Fuzzy Sets, Logic and Systems and Applications
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Lecture – 26
Parameterized T-norm Operators

So, welcome to lecture number 26 of a Fuzzy Sets Logic and Systems and Applications. So, in this lecture we will discuss parameterized T-norm as we all know parameterized T-norm is also known as parameterized S-conorm.

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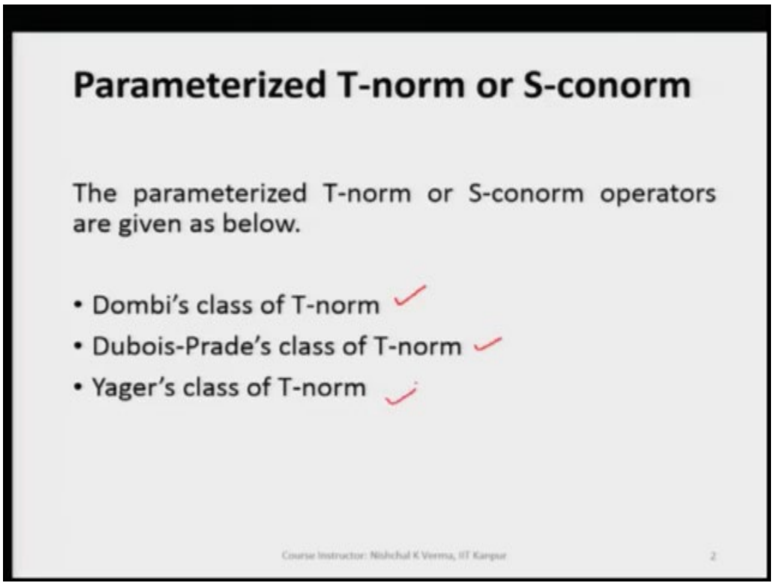

So, in parameterized T-norm operator, we have certain classes of T-norm like Dombi's class of T-norm, Dubois Prade's class of T-norm and Yager's class of T-norm.

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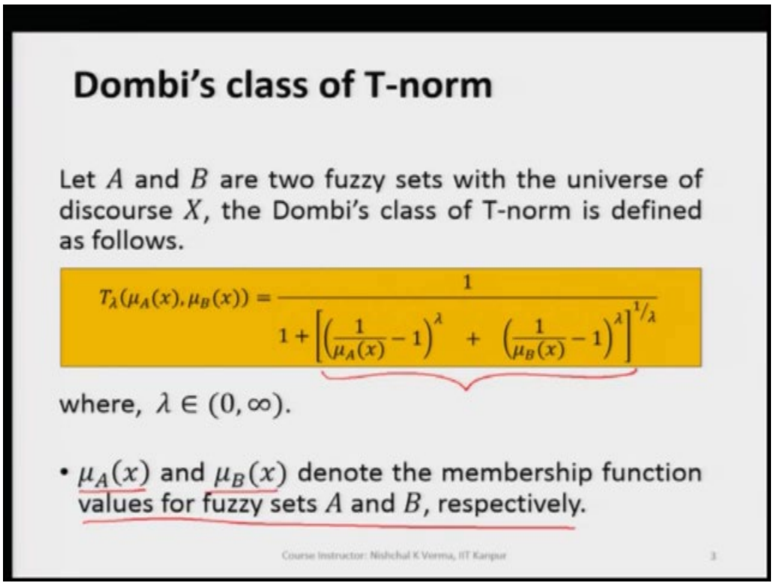

So, let us first discuss Dombi's class of T-norm here if we have 2 fuzzy sets A and B with the universe of discourse capital X. So, the Dombi's class of T-norm is defined as T lambda of $\mu_A(x)$ and $\mu_B(x)$. Here, T lambda is for Dombi's class and this lambda here is a parameter. So, this will appear in the right hand side. So, $T_\lambda(\mu_A(x), \mu_B(x)) = \frac{1}{1+\left[\left(\frac{1}{\mu_A(x)}-1\right)^\lambda+\left(\frac{1}{\mu_B(x)}-1\right)^\lambda\right]^{1/\lambda}}$, where this lambda is in between $0\ and\ \infty$.

So, the lambda values can take any value in between $0\ and\ \infty$. This $\mu_A(x)$ and $\mu_B(x)$ denote the membership function values for fuzzy sets A and B respectively, you can see here. So, it is very clear from this expression of Dombi's class of T-norm that this T lambda applies to any two values of membership functions.

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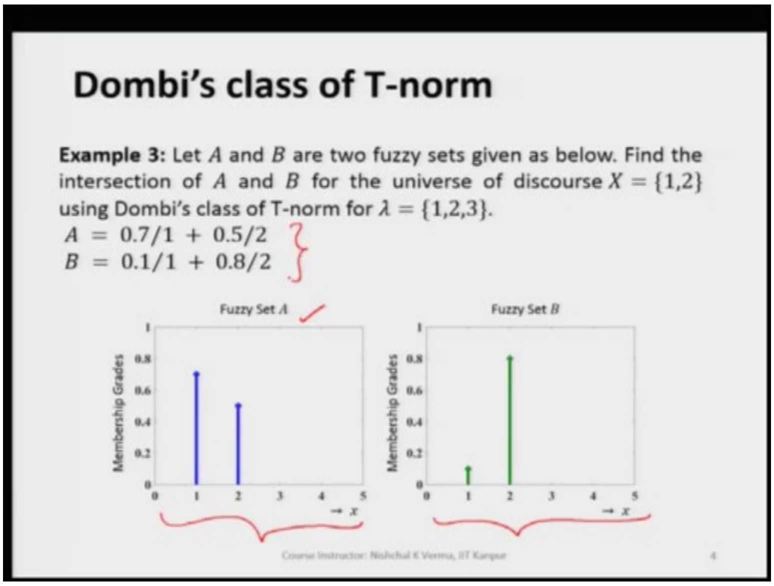

So, this can be very well understood by this example here. So, if we take an example here where we have two fuzzy sets A and B in this case we have fuzzy sets A and B are discrete fuzzy sets, then let us use Dombi's class of T-norm for the intersection of A and B under the universe of discourse $X = \{1,2\}$.

