Fuzzy Sets, Logic and Systems and Applications
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Lecture – 39
Fuzzy Tolerance and Equivalence Relations- II

So, welcome to lecture number 39 of Fuzzy Sets, Logic and Systems and Applications. And in this lecture we will discuss Fuzzy Tolerance and Equivalence Relations and this lecture is in continuation to our previous discussions on this topic.

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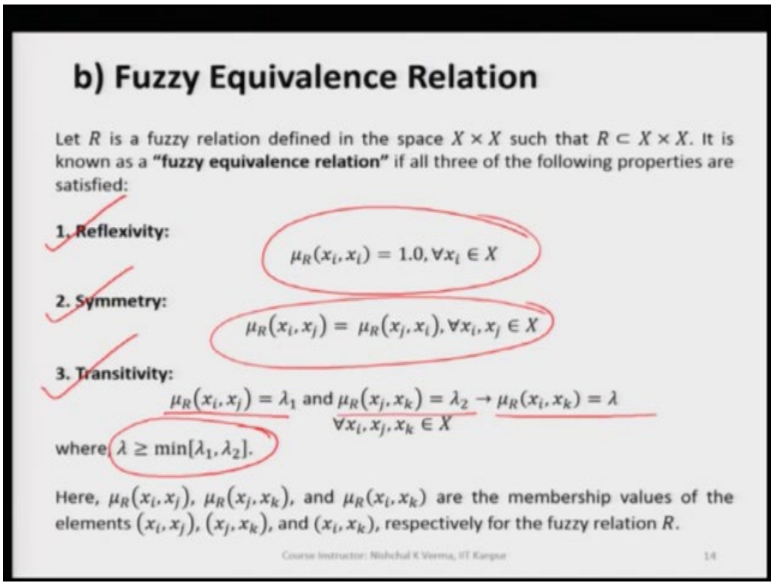

Now, let us move ahead and talk about fuzzy equivalence relation. So, like fuzzy tolerance relation we have fuzzy equivalence relation. So, here we have gone one step ahead. So, it is quite interesting to understand here that any fuzzy tolerance relation can be a fuzzy equivalence relation.

So, if any fuzzy relation set which is not a fuzzy relation fuzzy tolerance relation cannot be a fuzzy equivalence relation because for qualifying for any R to qualify to be called as a fuzzy equivalence relation. First this R has to be a fuzzy tolerance relation. It is because for fuzzy equivalence relation we have three properties that needs to be satisfied. So, apart from the reflexivity, symmetry we have an additional property which is transitivity and these three properties need to be satisfied before any fuzzy relations set R can be called as fuzzy equivalence relation.

So, since we have already discussed in detail the reflexivity, the symmetry. Now, we see what is transitivity the third property, so transitivity is basically it follows this condition which I am just going to explain. So, from the fuzzy relation matrix if we have the membership values corresponding to $\mu_R(x_i, x_j)$ let us say which is equal to λ_1, and $\mu_R(x_j, x_k)$ which is equal to λ_2, and $\mu_R(x_i, x_k) = \lambda$. So, if this is the case, then we need to check whether the $\lambda \geq \min[\lambda_1, \lambda_2]$.

So, for transitivity condition to be satisfied this $\lambda \geq \min[\lambda_1, \lambda_2]$ which is here. So, if this is the case then we can say that the transitivity condition is also satisfied. So, this way if all the three conditions as mentioned here. The first condition is reflexivity, second condition is symmetry, third condition is transitivity. If all these three conditions are satisfied for any fuzzy relation set we can say this fuzzy relation set is a fuzzy equivalence relation set.

