Fuzzy Sets, Logic and Systems and Applications
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Lecture – 22
Arithmetic Operations on Fuzzy Numbers

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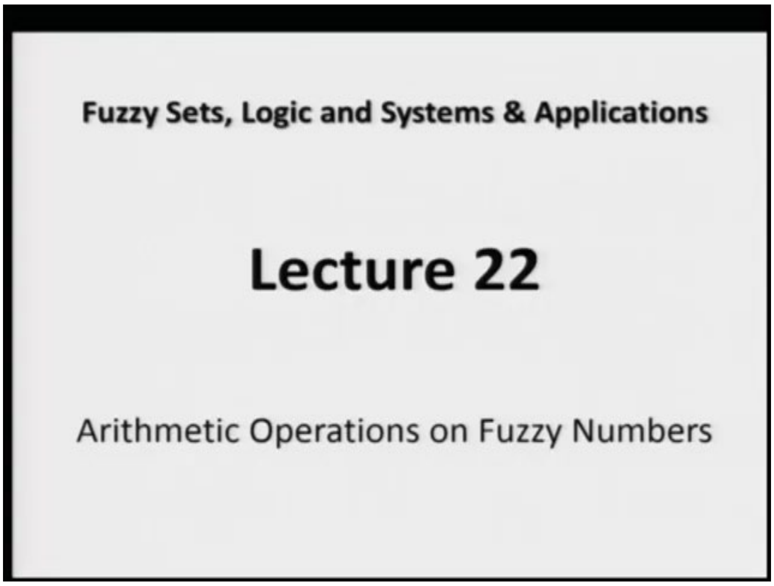

So, welcome to lecture number 22 of Fuzzy Sets Logic and Systems and Applications. So, this lecture is in continuation to our previous lectures, where we discussed Arithmetic Operations on Fuzzy Numbers. So, before we move ahead, we need to know that a fuzzy number is a fuzzy set which satisfies the condition of normality as well as the condition of convexity.

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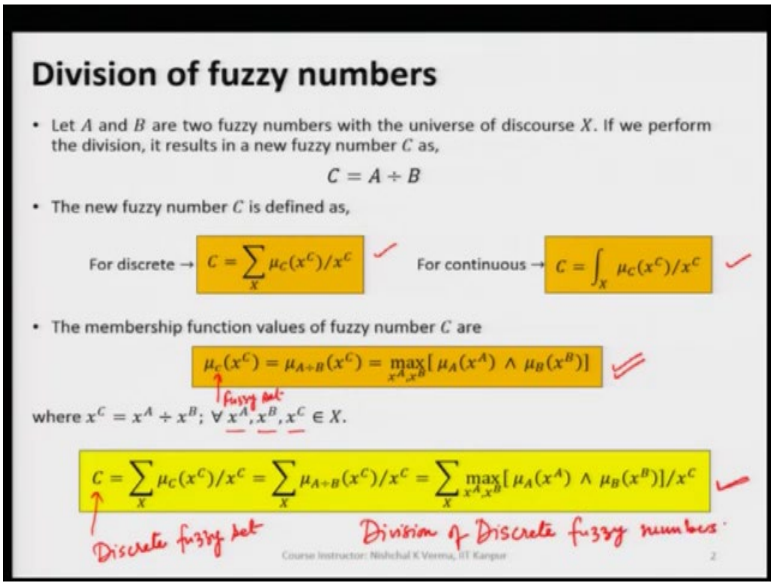

So, before this arithmetic operation of division, we had already discussed the other arithmetic operations like addition of fuzzy numbers, subtraction of fuzzy numbers, multiplication of fuzzy numbers. So, now, today here, we will be discussing the division of fuzzy numbers.

So, if we have two fuzzy numbers A and B within the universe of discourse capital X and if we are supposed to divide the fuzzy number A by fuzzy number B, let us see what happens. So, this is expressed by this formula here and this formula is for discrete fuzzy numbers, means when we have, when we are dealing with discrete fuzzy numbers, we use the formula for division here as

$$C = \sum_X \mu_C(x^C)/x^C$$

Similarly, if we are dealing with the continuous fuzzy numbers A and B instead of using the summation sign, we are using the integration sign.

And this is needless to say here that, the summation and integration both the signs are just symbolic representation; we are not supposed to add any of the elements or integrate the membership function here, so this needs to be understood. Now next is when we are dividing a fuzzy number A by another fuzzy number B, we are going to get a resultant which is again a fuzzy set. I am not saying here a fuzzy number, because the resultant is not going to be a fuzzy number because resultant fuzzy set is not going to be normal always and the convex. And since this is a fuzzy set, so we'll have its membership values and the corresponding generic variable values of the resultant fuzzy set.

So, let us first find the membership values and, this is represented by the μ_C because C here will be the resultant of $A \div B$. So, C has been designated here as the fuzzy set. So, the membership value of this fuzzy set corresponding to x^C is designated as $\mu_C(x^C) = \mu_{A \div B}(x^C) = \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$ where this x^C is nothing but it is $x^A \div x^B$. What are these x^A, x^B and x^C? These x^A, x^B, x^C's are the generic variable values coming from the fuzzy number A, B and fuzzy set C respectively and these x^A, x^B, x^C will be part of the universe of discourse.

So, we can finally write here as shown here in this equation in this expression, if we write together the membership value of fuzzy set C and the corresponding generic variable values. So, when we take discrete fuzzy numbers, we get the discreet fuzzy set here and A, B are the two discrete fuzzy numbers fuzzy numbers, whereas the μ_A, μ_B are the corresponding membership values. So, this expression

$$C = \sum_X \mu_C(x^C)/x^C = \sum_X \mu_{A \div B}(x^C)/x^C = \sum_X \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$$

So, this expression is for discreet or we can say division of discreet fuzzy numbers. We will replace this summation when we are taking the continuous fuzzy numbers instead of discrete fuzzy numbers, we will replace the summation sign by the integration sign. So, I hope now this is understood clearly.

