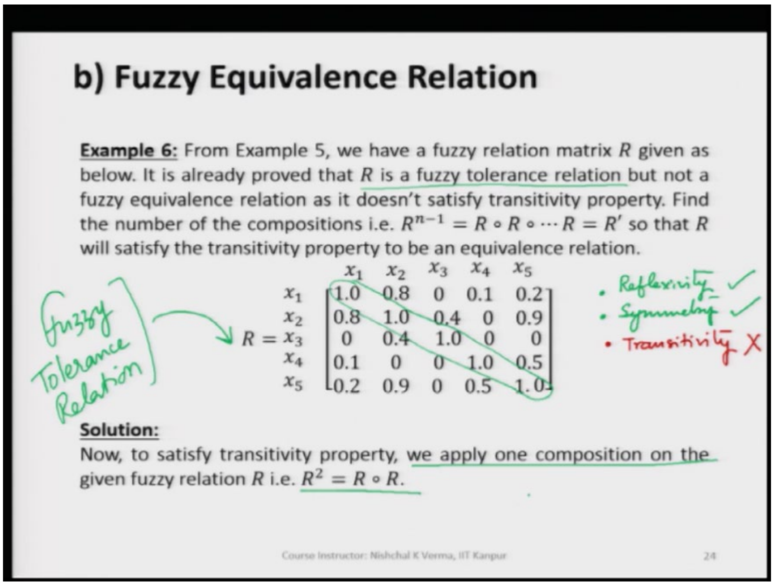

So, let us take an example here to understand this concept also. So, here we have taken fuzzy relation set in form of matrix we can call this as the fuzzy relation matrix R. And if we look at the fuzzy relation matrix we can quickly comment on the reflexivity property, all the diagonal elements are 1. And not only this the symmetry property is also satisfied because we have already done this in the previous example. Example number 5 where we had a fuzzy relation R which is here and we have already proved that R is a fuzzy tolerance relation.

So, this is a fuzzy tolerance relation. So, R is a fuzzy tolerance relation, why because R satisfies the reflexivity property and symmetry property, now if we are interested in checking whether this fuzzy tolerance relation is qualified to be called as fuzzy equivalence relation or not. So, the third property that is transitivity property it needs to be satisfied.

Now, in the example number 5 we have checked this that the transitivity property is not satisfied. So, this fuzzy relation set R is not a fuzzy equivalence relation. So, here I am just mentioning that I am just writing that the what are the properties that are satisfied. So, reflexivity, then symmetry, then transitivity.

So, transitivity criteria transitivity property is not satisfied for this R, however the reflexivity and symmetry both the properties are satisfied. So, that is why the given fuzzy tolerance relation are given fuzzy relation matrix R is not qualified to be called as fuzzy equivalence relation. So, now, the question is how to make this a fuzzy equivalence relation by having the compositions as we have just discussed. So, let us proceed for that.

So we apply now the composition of R on R; so that is R^2. So, R^2 is nothing, but the $R \circ R$. So, we have the fuzzy tolerance relation. Now, R and we are now having the $R \circ R$ and let us see what happens.

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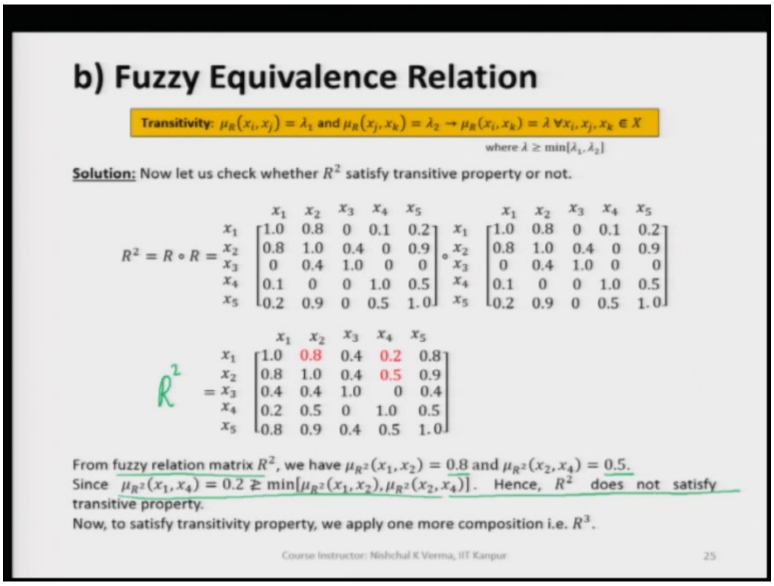

So, we have the R that is given again and the same R is here. And when we take the composition of these two and here we have $\circ$ sign, $\circ$ is for the composition and here we are taking the max-min composition. So, when we do that we see that this is the outcome.