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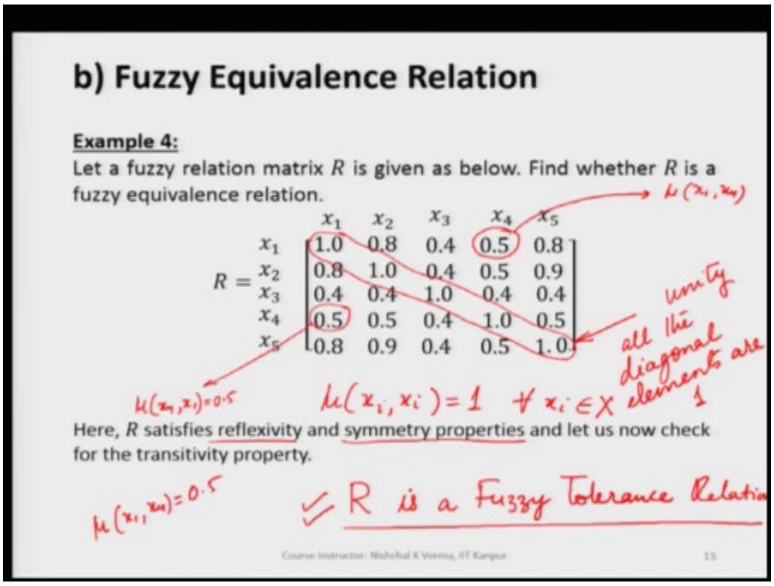

So, for this also let us take an example here to understand this concept better. Here we have taken a fuzzy relation set which is represented in the form of matrix here. And as we have already seen that all its elements are nothing but its membership values corresponding to its rows and columns. So, if we need to check whether this fuzzy relation is qualified to be called as fuzzy equivalence relation or not, we need to check for the three conditions.

First condition is reflexivity, second condition is symmetry and the third condition is the transitivity.

So, let us quickly go ahead and check for that. So, when we do that when in order to check for the reflexivity first. So, as I have already mentioned that all its diagonal elements should be unity, should be equal to 1. So, I am quickly going through the diagonal elements here. And we see that all its diagonal elements, all the diagonal elements of the fuzzy relation set are unity or equal to 1 or another words we can say right here all the diagonal elements are 1. So, if this is the case we can quickly comment on the reflexivity property because here $\mu(x_i, x_i) = 1, \forall x_i \in X$.

So, here since this is satisfied because diagonal elements are nothing but the diagonal elements are the elements are corresponding to the same row and same column. So, we can say that $\mu(x_i, x_i) = 1$. And since this is the case all the diagonal elements are 1, so we can quickly say that the reflexivity property is satisfied. So, first property is satisfied first property that is reflexivity is satisfied. Now, let us check for the symmetry property. So, when we see that here if we take up this element, what is this element? This element is $\mu(x_4, x_1)$ and this is 0.5.

Now, let us interchange the rows and column here. And if you change the rows and columns, we see that $\mu(x_1, x_4)$. So, $\mu(x_1, x_4)$ is here, $\mu(x_1, x_4)$ and we see that this is also 0.5. So, this way for this element for x_4, x_1 and x_1, x_4 both the elements are same. And this has to be checked for all the elements in the fuzzy religion set. So, when we do that we see that all the elements are satisfying this criteria.

So, we can quickly say that the symmetry property is also satisfied. So, this way for the given fuzzy relation matrix, for the given fuzzy relation set *R* the reflectivity and symmetry property both are satisfied. So, here we can also make a comment that *R* is a fuzzy tolerance, fuzzy tolerance relation because both the properties are satisfied, reflexivity and symmetry. But here since we are interested in finding whether fuzzy relation set *R* is a fuzzy equivalence relation or not.

So, this is quite interesting to note that a fuzzy relation has to be first a fuzzy tolerance relation before it can qualify to become fuzzy equivalence relation. So, this step is must. So, since the given fuzzy relation set capital *R* is a fuzzy tolerance relation.

