Fuzzy Sets, Logic and Systems and Applications
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Lecture – 21
Arithmetic Operations on Fuzzy Numbers

So, welcome to lecture number 21 of Fuzzy Sets, Logic and Systems and Applications. So, this lecture is in continuation to our discussions on Arithmetic Operations and this arithmetic operations are on Fuzzy Numbers. We have already covered the arithmetic operation that is the addition on fuzzy numbers in our last lecture.

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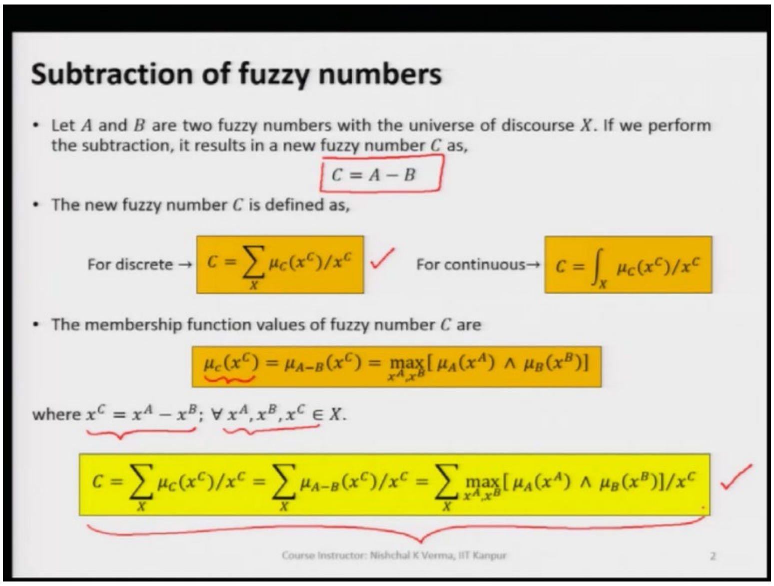

Today we will be discussing the subtraction of fuzzy numbers. So, when we have two fuzzy numbers A and B and these two fuzzy numbers are within some universe of discourse capital X. And if we perform the subtraction it results in a new fuzzy number let's say C and C this can be defined as $C = A - B$.

So, please note here, that A and B both are fuzzy numbers. So, what does this mean here is if we say A is a fuzzy number means A is a fuzzy set which satisfies the criteria of normality as well as the convexity. So, we can say A is a normal fuzzy set and A is a convex fuzzy set. Similarly, B also is a normal fuzzy set and convex fuzzy set.

Now, when we subtract B from A fuzzy numbers we have a C fuzzy number which is again convex and normal fuzzy set. So, for discrete fuzzy sets the subtraction will govern by this equation here. So, C is nothing but C will be equal to

$$C = \sum_{X} \mu_c(x^C)/x^C$$

So, this is for discrete fuzzy sets and for continuous fuzzy set this summation will be replaced by the integration sign. Please note that this summation is a symbolic representation here. Similarly the integration is also a symbolic representation, we do not have to either sum or integrate.

So, these have to be there these values these terms in summation we have to just separate by plus signs and in integration we do not have to integrate these the whatever is the outcome because of course, here there is no dx or something like that.

So, the resultant fuzzy set that is coming out of $A - B$ at C, so C will have a membership function and its generic variable. So, μ_C here can be found by $\mu_{A-B}(x^C)$ where C is nothing but the resultant fuzzy number and this is equal to $\max_{x^A, x^B}(\min(\mu_A(x^A), \mu_B(x^B)))$.

So, it is clearly understood that $x^C = x^A - x^B$ for subtraction purpose and for all x^A, x^B, x^C they are belonging all values are belonging to the universe of discourse. So, we can write here this C is nothing but C is represented by this expression.

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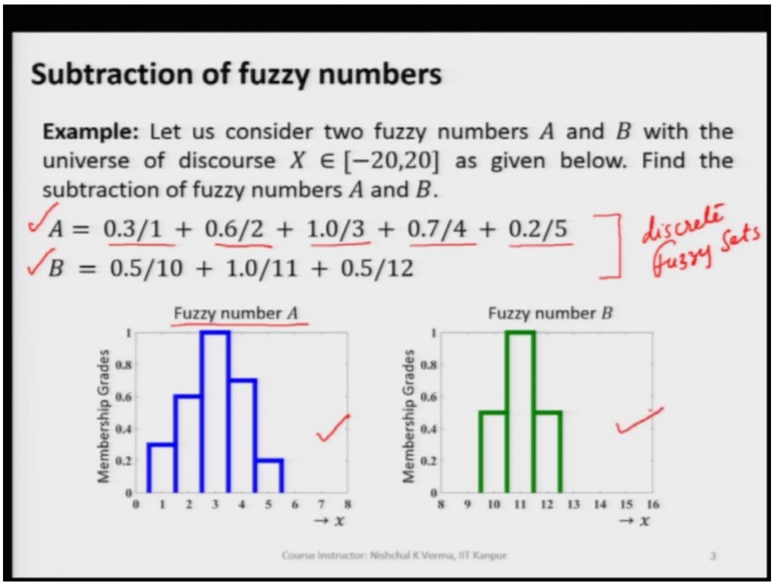

So, let us now take an example of subtraction of fuzzy numbers A and B and understand the operation. So, if we have two fuzzy sets A and B, $A = 0.3/1 + 0.6/2 + 1/3 + 0.7/4 + 0.2/5$. B is represented as $B = 0.5/10 + 1/11 + 0.5/12$ and both these A and B sets are discrete fuzzy sets. And these are represented here as the fuzzy numbers A and B.

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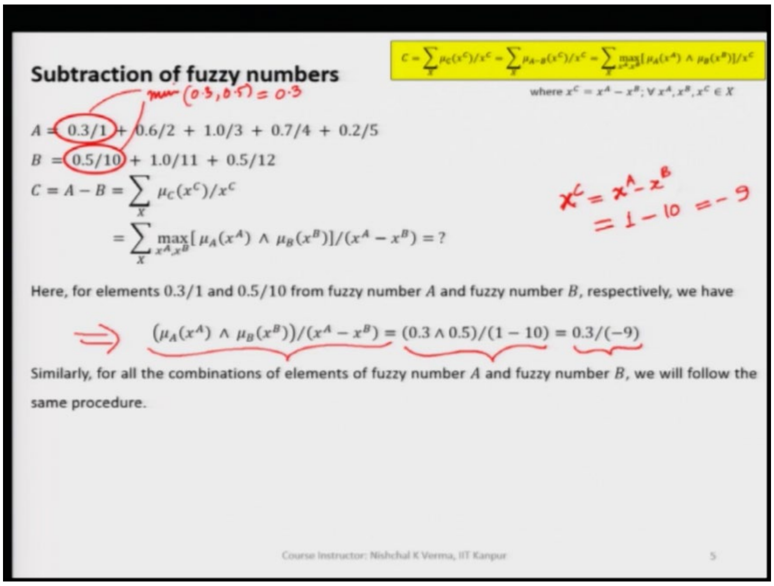

So, let us now apply the formula that we have just seen what is resulting. So, $C = A - B$, C is resulting fuzzy number here and this is equal to the fuzzy number $A - B$. And when we apply this formula let's see what is happening. So, first let us compute this value for one combination of the elements from A and B.