So, let us now find out this the Dombi's class of T-norm for the $A \cap B$. So, you can see here that we have one fuzzy set here which is a discrete fuzzy set as I mentioned and this is represented by you see here by this figure and similarly we have another fuzzy set B which is you can see here represented by this figure. So, we have two discrete fuzzy sets A and B and you see here these are defined by these two expressions.

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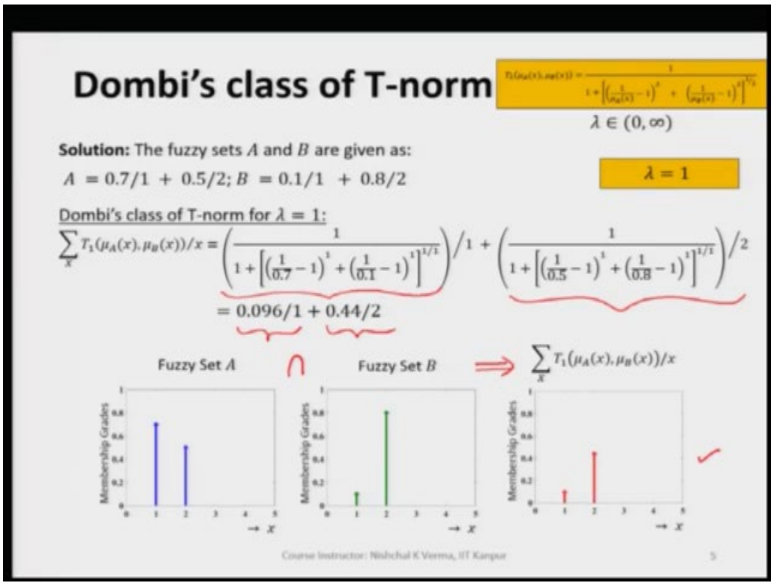

Now, let us apply the Dombi's class of T-norm for finding the intersection of A and B. So, it's very simple, we will have to take the membership values of the fuzzy sets A and B respectively and then we'll apply the Dombi's class of T-norm formula and this where we get the result as you see here when we apply. So, we see that, we get two terms as a result of Dombi's class of T-norm for $\lambda = 1$. And when we simply these two terms, we are going to get these two terms as the element of the resultant fuzzy set which is nothing but the intersection of fuzzy set A and fuzzy set B based on the Dombi's class of T-norm.

So, this is coming out to be $0.096/1 + 0.44/2$. So, we have two elements of the resultant fuzzy set. So, we can say when we take the intersection of these two fuzzy set using the Dombi's class of T-norm, we are going to get this thing and you can see here this is plotted and please understand here we have taken the value of the parameter $\lambda = 1$. So, here we can change the value of a λ, let us take some other values of λ and see how the results vary.

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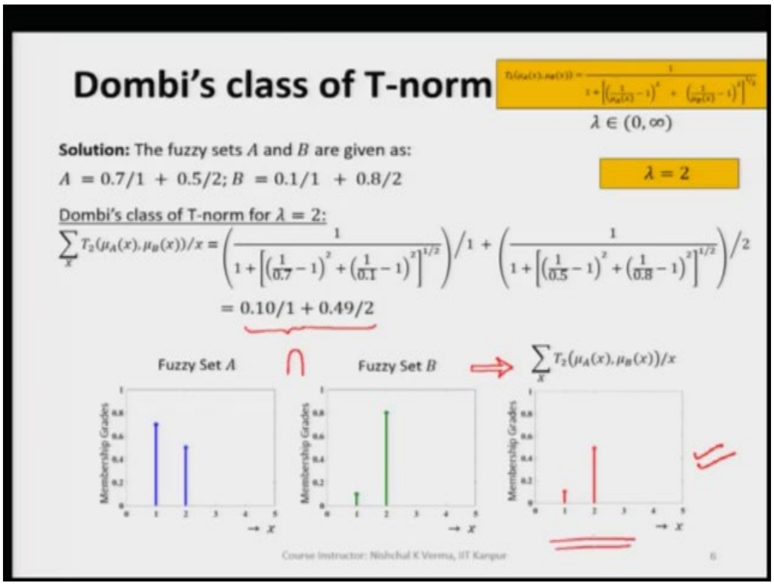

So, when we take $\lambda = 2$, we see that we are getting the intersection of A and B as a fuzzy set whose elements are $0.10\ /\ 1 +\ 0.49\ /2$. So, this way here, the intersection of fuzzy set A and fuzzy set B, obviously both the fuzzy sets A and B are the discrete fuzzy sets. And when we take the intersection of these two based on the Dombi's class of T-norm, we get this as the output which has been plotted here. So, similarly we can now increase the value of λ further and see what is happening.

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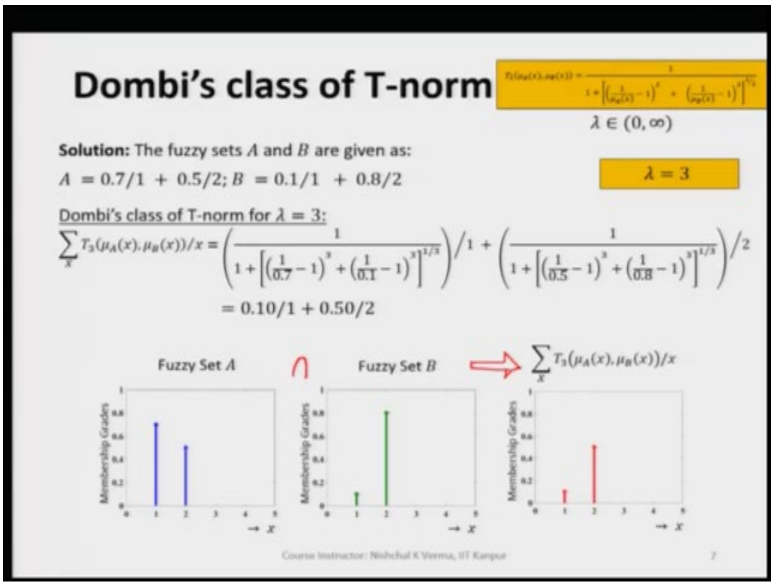

So, we see that on applying the Dombi's class of T-norm for $\lambda = 3$, we are getting here the elements of the resultant fuzzy set and this resultant fuzzy set is the intersection of the fuzzy set A and fuzzy set B which is giving us this the fuzzy set. So, here also when we plot the fuzzy set because it is the discrete fuzzy sets, so we are getting the plots for the discrete points and we see this as the result of the intersection of the two discrete fuzzy sets A and B.

