Fuzzy Sets, Logic and Systems and Applications
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Lecture - 38
Fuzzy Tolerance and Equivalence Relations-I

So, welcome to lecture number 38 of Fuzzy Sets, Logic and Systems and Applications. In this lecture we will discuss Fuzzy Tolerance and Equivalence Relations.

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So, as I mentioned that in this lecture, we will discuss two relations; fuzzy tolerance and fuzzy equivalence.

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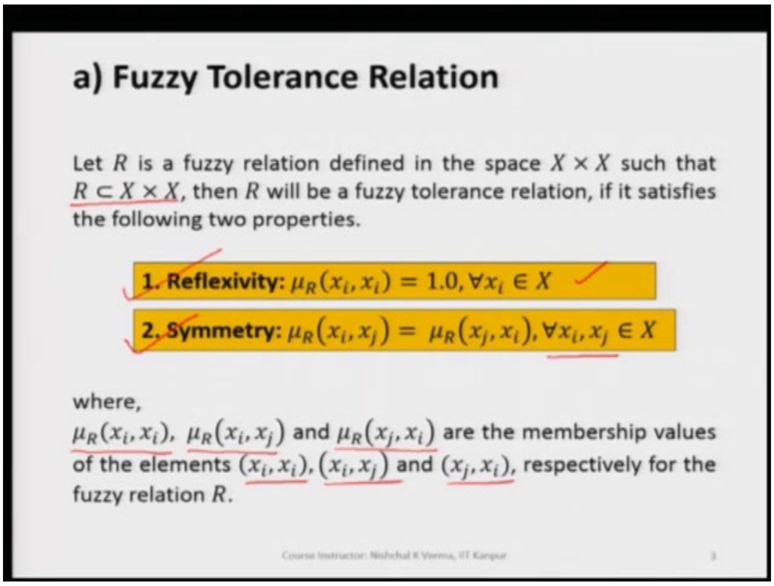

So, let us first understand what is a fuzzy tolerance relation. So, here if we have a set, a relation set R. And if it is defined in the universe of discourse $X \times X$, such that the fuzzy relation set $R \subset X \times X$. You can see here.

Then, the fuzzy relation set R will be a fuzzy tolerance relation if it satisfies the following two properties. What are these two properties? The first property is the reflectivity, you can see here. And then, the second property is symmetry. So, reflexivity property, what does it says is that from the fuzzy relation matrix we have here the strength values or in other words we can say the membership values of the fuzzy relation set. So, here $\mu_R(x_i, x_i) = 1.0$.

So, what does this mean? This means that if we have a fuzzy relation set, if we represent this fuzzy relation set in the form of a matrix as we have already seen in the previous lectures. So, all diagonal elements here will be 1. So, in other words we say the $\mu_R(x_i, x_i) = 1.0, \forall x_i \in X$. Is very simple to understand that the all the corresponding elements all the elements corresponding to the same row and same column will be equal to 1 or will be unity in other words.

So, this is called the reflexivity. Now symmetry property. So, symmetry property is also very simple to understand here that if we have the membership values corresponding to x_i, x_j; that means, $\mu_R(x_i, x_j)$ and this should be equal to $\mu_R(x_j, x_i)$. So, if this satisfies, then we can say the matrix which is corresponding to the fuzzy relation set R is symmetric.

R which is which follows the symmetry property. And here of course, this is for every x_i, x_j belonging into X as the universe of discourse.

And here this is needless to say that $\mu_R(x_i, x_i)$, $\mu_R(x_i, x_j)$ and $\mu_R(x_j, x_i)$ are the membership values of the elements x_i, x_i, x_i, x_j and x_j, x_i respectively for any fuzzy relation R. So, basically, fuzzy tolerance relation satisfies two properties. First property is reflexivity and the second property is symmetry. So, if these two properties for any fuzzy relation set R is satisfied then, we can say fuzzy relation set is a fuzzy tolerance relation.

