Fuzzy Sets, Logic and Systems and Applications
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Lecture - 23
Complement of Fuzzy Sets

Welcome to lecture number 23 of Fuzzy Sets, Logic and Systems and Applications. So, in this lecture, we will discuss the Complement of Fuzzy Sets. Although, we have already discussed in one of our previous lectures, the basic type of complement of fuzzy sets, but here, we will discuss the complement of fuzzy sets in more detail. So, let us now take up a fuzzy set, whose complement we say is A bar. So, the complement A bar is going to be obtained by operating a complement function.

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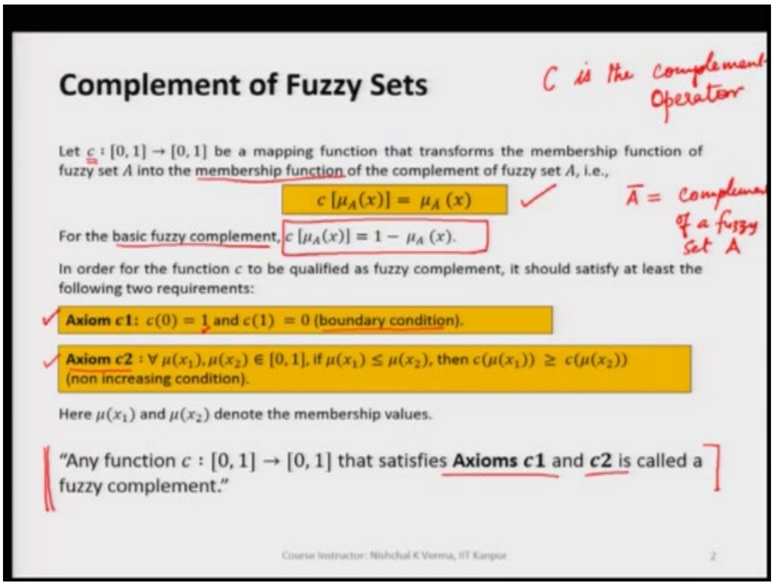

So, here the c, you can see here. The c is such that this is going to give us when it is operated on any membership value; we are going to get all those values in between or right from 0 to 1. So, what does this mean? This means that, if we operate the c which is a complement which is, c is a complement operator and if this is operated on any membership value and this membership value will be anywhere from 0 to 1 and this is going to give us again the value which is from 0 to 1 somewhere in between. So, we can say let c which is a complement operator is such that interval 0 to 1 is returning us the value in between 0 to 1. So, this be a mapping function that transforms the membership function of a fuzzy set A into the membership function of a complement of fuzzy set.

So, here we have written membership function. So, this is applicable when we are dealing with a continuous fuzzy set. But if we are dealing with the discrete fuzzy set, then we will use the membership values or membership grades. So, we can see here that if we apply the complement operator c. So, let me right here c is the complement operator. So, c is a complement operator, when this c is applied to the membership value or membership function in case of continuous fuzzy set, it is returning us $\mu_{\bar{A}}(x)$. This means that this going to give us the membership value corresponding to the complement of fuzzy set and this complement of fuzzy set is denoted as $\bar{A}$.

So, $\bar{A}$ is nothing but I will I will write it here. $\bar{A}$ is nothing but it is the complement of fuzzy set A. And what we have studied so far for the complement is the basic complement of a fuzzy set is here like if we have membership function or membership value and when we want to have the complement for finding the complement of a fuzzy set, we are going to simply subtract all these membership values, membership function from 1 to get the membership values or functions of the complement of fuzzy set A.

So, this is simply denoted by I will just make a box here so that, this can be clearly understood as to what is the basic fuzzy complement. So, but apart from basic complement we have so many other complement operators. So, in order for the function c that is the fuzzy complement to be qualified as fuzzy complement, it should satisfy certain criteria, certain axioms. So, here we have two axioms. Axiom number 1, Axiom number 2, these are designated by $c1$ and $c2$. So, let us now look at $c1$.

So, c 1 says that the axiom c 1 says that if we apply the complement operators c on 0, we are going to get 1. We should be getting one and if we apply this complement operator to 1, where we are we should be getting 0 and this is also called as the Boundary condition. So, this is what is the axiom $c1$. Now, let us now look at the $c2$, axiom $c2$. So, axiom $c2$ says that if we have membership values μ_{x_1}, μ_{x_2} and of course, these x_1 and x_2 are from the universe of this course and μ_{x_1}, μ_{x_2} should belong to the range 0 to 1. So, if $\mu_{x_1} \leq \mu_{x_2}$, then the c of μ_{x_1} should be greater than or equal to c of μ_{x_2}. So, c here is the complement operator.

So, what does this mean? This means that a complement operator or I would say the c is some operator. So, if this c is following these two axioms $c1$, $c2$ then, we can say the c is a complement operator. So, c for c to be called as the complement operator, the c should satisfy axiom $c1$ and axiom $c2$. So, if these two axioms are satisfied, we can say the operator that we are taking as c here is a complement operator. So, in other words we can also say as its mentioned here that any function c, either we say function or operator both conveys the same meaning.

So, any function c such that range 0 to 1; this for the membership value. So, if it is applied to some membership function or membership value which is right from the 0 up to 1 is going to be again the resulting membership value for the complement fuzzy set right from 0 to 1. And of course, the function c should satisfy the axiom $c1$ and $c2$. So, this needs to be clearly understood that if we take any complement, if we take any function or any operator say that this is a complement operator, then we need to first check the axiom $c1$ and $c2$. And if these 2 axioms are satisfied, then this function or this operator will be regarded as the complement operator. And this is going to give us the membership values or membership function for the corresponding complement of the fuzzy set that has been taken.

