Fuzzy Sets, Logic and Systems and Applications
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Lecture – 27
Parameterized S-norm Operators

Welcome to lecture number 27 of Fuzzy Sets, Logic and Systems and Applications. Today we will be discussing the Parameterized S-norm Operators.

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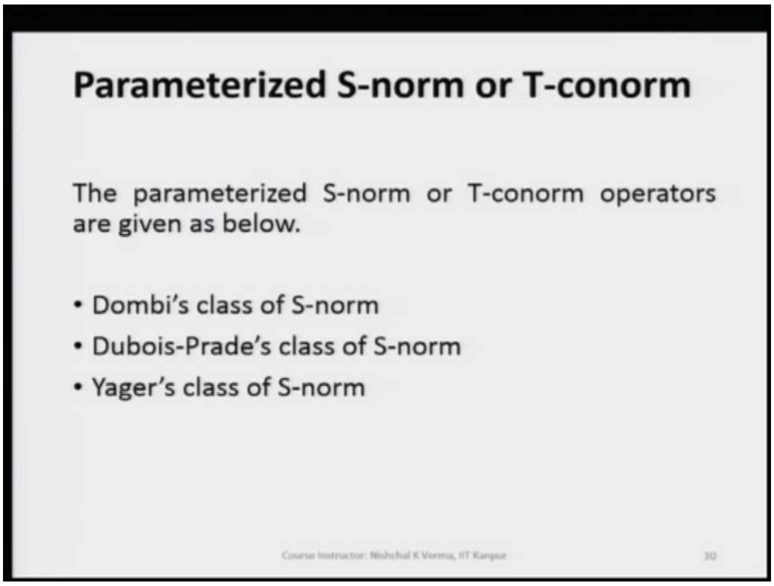

And here we have Dombi's class of S-norm, Dubois-Prade's class of S-norm and Yager's class of S-norm. The parameterized S-norm we can call this as parameterized T-conorm.

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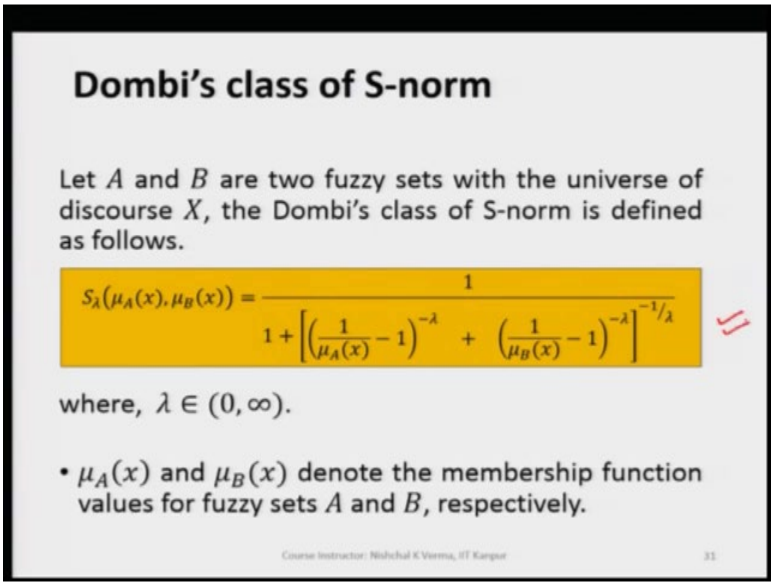

So, let us go one by one and discuss the first Dombi's class of S-norm. So, if we have two fuzzy sets let us say A and B within universe of discourse capital X. So, the Dombi's class of S-norm is defined by this expression where S_λ is the Dombi's class of S-norm operator. So, $S_\lambda\big(\mu_A(x), \mu_B(x)\big) = \frac{1}{1+\left[\left(\frac{1}{\mu_A(x)}-1\right)^{-\lambda}+\left(\frac{1}{\mu_B(x)}-1\right)^{-\lambda}\right]^{-1/\lambda}}$.

So, this is defined by this and the lambda values here, will be in between 0 and infinite. So, as I have already mention that the $S_\lambda\big(\mu_A(x), \mu_B(x)\big)$ here is basically termed as you know the S-norm operated Dombi's class of S-norm operator applied on the 2 membership values. And these 2 membership values are from 2 different fuzzy sets whose union are to be found.

So, here this has to be noted that normally we use S-norm, when we are interested in finding the union of two fuzzy sets. So, here also we take two fuzzy sets and if we are interested in union of these two fuzzy sets. So, we can either use the basic S-norm where we take the max where we take just the union by taking max, but in parameterized norm we have the expression here, which is given by Dombi's class of S-norm and Dombi's class of S-norm as I have mentioned here and this involves a parameter lambda and this lambda will be in between 0 and ∞.