So this way we have understood that the Dombi's class of T-norm can be found by applying this expression $T_\lambda(\mu_A(x), \mu_B(x)) = \frac{1}{1+\left[\left(\frac{1}{\mu_A(x)}-1\right)^\lambda+\left(\frac{1}{\mu_B(x)}-1\right)^\lambda\right]^{1/\lambda}}$. And this way by substituting $\lambda = 3$, we are getting the expression of the resultant fuzzy set and please understand this expression is applied for the membership values of the corresponding fuzzy set for which we are interested in taking the intersection.

So, since this is the discrete fuzzy set, we first take all the elements and then we take corresponding numbers of values of the fuzzy sets A and fuzzy set B and then apply the T-norm of the Dombi's class. So, if we look at all the results we see here for $\lambda = 1,\ \lambda = 2,\ \lambda = 3$, we see that the membership values of the resultant fuzzy set increases as we increase the values of λ. So, when we increase the value of λ, the resultant membership value also increase.

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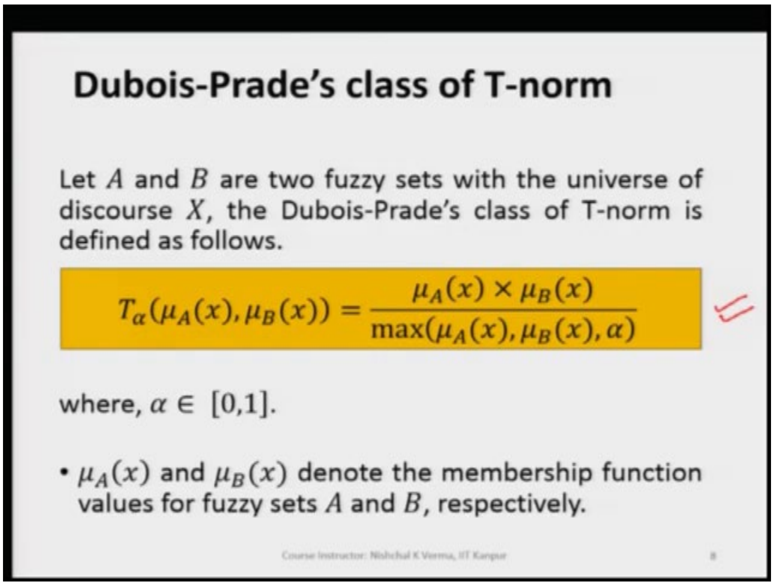

Now, let us go to Dubois Prade's class of T norm. So, if we take A and B fuzzy sets again and with the universe of a discourse X. The Dubois Prade's class of T-norm is defined by this expression you can see here and here we have another parameter that is α. So, we write this Dubois Prade's class of T-norm as T_α and α value can be from 0 to 1. So, Dubois Prade's class of T-norm is defined by the $T_\alpha(\mu_A(x), \mu_B(x)) = \frac{\mu_A(x)\times\mu_B(x)}{\max(\mu_A(x),\mu_B(x),\alpha)}$.

So this is very simple expression here and Dubois Prade's class of T-norm is found by using this expression. Again the Dubois Prade's class of T-norm is used for taking the intersection of any two fuzzy sets, here we have two fuzzy sets A and B whose intersection is found by using Dubois Prade's class of T-norm. So, when we use this formula here we have as I mentioned T_α, α here is a parameter and α this parameter value can be any value from 0 up to 1, as I mentioned. And when we apply this, we get the resultant membership value or I would say the membership value of the fuzzy set which is the intersection of fuzzy set A and fuzzy set B.

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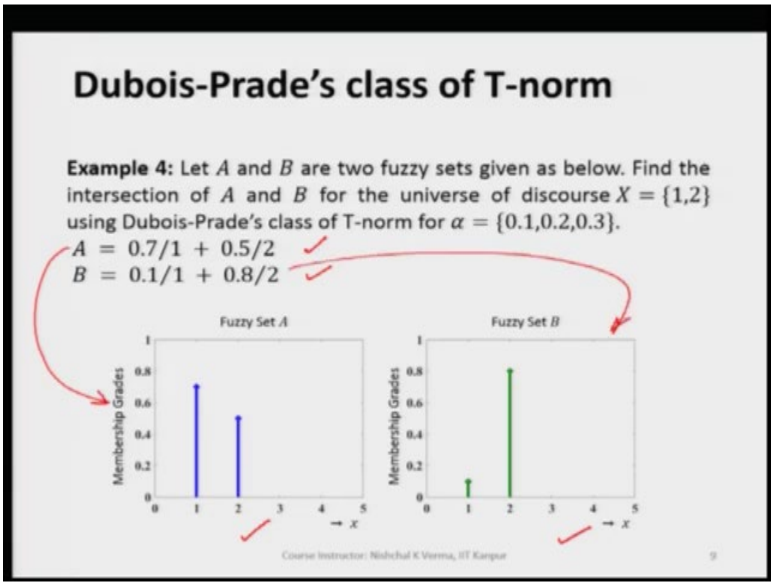

So, this can also be understood by an example here. So, we have two discrete fuzzy sets fuzzy set A and fuzzy set B, here you can see. So, fuzzy set $A = 0.7 / 1 + 0.5 / 2$ and then $B = 0.1/1 + 0.8 / 2$. So, these two fuzzy set we have and we are here interested in finding out the intersection of these two fuzzy sets for different values of α and these values are α are 0.1, 0.2, 0.3 and this is again through the intersection is found through the Dubois Prade's class of T-norm.

So, if we take two fuzzy sets as shown here in these two plots these two fuzzy sets are the discrete fuzzy sets A and fuzzy set B. So, A is this fuzzy set and B is this fuzzy set.