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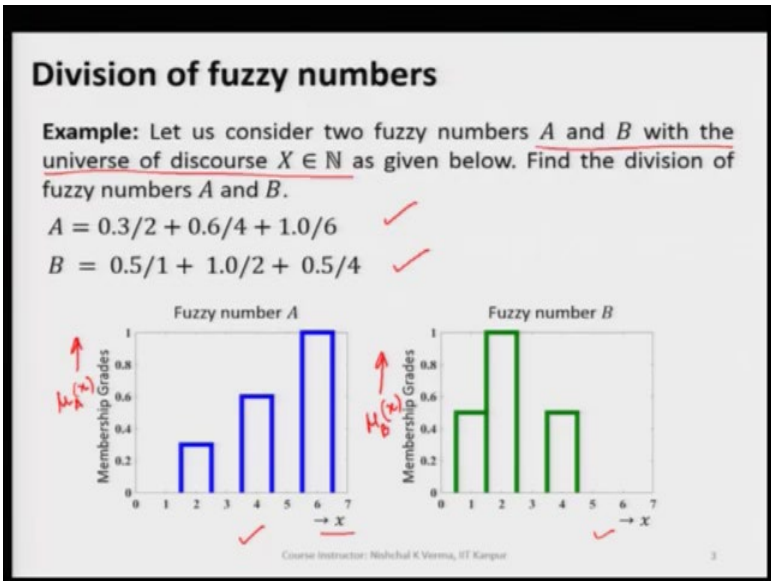

Now, let us take an example and understand the division of fuzzy numbers. So, here we are having two discrete fuzzy sets and of course these two fuzzy sets are qualified to be fuzzy numbers. So, we can say these two fuzzy numbers A and B are there and we are supposed to divide A fuzzy number by B fuzzy number. And it is given here that, A and B are within the universe of discourse X belonging to the natural number, is given.

So, let us first plot the fuzzy number A here and the fuzzy number B on the same generic variable value X here as the x axis and the membership grades here as the y axis. So, I can write here $\mu_A(x)$, this is $\mu_A(x)$; here we have $\mu_B(x)$. So, these two fuzzy numbers are shown and these two fuzzy numbers are discrete fuzzy numbers.

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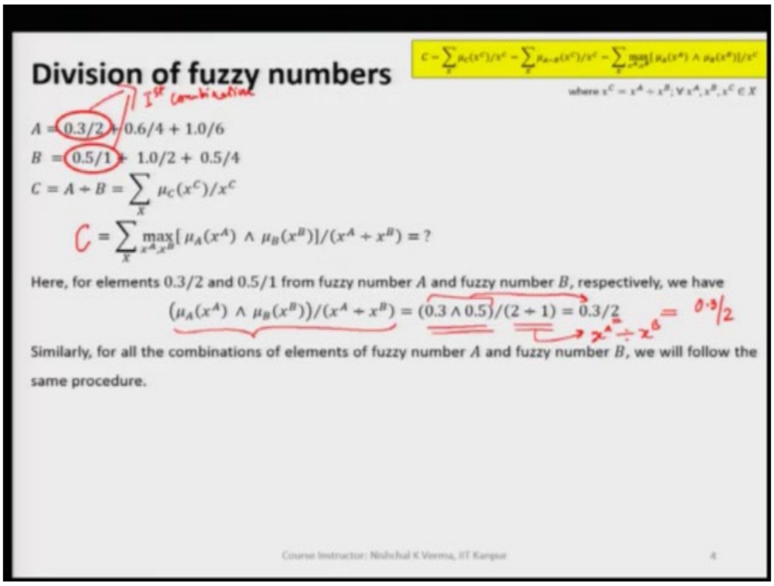

Let us see how we proceed for the division of A by B and this is going to result as another fuzzy set which is C. So, for this we need to take the first element from A and the first element from B fuzzy numbers, and let us take their corresponding membership values and the generic variable values and let us find the min of the membership values of the first combination that is the combination is here. So, this is the first combination.

So, likewise we will have so many combinations, we will have all the combination; means we will take all the elements of A and B, we will be combine them together and then we will process this, as we will see here in this couple of slides. So, when we substitute these values here, we find the $\min(0.3, 0.5)/2 \div 1$; this comes here 2 divided by 1, because we have $x^A \div x^B$. And x^A, x^B are nothing, but the generic variable values from the fuzzy numbers A and B respectively. When we are taking the min of this, we are getting 0.3 and then when we divide x^A by x^B, we are getting 2.

So, this way we are getting the one element of C fuzzy set as $0.3/\ 2$. Please note that, we are not supposed to divide here $0.3/\ 2$, so this is just the representation, we I have to keep this element as it is, where 0.3 of this element represents the membership value corresponding to the generic variable value of 2 for the resulting fuzzy set C. So, similarly let us go ahead and take up all the combinations here.

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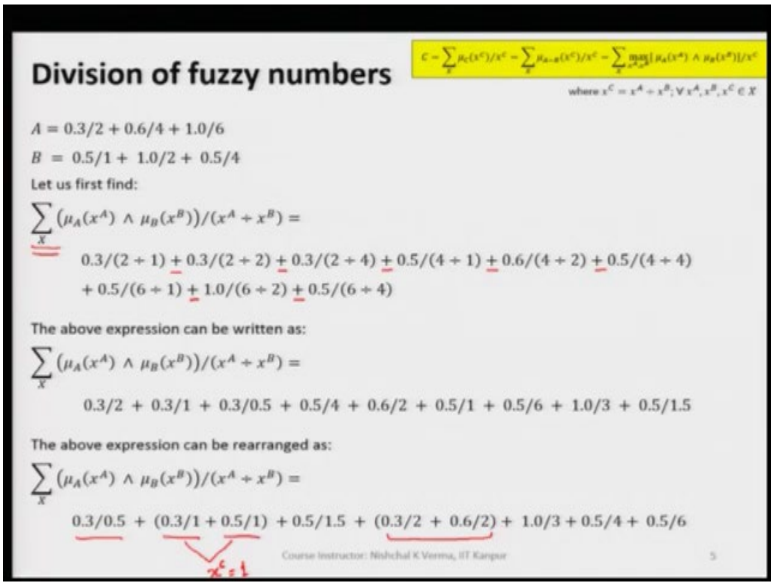

We see all the combinations because summation is here, so we are taking all the combinations separated by the plus signs, we see here all these plus signs are there, again we need not add these values together this is just for the representation purpose. So, these values have been shown here and now let us rearrange these values and we find that we have some elements, we have same generic variable values, like for these two elements, we have the same generic variable, means the x^C here is one for both the terms.

So, for such cases what we do here is, we avoid the conflict and by taking the max of the membership values. So, we will take max in the next step, so but what we need to understand that all such elements need to be written together.