So, after taking the max min composition of $R \circ R$, we get a new fuzzy relation which is represented by R^2. So, this is the first composition, this is you know the first stage of the composition. So, the fuzzy relation matrix if we see if we look at we see that we have $\mu_{R^2}(x_1, x_2)$ which is equal to 0.8 and $\mu_{R^2}(x_2, x_4)$ is 0.5.

And we see that here also the transitivity condition is not satisfied because the $\mu_{R^2}(x_1, x_4)$ equal to 0.2 which is λ. And this is not either equal to or greater than the mean of the 0.8 and 0.5. So, this way we can clearly say that the transitivity condition is still not satisfied.

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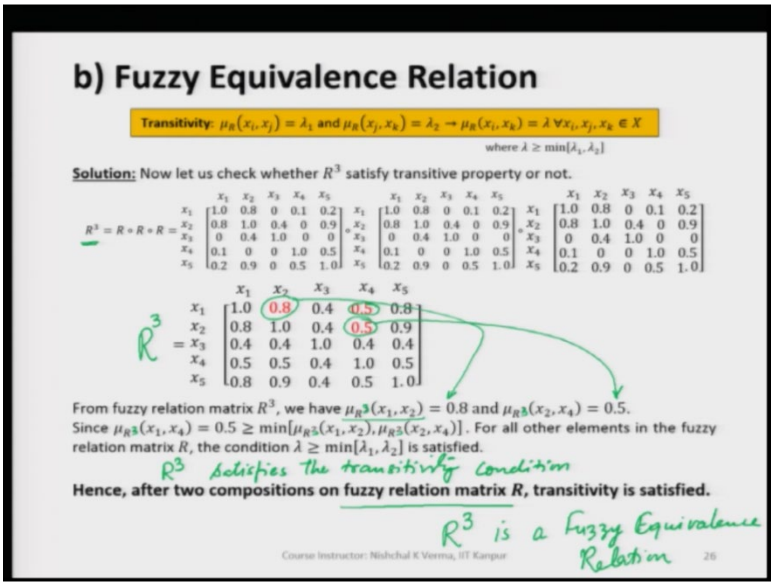

Now, let us go one step further and let us have one more composition. So, we already have R^2 and now let us have a composition of $R^2 \circ R$ which is R^3 here. So, when we have the max-min composition of R^2 on R we have this as the outcome, we have a new fuzzy relation here that is R^3.

New fuzzy relation matrix and of course, this fuzzy relation matrix is a fuzzy tolerance relation because we have already seen that this satisfies the reflexivity and symmetry. We can again once again we can check this and here also the condition of reflexivity and symmetry both are satisfied.

Now, when we check for transitivity we see that a μ_{R^2} here that is $\mu_{R^3}(x_1, x_2)$ and $\mu_{R^3}(x_2, x_4)$ values are 0.8 and 0.5. So, $\mu_{R^3}(x_1, x_2)$; x_1, x_2 is this, and $\mu_{R^3}(x_2, x_4)$value is 0.5 here.

Now, when we check for $\mu_{R^3}(x_1, x_4)$, which is 0.5 again. And if we see that this 0.5 is either greater than or equal to the mean of these two the 0.8 and 0.5. So, here for this case the transitivity condition is satisfied, but we have to go further and we have to check this condition for all its elements.

So, when we have checked for all its elements all the membership values of the fuzzy tolerance relation R^3. Then we see that all the elements are satisfying the transitivity condition and this way we can comment here that the R^3 is satisfies the transitivity condition.

So, now when the R^3 is satisfying the transitivity condition and we have already seen that R^3 is reflexive and symmetric. So, this means all the three conditions of fuzzy equivalence relations satisfied. So, we can now say that the fuzzy relation matrix R is converted into R^3. And R^3 is a fuzzy equivalence relation.

So, what does this mean? This means that if we have any fuzzy tolerance relation which is not a fuzzy equivalence relation. So, with this with the help of this fuzzy tolerance relation we can use the fuzzy tolerance relation and we can convert this into fuzzy equivalence relations by taking the suitable compositions.