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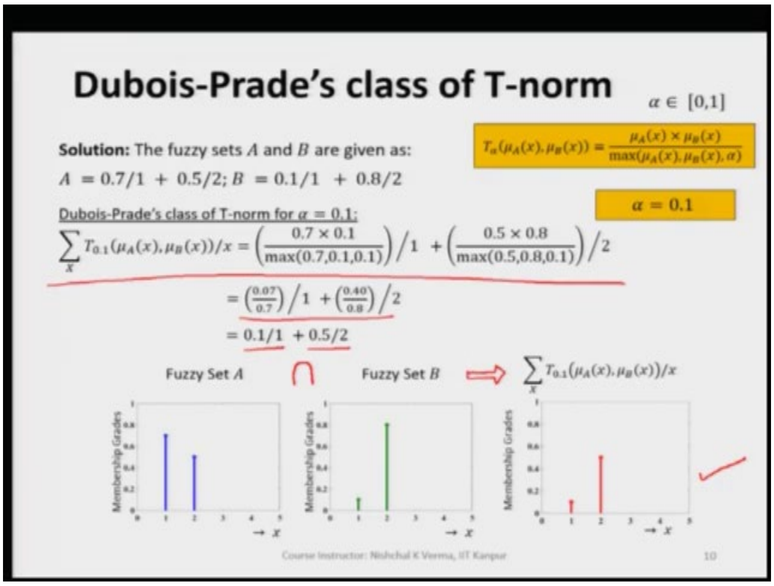

And when we use the formula that I have already discussed. Just now. the formula here is $T_\alpha(\mu_A(x), \mu_B(x))$, this is giving us the $\frac{\mu_A(x) \times \mu_B(x)}{\max(\mu_A(x), \mu_B(x), \alpha)}$. So, let us now take $\alpha = 0.1$ and see what is the membership value of the corresponding resultant fuzzy set and this again this resultant fuzzy set is there a intersection of fuzzy set A and fuzzy set B. So, when we apply this, we find here the resultant fuzzy set which is the intersection of fuzzy set A and fuzzy set B and this resultant fuzzy set has two elements. So, we have here $0.1 / 1 + 0.5 / 2$ as two elements of the resultant fuzzy set.

So, when we plot this the fuzzy set looks like this. So, here I can write that we are taking the intersection of fuzzy set A and fuzzy set B and this is giving us the resultant fuzzy set here and this fuzzy set has again two elements and this looks like as it is shown here in this slide as a result.

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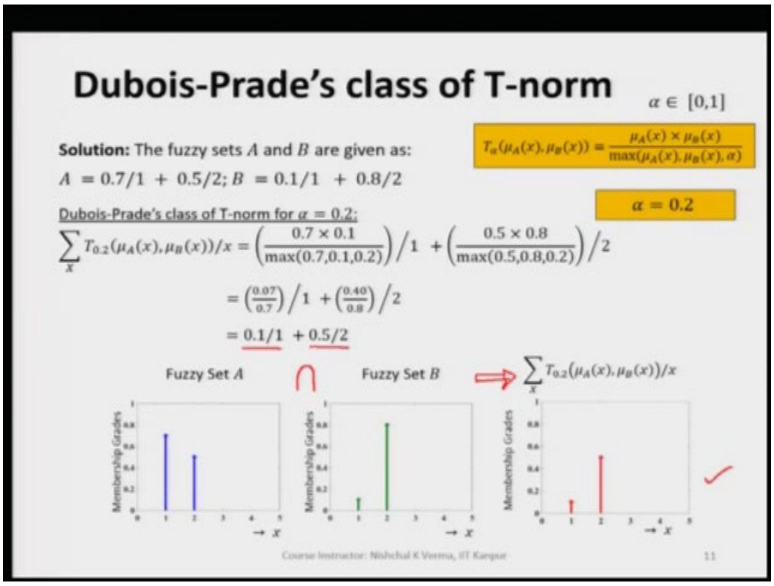

Now, when we take some other value of alpha, so here we are now increasing the value of alpha. In earlier example we have $\alpha = 0.1$, in this example we have $\alpha = 0.2$. And let see what is happening how the result is varying, how the elements are changing. I mean these elements are from the intersection of fuzzy set A and fuzzy set B.

So, again I can write here that this is the intersection of fuzzy set A and fuzzy set B and here we are getting the resultant fuzzy set which again is having two elements as $0.1\ /\ 1 +\ 0.5\ /\ 2$ and when it is plotted it looks like as it shown here.

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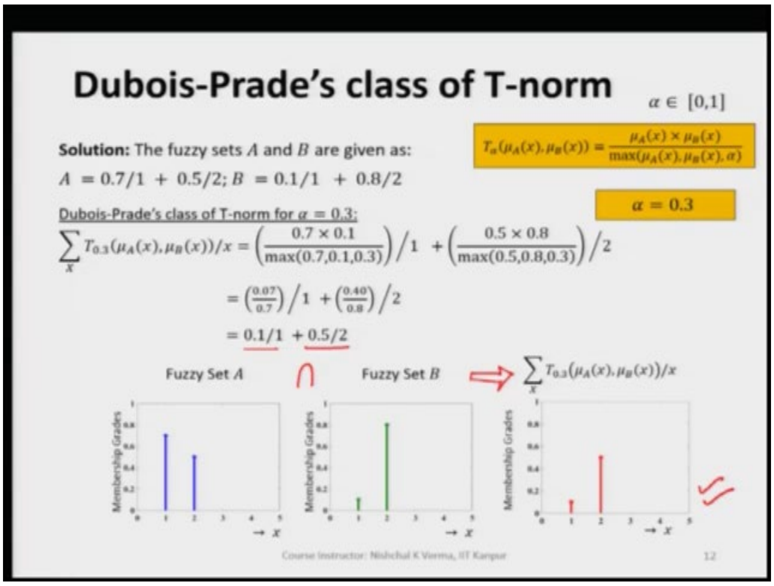

Now, again we change the value of alpha and we increase the value of alpha here. Now for $\alpha = 0.3$, when we substitute in the expression that is given for Dubois Prade's class of T-norm. We see that the resultant fuzzy set again has two terms two elements and this these elements are $0.1\ /\ 1 +\ 0.5\ /\ 2$.