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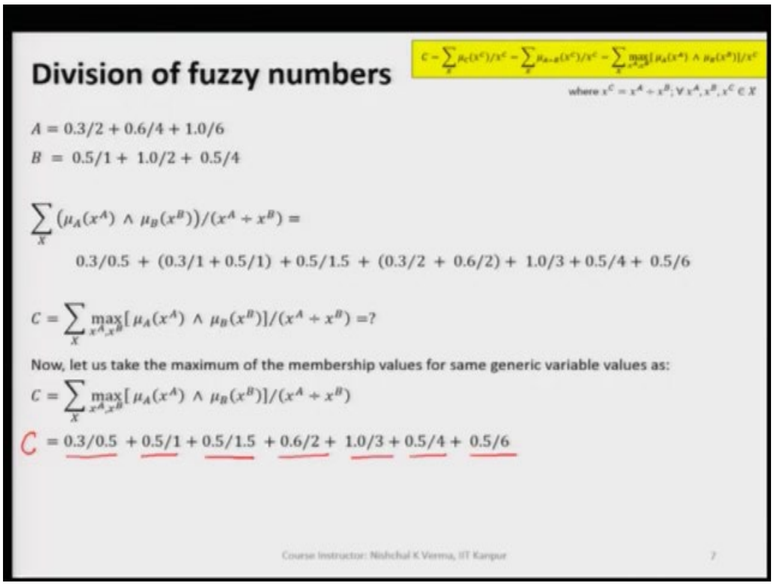

So, we have written here and then we see that here, we see that; when we apply the max criteria, we are getting $C = 0.3/0.5 + 0.5/1 + 0.5/1.5 + 0.6/2 + 1.0/3 + 0.5/4 + 0.5/6$. So, here we see that, we have as a result of fuzzy number A divided by a fuzzy number B, we are getting another fuzzy set C. So, here I am saying a fuzzy set C, because this fuzzy set need not be a normal fuzzy set and a convex fuzzy set always.

This can be possible, but it may not be necessary that you are always getting a fuzzy number as a result of the division of one fuzzy number by the another fuzzy number.

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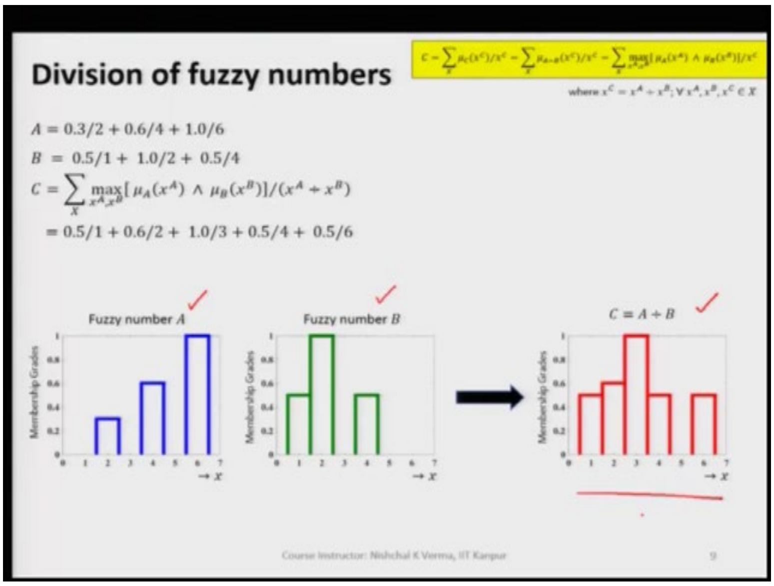

So, here when we see the fuzzy set which has been plotted. So, we see that we have a fuzzy number here A and B that was given to us as a result of fuzzy number A divided by fuzzy number B, we got another fuzzy set C and we can check if this satisfies the criteria of normality and convexity. And if it this satisfies, we can say we are getting a fuzzy number C; but I as I said as I mentioned that, this fuzzy set C which is a result of the division of fuzzy A and B, we do not always get the fuzzy number. So, that is why we need to check before we say that C is a fuzzy number.

So, here in this case it looks like it is a fuzzy number. So, C is a fuzzy number, because you see this is a normal fuzzy set and then we can check with convexity and if it this follows the convexity, we can say C fuzzy set is a fuzzy number.

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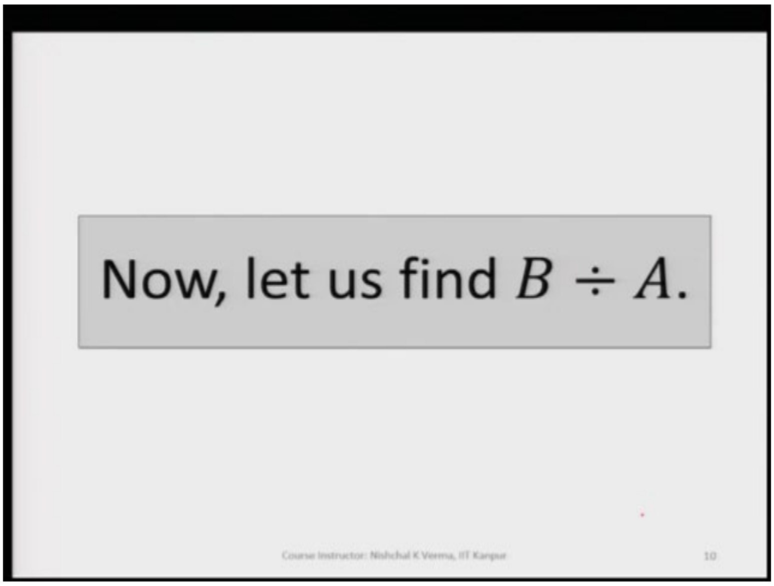

So, now let us find the $B \div A$. So, let us see whether we are getting the same result as we have gotten for the fuzzy number A divided by fuzzy number; of course we will not, because in crisp number also when we do like this, we are not getting the same. So, here also let us now divide fuzzy number B by another fuzzy number A and see what we are getting.

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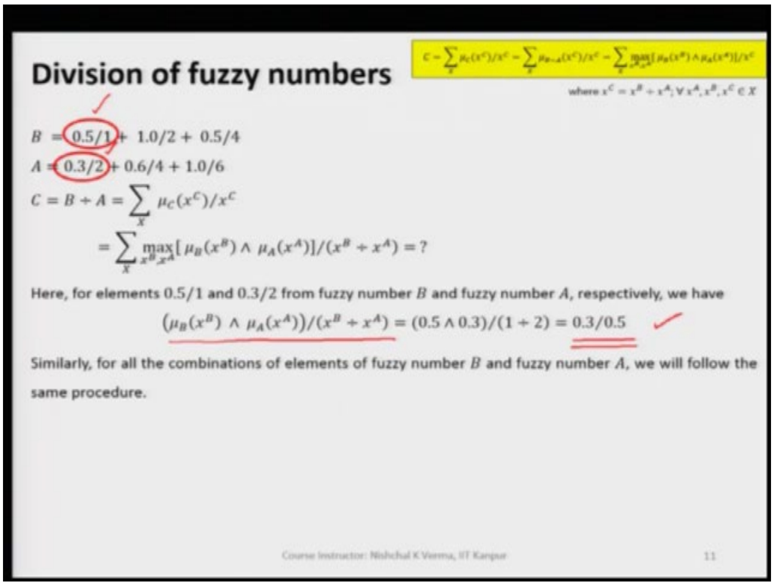

So, if we do the same exercise here, we get the first element of the resultant fuzzy set as $0.3/0.5$. And we know how are we getting this; we are taking the first element of the fuzzy number B and we take first element of fuzzy number A and with this combination, we get all $\mu_B(x^B)$ and $\mu_A(x^A)$ and their corresponding generic variable values. And this way we are getting $0.3/0.5$ as the first element or I would say one of the elements of the resultant fuzzy set C.