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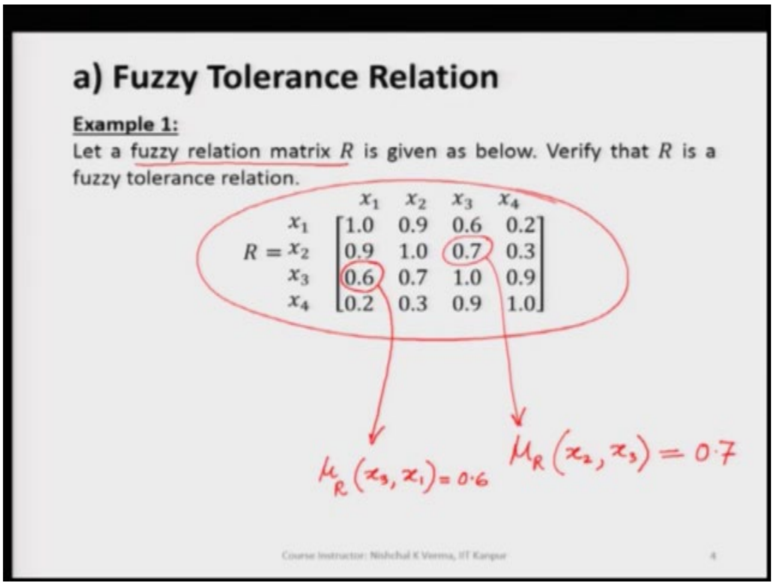

So, let us take an example here to understand the fuzzy tolerance relation better. So, here we are taking a fuzzy relation matrix R. You can see here this is the fuzzy relation set R and this fuzzy relation set is given in the form of a matrix. So, we see here the elements as part of this matrix and these elements are nothing but the membership values.

And these membership values can be represented in terms of μ and its corresponding rows and columns. For example, if we take 0.7 so this 0.7 of fuzzy relation matrix is nothing but this is μ_R and since this 0.7 is corresponding to row number x_2. So, we write here x_2 and then this is corresponding to the column x_3. So, we write here x_3.

So, $\mu_R(x_2, x_3)$ is 0.7. Similarly, if we take another element here let us say 0.6. This is in terms of its membership value of the fuzzy relation set that we have taken is $\mu_R(x_3, x_1)$. Why (x_3, x_1)? Because, 0.6 is corresponding to the third row and which is designated as

x_3. So, here we have written x_3 and then separated by comma and we have written x_1. So, x_1 is basically the column corresponding to which 0.6 is.

So, 0.6 can be represented here as μ_R. And please understand that R here is nothing but the name of the fuzzy relation set. So, that is why this R is and so 0.6 here is nothing but this is corresponding $\mu_R(x_3, x_1)$. So, this way we understand that all the elements of the fuzzy relation matrix can be understood.

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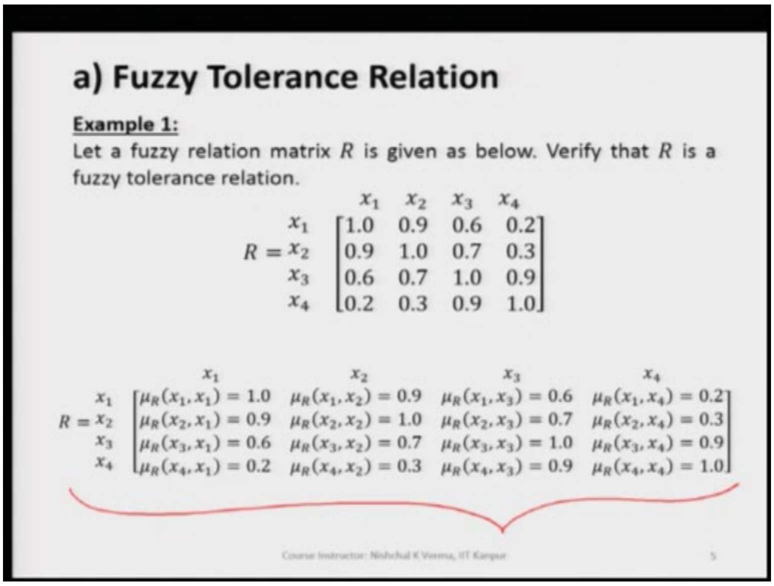

And now here we have all the elements of the fuzzy relation matrix is represented in terms of its mu membership values. So, you can see here that as to how all these elements are with respect to its rows and columns and these values are the membership values of fuzzy relation set R.