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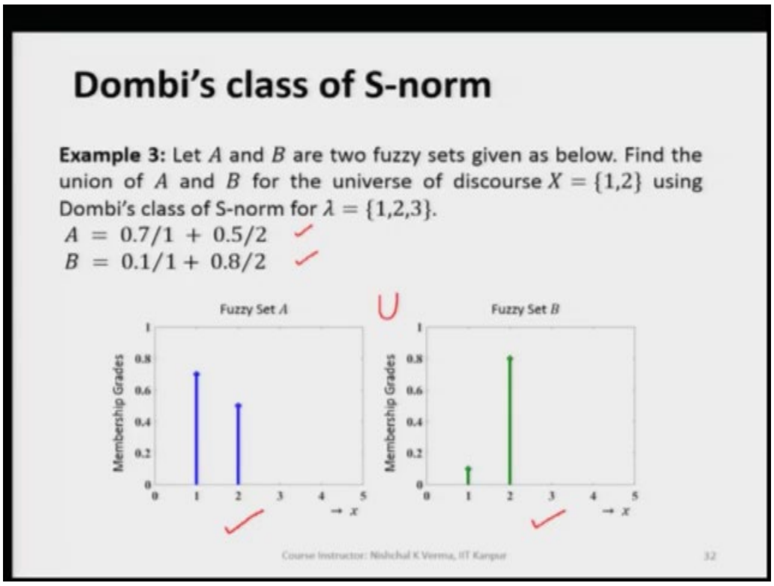

So, let us take an example here where we have two discrete fuzzy sets A and B you can see here discrete fuzzy set A is shown here and discrete fuzzy set B is also shown here we are interested in the union of these two fuzzy sets A and B. And the discrete fuzzy set A has 2 elements here discrete fuzzy set B also has 2 elements here and both the elements you can see have the membership values for corresponding generic variable values.

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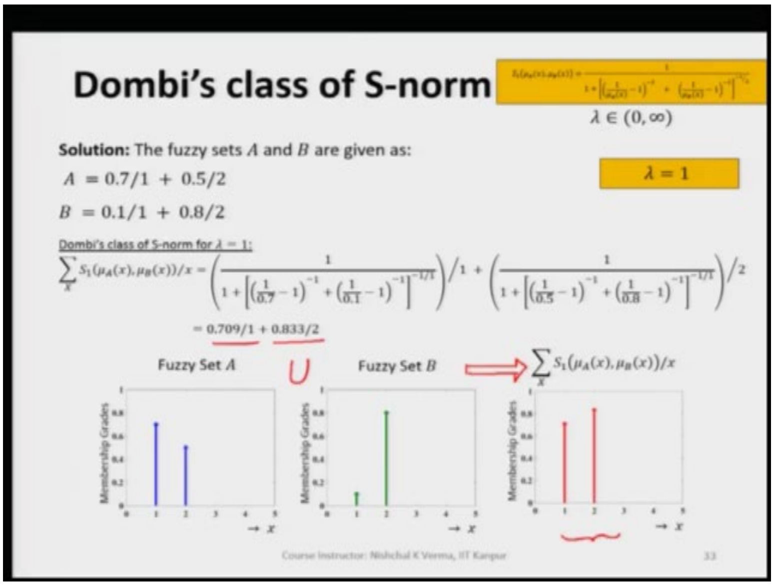

So, when we take the union of these two we can use the S-norm and that to with the Dombi's class of S-norm. So, when we use Dombi's class of S-norm we apply the expression that I just discussed and this expression is nothing but the $S_\lambda\big(\mu_A(x), \mu_B(x)\big) = \frac{1}{1+\left[\left(\frac{1}{\mu_A(x)}-1\right)^{-\lambda}+\left(\frac{1}{\mu_B(x)}-1\right)^{-\lambda}\right]^{-1/\lambda}}$. So, when we apply this expression for all possible pairs of the membership values of fuzzy set A and fuzzy set B.

So, we see that the resultant fuzzy set which is here. So, the first term is like this it is $0.709\ /\ 1 +\ 0.833\ /\ 2$. So, have two terms and this basically is a fuzzy set of the resulting fuzzy set out of the union of fuzzy set A and fuzzy set B. So, we can clearly see that we had two discrete fuzzy sets A and B and both the fuzzy sets have two elements and hence the union of these two will also have two elements.

So, here we take the union of fuzzy set A and fuzzy set B and this is what is resulting here as the fuzzy set. And we see that when we take the parameter $\lambda = 0$. So, please note that we can take any value of λ in between 0 and ∞ depending upon our suitability or requirement these values are used for finding the different sets of the union of the fuzzy sets A and fuzzy set B.

So, we see that when we take lambda is equal to 1 the resulting fuzzy set here we get as which has 2 terms and which has been shown here.

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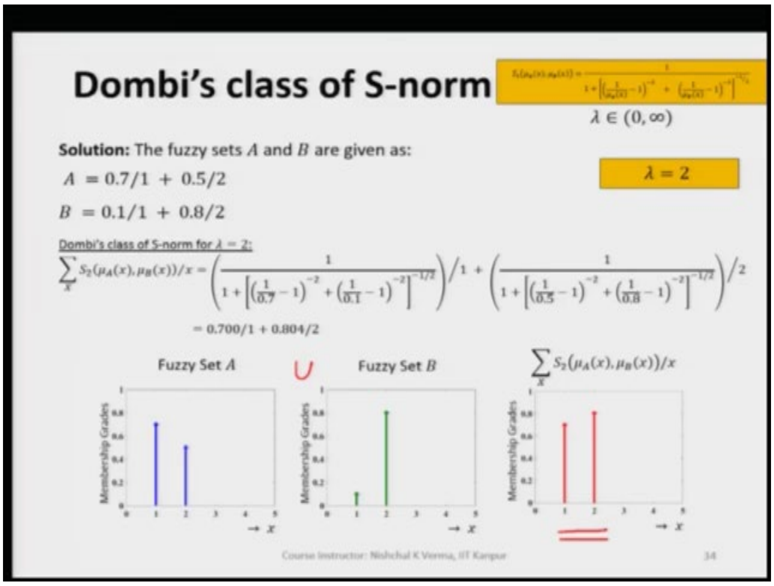

And now if we change the value of λ to 2 so, let us see what happens. So, we see that when we increase the value of λ from 1 to 2 the membership values of the corresponding generic variable values are reducing or I would say decreasing. So, when we take the union of fuzzy set A and fuzzy set B for the value of lambda is equal to 2 we see the fuzzy set which has the increased value of the membership for corresponding generic variable values.