So, let us take min of these two, this term and this term. So, $0.3/1$ and $0.5/10$ we have two elements and let us take min of these. So, $min(0.3, 0.5)$ we are going to get 0.3. And then here for generic variable value since we are subtracting, so we'll simply subtract so, the generic variable value will be the x^C here for this case and x^C will be here nothing but $x^A - x^B$; $x^C = x^A - x^B$ and this is here in this case $x^A = 1$ and $x^B = 10$. So, what we are going to get here is -9.

So, that is how we have computed here and the value of this term here the when we have taken the one combination and this combination; the first combination is giving us $0.3/-9$ for $A - B$. So, similarly for all the combinations of elements of fuzzy number A and fuzzy number B. So, we will follow the same procedure for finding the other elements for corresponding to the other combinations.

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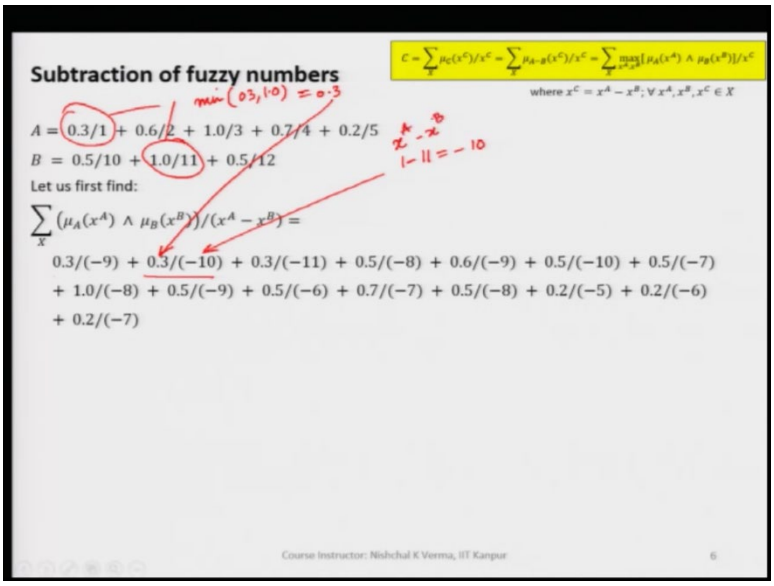

So, let's quickly go through this and we find the other elements here as $0.3/-10$. So, this we will get out of this, so when we have these two pairs we clearly see that the $min(0.3, 1)$ we are going to get 0.3. So, that's for how this 0.3 is coming. And then this is the outcome of the generic variable valued subtraction, so $1 - 11$. So, this is going to give us -10 which is here.

So, on the same lines here we go ahead and find all the combinations. So, this way we take all these combinations of pairs we get these values coming you can see here this is shown by the outcome of each and every pair is coming and shown in the red color you see here.

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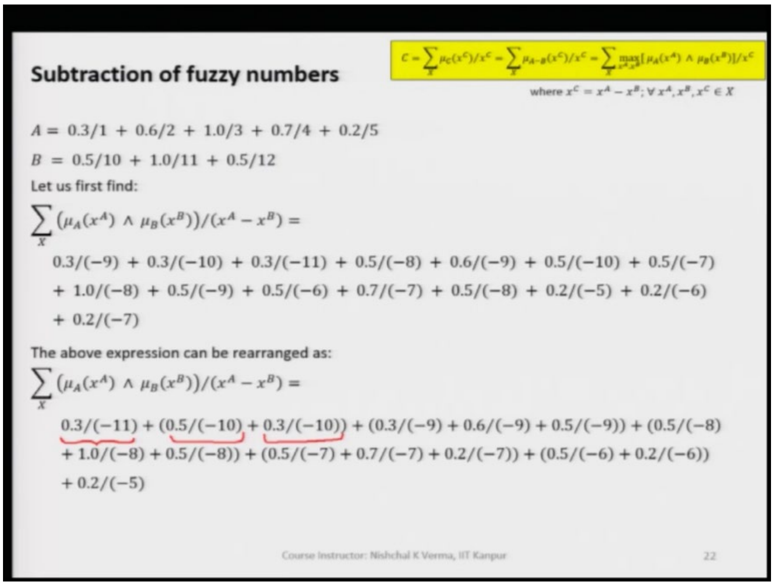

So, that's how we are getting the subtraction of all these pairs. So, that's how we get the complete total terms the total elements, the total outcomes are listed.

Now, what we have to do is, we rearrange all these terms means we form a group of all these elements which are having same generic variable value. So, for generic variable value -11 we have only one term, so that is how it is written here. Now, for generic variable value minus 10 we have two values, the first one is this and the second one is this.

So, we see that we have two terms for generic variable value -10. Similarly we go ahead with all the generic variable values that are listed here and we form a group of same generic variable values. And then we apply the max criteria, if we take the max of membership values of the same generic variable values.

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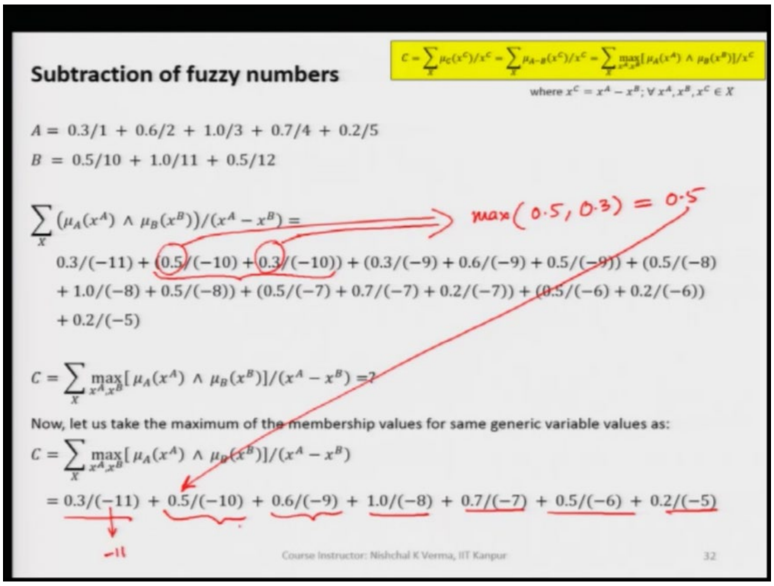

So, let us now go ahead and see what is happening here and when we do that we see, so here we are now applying the max and when we apply the max we see here this is the term which is having two membership values. First membership is 0.5 and the second membership here is 0.3 for the same generic variable value -10.