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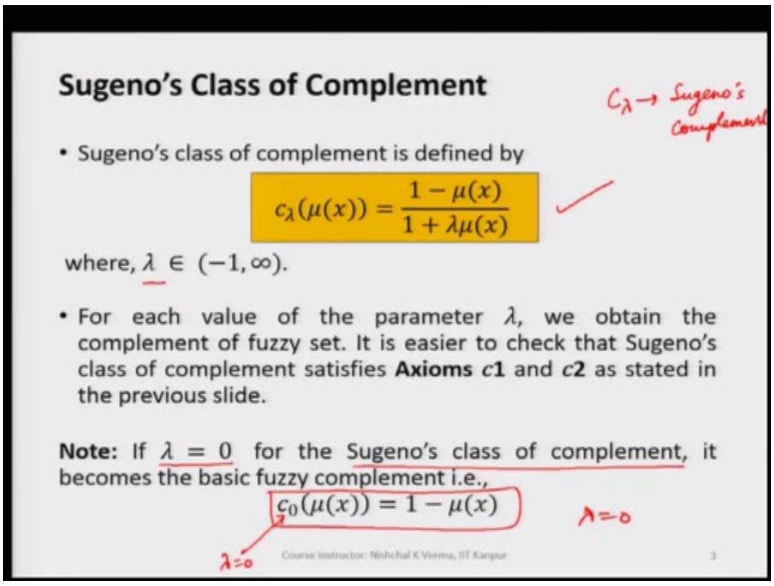

Then, we have some other complements. We have a Sugeno's class of complement here and which is written here which is mentioned as the $c_\lambda(\mu(x))$, which is nothing but is equal to $\frac{1-\mu(x)}{1+\lambda\mu(x)}$, where, c_λ here is the Sugeno's complement. So, c_λ is Sugeno's complement. So, Sugeno's class of complement is $c_\lambda(\mu(x)) = \frac{1-\mu(x)}{1+\lambda\mu(x)}$, where λ values will be in the range $-1\ to\ \infty$. So, this has to be understood. And please understand here that since this is a complement, so this Sugeno's class of complement is also satisfying the axioms $c1$ and $c2$ and that is how it is regarded as it is called as the Complement operator.

Now, if we take $\lambda = 0$. So, let us see what do we get. So, when we take $\lambda = 0$, the value of lambda 0, then the Sugeno's class of complement becomes the basic fuzzy complement. That is another interesting thing to note. So, here the lambda is equal to 0 means c_0. So, you can see 0 here, this is for lambda is equal to 0. This is this becomes c is $c_0\mu(x) = 1 - \mu(x)$. So, when lambda is 0, the Sugeno's class of complement gives us the basic complement, the basic fuzzy complement that we have studied earlier.

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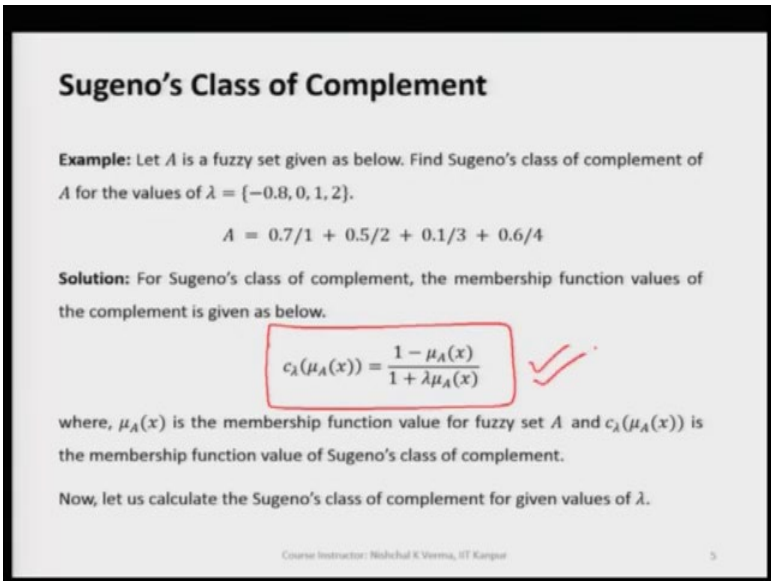

Let us take an example here to understand better the Sugeno's class of complement. So, here we have an example, where we have a fuzzy set, discrete fuzzy set A as $0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$. So, we have this fuzzy set and we need to find Sugeno's class of complement of this fuzzy set for the values which are listed here. So, values of lambda are minus 0.8, 0, 1 and 2. So, let us first write the Sugeno's class of complement here which is nothing but $c_\lambda(\mu_A(x)) = \frac{1-\mu_A(x)}{1+\lambda\mu_A(x)}$ which is here.

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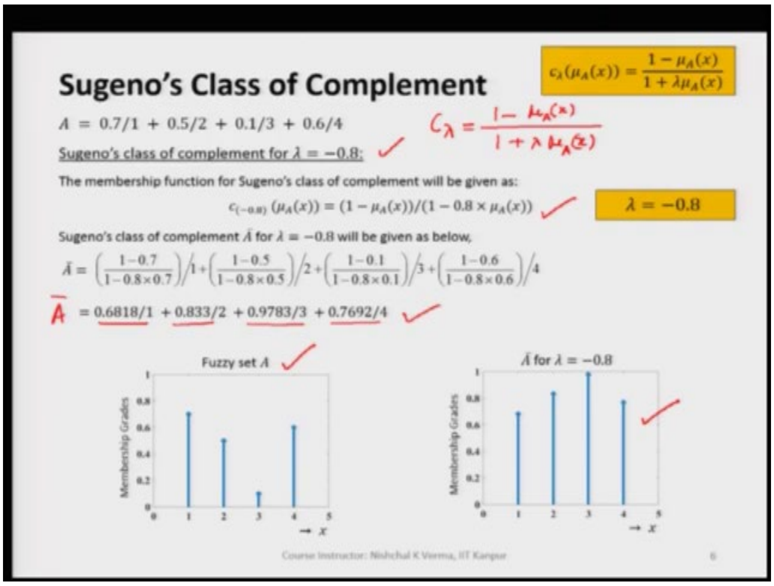

And let us now quickly take the value of lambda is equal to minus 0.8, which is here and substitute this in the expression. The expression of the $c_\lambda = 1 - \mu(x)$. If it is A then, we write here A and then, the we have here $1 + \lambda\mu_A(x)$. So, when we use this, we see that, we are getting for the value of lambda is equal to -0.8, we are getting A complement as $0.6818/1 + 0.833/2 + 0.9783/3 + \ 0.7692/4$.