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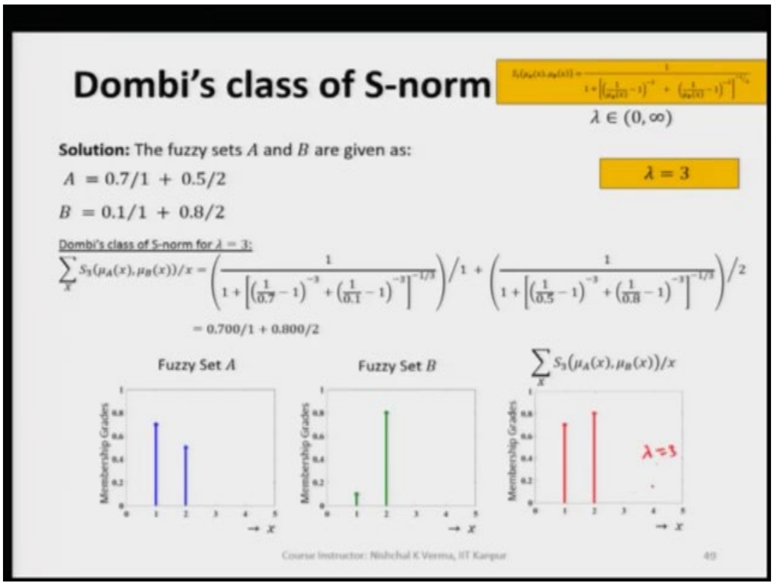

So, when we take the value of $\lambda = 3$. Let us see what we are getting here we again see that once again I would like to tell you that when we increase the value of λ the membership values are reducing. You can see here in the previous example where we took $\lambda = 1$, the union of the fuzzy sets A and fuzzy set B. We have the membership values that are reducing when we increase the value of λ.

So, for value of $\lambda = 3$ here also we see the decrease in the membership value. So, we can say when we increase the value of λ for the same set as input the resulting fuzzy set will have reduced membership values of the corresponding generic variable values.

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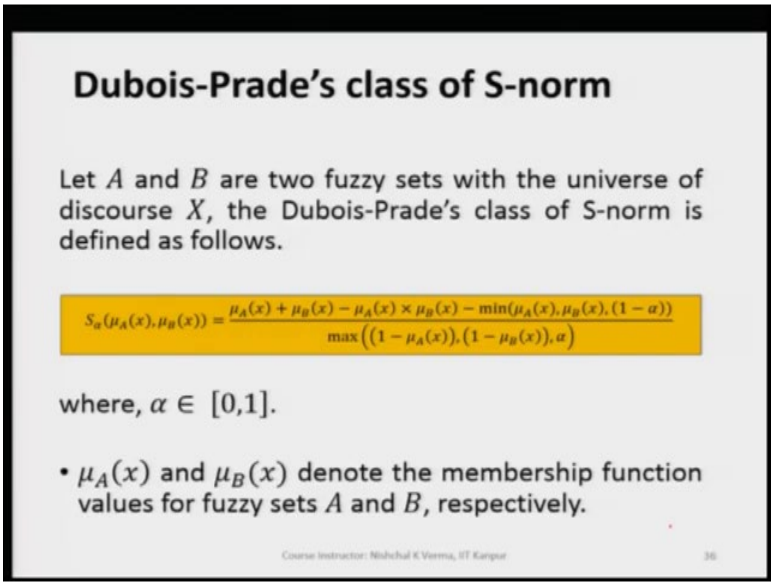

Now, let us discuss the Dubois-Prade's class of S-norm. So, here we have S_α and this Dubois-Prade's class of S-norm is represented by S_α and on the same lines as we have discussed in the previous class where we took the Dombi's class of S-norm. So, here we use $S_\alpha = \frac{\mu_A(x)+\mu_B(x)-\mu_A(x)\times\mu_B(x)-\min(\mu_A(x),\mu_B(x),(1-\alpha))}{\max((1-\mu_A(x)),(1-\mu_B(x)),\alpha)}$. Please understand that the α takes the values anywhere from 0 up to 1.

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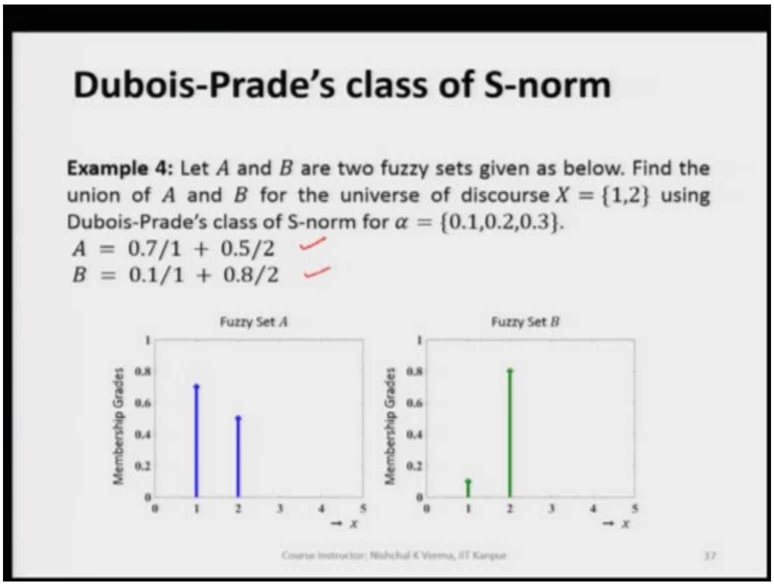

So, let us now take the same fuzzy sets as we have taken and in the previous example and let us find the union of two fuzzy sets using Dubois-Prade's class of S-norm. So, have two fuzzy sets here fuzzy set A and fuzzy set B. A is defined by this expression and B is also defined by here, these expression two these two fuzzy sets are discrete fuzzy sets.