So, for -10 we will have to have only one membership value which is 0.5 when we take $max(0.5, 0.3)$. So, that is how we are getting here 0.5. So, we here we can say here that we are applying the max in between or $max(0.5, 0.3)$. So, that is how we are getting 0.5 and this 0.5 is coming over here.

And on the same lines we are obtaining all these terms. So, far corresponding to -9 we are getting $0.6 - 8$ we are getting $1 - 7$ we are getting $0.7 - 6$ we are getting $0.5 - 5$ we are getting 0.2. So, all these terms we have got here.

And now we can clearly see that there is no duplication or in other words we can say there is no duplication in the sense that we do not have here more than two values of generic variables.

Means we have only the generic variable -10, we do not have other places -10. So, it means we have the generic variable value -11 and then $-10,\ -9,\ -8, -7, -6, -5$ like that. So, this way we have obtained the expression for C and we can clearly see this is a fuzzy number.

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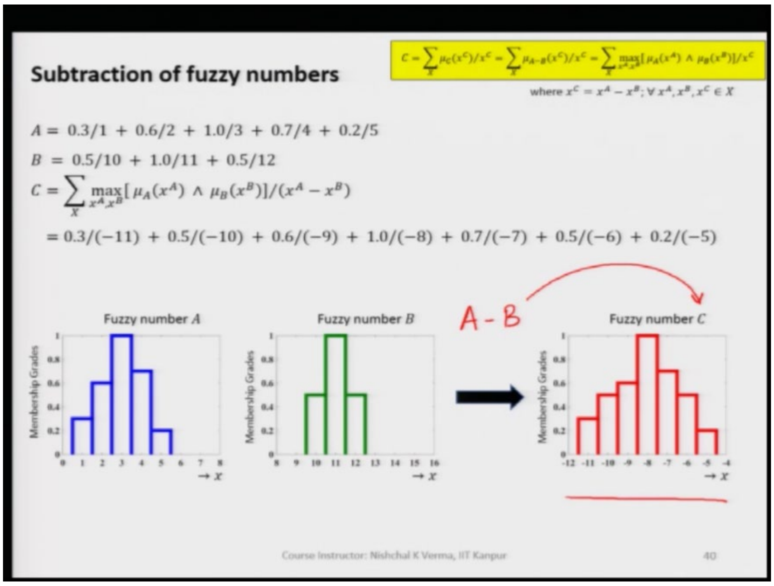

And now let us plot this fuzzy number C which is the outcome of $A - B$ here. So, fuzzy number A minus fuzzy number B we have fuzzy number C and C is you see here also in this case comparatively we have more spread than A and B. So, this is because the uncertainty level in the fuzzy number C is increased.

So, when we have a fuzzy number A which has some uncertainty level and again we have fuzzy number B which also has some uncertainty level. And when we take $A - B$ or $A + B$. So, the outcome fuzzy set which is a fuzzy number in both the cases will have comparatively more spread. So, it means the uncertainty level gets increased. And please understand that this uncertainty that I am talking of is the uncertainty due to ambiguity, imprecision and vagueness. So, in this category these uncertainty is normally accounted in the fuzzy sets.

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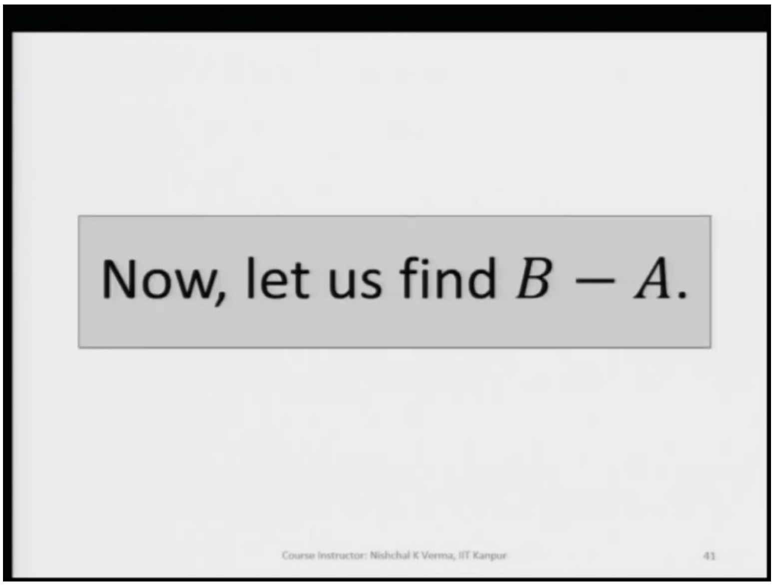

Now, let us find the subtraction $B - A$. So, let us now subtract fuzzy number A from fuzzy number B and see what is happening.

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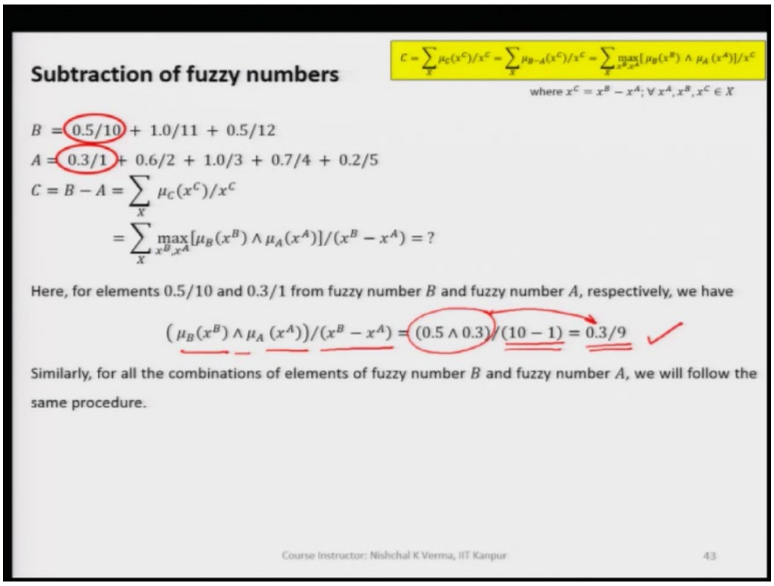

So, on the same lines as we have discussed $A - B$, so we have seen that we have first computed the min of the membership values of corresponding terms, corresponding pairs. And then when we have all these pairs we have found then we moved ahead and we took max of these and then we moved ahead. So, we can quickly go through this and see what is happening here when we subtract fuzzy number A from fuzzy number B. So, we can directly here go through this.