So, let us now see what we getting? We have taken this fuzzy set A and here, we have plotted all these values means we have shown this discrete fuzzy set A and let us see what we are getting as the complement of this fuzzy set A for lambda is equal to minus 0.8. So, we are getting this fuzzy set here which is shown here as the complement of fuzzy set A that is A bar and this is for lambda is equal to -0.8.

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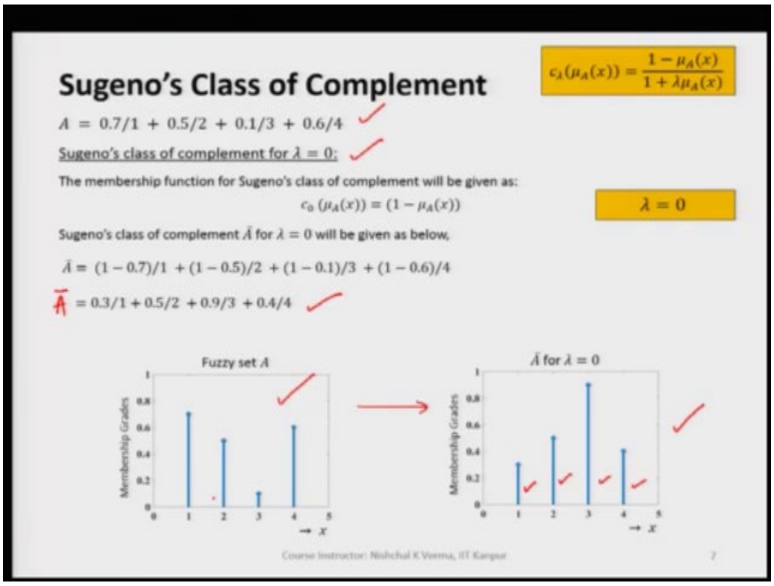

Now, let us take the another value of lambda, which is λ is equal to 0 and let us see what we are getting. So, here we have the fuzzy set A which was given to us and then here the fuzzy set A and then, we have $\bar{A}$ here, the complement of A here for λ is equal to 0 and we see that when we plot, we see that this is basic complement, we can clearly see that each and every membership value here of the complement fuzzy set, we are getting from A by subtracting with the respective membership values from 1.

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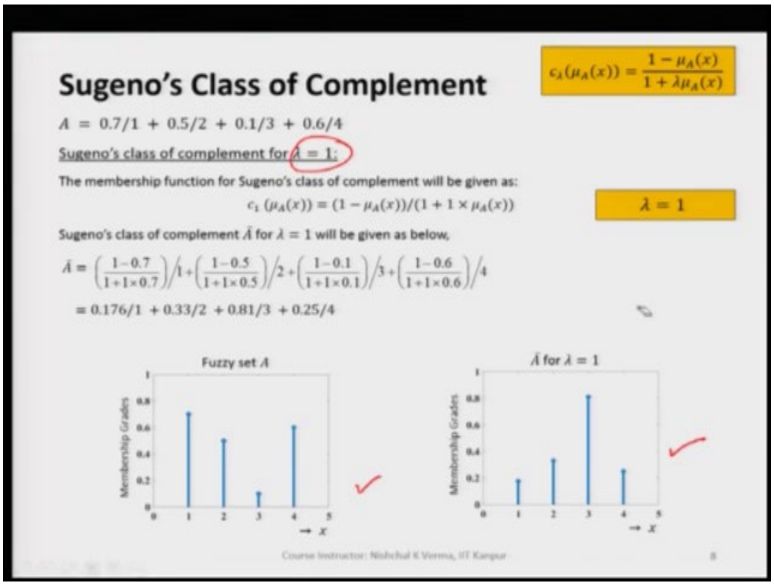

So, similarly when we take the λ is equal to 1, let us see what we are getting here. So, this is also shown here that $\bar{A}$ is the complement of fuzzy set A for λ is equal to 1. So, this is c_λ is the complement operator here. The Sugeno's complement for λ is equal to 1 in other way we can say and then here for λ is equal to 2. So, we are getting the complement Sugeno's complement for λ is equal to 2.

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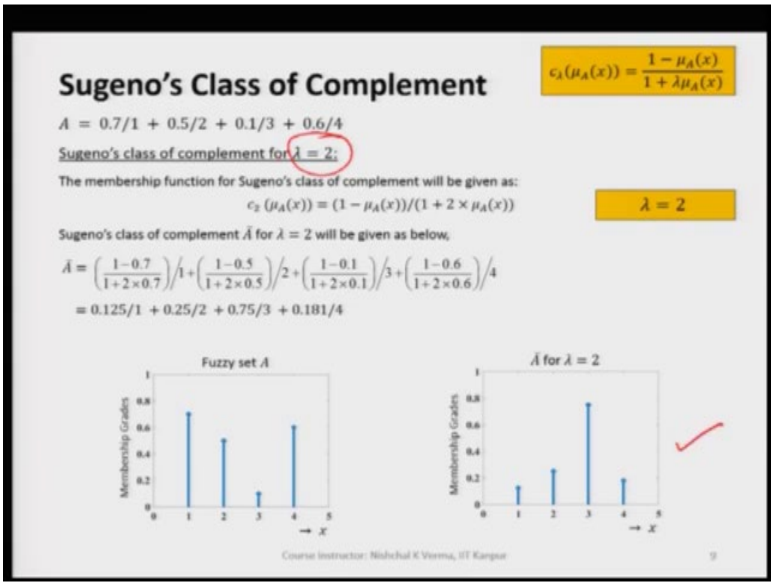