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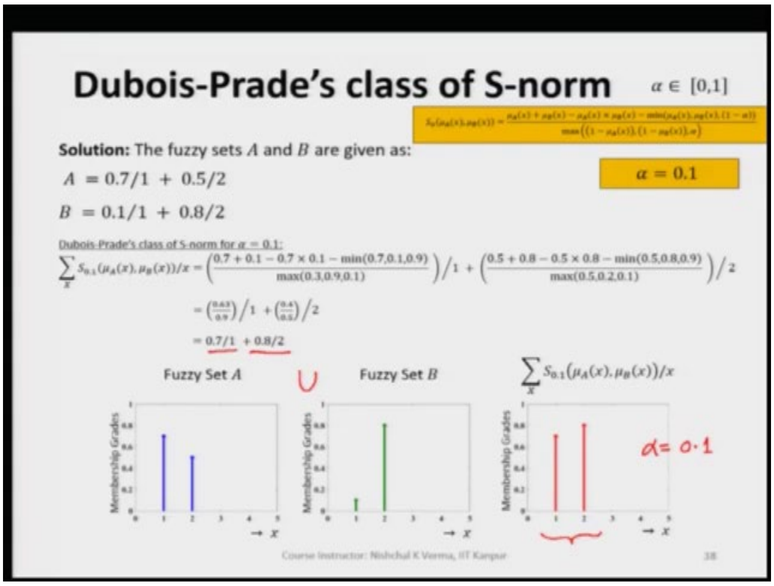

So, when we apply the expression that I just discussed for S_α which is for Dubois Prade's class of S-norm. So, when we use this for finding the membership values of the resulting fuzzy set, which is out of the union of fuzzy set A and fuzzy set B. So, we are getting two terms here in the resulting fuzzy set. So, we see that we get two terms. And these two terms will have its membership values for corresponding generic variable values $0.7/1 + 0.8/2$.

So, here if we take union of these two fuzzy sets we are getting a fuzzy set which will have two elements which is shown here and this is when you take the alpha is equal to 0.1. Similarly, when we increase the value of α let's see what we are getting.

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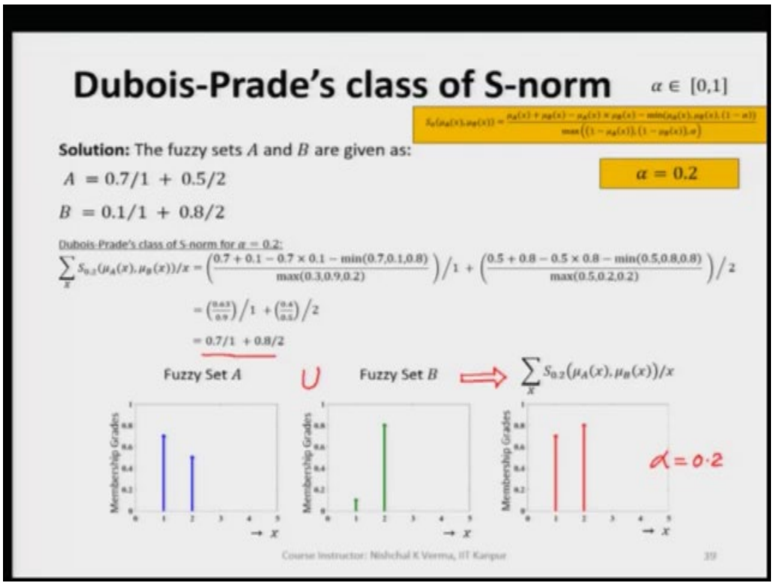

So, when we take the union again of fuzzy set A and fuzzy set B and here we get resultant of this union of fuzzy set, which is again a discrete fuzzy set which has two terms and we see that there is not any significant change here in the membership values of the resulting fuzzy set which is out of the union of fuzzy set A and fuzzy set B. So, here this case is when we have taken $\alpha = 0.2$. So, this result is same as the $\alpha = 0.1$.

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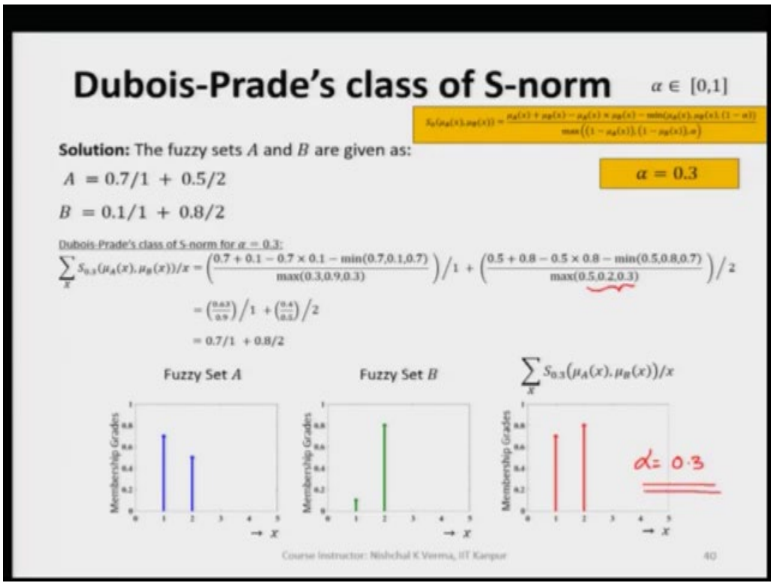

And then when we again go ahead and we use $\alpha = 0.3$ we see that again we do not see any significant increase in the membership values of the resulting fuzzy set. So, of course here alpha is not playing a big role or I would say α is here with the membership values of the fuzzy sets that we have taken.