And here also when we subtract fuzzy number A from fuzzy number B we see that we have $(\mu_B(x^B) \wedge \mu_A(x^A))/x^B - x^A$ is because we are subtracting fuzzy number A from fuzzy number B. So, here when we take the min of the corresponding membership values. We see here that we get 0.3 and then when we subtract here 1 from 10 we are getting 9. So, 0.3/ 9 we are getting as one of the terms of fuzzy number C which is the outcome of $B - A$.

So, similarly for all the combinations of elements of fuzzy number B and fuzzy number A we will follow the same procedure.

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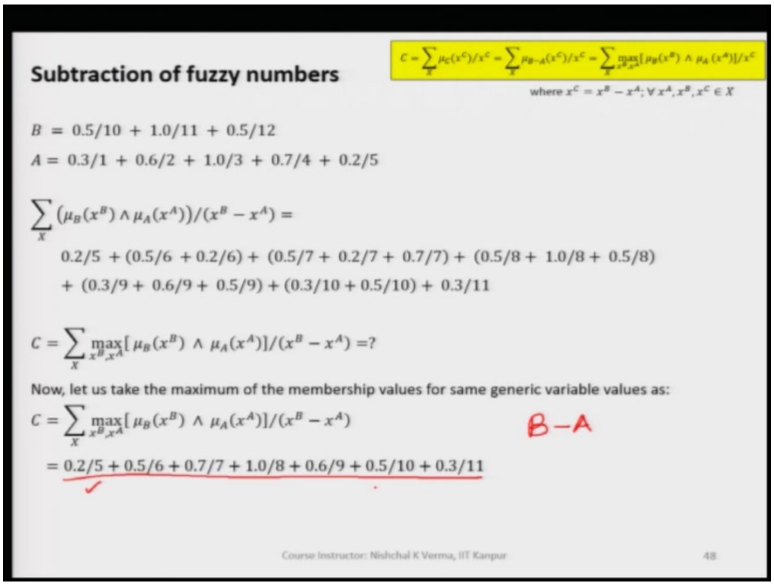

So, we see here that we are getting C as if we see here we are getting here result of $B - A$ fuzzy numbers we get $0.2/5 + 0.5/6 + 0.7/7 + 1.0/8 + 0.6/9 + 0.5/10 + 0.3/11$.

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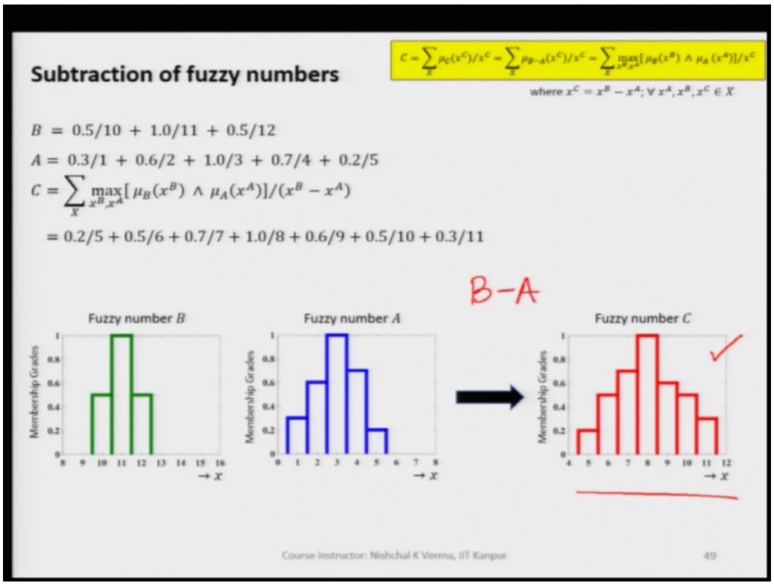

Now, our $B - A$ is coming here as fuzzy number C which is plotted here and when we clearly see that this is very similar to the outcome which we got in the earlier case of $B - A$.

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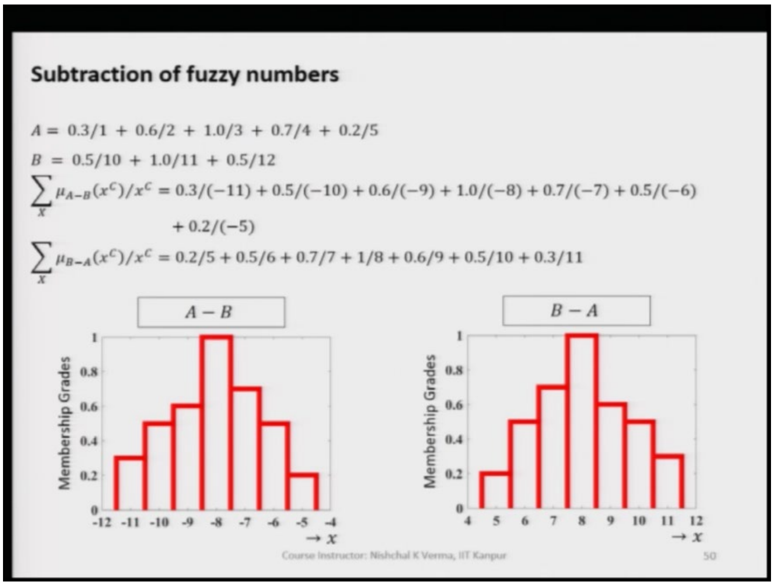

So, when we bring these two outcomes $A - B$ and $B - A$ together we see that these two are symmetric. So, or in other words we can say that $A - B$ and $B - A$ both are the mirror images of each other. Of course, here we see when we take $A - B$ and $B - A$. We see that these two are not equal to each other. So, it means commutativity property doesn't hold good with respect to the subtraction.