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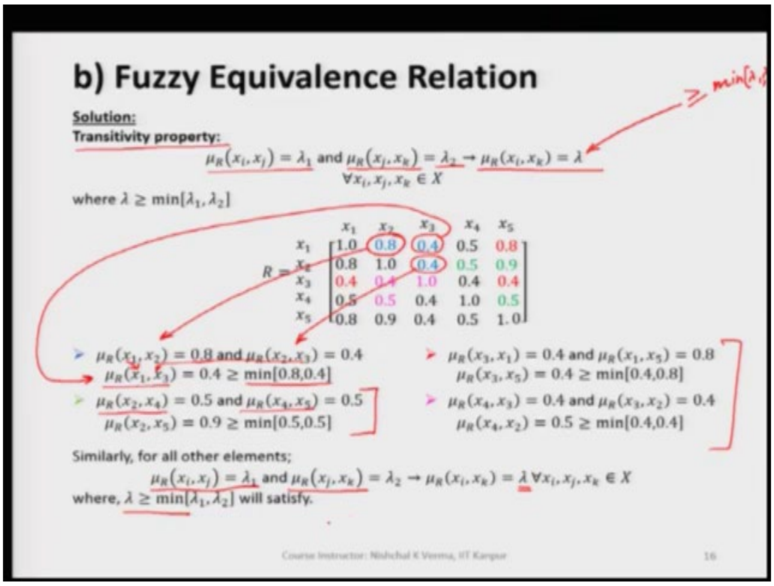

Now, we can move forward and check for the third property that is transitivity property. And transitivity property says that the $\mu_R(x_i, x_j)$ if it is some value that is λ_1, and $\mu_R(x_j, x_k)$ here, it is also some value say λ_2. Then $\mu_R(x_i, x_k)$ if it is λ, then this $\lambda \geq \min[\lambda_1, \lambda_2]$. So, now, let us quickly check that and in order to do that we have here let's say $\mu_R(x_1, x_2)$ and this we have found as 0.8 from this given fuzzy relation matrix, this is here, this is the value $\mu_R(x_1, x_2)$.

And now when we find $\mu_R(x_2, x_3)$ let us say this value $\mu_R(x_2, x_3)$. So, x_2 is this and then x_3 is this. So, $\mu_R(x_2, x_3)$ this is nothing, but this value. Now, as per the criteria that has been given for transitivity property we need to take $\mu_R(x_1, x_3)$ because you see x_1 is coming from here, x_1 is coming from here and x_3 is coming from here.

So, this $\mu_R(x_1, x_3)$ if we check we are finding you see here $\mu_R(x_1, x_3)$ this is $\mu_R(x_1, x_3)$. So, we are finding here this is 0.4, and this 0.4 is when we take min of the previous values the 0.4, 0.8 here. So, we see that this is equal to the min of 0.8 and 0.4, so this means that this condition is satisfied. Similarly, we check for $\mu_R(x_2, x_4)$ and $\mu_R(x_4, x_5)$. So, we see that this criteria, the transitivity criteria is satisfied.

Similarly, for all the other elements we have checked you can also verify this. And we find here $\mu_R(x_i, x_j)$ which is λ_1 and $\mu_R(x_j, x_k)$ which is λ_2. And with this if we find $\mu_R(x_i, x_k)$. Then in that case this value the $\lambda \geq \min[\lambda_1, \lambda_2]$, for this given fuzzy relation matrix. So, since we have checked for all the elements the transitivity condition, the transitivity property is satisfied we can say here that the transitivity property for the fuzzy relation set is satisfied here.

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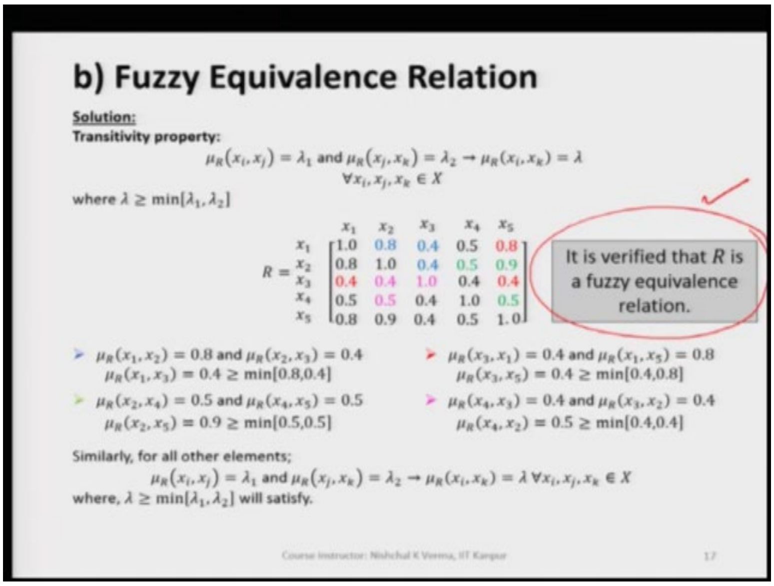