And it depends upon the combination of the membership values and then the α. So, here in our case even when we increase the value of α we see that we do not have any significant increase in the values of the membership of the resulting fuzzy set. So, this way even when we increase the values of α we do not see any significant change.

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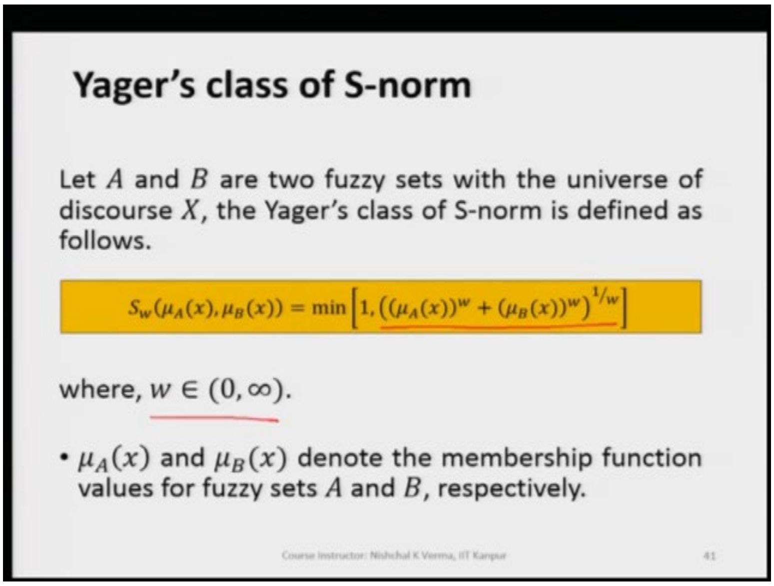

Now, let us discuss the another class which is Yager's class of S-norm. So, we have for Yager's class of S-norm for finding the union of the two fuzzy sets. So, by using this is Yager's class of S-norm we can operate this Yager's class on any two membership value of the fuzzy sets that we are interested in finding out the union of and if we have a fuzzy set A and fuzzy set B.

So, we will have the corresponding membership values as $\mu_A(x), \mu_B(x)$ so when we apply Yager's class of S-norm on these two membership values $\mu_A(x)\ and\ \mu_B(x)$ and this will be equal to $min\left[1, \left((\mu_A(x))^w + (\mu_B(x))^w\right)^{1/w}\right]$. Here w is a parameter of a Yager's class of S-norm and this w is going to be in between 0 and ∞. So, the w can take any value in between 0 and ∞.

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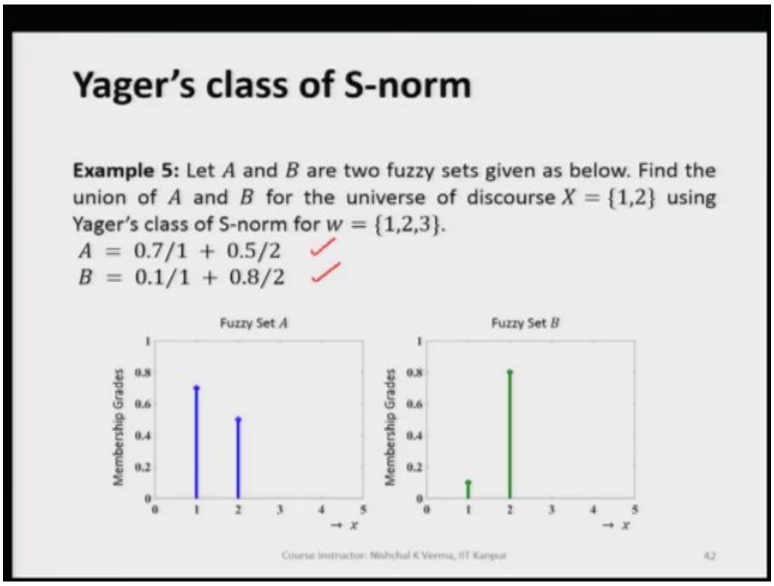

Let us, now understand this also Yager's class of S-norm also by taking an example. So, here we take two discrete fuzzy sets A and B and here we will be taking the union of these two fuzzy sets and we will be using the Yager's class of S-norm to find the union and let's see what happens when we use Yager's class of S-norm.