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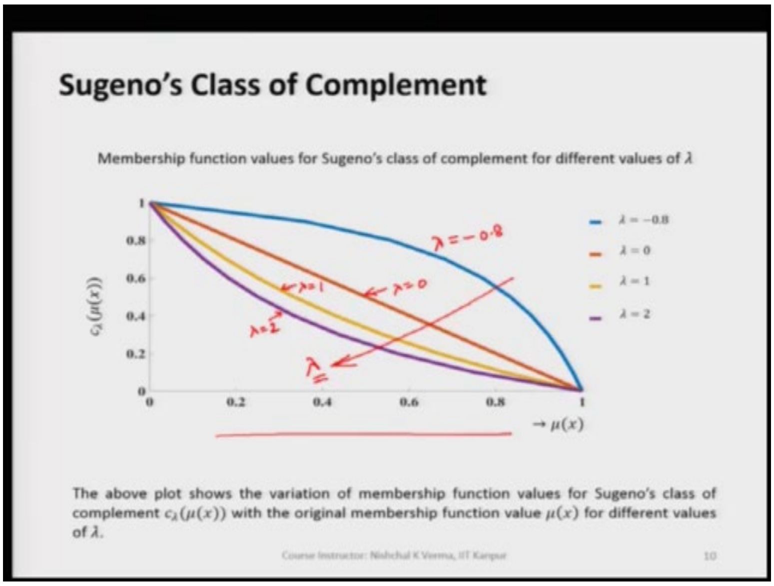

So, this way we are getting four different fuzzy sets as complements Sugeno's complement for different values of λ. So, let us now plot all these outcomes, all these complement fuzzy sets or the complement of fuzzy set A. And when we plot membership values here, when we plot the membership values as to how they are changing with the various values of λ is equal to minus 0.8, λ is equal to 0, λ is equal to 1, λ is equal to 2.

So, when we plot these membership values and its complement membership values we see that here we have this as λ is equal to -0.8 is and this is for λ is equal to 0 and this curve we are getting as the λ is equal to 1 and then, we are getting this as the λ is equal to 2. So, this means what? What we are trying to relate? So, we are trying to relate here like if we increase the value of lambda, when we increase the value of lambda, we are going to get the changes like this. We can clearly see here that the relationship is changing here the in between the membership values and its corresponding complement of membership values.

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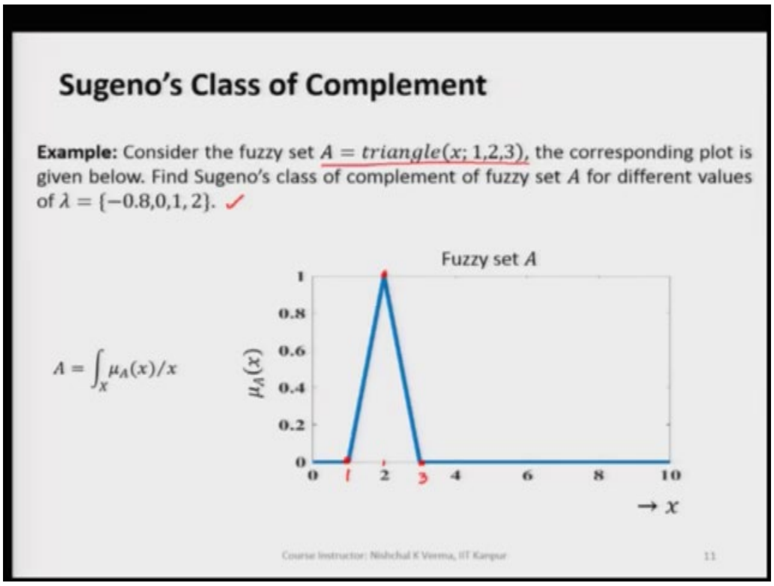

Now, let us take another example here. Earlier we took the example of discrete fuzzy set, now we take the example of continuous fuzzy set which is a triangular fuzzy set here whose vertices are at 1 and 2 and then 3; this is 3. So, now let us find the Sugeno's complement of this fuzzy set for lambda values and lambda values again are from this set here. And this means that lambda will take the value minus 0.8 and lambda is equal to can be again the λ is equal to 0, λ is equal to 1, λ is equal to 2. So, like that. Let us now quickly go ahead and see what we are getting when we take the λ is equal to -0.8 and what we are getting here as the complement of this fuzzy set Sugeno's complement of this fuzzy set.

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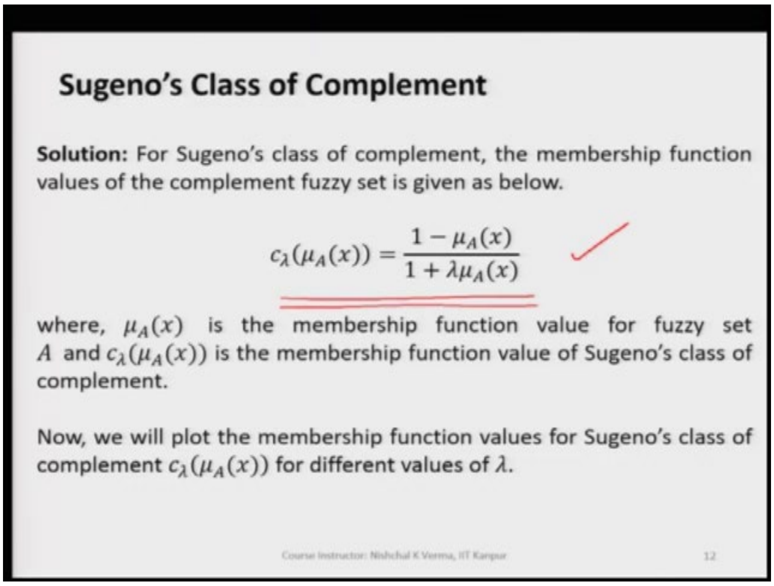

So, again, we have this expression for computing the Sugeno's complement of Sugeno's complement of membership function, membership values, because these membership values will be needing for expressing the complement of the fuzzy set A. So, that is how we use this expression, we use this formula for computing the membership values, membership function for the complement of the fuzzy set A.