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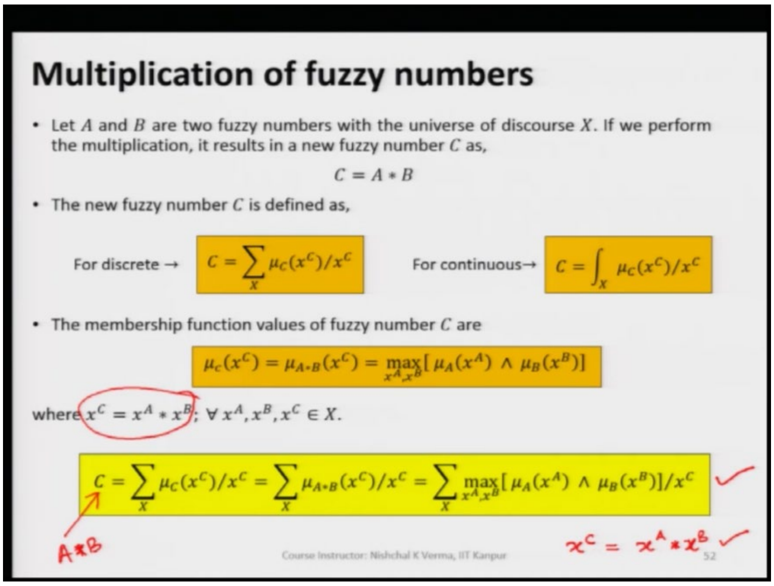

Now, let us discuss the multiplication of fuzzy numbers. So, multiplication can be defined here as $C = A * B$. So, star here signifies the multiplication. So, and A, B, C are fuzzy sets where A and B both are the fuzzy numbers. So, once again I would like to mention that when we say fuzzy number it means it is fuzzy set which is satisfying both the properties, the convexity as well as the normality.

So, when we say here A and B both are fuzzy numbers it means both the sets A and B are the normal fuzzy sets as well as convex fuzzy sets. So, here we use this formula for finding out the fuzzy set the multiplication of A and B and which is resulting let us say C fuzzy set is the $A * B$. So, of course, when we multiply any two fuzzy number we are going to get a fuzzy set out of it.

This fuzzy set here may not be fuzzy number because this may not satisfy the criteria of the normality as well as the convexity. So, we will see ahead what is happening when we multiply two fuzzy numbers.

So, let us quickly go ahead and see how do we multiply two fuzzy numbers. So, formula remains the same if we see the only difference that we see here is this x_C comes out to be x_A multiplied by x_B which is here as well which is mentioned. So, other than this there is no difference at all. So, all min and max remains intact, so we need to be careful while finding the multiplication of two fuzzy numbers A and B. So, in addition this $x_C = x_A + x_B$, in subtraction the $x_C = x_A - x_B$. So, in multiplication here we have $x_C = x_A * x_B$.

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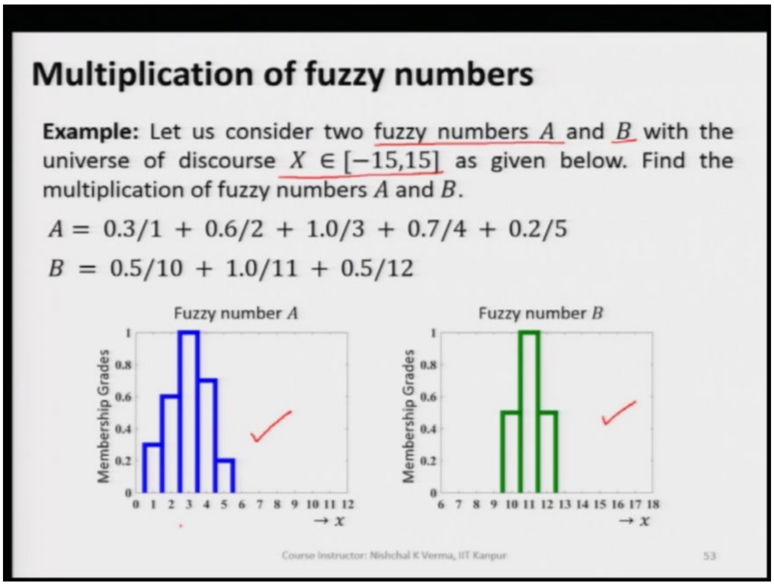

So, let us quickly go through an example here which will make us understand little better the multiplication of two fuzzy numbers. So, here we have the fuzzy numbers A and B and this fuzzy numbers here are within the universe of discourse -15 to 15. So, here we have to consider the universe of discourse as well.

Now, let us find the multiplication of these two fuzzy numbers. So, when we plot the fuzzy number A this will look like this, when we plot the fuzzy number B this will look like this here it is shown by the green color and fuzzy number A is shown by the blue color.

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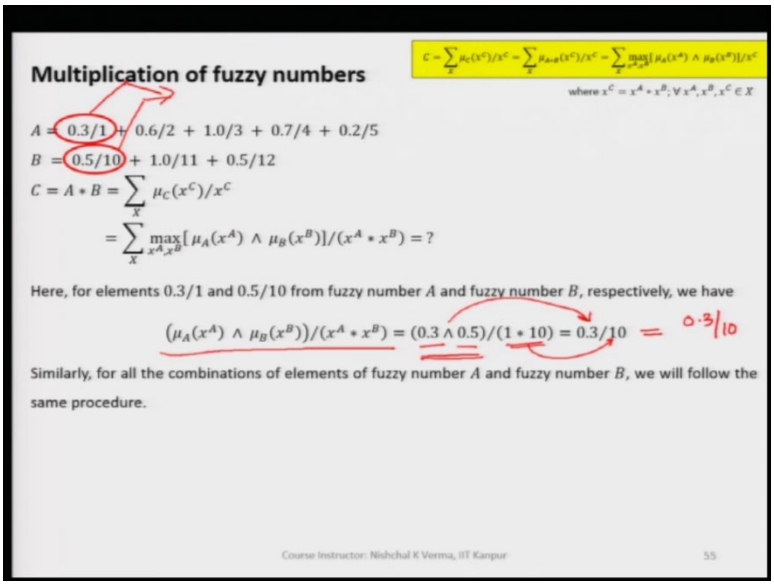

So, let us now apply the formulation that we have just discussed and when we apply we first try to find this term which is $(\mu_A(x^A) \wedge \mu_B(x^B))/(x^A * x^B)$. So, let us quickly find out these values $\mu_A(x^A), \mu_B(x^B)$ from this pair the first combination. So, we have first combination here and from here we get all these values $\mu_A(x^A), \mu_B(x^B)$ and also we get x^A and x^B as generic variable values, 0.3 is $\mu_A(x^A)$, 0.5 is $\mu_B(x^B)$.