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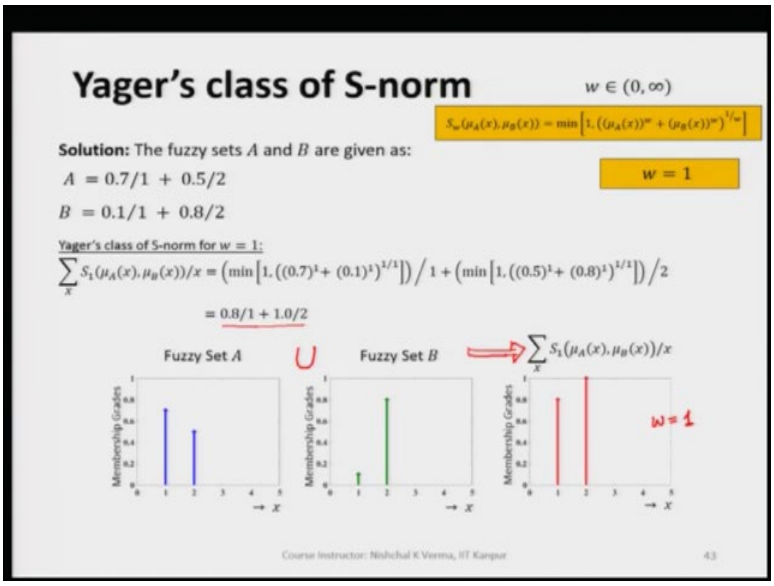

So, we have two fuzzy set as I mentioned and when we take union of the fuzzy set A and fuzzy set B so obviously the membership values of fuzzy set A membership values of fuzzy set B we'll collect. And then we will use Yager's class of S-norm on these the pairs of the membership values that are coming from fuzzy set A and fuzzy set B. So, when we do that, we use Yager's class of S-norm.

We find another fuzzy set as a result which will have two elements. So, one of the elements will be $0.8 \,/\, 1 + \ 1.0 \,/\, 2$. So, we see here that when we take the union of fuzzy set A and fuzzy set B. We get another fuzzy set which is the resultant of the fuzzy set of the union of fuzzy sets A and fuzzy set B we have two elements in this fuzzy set.

And this element has 0.8 as the membership value corresponding to the generic variable value as 1 plus we have 1 as the membership value corresponding to the generic variable value 2 and both of these have been plotted here you can see and this is for w is equal to 1. So, w is an important parameter here, in Yager's class of S-norm and when we change the value of w let's see what happens and obviously as I mentioned the w can be any value in between 0 to ∞.

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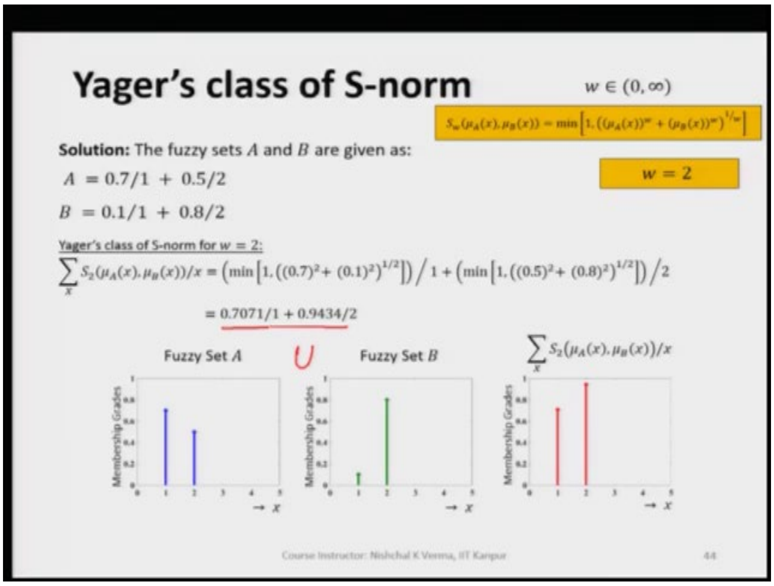

So, let us now use again both the fuzzy sets and let us find the union of these two fuzzy sets A and B and we take the value of w, 2. So, $w = \ 2$ we are going to get here some change in the membership values of the resulting fuzzy set which is coming out of the union of fuzzy set A and fuzzy set B. So, we can clearly see that the values of the membership are changing as we are changing w means the values of the membership are reducing here.

So, earlier we had 0.8 and then now we have 0.707 for the generic variable 1 when we have changed the value of w from 1 to 2. So, this means that when we increase the value of w the membership values of the resulting fuzzy set are reducing.

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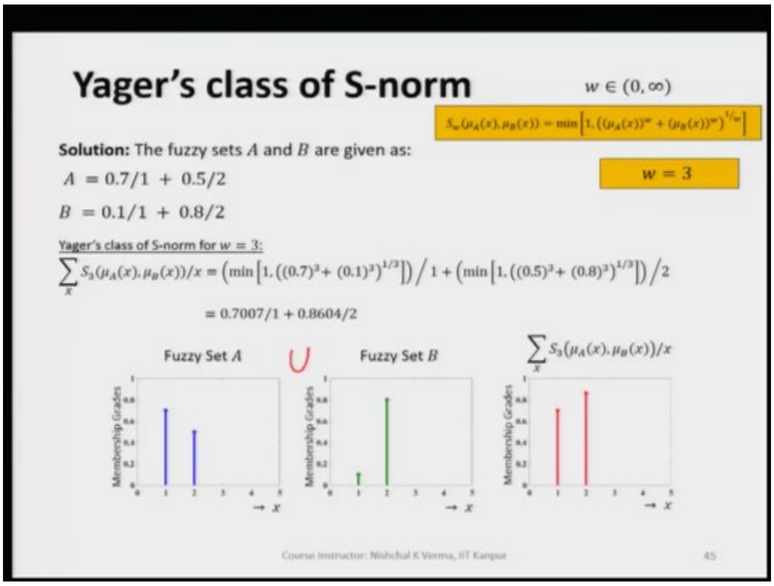

So, similarly when we again take the value of w as 3 here you can see the $w = 3$. So, when we take $w = 3$ again there is some decrease in the values of the membership of the corresponding generic variable values of the resulting fuzzy set which is coming out of the union of fuzzy set A and fuzzy set B.