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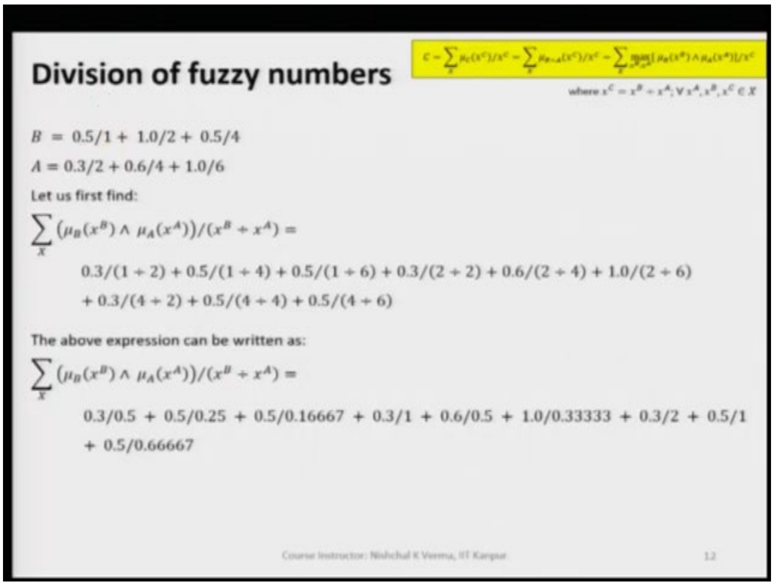

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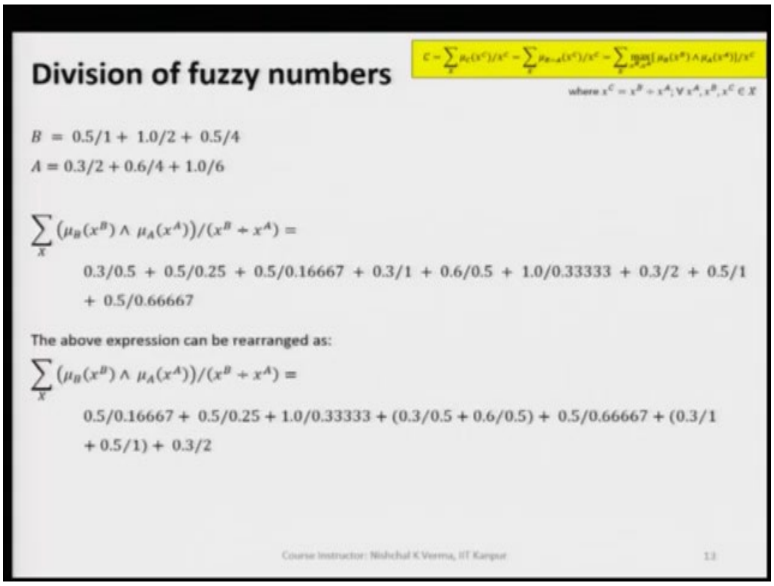

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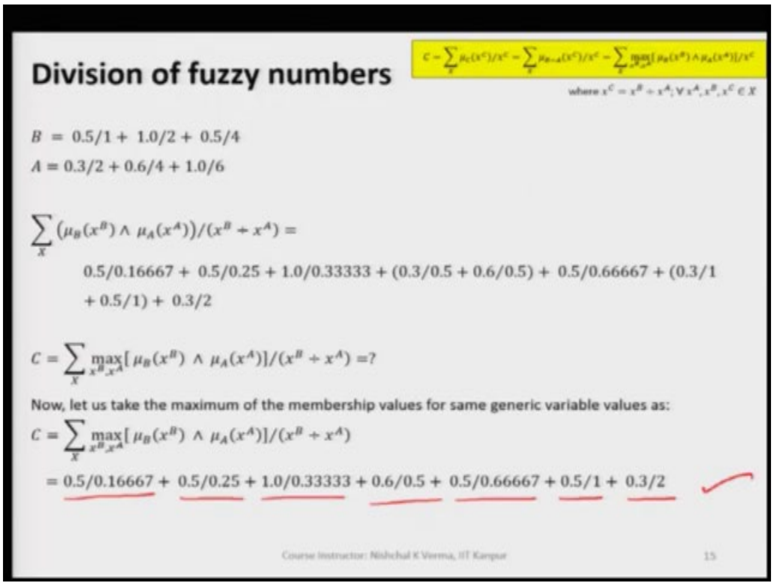

So, likewise we find all such elements with the same exercise and we see that, finally we are getting after applying the max criteria to avoid the conflicts for the same generic variable values. So, we see that we are getting $0.5/0.16667$.

Similarly, $0.5/0.25$, then we get $1.0/0.333$ here and then we get $0.6/0.5$, then we get here $0.5/0.6667$, here we get another term which is $0.5/1$ and then we get $0.3/2$. So, we have got here this out of the division of fuzzy number B by fuzzy number A.

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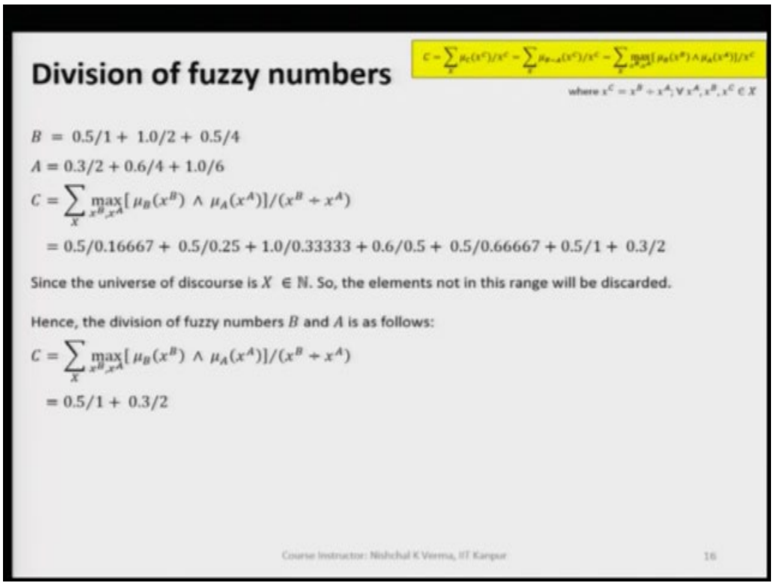