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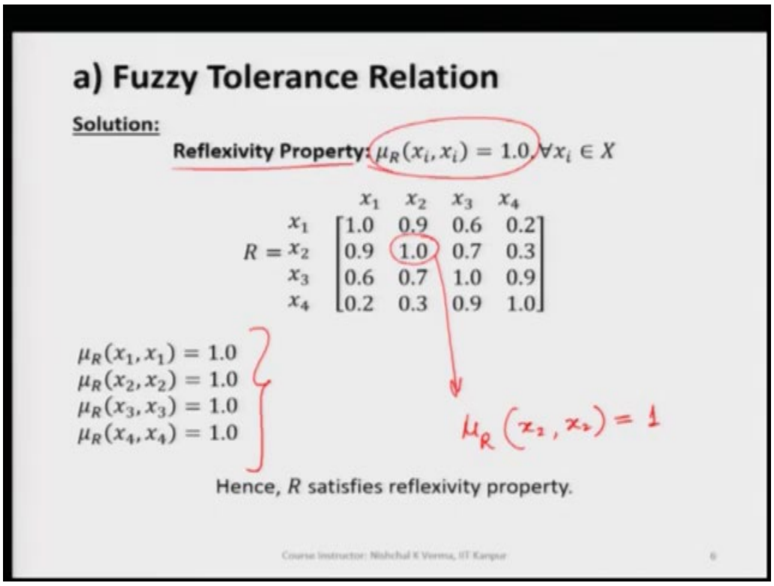

So, once this is clear, now since we are interested in checking that the given fuzzy relation set R is a fuzzy tolerance relation or not. So, in order to do that we need to check whether the two properties that was just mentioned or satisfied or not. So, these two properties are the reflexivity property and then another property here is the symmetry property.

So, let us now go through the first property and check whether the given fuzzy relation matrix R satisfies reflexivity property or not. And we know that we need to check this condition. If this condition is there then we can say that a given fuzzy relation set R satisfies reflexivity property. So, as I have already mentioned that reflexivity property is satisfied when its diagonal elements are 1. Why diagonal elements are 1? Because you know in therefore, diagonal elements we have this corresponding elements are having same row and same column.

So for example, here if we take this element we see that we have this as this element is nothing but this is $\mu_R(x_2, x_2)$ and this is 1. Similarly, we write all these elements where these elements are corresponding to the same row and same column. So, we see that $\mu_R(x_1, x_1)$ is 1, $\mu_R(x_2, x_2)$ is 1, $\mu_R(x_3, x_3)$ is 1, $\mu_R(x_4, x_4)$ is 1 which we can see very clearly.