So, this way the fuzzy relation that has been given fuzzy relation R that has been given is a fuzzy equivalence relation. And we all know why we are making this comment because we have already checked all the three properties. The first one is the reflexivity, second one is the symmetry and the third one is the transitivity. All these three conditions are all these three properties are satisfied. So, the given R is qualified to be called as a fuzzy equivalence relation.

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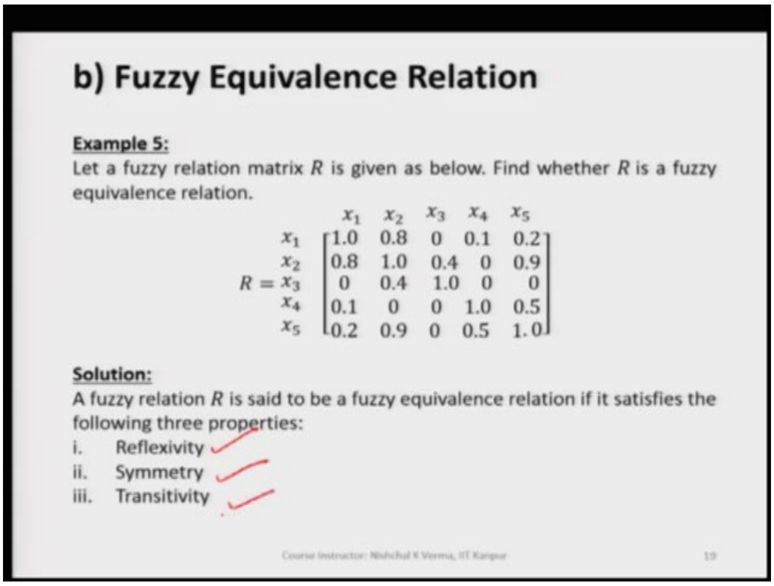

Now, let us take another example and see whether in this example the fuzzy relation matrix R is a fuzzy equivalence relation are not.

So, this is the fuzzy relation R that has been given and again we have to check for the three properties all the three properties must be holding good before we can say that R is a fuzzy equivalence relation. So, let us quickly go through the reflexivity property, symmetry property, transitivity property for the given fuzzy relations set and comment on it.

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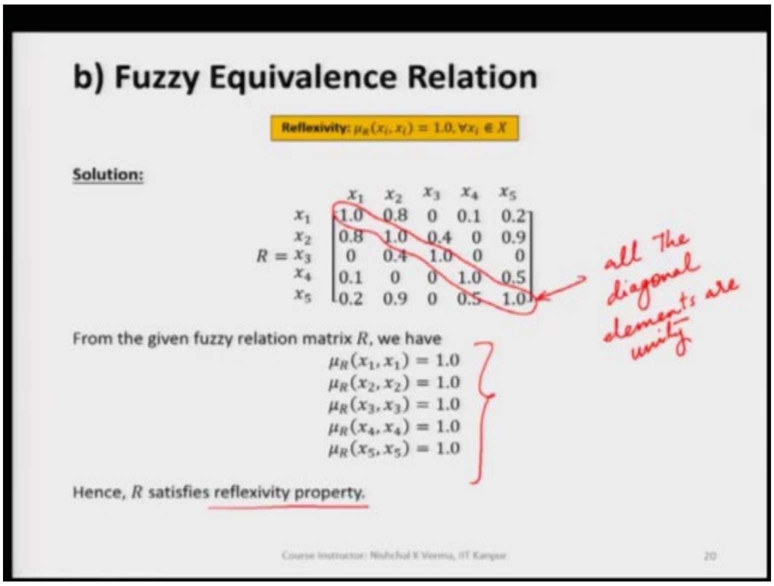

So, we see that the given fuzzy relation set R has its all its diagonal elements as unity. So, all the diagonal elements are unity.