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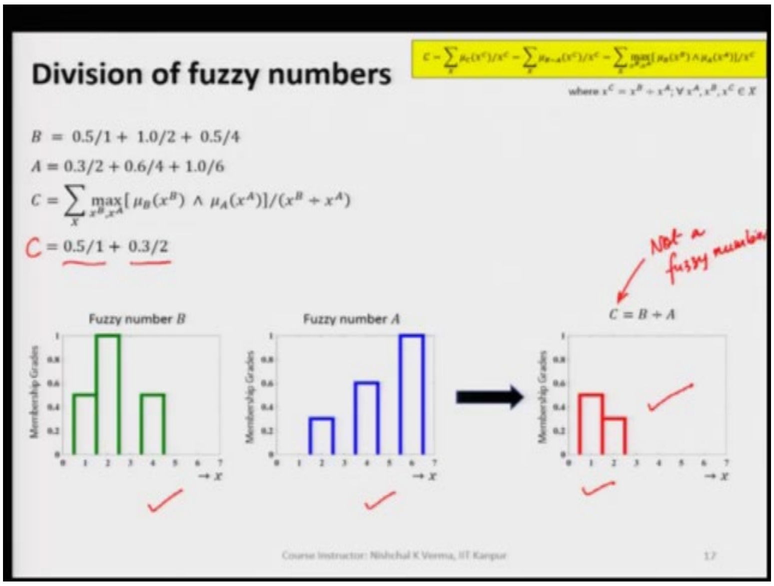

So, now let us quickly go ahead and plot these fuzzy numbers. So, we had taken the fuzzy number B first and then fuzzy number A and then when we divide it the fuzzy number C which is the result of the fuzzy number B divided by fuzzy number A, so we are getting this as a plot.

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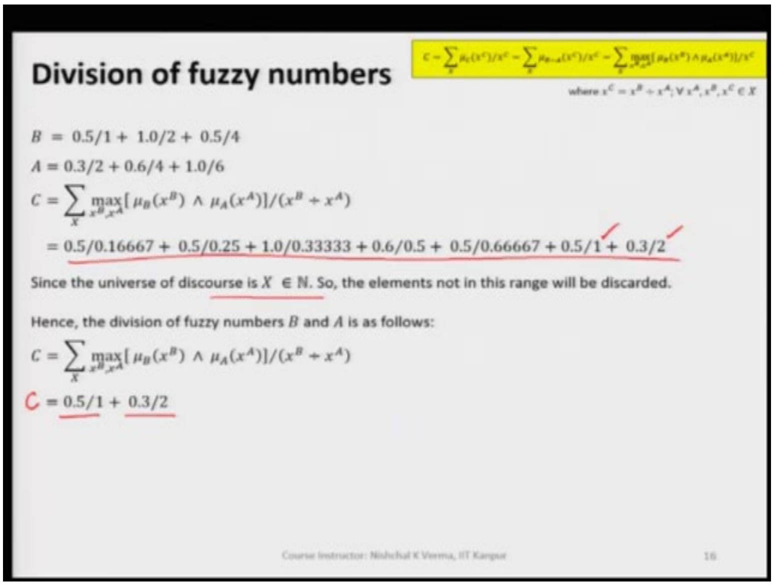

But you see why are we getting this plot here? Here we had so many values as we have seen here as I mentioned. So, please now let us look at the universe of discourse. So, the universe of discourse that is given to us here in the, this example is the natural number. So, we see that we have only out of these elements, we have only two natural numbers here as the generic variable values, rest others do not qualify. So, that is how we discard other elements and only we are keeping here $0.5/1$ and $0.3/2$ as the elements of the resultant fuzzy set

So, let us now quickly see that, here we have only two elements of C, we are dealing with the universe of discourse which is natural number here. So, that's how other elements do not qualify and then we are only left with only two elements. And when we have plotted this we see that, we have getting this as a result which is not a normal discrete fuzzy set. So, we can say that, the C is not a fuzzy number here. So, this is not a fuzzy number, because the normality condition is not satisfied.

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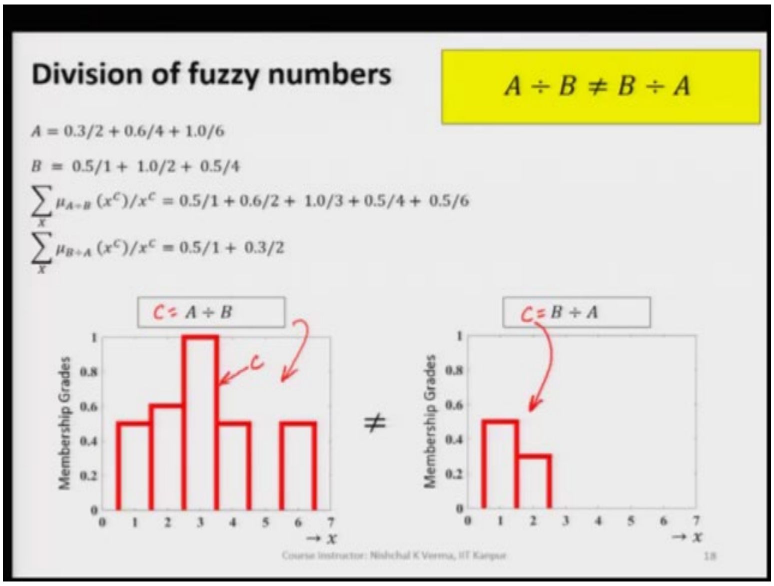

So, this way we have been able to know that, the fuzzy set that is we see that is we are going to get out of the division is not always a fuzzy number.

And another point that I would like to make here is, if you recall what we have gotten when we divided fuzzy number A by fuzzy number B, we have got a fuzzy number C which is shown here. So, this is fuzzy number C, this is a full C fuzzy set and here in this case when we have divided fuzzy number B by fuzzy number A, we have gotten C here like this. So, we can clearly say that, $A \div B \neq B \div A$ when we have A and B as fuzzy numbers and the same is true for the crisp sets as well.