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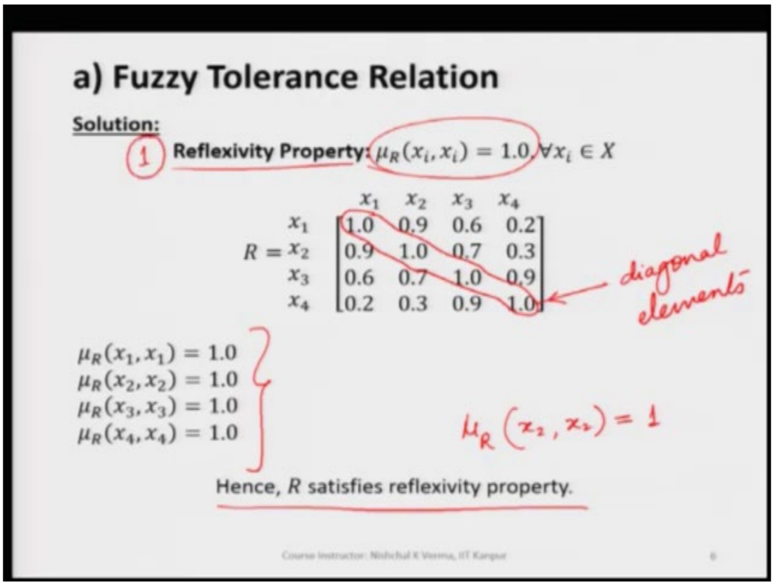

So, finally, we see that the diagonal elements here. All these diagonal elements are 1. Just by looking at the fuzzy relation matrix that is given to us, we can just check the diagonal elements and if all the diagonal elements are 1 then we can say the reflexivity property for the fuzzy relation set R is satisfied. So, now we can clearly see that R satisfies the reflexivity property. So, the first property is satisfied.

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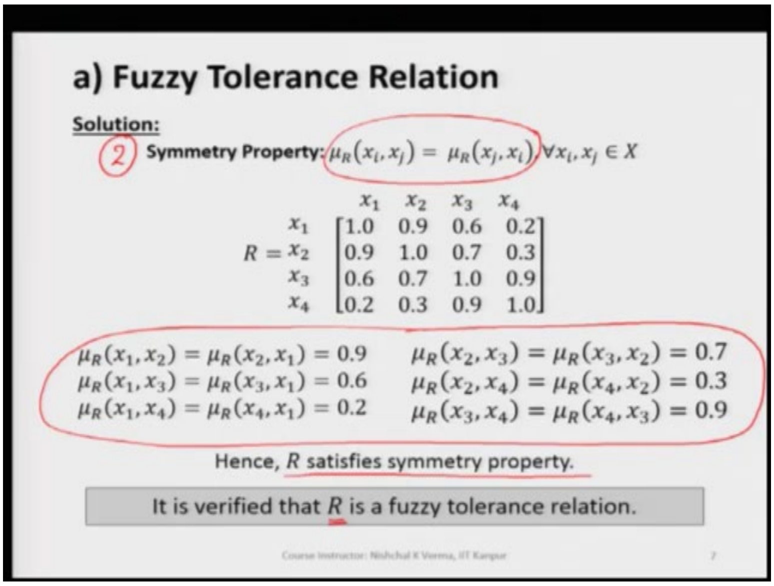

Now, let us look for the second property, that is symmetry property. So, in symmetry property we have $\mu_R(x_i, x_j) = \mu_R(x_j, x_i)$, which is here. And again this is for every x_i, x_j belonging into X.

So, we have the fuzzy relation set R the same fuzzy relation set R that is given here is this and let us take this and verify this. So, when we check for this condition we find that $\mu_R(x_1, x_2) = \mu_R(x_2, x_1)$ and here we see that both of these elements is equal to 0.9.

Similarly, when we see for all combinations we find that the symmetry property is very well satisfied here. So, what does this mean? This means that any element corresponding to a particular row and column is same if the row and columns are interchanged. So, here in this case if fuzzy relations set R that has been given satisfies the symmetry property as well.

So, when both of these properties are satisfied we can clearly say that the fuzzy relation set R is a fuzzy tolerance relation please understand that a fuzzy relation set R can only be called as fuzzy tolerance relation when both of these properties are satisfied. So, here in this case in the example that we have taken. We have taken fuzzy relation set R and for this fuzzy relation set R, both of these properties the reflexivity symmetry are satisfied. So, this R is qualified to be called as fuzzy tolerance relation.