So, this way we can say that $\mu_R(x_i, x_i)$ is 1 and you can see here all the elements all the x $\mu_R(x_1, x_1)$, $\mu_R(x_2, x_2)$, $\mu_R(x_3, x_3)$, $\mu_R(x_4, x_4)$, $\mu_R(x_5, x_5)$ all the elements are 1. So, we can say that R is satisfying the reflexivity property. So, the first property is satisfied.

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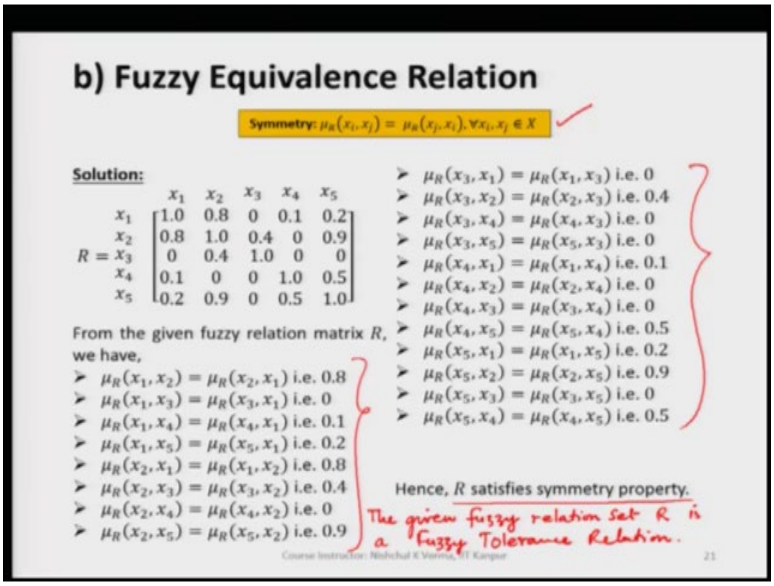

Now, let us check for the symmetry property. Symmetry says that $\mu_R(x_i, x_j) = \mu_R(x_j, x_i), \forall x_i, x_j \in X$. So, when we have checked for this here we see that this condition is also satisfied for fuzzy relation set that has given to us. And this way we can say that the R is satisfying the symmetry property.

So, now since we have checked for so far we have checked for two properties. First one is reflexivity and the second one is symmetry. So, at this juncture we can say that fuzzy relation matrix R is a fuzzy tolerance relation. So, here I would like to make a comment for fuzzy relation R. So, I can write here that the given fuzzy relation set R is a fuzzy tolerance relation. So, up to this is very clear. Now, this R before it can be called as fuzzy equivalence relation should satisfy the third property, that is a transitivity.

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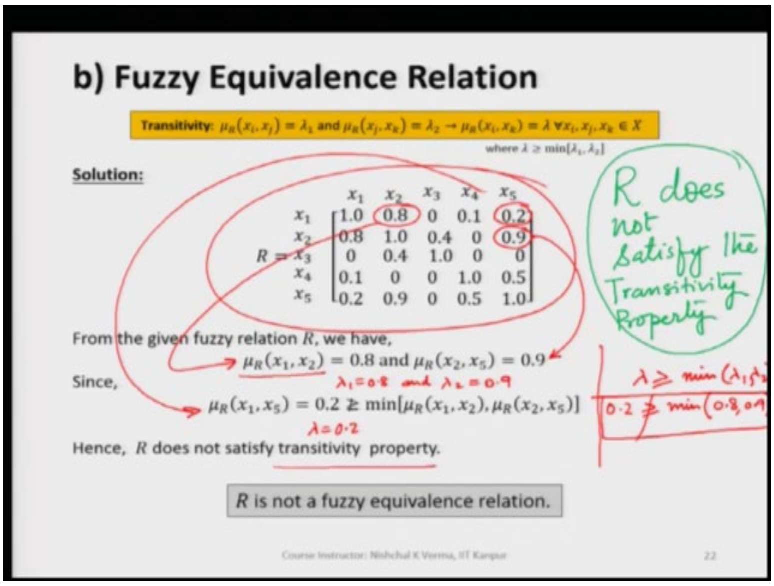