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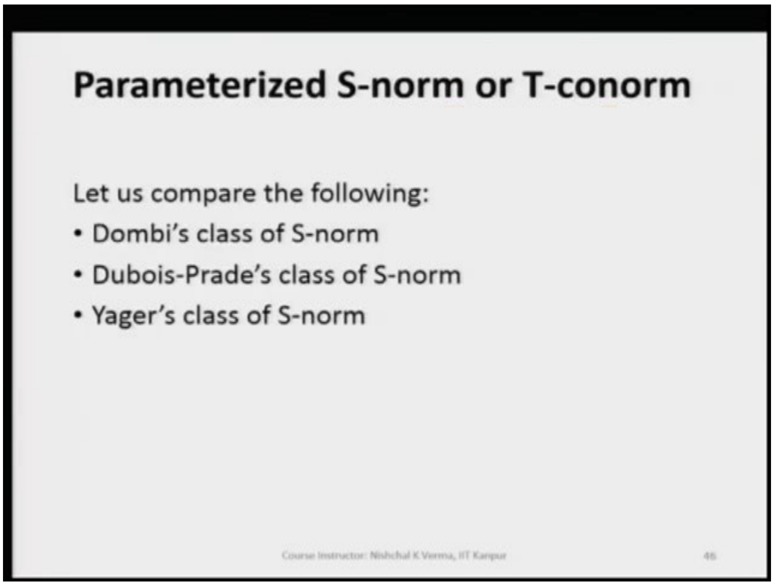

So, this way we can say that we have the parameterized S-norms this is also called as T-conorm so, we can either say we can either call the parameterized S-norm or parameterized T-conorm And we have 3 parameterized S-norms as we discuss Dombi's class of S-norm, Dubois Prades class of S-norm and Yager's class of S-norms. And let us compare these S-norms for the parameters that are involved here in this three parameterized S-norms.

So, let us see what happens when we are increasing the values of λ in case of Dombi's class of S-norm and α in case of Dubois-Prade's class of S-norm and w when we take the case of Yager's class of S-norm.

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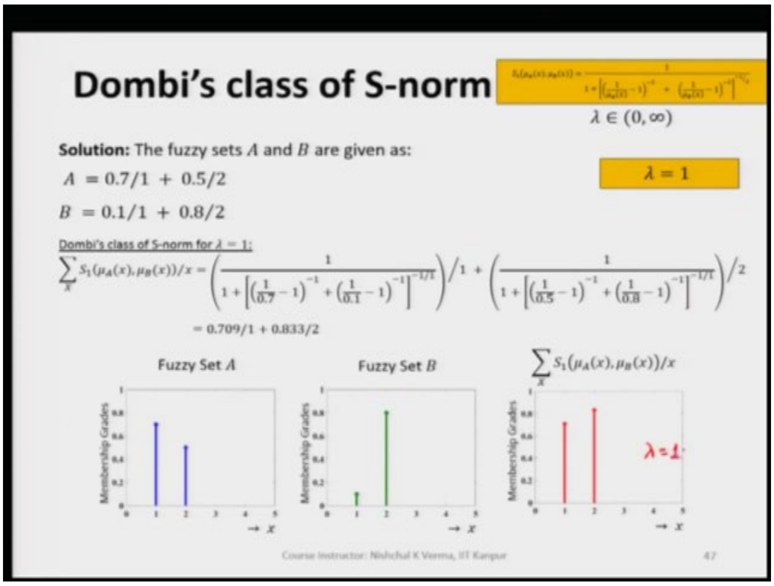

So, this we have already discussed, but I would like to just summarize and conclude here that you see when we increase the value of λ here. So, we can see here this for λ is equal to 1.

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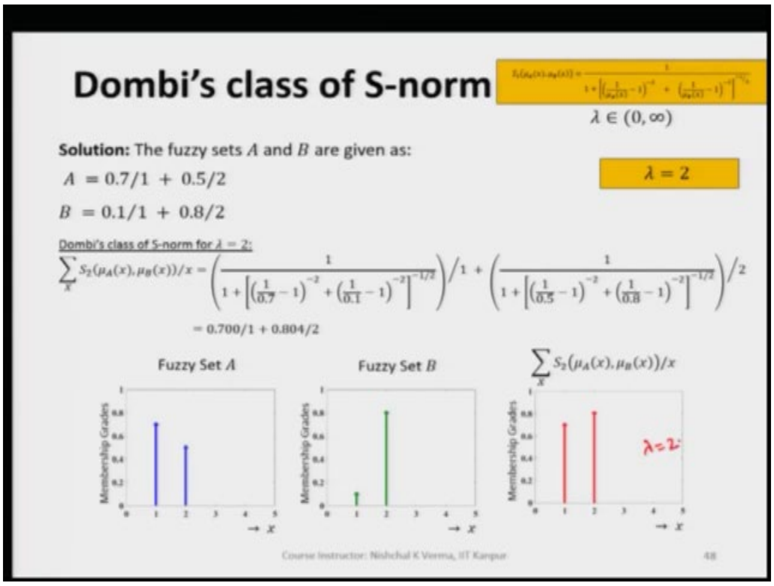

And then this λ is equal to 2 this for the Dombi's class of S-norm and then when we take the λ is equal to 3. So you see the output. So we see that the output the resulting fuzzy set that is you know out of the fuzzy set A union fuzzy set B. So, we see that as we increase the value of λ the membership values of the resulting fuzzy set for the corresponding generic variable values are reducing.