So, when we substitute here and take the min of these we find the min as 0.3 and $1 * 10$ which are the generic variable values of A and B respectively as x^A and x^B, so when we multiply this we are going to get 10. So, this is going to result $0.3/10$. So, similarly for all the combinations of elements of fuzzy number A and fuzzy number B we will follow the same procedure.

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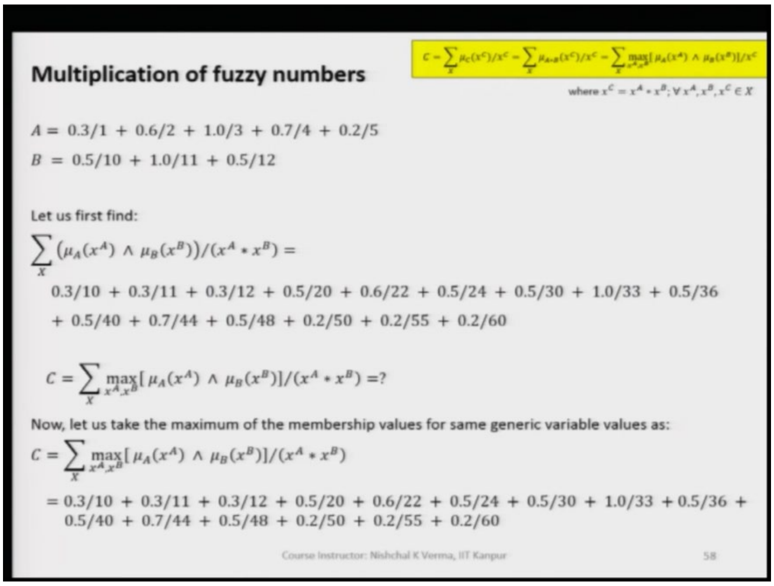

And that is how we are going to get 0.3 all the elements as $0.3/10$,$0.3/11, 0.3/12, 0.5/20, 0.6/22, 0.5/24, 0.5/30, 1.0/33, 0.5/36, 0.5/40, 0.7/44, 0.5/48, 0.2/50, 0.2/55, 0.2/60$.

So, all these elements we have got after computing minimum of $\mu_A(x^A)$ and $\mu_B(x^B)$ oblique x^A into x^B from all the combinations that are possible from fuzzy number A and B. So, when we have done that that will result here these terms. So, when we rearrange; when we write all these terms we see that we have all these terms are; some of the terms that are there which are not within the universe of discourse. So, let’s see what we need to do further.

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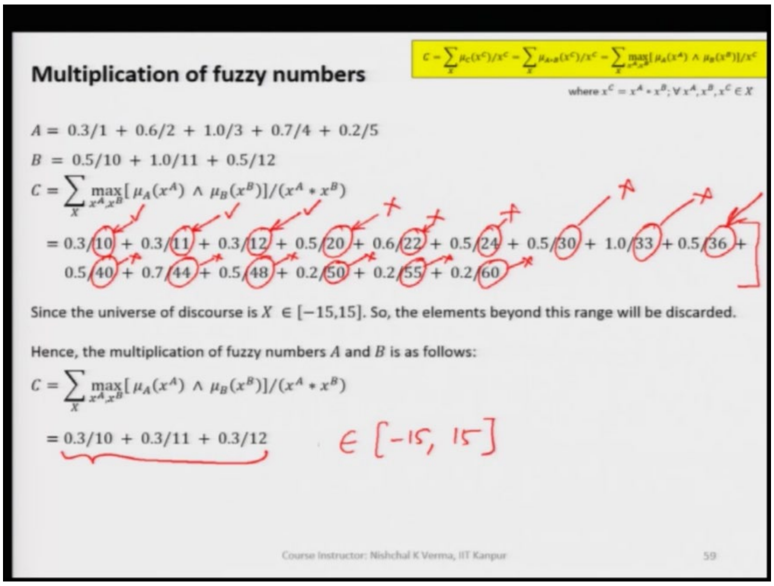

Since the universe of discourse that has been given to us is -15 to 15, it means whatever elements that we will be having or that will be getting here should be within the universe of discourse.

So, this means what? This means we have to consider only those terms for which we have the generic variable values within this universe of discourse. So, this way if we will look here in this expression. So, we see that we have generic variable value 10 which is within the universe of discourse, 11 which is within the universe of discourse, 12 which is within the universe of discourse. So, like that these are within the universe of discourse these values these generic variable values. Now, what about this 20? So, 20 is not within the universe of discourse.

Similarly, 22 also is not within the universe of discourse that has been given to us. Similarly, 24 which is also not within the universe of discourse, similarly 30 is also not within the universe of discourse, 33 is also not within the universe of discourse, 36 is also not within the universe of discourse, 40 is also not within the universe of discourse, 44 is also not within the universe of discourse, 48 is also not within the universe of discourse, 50 is also not within the universe of discourse, 55 is also not within the universe of discourse, 60 is also not within the universe of discourse.

Only 10 is within the universe of discourse this 10 as the generic variable value. So, this generic variable value is within the universe of discourse. Then 11 is within the universe of discourse, 12 is also within the universe of discourse. So, when we have now known this thing that only 10, 11, 12 are the generic variable values present here are the part of the universe of discourse.

So, we will take up only those terms which are having the generic variable values within the universe of discourse. So, it means we are only considering these 3 terms and these 3 terms we see here that $0.3/10, 0.3/11, 0.3/12$ these 3 are lying within the universe of discourse which is -15 to 15.