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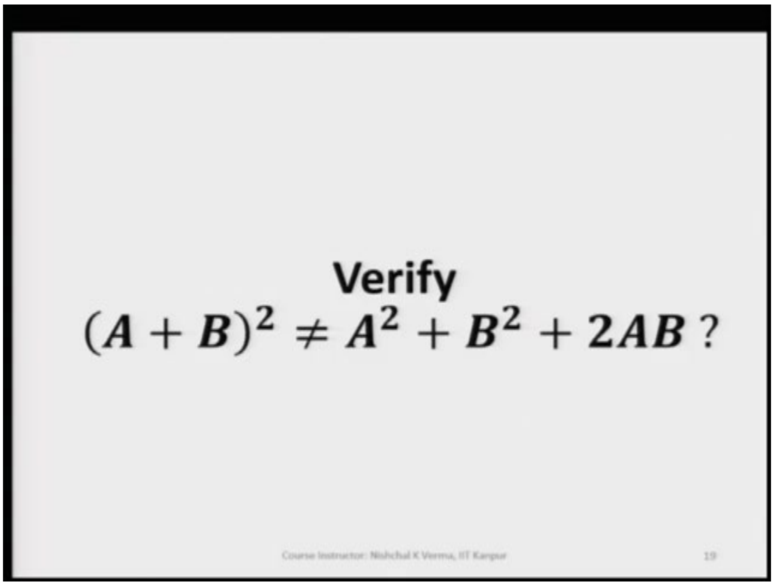

Now, by now we have finished all the arithmetic operations, the addition of two fuzzy numbers, subtraction of two fuzzy numbers, multiplication of two fuzzy numbers, division of two fuzzy numbers. So, I would like to take up this example here, because by now we have understood as to how we undertake the arithmetic operations on fuzzy numbers.

So, let us take this example, where we have $(A+B)^2 \neq A^2 + B^2 + 2AB$, as we have this true for crisp numbers. So, when we deal with crisp numbers, $(A+B)^2 = A^2 + B^2 + 2AB$, but here we will see, when we take fuzzy numbers, this is not going to be the true.

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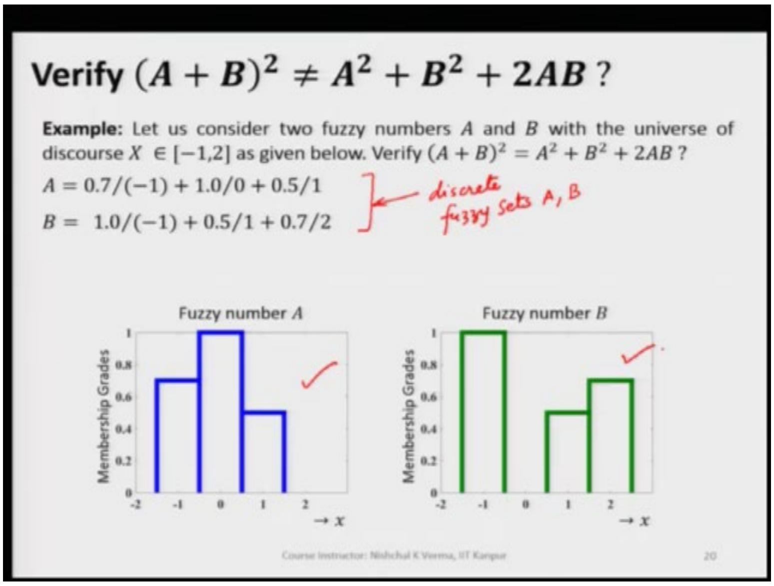

So, let us now first try to get $(A+B)^2$. So, here we have taken discrete fuzzy set here. So, we have two discrete fuzzy sets A and B, A and B and these fuzzy sets are fuzzy numbers as you can see. So, these two are the fuzzy numbers and let us now find the addition of these two fuzzy numbers.

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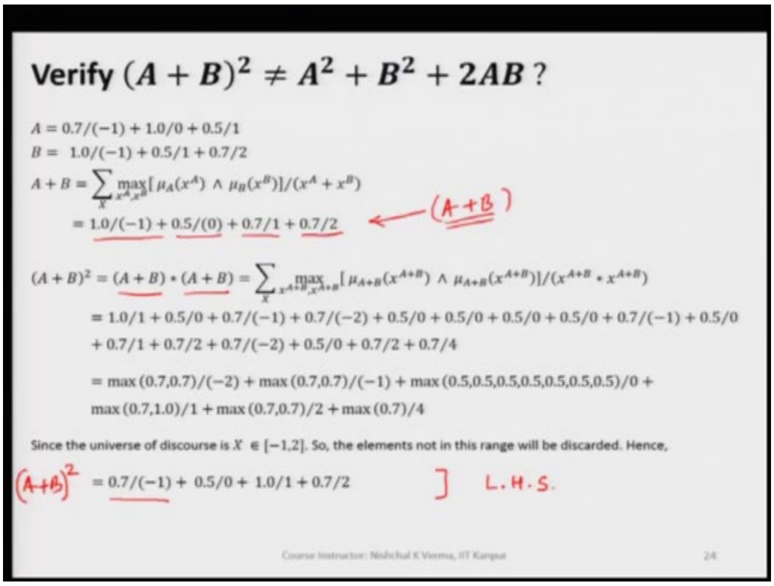

So, first what we are going to get here as $A+B$ is here, I am not going to explain each and every step of this addition, because we have already done this in detail. And here I am quickly going to show you the $A+B$ which is here, you can see. So, when we do the addition; when we add $A+B$, we are going to get this as the result. So, this is our $A+B$. So, when we have $A+B$, now we need to get the $(A+B)^2$. So, let us now multiply $A+B$ by $A+B$ and see what is happening. So, when we do that here finally we are getting as a result $0.7/(-1) + 0.5/0 + 1.0/1 + 0.7/2$.

So, this way you see here that, we are able to find $(A+B)^2$, when A and B are the fuzzy numbers.

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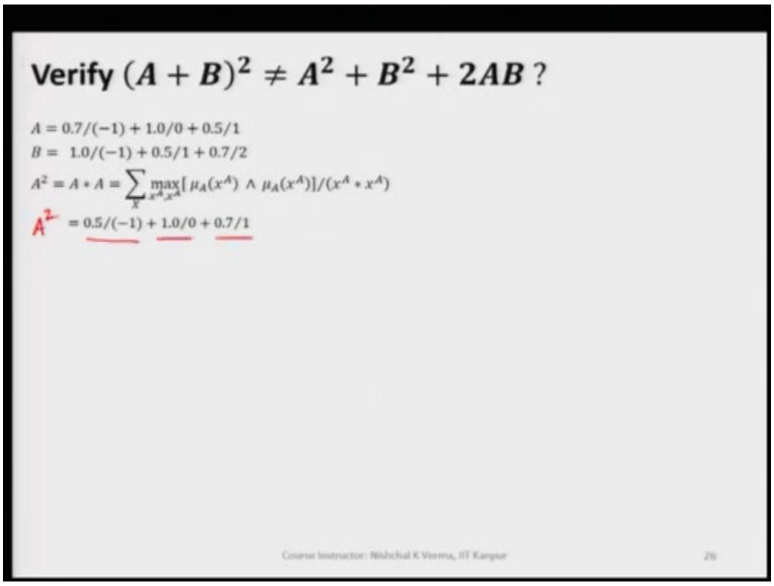

Now, let us go to the right hand side of the expression. So, earlier was the LHS, the left hand side of the expression. So, this was the LHS, I will write here LHS.