So, when we take intersection based on the Dubois Prade's class of T-norm, we are getting here the resultant fuzzy set which is shown here like this. So, what we have done in this Dubois Prade's class of T-norm, we apply Dubois Prade's class of T-norm for taking the intersection of fuzzy set A and fuzzy set B and then the resultant fuzzy set B we have obtained based on the Dubois class of T-norm. And we see that when we increase the value of α like in the first case we took $\alpha = 0.1$, the second case we took $\alpha = 0.2$ and then we took $\alpha = 0.3$.

And we see that the values of the membership we can see here the values of the membership. So, let us now quickly go to the first case where we took $\alpha = 0.1$. So, you see here 0.1 and then when we change the value of α to 0.2, we see that there is no change in the output, there is no change in the membership value even when we change the value of α.

So, this way we see that for the values of alpha, here the change is not very significant in the values of α. So, because of that the membership value of the resultant fuzzy set is also not changing significantly.

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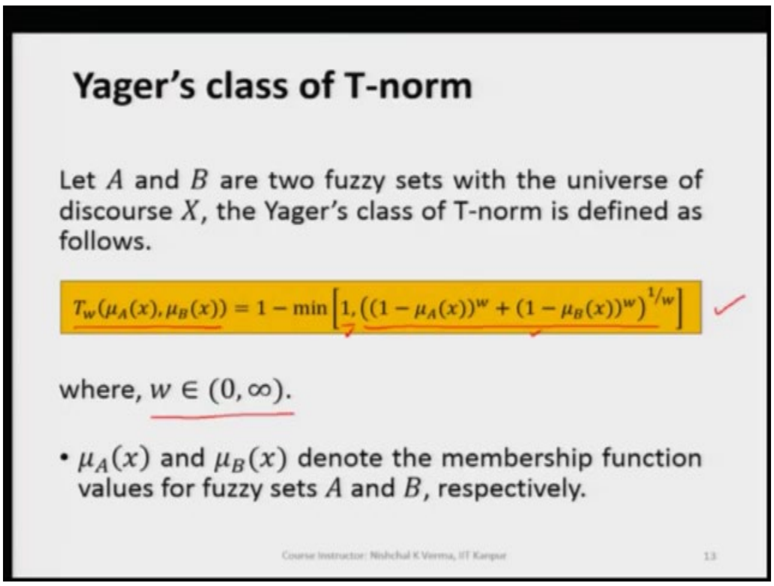

Now, let us talk about the Yager's class of T-norm which is the another class. So, in Yager's class we see that if we have two fuzzy sets A and B and within the universe of discourse capital X. So, Yager's class is defined by the expression which is given here, the Yager's class operator is represented by T_w. So, T_w is operated on a pair of membership values, here we have this pairs as $\mu_A(x)$ and $\mu_B(x)$.

So, we can say that $T_w(\mu_A(x), \mu_B(x)) = 1 - min\left[1, ((1 - \mu_A(x))^w + (1 - \mu_B(x))^w)^{1/w}\right]$, you can see here. So, we will take min of this and this. And here the value of w lies in between 0 and ∞. So, when we are interested in finding the intersection by using the Yager's class of T-norm, so let us see how it works.

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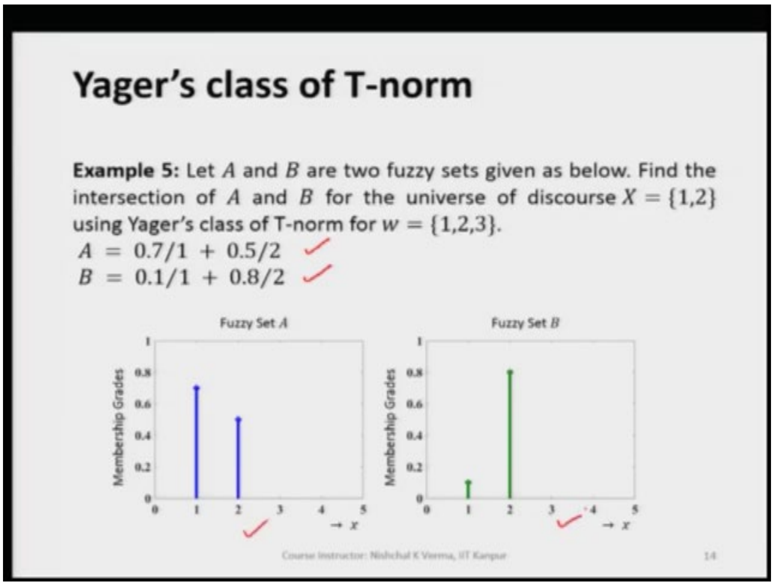

We have this example here where we have taken two discrete fuzzy sets A and B, the same fuzzy sets same discrete fuzzy sets as we have taken in the previous example. So, here when we apply the Yager's class of T-norm on these two fuzzy sets A and B.

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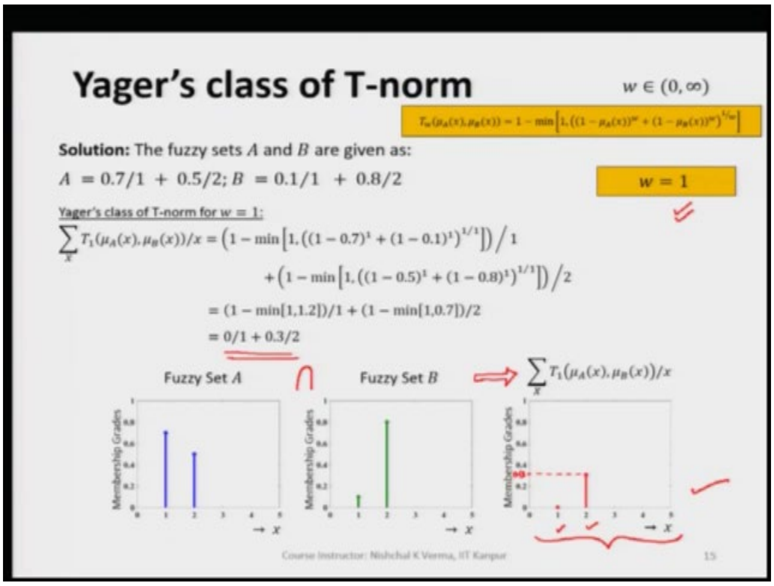

When we apply the expression for the Yager's class of T-norm on the membership values of fuzzy set A and B. So, we get this expression finally, you can see all these steps are shown very nicely for understanding. So, we get again two terms where we have a taken $w = 1$ and for $w = 1$, when we apply the Yager's class of T-norm for finding the intersection of fuzzy set A and fuzzy set B, we are getting here two elements. In fact, we are getting only 1 element, because the two elements that are mentioned here, one has the 0 membership value. So, 0 membership value is normally not included when we represent a fuzzy set.