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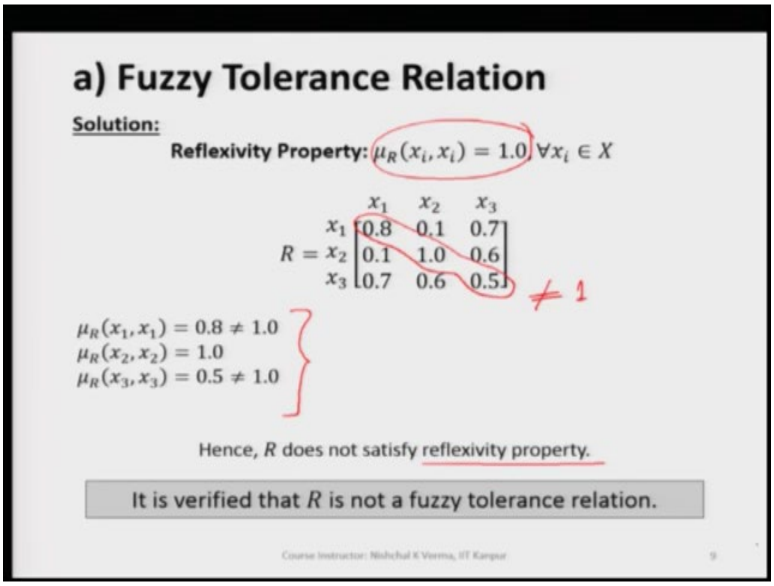

Now, let us take another example here where we check whether the given fuzzy relation set R is a fuzzy tolerance relation or not. So, on the same lines we move forward and see that whether the reflexivity property and symmetry property, both are satisfied for this fuzzy relation set are not. So, when we try to see for reflexivity property, we apply this criteria. So, as I mentioned that we can quickly just by looking at the fuzzy relation matrix we can quickly comment on this property. So, we see that all the diagonal elements are not 1.

So, this is not equal to 1. Only out of these three diagonal elements only 1 diagonal element is 1, but rest two elements are not 1. So, the condition for the reflexivity property is that all these diagonal elements should be equal to 1. So, just by looking at it we can say that the reflexivity property is not satisfied.

So, it is mentioned here you can see that $\mu_R(x_1, x_1)$ is equal to 0.8 which is not equal to 1. $\mu_R(x_2, x_2)$ is 1, $\mu_R(x_3, x_3)$ is 0.5 which is not equal to 1. So, this way we can say the fuzzy relation set that has been given to us is not satisfying the reflexivity property. Now let us go ahead and check for the symmetry property. Here I would like to tell you that since we are finding whether the fuzzy tolerance relation whether these R that is that has been given to us is a fuzzy tolerance relation or not.

So, please understand that since the reflectivity property is not satisfied, so we need not go ahead and check for the symmetry property because there is no use of it. It may be symmetric, but if it is not reflexive then we can say the given for the relation set R is not a fuzzy tolerance relation. So, the condition is that both the properties needs to be satisfied. So, here since the reflexivity property is not satisfied. So, which we need not go ahead and check for the symmetry property because just by checking any one of the properties we can say that whether we should go forward or not.

So, since reflexivity property is not satisfied we can clearly say that the given fuzzy relations set is not a fuzzy tolerance relation.

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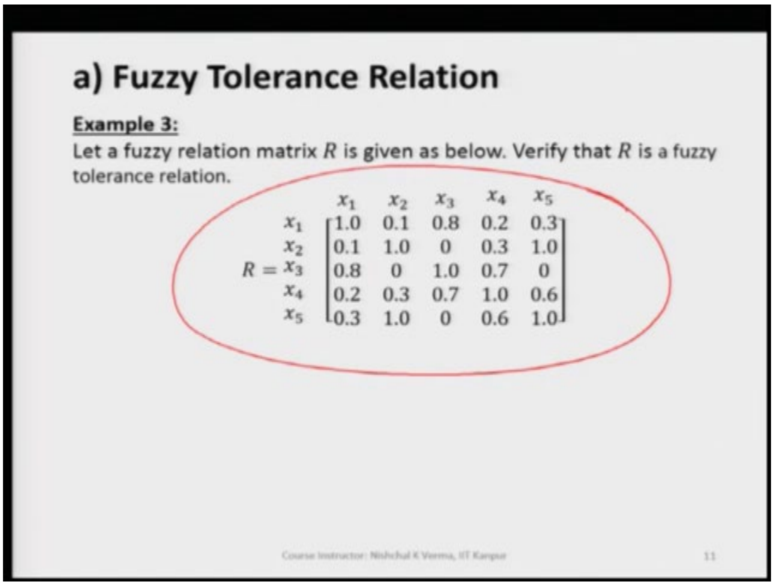