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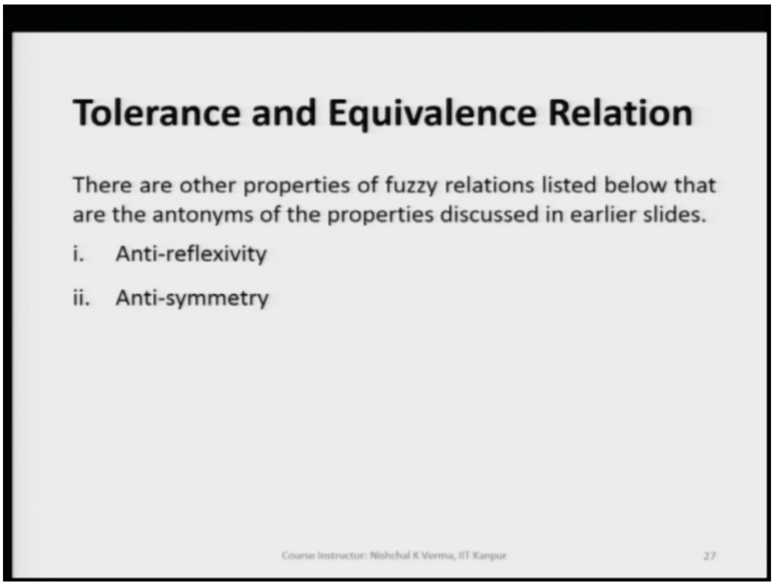

Now, the anti-reflexivity and anti-symmetry these are two other properties, we can just understand. So, at this stage the fuzzy relations for fuzzy relations we have anti-reflexivity anti-symmetry and these are nothing but the antonyms of the properties that we have already discussed. So, anti-reflexivity is the property where the reflexivity is not satisfied; reflexivity condition is not satisfied.

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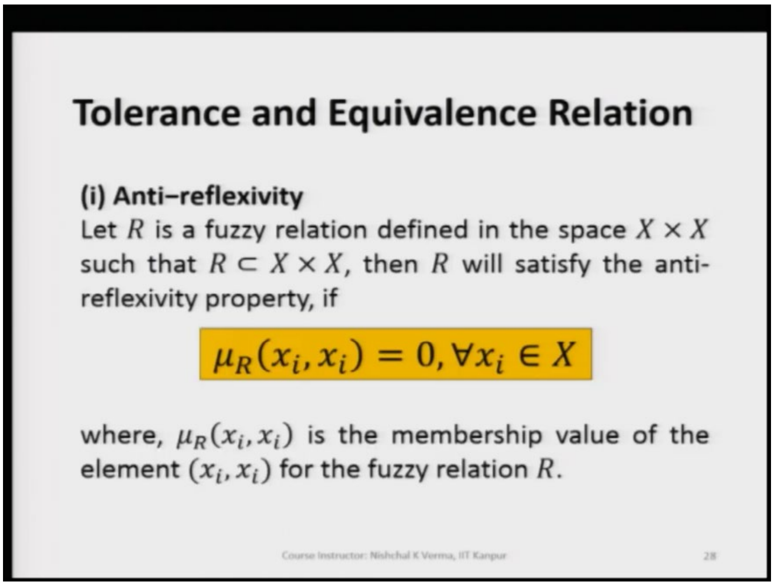

And similarly anti-symmetry is the property where the symmetry condition is not satisfied. So, let us now formally understand what is anti-reflexivity. So, if we have any fuzzy relation R which is defined in the universe of discourse $X \times X$ such that $R \subset X \times X$. Then R will satisfy the anti-reflexivity property.

If $\mu_R(x_i, x_i)$ is equal to 0 instead of 1 in the case of reflexivity. So, here $\mu_R(x_i, x_i)$ means, the diagonal elements so all the diagonal elements of the fuzzy relation set need to be 0. So, this condition if this is the case then we can say that the fuzzy relation has anti-reflexivity condition.