So, you see that when we take the Yager's class of T-norm, we get the resultant fuzzy set whose elements are $0 / 1 + 0.3 / 2$. So, this way as I already mentioned that one of these elements has 0 membership value. So, it is shown here, but it need not be included while writing the discrete fuzzy sets of the resultant of fuzzy set and fuzzy set B. So, it is shown here the resultant fuzzy set shown here which you can see at x is equal to 1, this x is equal to 1, we have 0 membership value and at x is equal to 2 as the generic variable we have membership value 0.3. So, you can see here, here 0.3.

So, this way we can find the intersection of any two fuzzy sets based on the Yager's class of T-norm for w is equal to 1. Now, for the same set of fuzzy sets we find the intersection based on the Yager's class of T-norm for $w = 2$.

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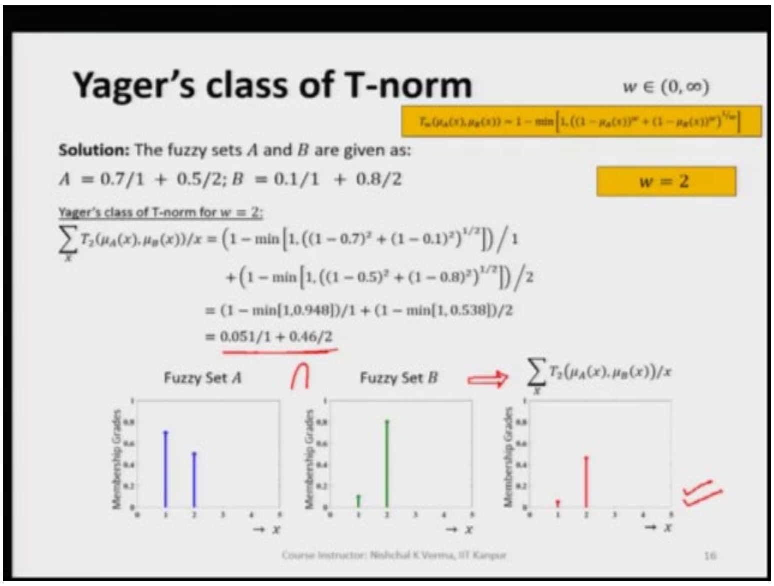

So, when we increase the value of $w = 2$, we see what is happening. You see that on applying the Yager's class of T-norm expression, we are getting here two elements of the resultant fuzzy set which is coming, because of the intersection here of the fuzzy set A and fuzzy set B, you can see here. So, this way these two elements if we see we find in the previous case where we had $w = 1$, this term had the membership value 0 whereas, here we see the membership value is not 0.

So, membership value for the first term here is 0.051 for x is equal to 1 and the other term has the membership value as 0.46 at x is equal to 2. So, you see here both the terms have been written here and this is the resultant of the intersection of fuzzy set A and fuzzy set B. So, we can clearly see that by increasing the value of w, we see that the membership values are also increasing. Now, let us take the value of $w = 3$ and see that this value is the membership value of the resultant fuzzy set is again increased.

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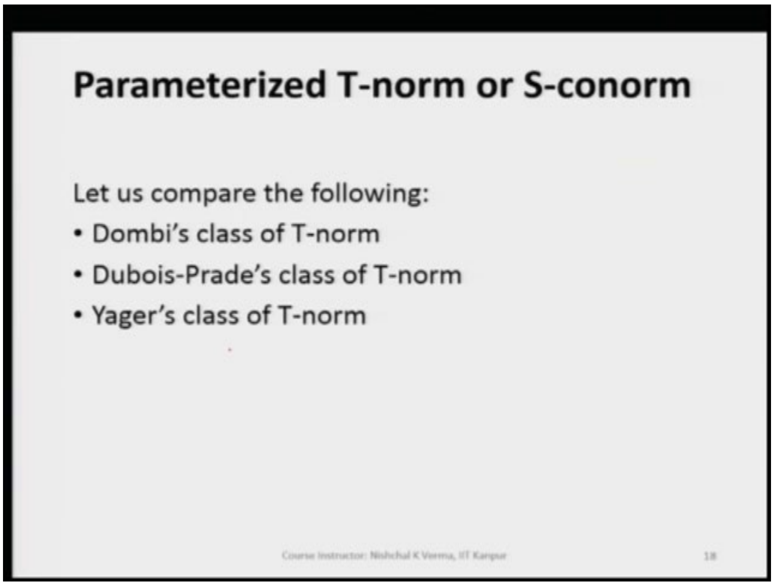

So, this way we see that when we increase the value of w, the membership values of the resultant of the intersection based on the Yager's class of T-norm also increases.