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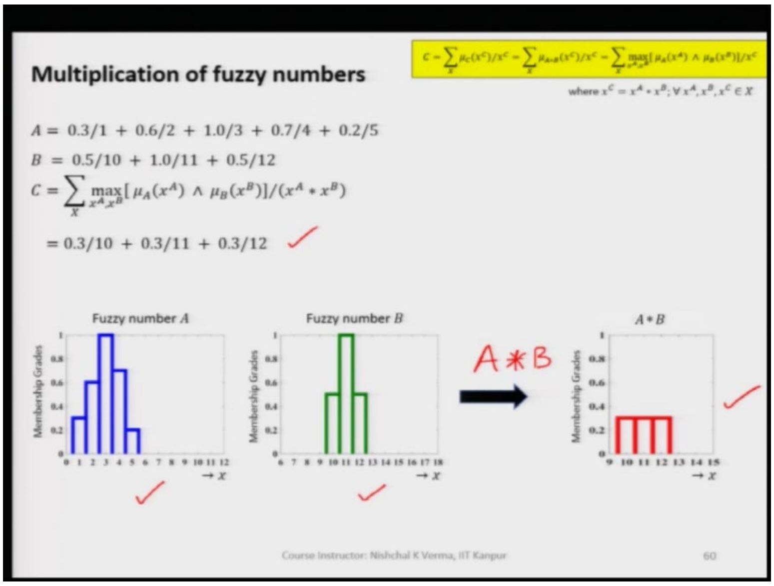

And when we plot this outcome here as fuzzy set C this is not a fuzzy number because you see this doesn't qualify to be a fuzzy number because this is not a normal set. So, this is a fuzzy set C this is not a fuzzy number please note that.

So, this way when we have A fuzzy number A and fuzzy number B when we multiply this as $A * B$ or in other words we can say when A and B both are fuzzy numbers and they are multiplied we are going to get a fuzzy set and this is not going to be the fuzzy number in our case here. So, in a more appropriate way I would say that the multiplication of two fuzzy numbers may not be a fuzzy number.

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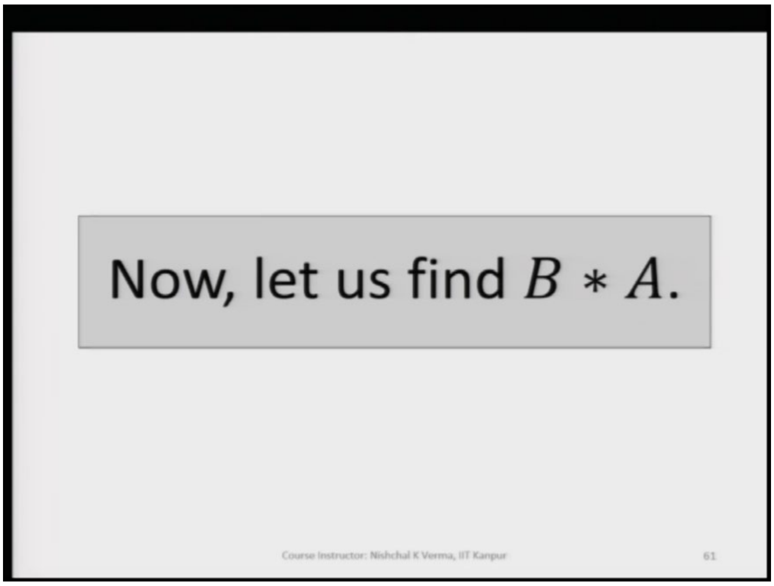

So, now let us find the $B * A$ means now let us take B first and then A.

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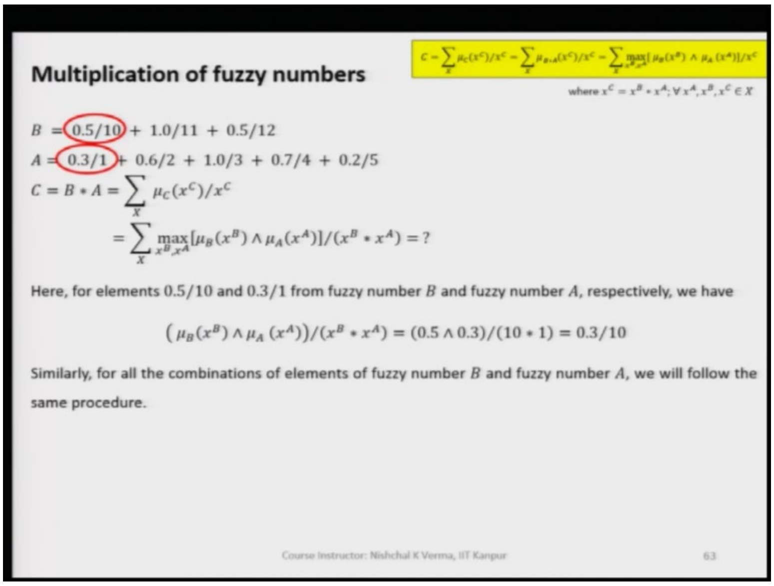

So, let's now see what is happening. So, when we do that and we apply the same procedure of computing B multiplication A.

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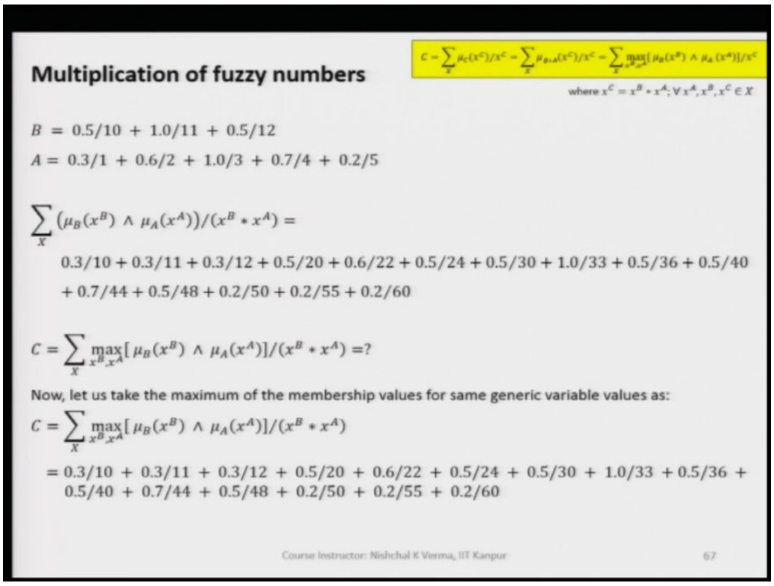

We see that we are getting here after doing all the computations and intermediate steps. So, we are landing up here.