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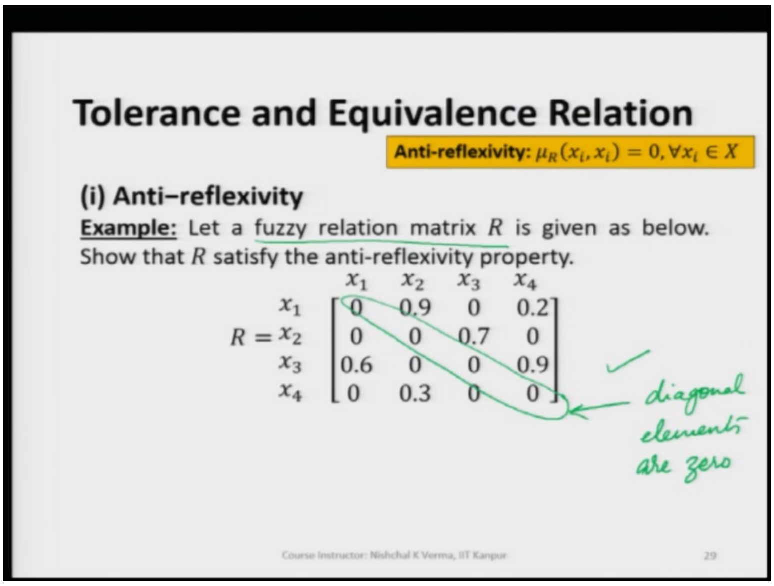

And similarly, now have an example to understand this better. So, if we have a fuzzy relation matrix R as shown here. And we see that the all the diagonal elements of it are zero. So, then in that case we can say that the anti-reflexivity criteria is satisfied. So, we can say that the fuzzy relation R is anti-reflexive.

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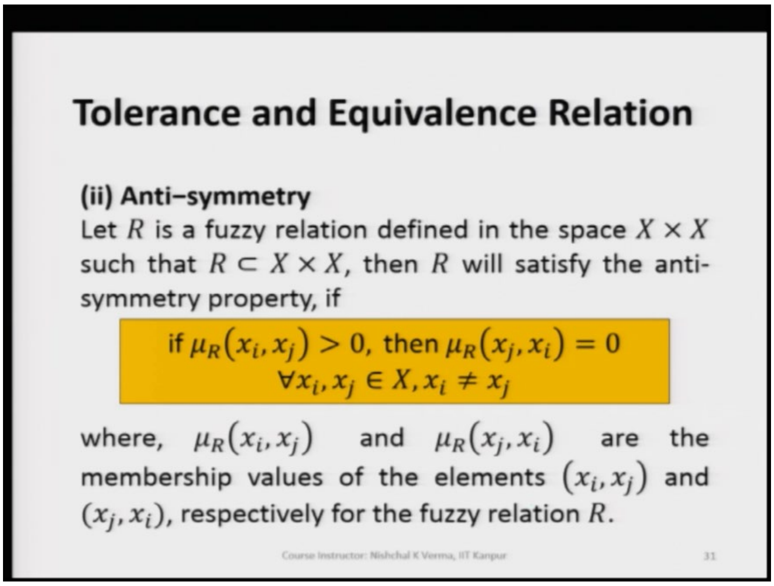

Now, anti-symmetry; so, for any fuzzy relation set R if we have this condition satisfied which is $\mu_R(x_i, x_j) > 0$, then $\mu_R(x_j, x_i) = 0$, $\forall x_i, x_j \in X, x_i \neq x_j$.

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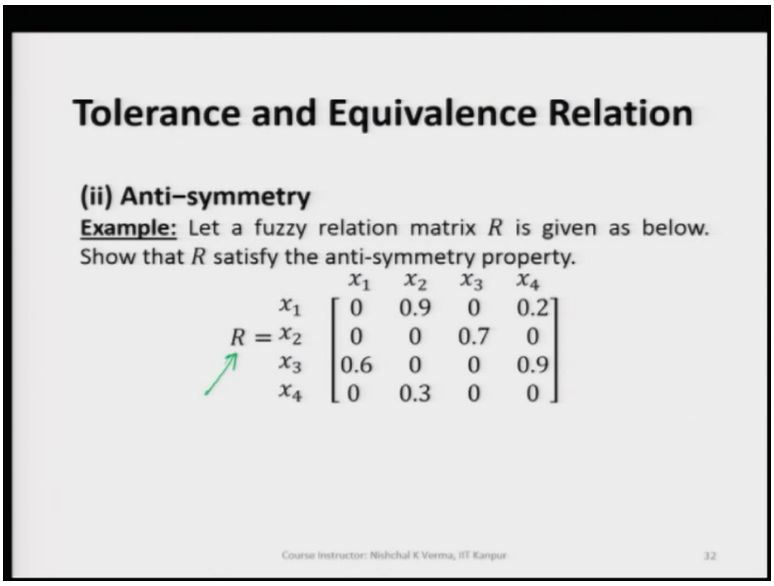

So, let us now understand this property the anti-symmetry property by taking an example. So, we have any fuzzy relation set R which is represented in the form of matrix here.