Now, let us take another example here of fuzzy tolerance relation here also let us check for the given fuzzy relation matrix fuzzy relation set *R* and let us check for this fuzzy relation matrix R whether this R is a fuzzy tolerance relation or not.

So, the fuzzy relation matrix that is given to us is here and you can see. And if we are interested in checking the fuzzy tolerance relationship or whether *R* is a fuzzy tolerance relation or not. Again we have to proceed on the similar lines and we need to check for the two properties first one is reflexivity and the second one is the symmetry.

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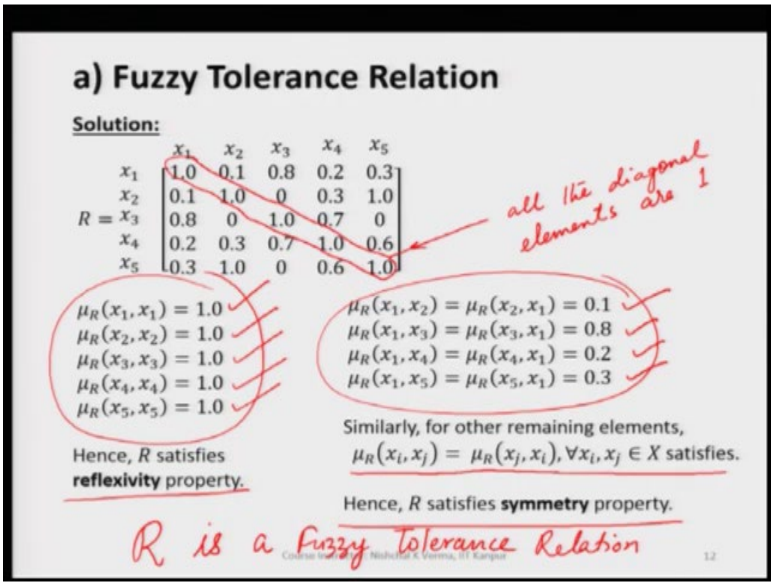

So, when we see here the given fuzzy relation matrix. So, just by looking at the diagonal elements here. We can see that all the diagonal elements are 1. So, when as I have already mentioned when all the diagonal elements are 1, then it is obvious that $\mu_R(x_i, x_i)$ is 1. And if this is the case then we can say that the reflexivity property is satisfied.

So, from the fuzzy relation matrix we can write all mu values in $\mu(x_i, x_j)$ format and then we see that $\mu_R(x_i, x_i)$ is 1. That means $\mu_R(x_1, x_1)$, 1, $\mu_R(x_2, x_2)$, 1, $\mu_R(x_3, x_3)$, 1, $\mu_R(x_4, x_4)$, 1, $\mu_R(x_5, x_5)$, 1. So, this way we see that the reflexivity property is satisfied.

Now let us quickly go ahead and check for the symmetry property. And for symmetry property also if we see that $\mu_R(x_1, x_2) = \mu_R(x_2, x_1) = 0.1$. $\mu_R(x_1, x_3) = \mu_R(x_3, x_1) = 0.8$. $\mu_R(x_1, x_4) = \mu_R(x_4, x_1) = 0.2$ and $\mu_R(x_1, x_5) = \mu_R(x_5, x_1) = 0.3$. So, similarly, all other values can also be written here. So, what does this mean here is that $\mu_R\left(x_i, x_j\right) = \mu_R(x_j, x_i)$. For this fuzzy relation matrix, for this fuzzy relation set which has been given to us.