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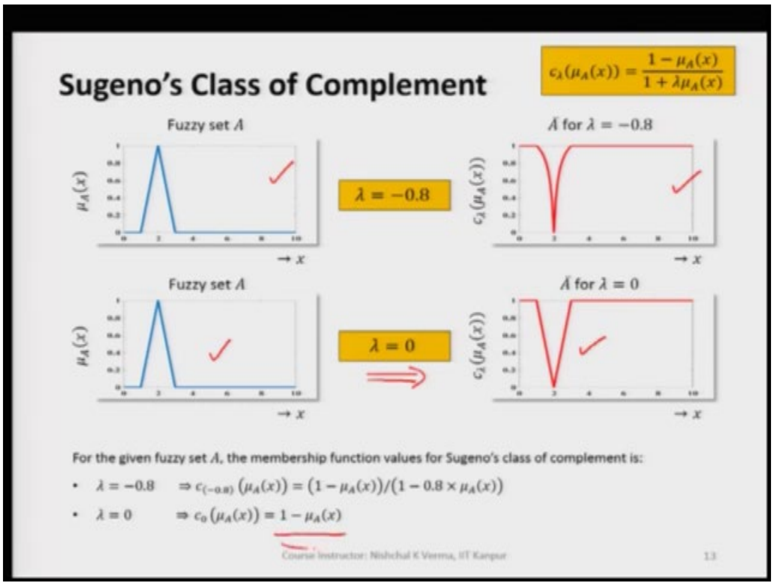

So, let us now quickly see as to how we are getting it. So, we have a fuzzy set A as given. This fuzzy set is a triangular fuzzy set and when we take λ is equal to -0.8, we are getting the Sugeno's complement you can see. And then, when we take the lambda is equal to 0, the complement of this fuzzy set which is expressed by the inverted triangular membership function.

So, here for lambda is equal to 0, we see that we are taking the basic complement. We are applying the basic complement, which is nothing but $1 - \mu(x)$ means the all the corresponding membership values we get by just subtracting from highest membership value that is 1. So, all the corresponding membership values of the complement of fuzzy set, we get by just subtracting the values of the fuzzy set we are taking from 1 which we can see here.

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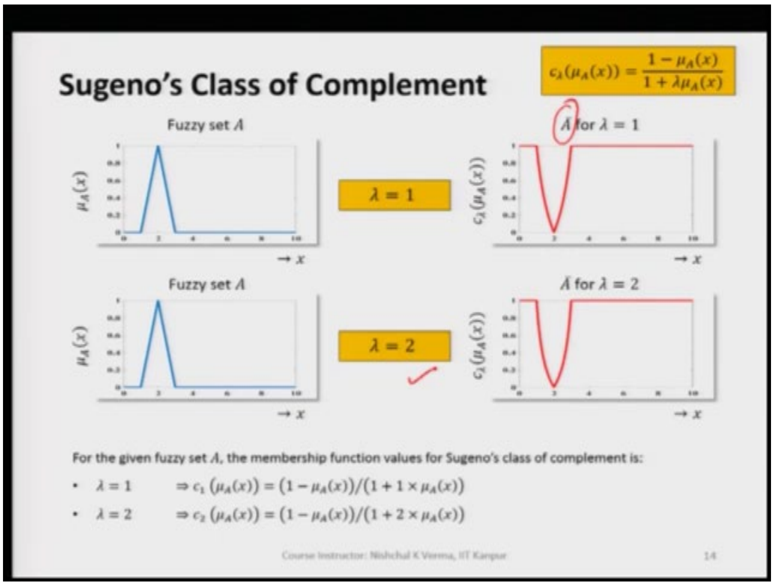

Now, similarly we can go ahead and we can use lambda is equal to 1 and we can get the membership values accordingly and then, we can plot the A complement by taking all the respective membership values or membership function here in case of a continuous because this is continuous. So, we can we can use the term membership function. Similarly, for lambda is equal to 2, we are getting A complement like this. So, we see that these the complement of the fuzzy set that we are getting for different values of lambda, we verify two axiom $c1$ and $c2$ for complement are satisfied.

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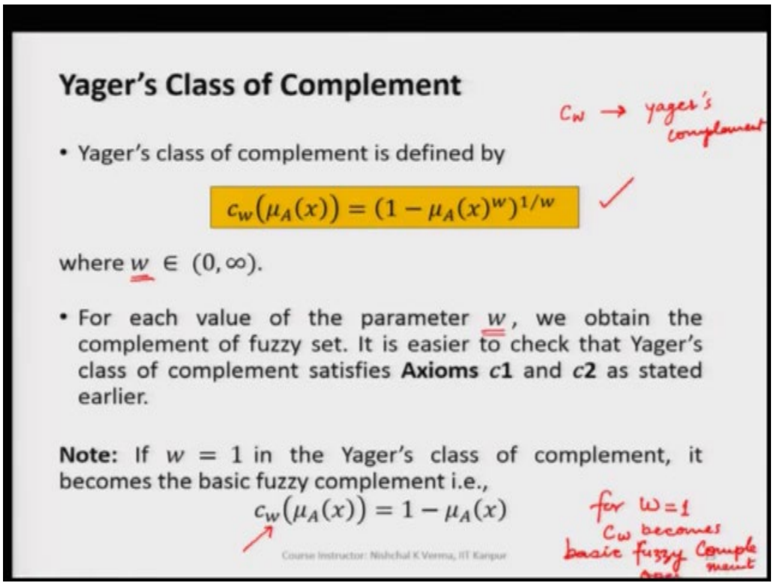

Now, here we have another class of complement which is Yager's class of complement. So, Yager's class of complement is defined by the first of all Yager's class of complement a complement operator here is c_w. So, this is called Yager's complement. Yager's complement and this is defined as $c_w \mu_A(x) = (1 - \mu_A(x)^w)^{1/w}$. Here, the w values can be anywhere in between $0\ to\ \infty$. So, or in other words, we can say the w can take any value from $0\ to\ \infty$.

So, from for each value of the parameter w, we obtain the complement of fuzzy set and it is easier to check that Yager's class of complement satisfies the axioms $c1$ and $c2$ that we have just discussed. And here also if we take w is equal to 1, we are landing up in the basic fuzzy complement. So, we can see here if we take w is equal to 1, what we are getting here is the $c_w \mu_A(x) = 1 - \mu_A(x)$. So, c_w becomes here, c_w becomes the basic fuzzy complement operator. So, when it becomes basic complement operator, when you have w is equal to 1. So, for w is equal to 1. So, for w is equal to 1, c_w becomes basic fuzzy complement operator.