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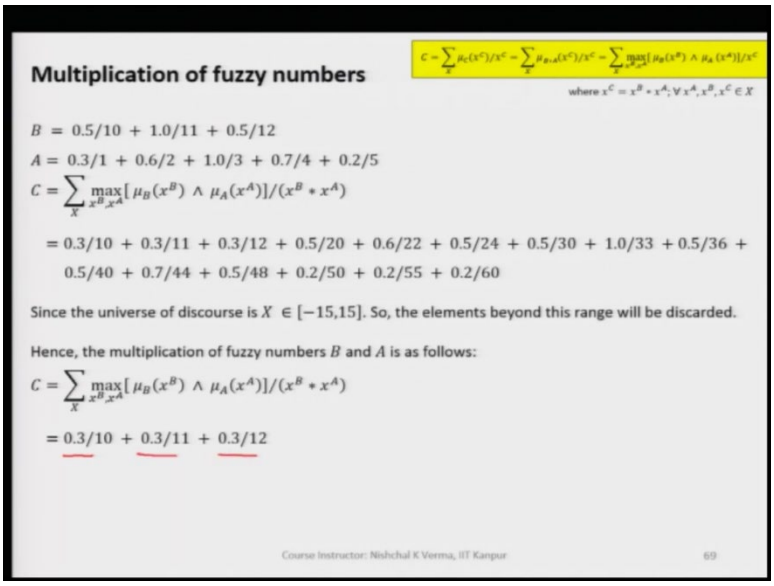

Again, we are landing up with only 3 elements means the same elements.

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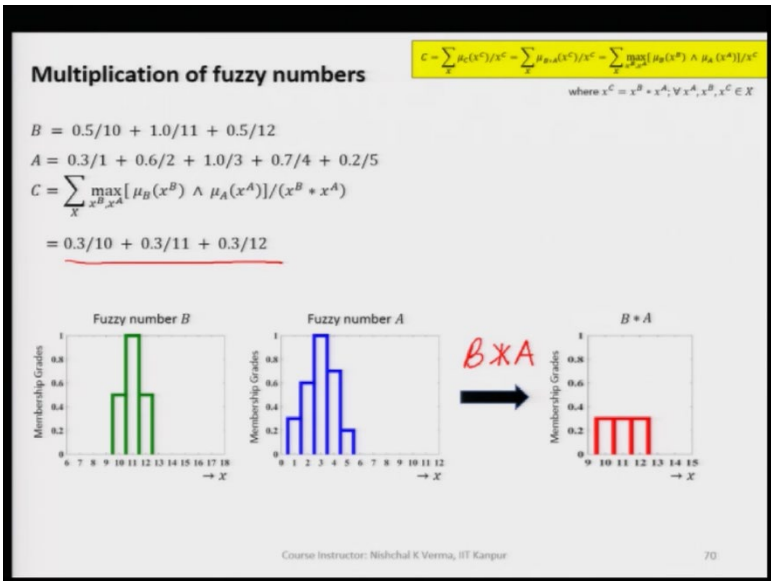

So, when we plot this $B * A$ means when we multiply B and A we get the same set. So, we can say the A multiplication B is equal to B multiplication A means A and B the multiplication of the fuzzy numbers A and B are commutative with respect to multiplication.

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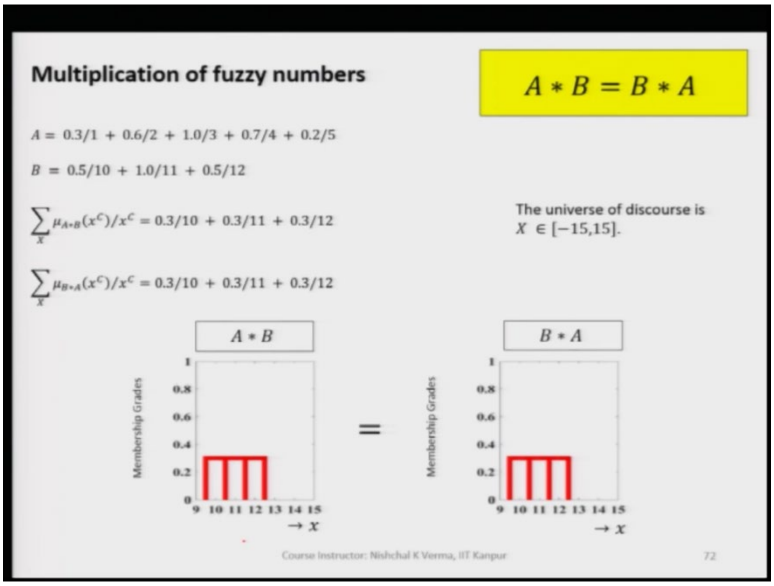

So, this way we see that here both the fuzzy sets are same and as I mentioned earlier that these are not the fuzzy numbers the result is not a fuzzy number here. Please understand that since we have limited the universe of discourse, we have shown only a portion of the actual multiplication. We could have seen some more terms which would have been spread probably the in wider universe of discourse, but here since the universe of discourse is very limited.

So, that is why only few terms are coming as are few terms are included within the universe of discourse few terms are included here and in fuzzy set C. So, that is why you can see here how these two look like within the universe of discourse -$15, 15$

And as I already mentioned A multiplication B is equal to B multiplication A. So, that's how we have seen as to how we can multiply two fuzzy numbers A and B. So, with this we will stop here.

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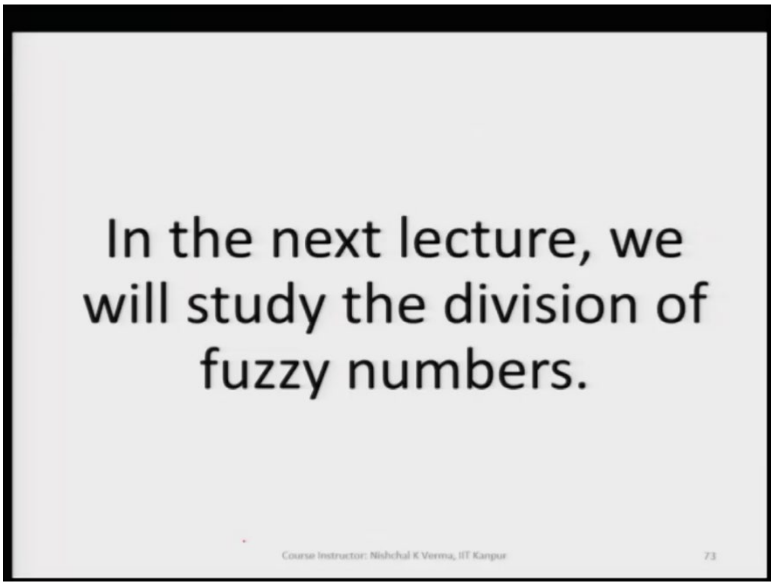

And in the next lecture we will study the division of fuzzy numbers and apart from these we will do some more examples on the addition, subtraction, multiplication and division of fuzzy numbers.

Thank you.