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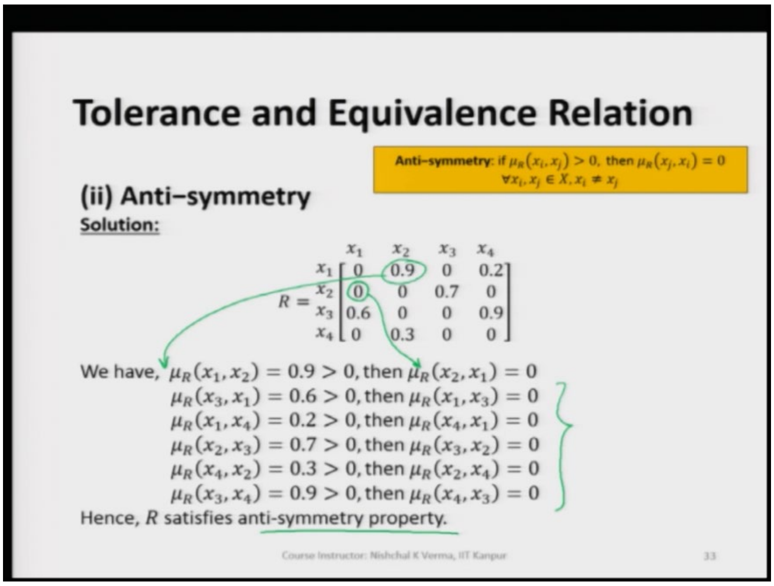

So, we see that the condition that is necessary for anti-symmetry is satisfied here. So, this means that if we have $\mu_R(x_1, x_2)$. So, $\mu_R(x_1, x_2)$ is 0.9 here. And this is greater than 0 of course, and then μ_R when we interchange the row and columns we see that $\mu_R(x_2, x_1)$ is equal to here 0.

So, this way we can say that the anti-symmetry for this element is satisfied. Similarly, when we check or for all the elements we see that the R satisfies the anti-symmetry property. And this way we can say that the fuzzy relation R that has been given satisfies the anti-symmetry property.

So, this way we have understood that a fuzzy relation can go through multiple tests and based on that we can comment on its whether it is the fuzzy tolerance relation or a fuzzy equivalence relation. And if a fuzzy tolerance relation is not a fuzzy equivalence relation let's say, then we can make a fuzzy equivalence relation by using a fuzzy tolerance relation with proper composition of it.

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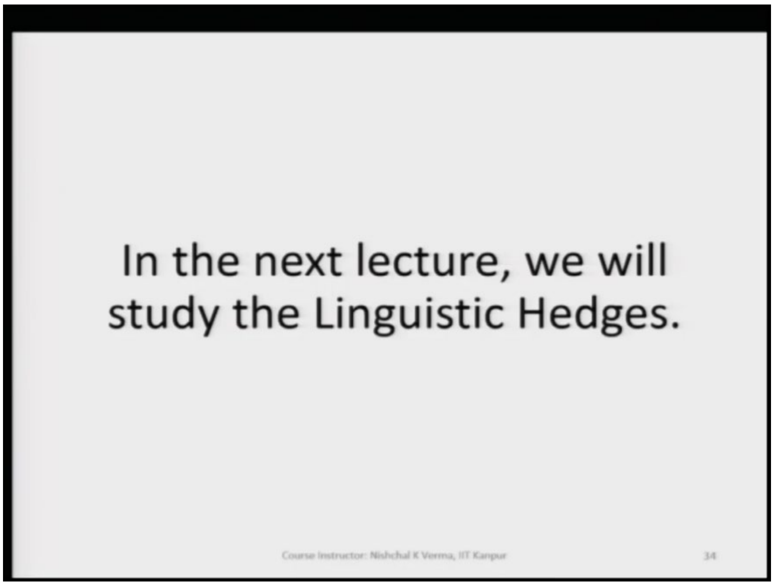

So, this way we will finish the lecture here. And in the next lecture we will study the Linguistic Hedges ahead.

Thank you.