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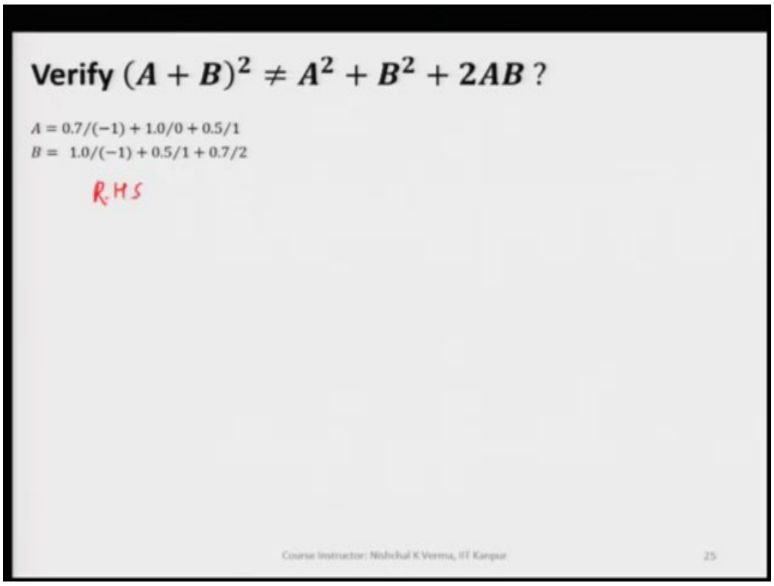

So, let us work for the RHS, the right hand side of the expression. So, RHS is here, in RHS we have the first term as A^2. So, A^2 is here, A multiplied by A. Once again, I would like to tell you that this A is a fuzzy number.

So, A multiplied by A is resulting as $0.5/(-1) + 1.0/0 + 0.7/1$. So, this is how we are getting these three terms as a square.

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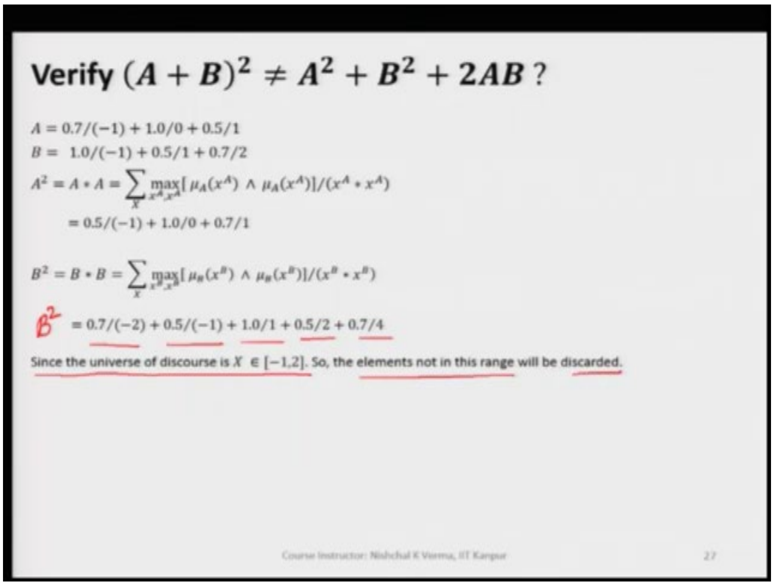

Similarly, we can find here the B^2 and as B^2 we are getting here, $0.7/(-2) + 0.5/(-1) + 1.0/1 + 0.5/2 + 0.7/4$; but understand here we need to know here that the given universe of discourse here is $-1,2$, means all the points that we will be dealing with should be right from -1 to up to 2.

So, this is an important point that we should always check the universe of discourse and all the results, all the generic variable values that are that we are using must be lying within the universe of discourse. So, when we see A^2 and B^2, so in A^2 we are fine, but in B^2 we see that, there are some elements, there are some terms that are not within the universe of discourse, means there are some terms whose generic variable values are not lying within the universe of discourse. So, now, what we do here is that, such elements we will be discarding.

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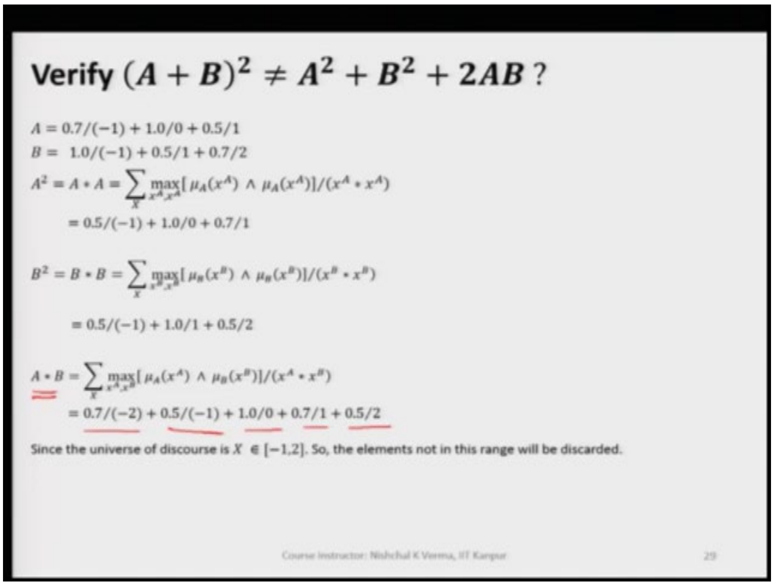

So, let us now rewrite the resulting fuzzy set B^2 and here we have only $0.5/(-1) + 1/1 + 0.5/2$.

Now, let us go ahead and find the AB? AB is nothing, but the fuzzy number A multiplied by fuzzy number B. So, when we do that here, we find the resulting fuzzy set as this and again when we apply the criteria of universe of discourse that all the generic variable values must be from the or must be lying within the universe of discourse.