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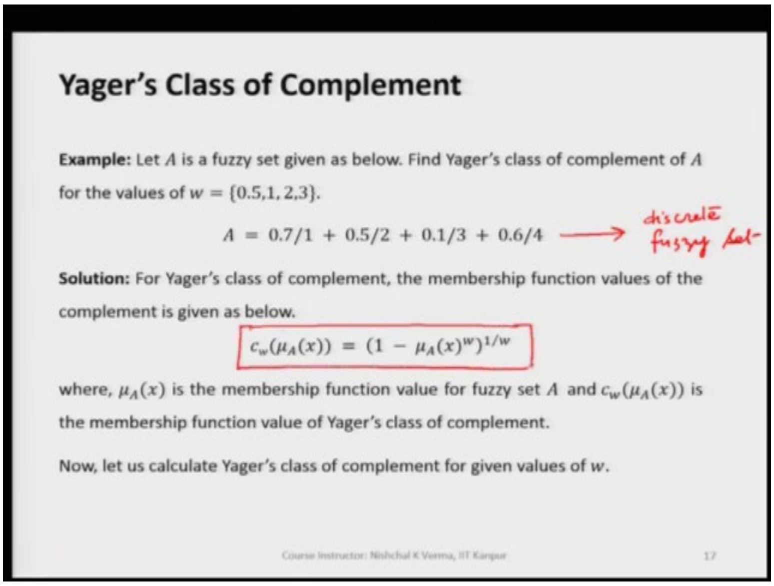

Now, let us understand Yager's class of complement better and let us take an example and see what is happening, how these complement of this fuzzy set A look like by going through different values of w's. So, we have the fuzzy set here. The discrete fuzzy set here again, so we have taken a discrete fuzzy set and we need to find its complement. So, as we are interested in Yager's complement, we have to use this, we have to use this formula here.

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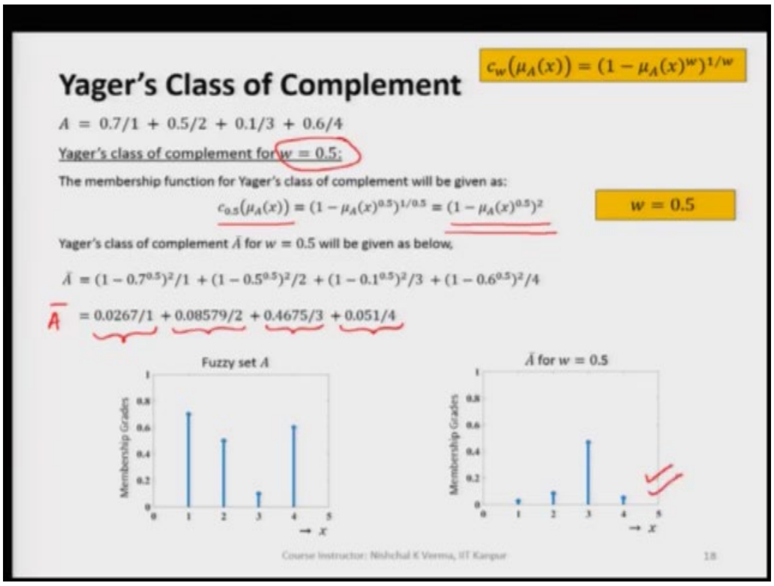

So, let us now apply this formula for w is equal to 0.5 which is here and when we apply this we get $c_{0.5} = (1 - \mu_A(x)^{0.5})^2$ which is here and this way when we compute this finally, we are going to get this value which is mentioned here that is $0.0267/1$. Similarly, other values other corresponding values of membership for w is equal to 0.5, we get here and then when we plot, we see that the complement of A for w is equal to 0.5 looks like this.

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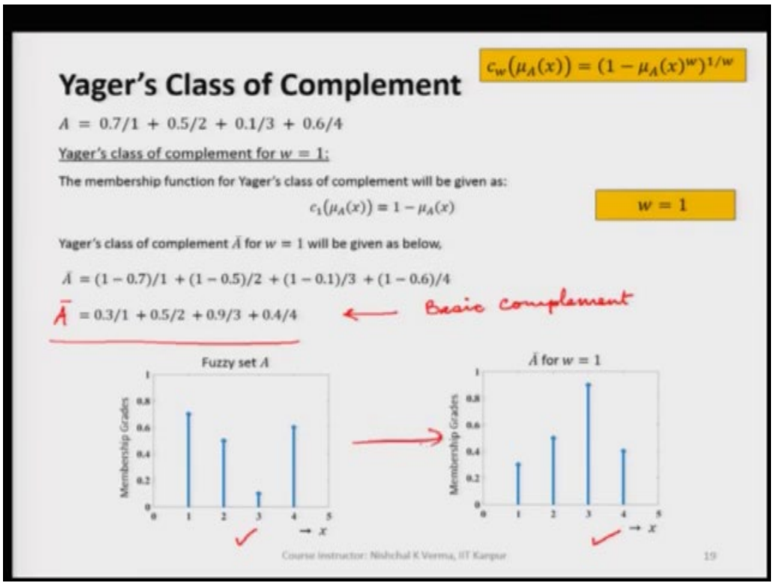

Next, when we take up the w is equal to 1. So, for w is equal to 1, we apply the same formula and the complement here complement of the fuzzy set A, we are getting as this. And since, we have taken here w is equal to 1, this is returning us the basic complement. So, this is after basic this basic complement. So, we can clearly see that these are the basic complement.