So, when this is satisfied we can say that the fuzzy relation set R satisfies the symmetry property. So, when this property is also satisfied, we can say the reflectivity and symmetry both of these properties are satisfied for the given fuzzy relation set. So, we can say that R is a fuzzy tolerance relation.

So, this way we can comment on R now and the given fuzzy relations set is fuzzy tolerance relation. Please note that it may also be possible that any given fuzzy relation set may not be a fuzzy tolerance relation always.

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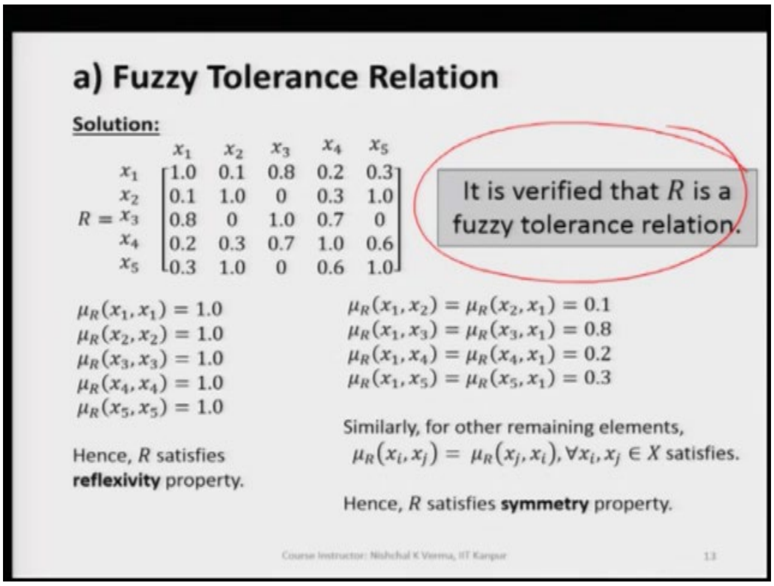

But in this case here, since both the properties for the *R* given is satisfied. We can say that the *R* is a fuzzy tolerance relation.

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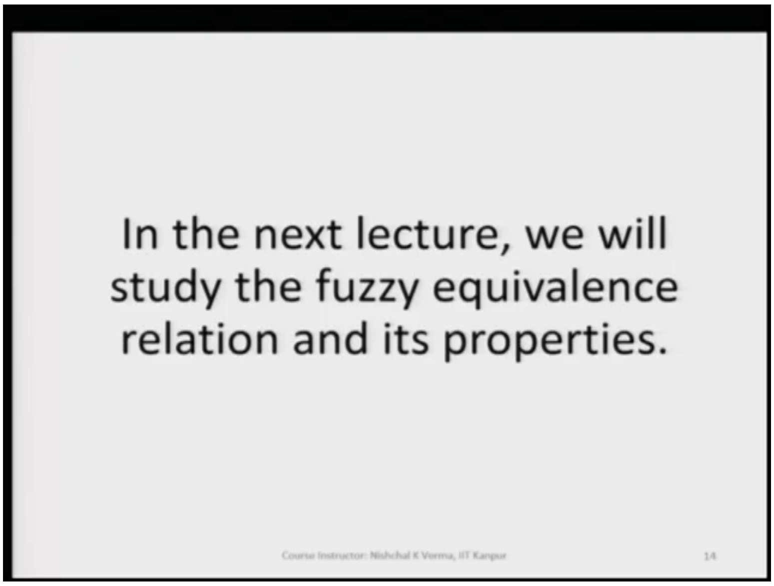

So, with this I would like to stop here and in the next lecture, we will discuss the fuzzy equivalence relations and its properties.

Thank you.