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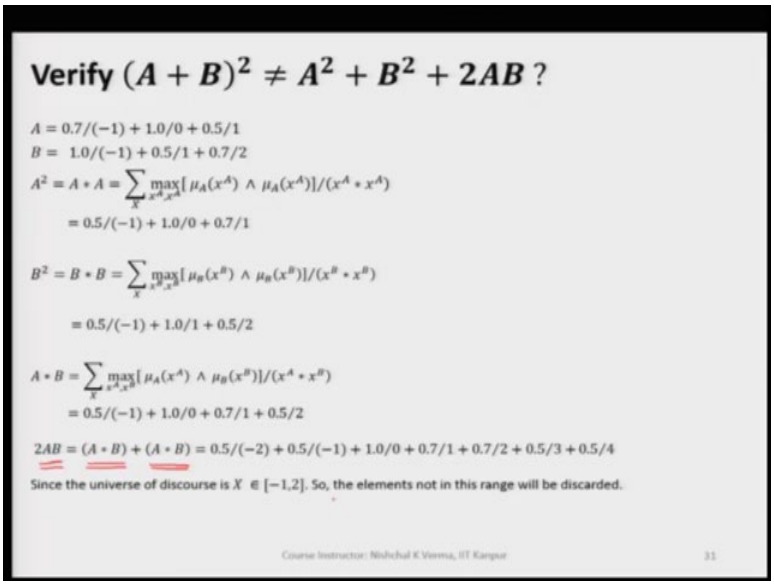

So, when we apply this, we see that we are getting the A multiplied by B has $0.5/(-1) + 1/0 + 0.7/1 + 0.5\ /2$. Now we have to find the twice of this thing. So, for twice of this thing, we will add these two fuzzy sets that we have just gotten as A into B. So, A into B plus A into B see here.

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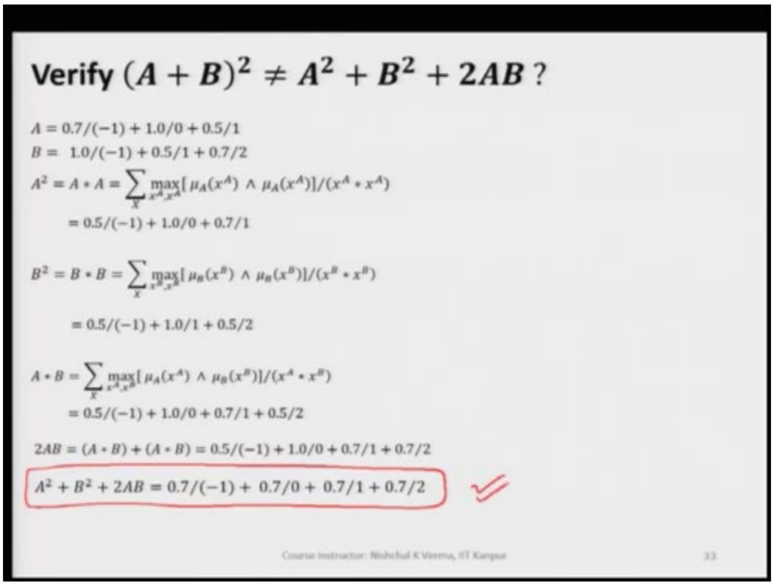

And when we apply the addition in between the AB and AB will become 2 AB. So, this 2 AB is going to give us a new fuzzy set which is here; this has four elements, so $0.5/-1 + 1/0 + 0.7/1 + 0.7/2$. So, this way we have you see result of A square plus B square plus 2 AB as, $0.7/(-1) + 0.7/0 + 0.7/1 + 0.7/2$.

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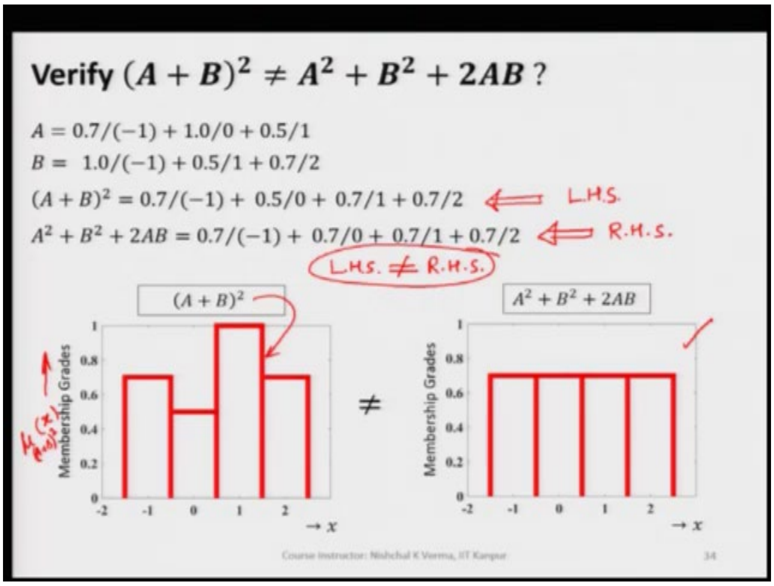

So, this is what is the expression that we are getting as a result of $A^2 + B^2 + 2AB$, which is nothing, but the RHS. So, we are writing here, this was our LHS and this was our RHS. And when we compare these two, we can clearly see that these two are not equal, means the RHS or LHS is not equal to RHS.

So, this we can better judge when we plot these two, the LHS and RHS and compare. So, here we have plotted the resultant fuzzy set, which is LHS which is $(A + B)^2$, which is coming like this. So, here x axis is the genetic variable and y axis the membership values are grades, we can write it like this and this is nothing but here A plus B whole square.

So, we can clearly see that these two, the LHS and RHS are not equal. So, by plotting these we can very clearly visualize. And this way we can say that, although $(A + B)^2 = A^2 + B^2 + 2AB$, but when we take fuzzy numbers $(A + B)^2 \neq A^2 + B^2 + 2AB$. So, you see the difference that, this does not hold good for fuzzy numbers and the same holds good for the crisp numbers.

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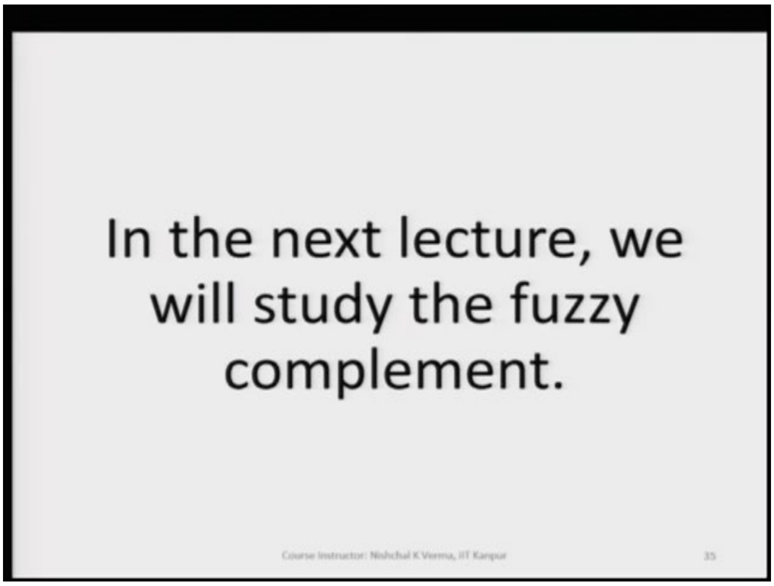

So, we will stop here in this lecture, and in the next lecture we will study the fuzzy compliments.

Thank you.