So, this is very clearly visible and when we see this in Dubois-Prade's class of S-norm here in Dubois-Prade's class of S-norm we have α as the parameter. So, when we increase the value of α we see that there is not much change or I would say not a significant change in the output even when we change the when we increase the values of the α. So, this is α is equal to 0.1 and then we have α is equal to 0.2 and then when we have α is equal to 0.3.

So, here once again I would like to mention that we have Dubois-Prade's class of S-norm and here even when we increase the value of α. We do not see any significant change in the membership values of the corresponding generic variable values of the resulting fuzzy set which is coming out of the union of fuzzy set A and fuzzy set B. Now in case of Yager's class of S-norm on the again on taking the union when we use Yager's class of S-norm we see that when we take w is equal to 1.

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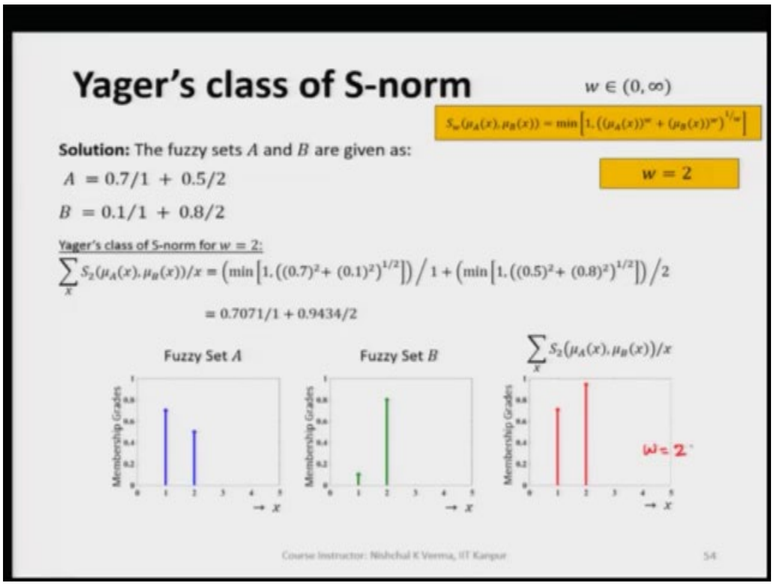

So, we see that there is the decrease in the membership value. So, as we increase the value of w here w is a parameter in Yager's class of S-norm.

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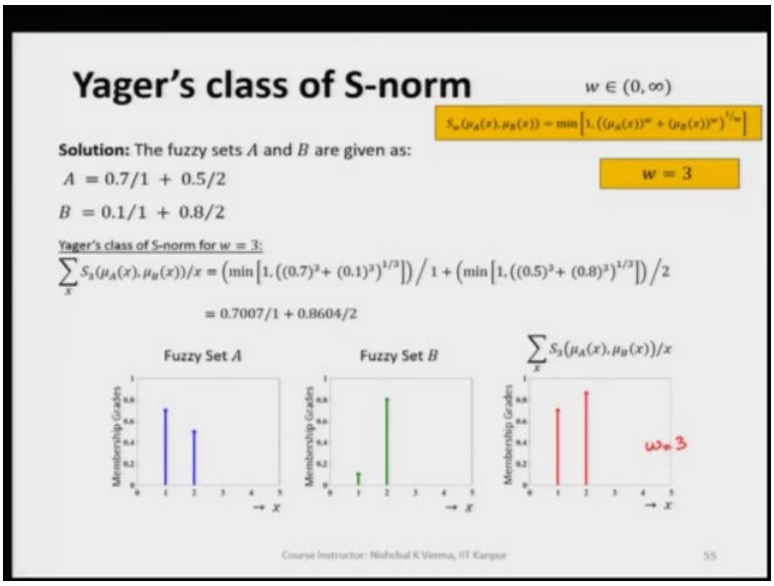

We see that the membership values are decreasing. So, these membership values of the corresponding generic variable values of the resulting fuzzy set which is coming out of the union of fuzzy set A and fuzzy set B. So, this way we are same as to how the parameters of the parameterized S-norms are playing an important role in the resulting fuzzy set.

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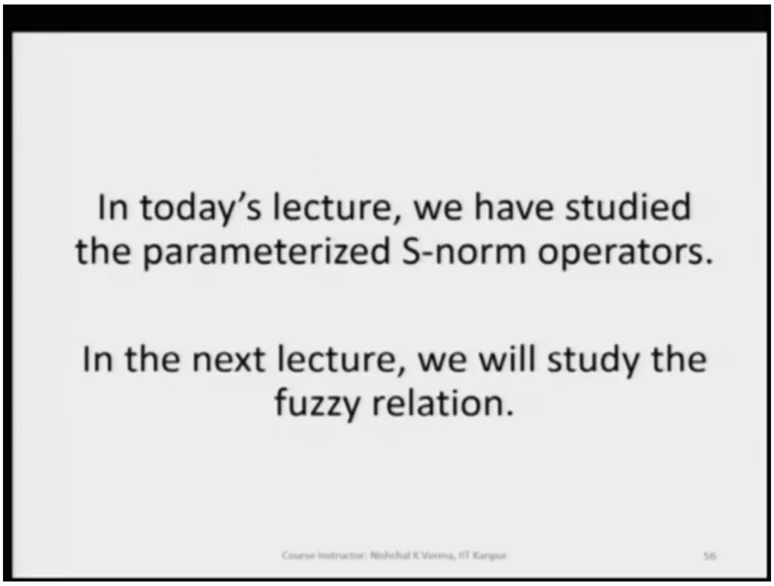

So, today we have discussed the parameterized S-norm in detail and in the next lecture we will be discussing the another topic that is fuzzy relation.

Thank you.