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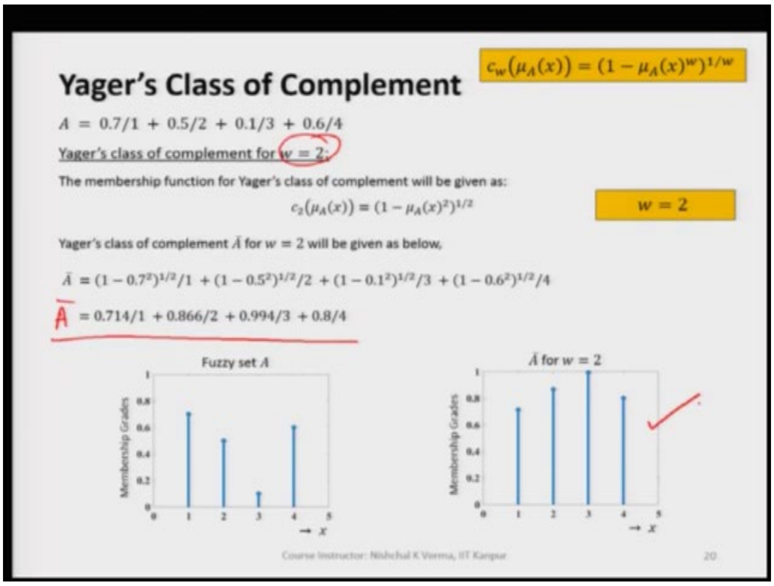

Now, when we take w is equal to 2 we are getting here A complement. Then when we plot, we are getting the $\bar{A}$ for w is equal to 2.

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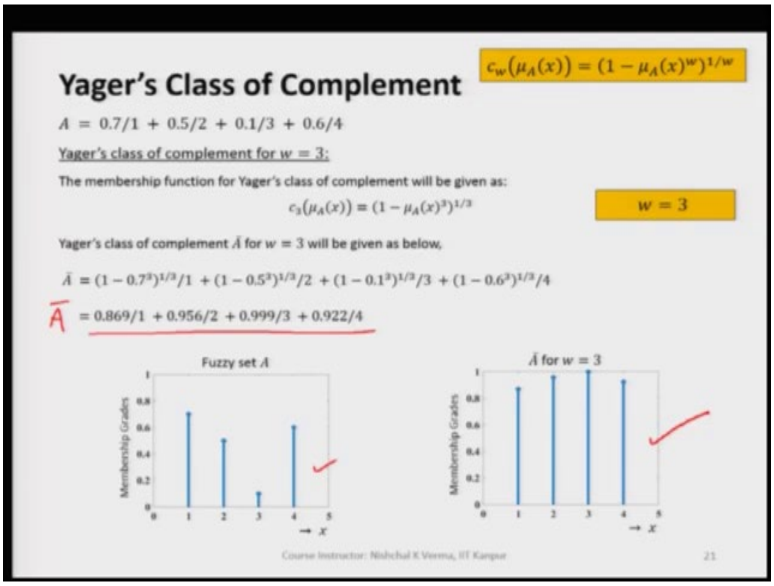

Similarly, for w is equal to 3, we are getting here A as the complement of fuzzy set A. So, $\bar{A}$; $\bar{A}$ is changed here.

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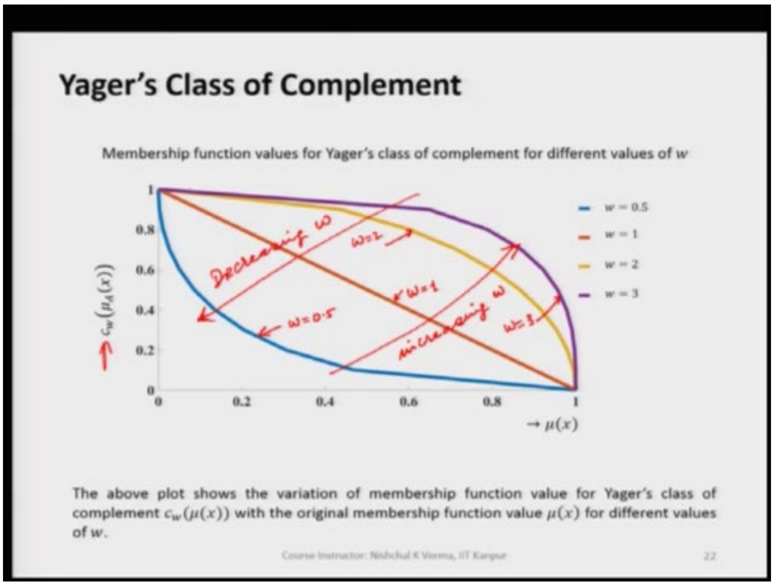

And again, when we plot when we try to plot the membership values of the set that we have taken and then, the membership values of the complement set, complement of the set A which is here, this is complement of the set A and yes, this has to be noted that this complement is Yager's class of complement or Yager's complement. So, when we do that we are when we plot, we are going to get this kind of relationship. So, this is for w is equal to this is for w is equal to 0.5 and then this for w is equal to 1 and this is for w is equal to 1, then this is for w is equal to 2 and this is for w is equal to 3.

So, what does this mean? This means that, when we are increasing the value values of w's, we are moving towards this and when we are this is increasing and I can say here that this is decreasing. So, this is how we are getting this kind of relationship in between the membership values, membership functions and its complement membership values and membership functions.

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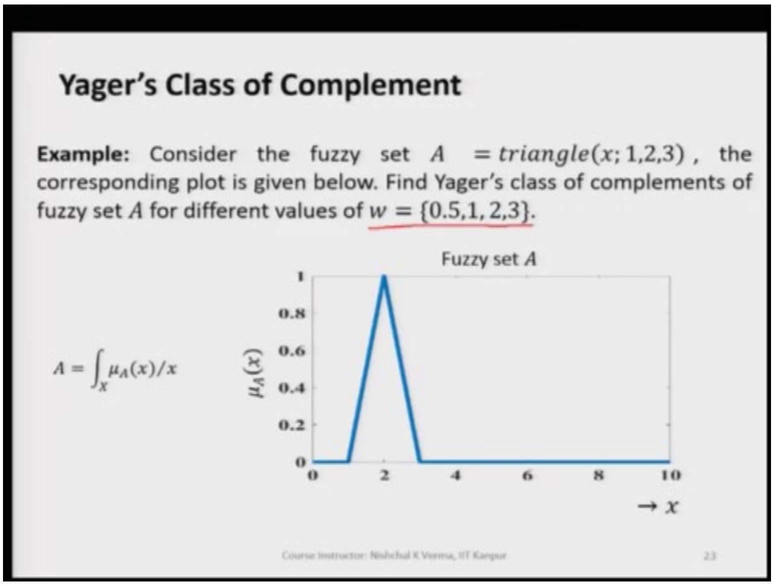

Let us take another example which is a continuous fuzzy set. Here we would like to find the Yager's class of complement on the continuous fuzzy set A. And we have here the w, set of w's that will be taking and let us see what we are getting, what we are getting as the output of the Yager's complement of the fuzzy set A.