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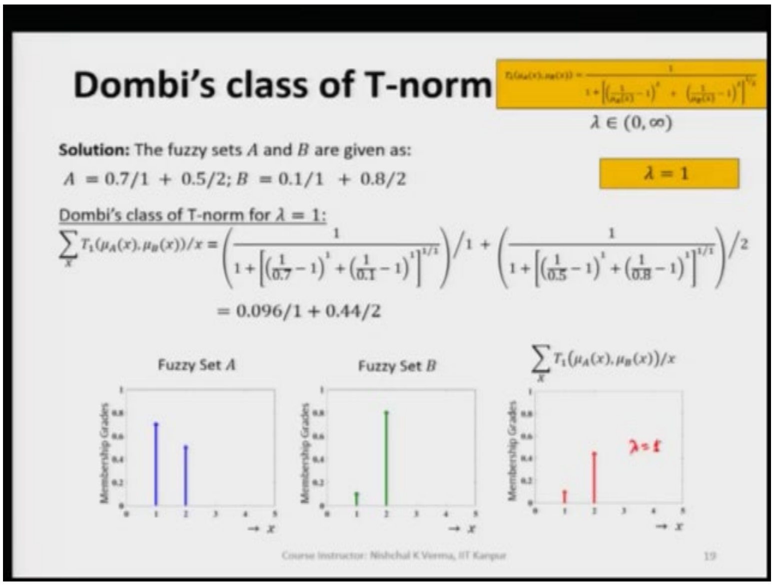

So, now let us compare this. So, I would just like to repeat what I have said again here. So, we see that while we took the Dombi's class of T-norm, we see that in Dombi's class of T-norm we have lambda and you see here this is for $\lambda = 1$.

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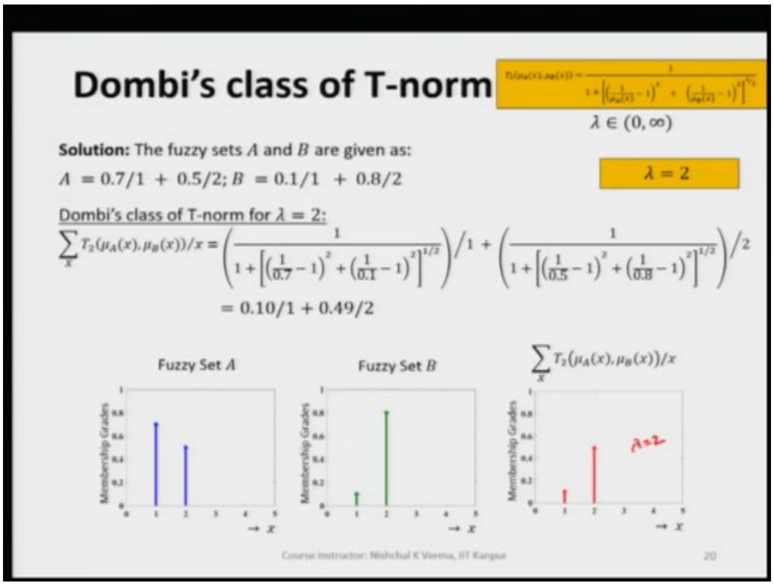

And when we increase the value of $\lambda = 2$. So, we see that the membership values of the corresponding terms of the resulting fuzzy set out of the intersection of fuzzy set A and fuzzy set B increase.

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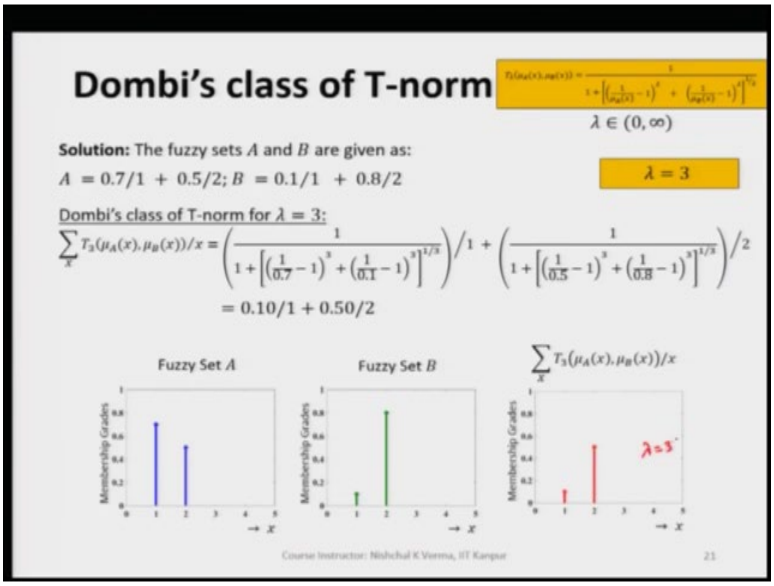

Similarly, here $\lambda = 3$, we have once again we see there is increase in the membership values.

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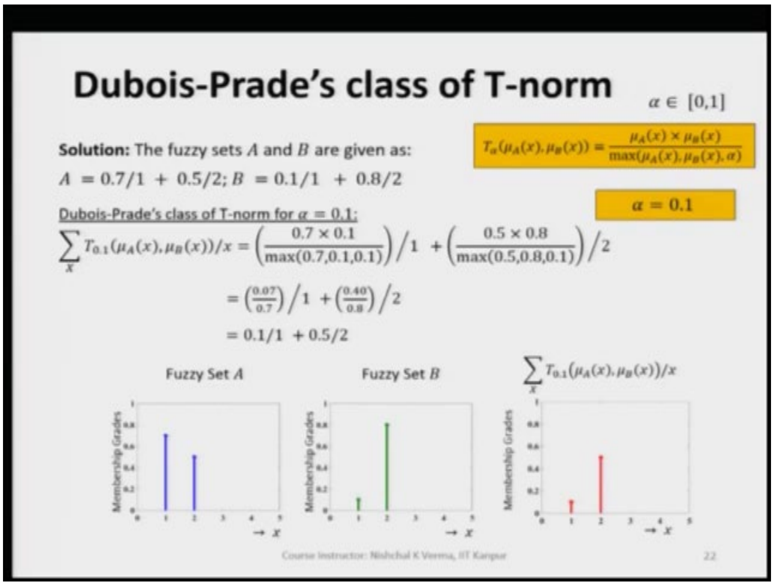

Similarly, when we talk of Dubois Prade's class of T-norm with the increase of the α as the parameter we increase here since the change is not very significant in alpha the so the significant change is also not seen. So, in the you see the membership values of the resulting fuzzy set out of the intersection of fuzzy set A and B. So, we don't see any significant change here for $\alpha = 0.1, 0.2, 0.3$.