So, when we take the R here and when we check for the transitivity condition. So, transitivity condition says that $\mu_R(x_i, x_j)$ is equal to λ_1 and $\mu_R(x_j, x_k)$ is equal to λ_2. Then the $\mu_R(x_i, x_k)$ should be the value of this should be say $\lambda \geq \min[\lambda_1, \lambda_2]$.

So, when we take $\mu_R(x_1, x_2)$ which is nothing but this element you can see this is the element. And this is nothing, but 0.8 and when we take $\mu_R(x_2, x_5)$ is this, this is $\mu_R(x_2, x_5)$. So, this means we have λ_1 to 0.8. In this case and λ_2 is equal to 0.9. Now, if we take μ_R of x_1 which is coming from here and x_5 which is coming from here.

So, μ_R of x_1 and x_5 if we see x_1, x_5 which is nothing but 0.2, so I can just mention this it is here. So, when we see this this is coming out to be 0.2 and this is let's say is λ, so λ is 0.2. So, now, let us put the condition of transitivity. So, $\lambda \geq \min[\lambda_1, \lambda_2]$. So, we have all the values. So, let us see whether this holds the condition. So, if we take the min here min of 0.8, 0.9 and then here we have lambda is 0.2. So, this is not holding means this is not true.

So, when this is not true then we can stop here and we can say that the transitivity property is not satisfied this for this fuzzy relation set that is given to us that is R. So, R does not, I can write here that R does not satisfy the transitivity property. So, now, in this case when we have the checked that the third condition is not satisfied we can quickly say that although the given fuzzy relation set is a fuzzy tolerance relation, but it is not fuzzy equivalence relation. So, we can now, quickly say that the given fuzzy relation set is not a fuzzy equivalence relation.

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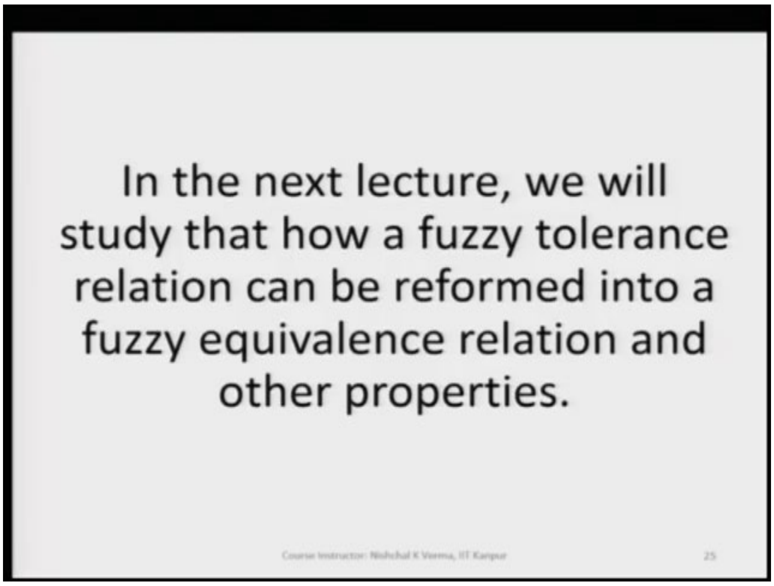

So, with this I would like to stop here. And in the next lecture we will discuss that as to how we can fuzzy tolerance relation transform or reformed into a fuzzy equivalence relation and we will discuss other properties as well.

Thank you.