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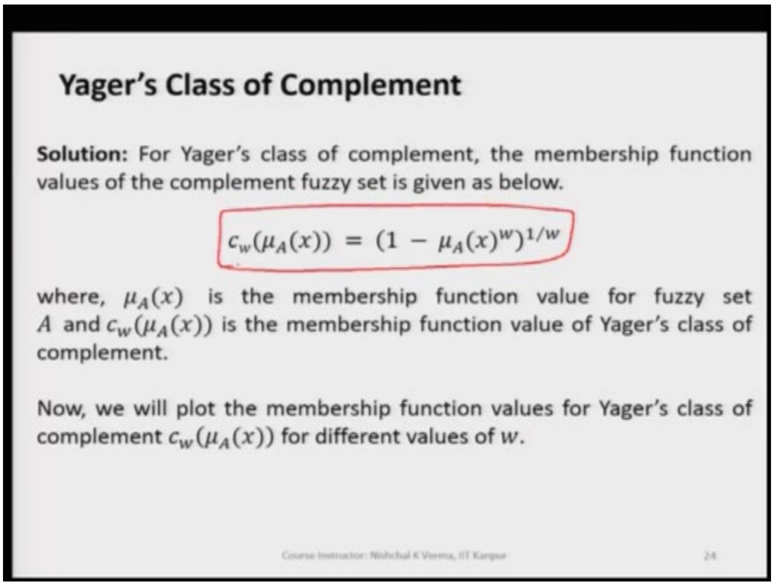

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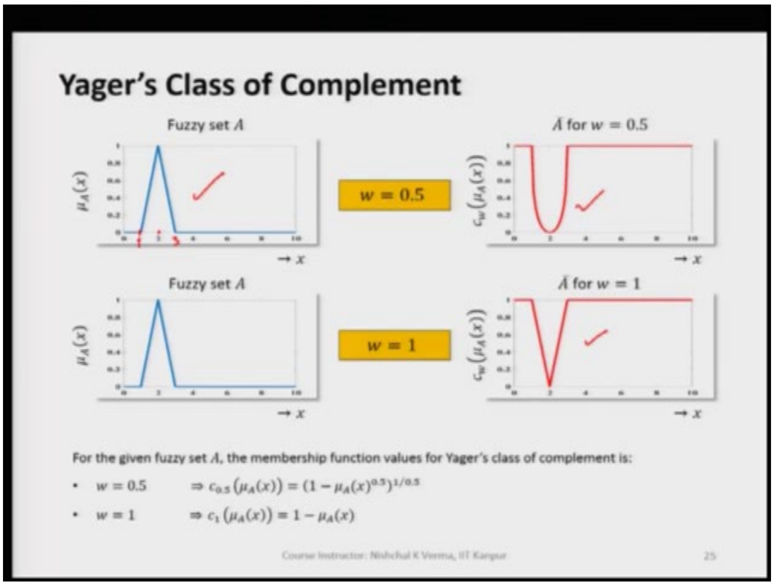

So, let us now apply Yager's complement and see what we are getting. So, when we have applied we first took the fuzzy set A's which is a continuous fuzzy set A, which is a triangular fuzzy set A whose vertices are at 1, 2 and 3. And then when we apply w is equal to 0.5, we are getting a complement fuzzy set of A, which is like this. Similarly, when we when we take w is equal to 1, we get completely inverted fuzzy set, inverted of the original fuzzy set and this is as I have already discussed that w is equal to 1 for Yager's class of complement is going to give us the basic complement. So, this is a basic complement basic complement of a fuzzy set that we have taken as A.

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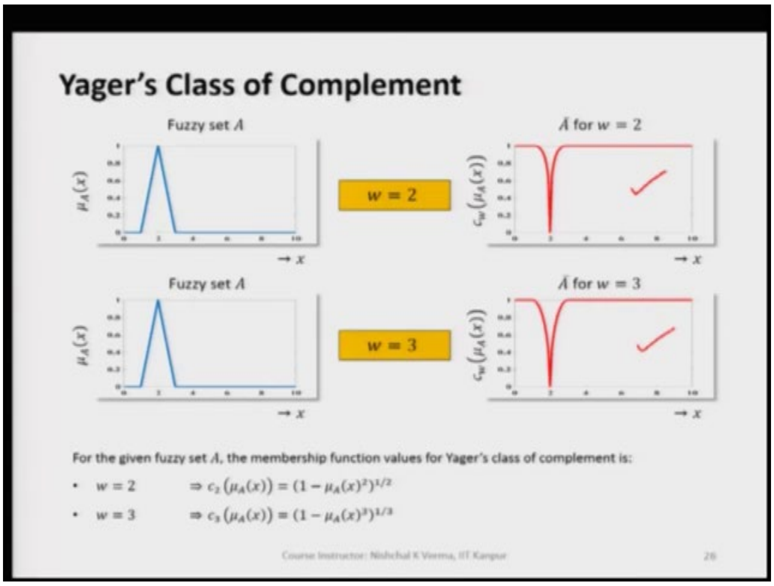

Next is for w is equal to 2, we are getting a bar like this the complement of fuzzy set like this, the Yager's complement of fuzzy like this. Here also we are getting Yager's complement of the fuzzy set A for w is equal to 3 and this way, we have seen that as to how we can manage to compute the Yager's class of the complement of the fuzzy set, any fuzzy set either continuous or discrete very easily.

So, this way we have seen Sugeno's class of complement and Yager's class of complement and we have also seen in Sugeno's class of complement for λ is equal to 0, we are getting this operators change to the basic fuzzy complement. And similarly, in Yager's class of complement, we have seen that for w is equal to 1, the Yager's complement changes to the basic fuzzy complement.

So now, with this we will stop here and in the next lecture, we will discuss t-norm operators on fuzzy sets.

Thank you.