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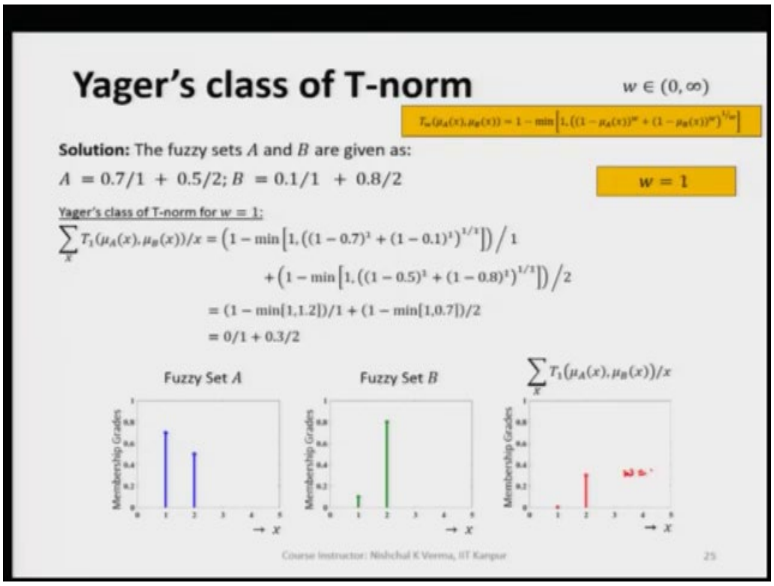

And when we see the Yager's class of T-norm, we see that when we increase the value of w, we see the change in the membership values of the resulting fuzzy set which is out of the intersection of fuzzy set A and fuzzy set B.

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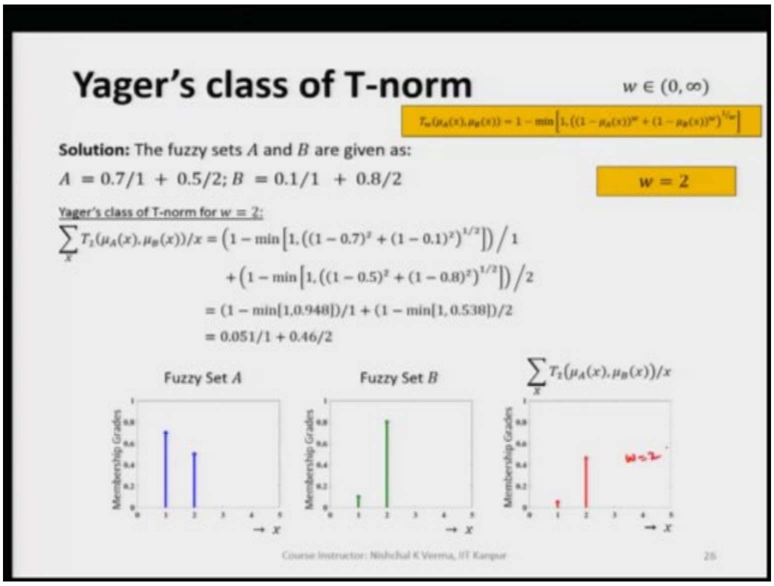

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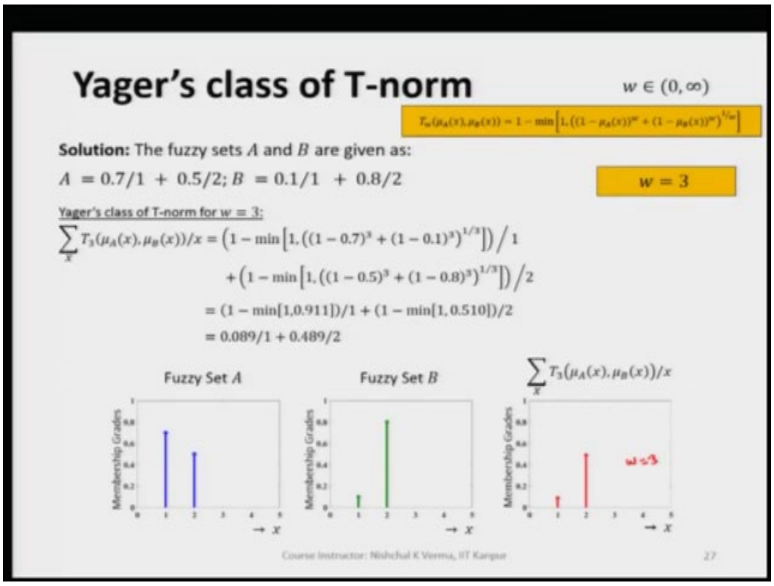

We can see here, this is for $w = 1$ and this is for $w = 2$ and here we have $w = 3$. And we can see that the membership values for the corresponding generic variable values are increasing.

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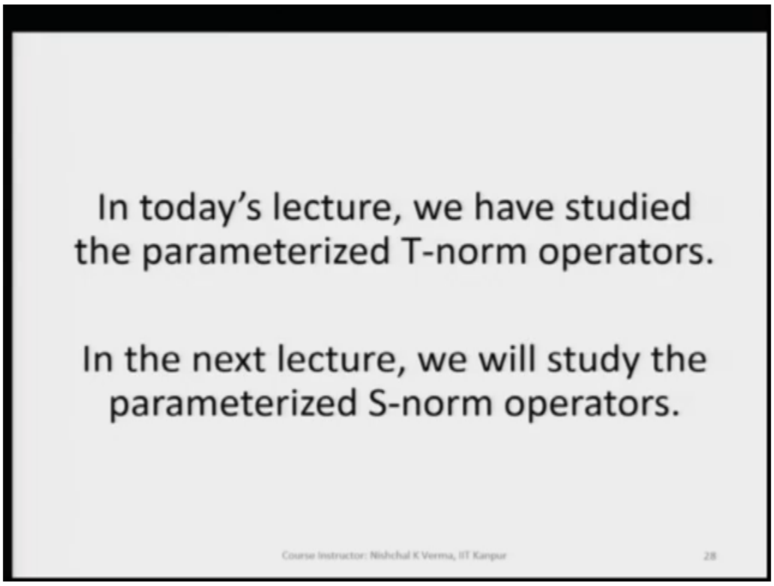

So, this way we see that we have covered the parameterized T-norms and in the next lecture we will discuss the parameterized S-norms are T-conorms.

Thank you.