Fuzzy Sets, Logic and Systems and Applications
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Lecture – 20
Arithmetic Operations on Fuzzy Numbers

Welcome to lecture number 20 of Fuzzy Sets Logic and Systems and Applications. So, in this lecture we will learn the Arithmetic Operations on Fuzzy Numbers. So, let us first understand what is a fuzzy number.

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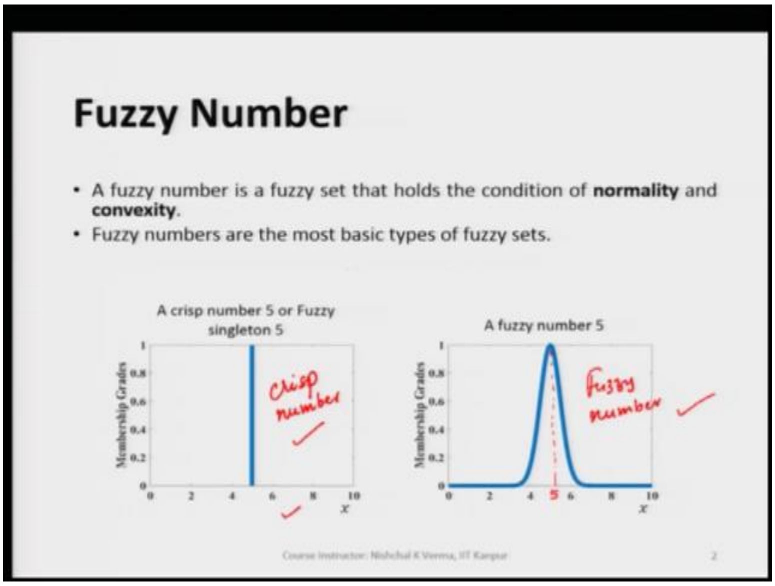

In one of the lectures we had already discussed what is a fuzzy number. So, a fuzzy number is a fuzzy set basically that holds the condition of normality and convexity. So, what does this mean? This means that any fuzzy set which satisfies the property of normality and convexity is qualified to be fuzzy number. So, all the fuzzy numbers are the most basic types of fuzzy sets.

So, here we have a fuzzy set the right side of this diagram. So, we have a plot of fuzzy number, we call this as fuzzy number 5. So, if we see a fuzzy number 5, we see that we have a fuzzy set which has its highest membership value that is 1 at 5, but at around 5 also we have some membership values and that’s how this is a fuzzy number.

So, when we talk of fuzzy number; obviously, let us also understand the crisp number. A fuzzy number is a fuzzy set, now if we see what is a crisp number. So, crisp number can also be regarded as a fuzzy set, but this fuzzy set is a fuzzy singleton. So, this is a special kind of fuzzy set that is singleton fuzzy set 5. And, it is because if we look at this diagram plot of fuzzy singleton, we see that at the generic variable $x = 5$ we see that we have only one membership value corresponding to 5 that is the highest; that means, the core.

So, we have only one core at 5. So, it is because this number is crisp number and we have at 5 its belongingness or it is membership value 1. So, no other membership values are possible around 5. So, that is why it is crisp. So, either we call a crisp number or in fuzzy systems we call this crisp numbers or we represent these crisp numbers by fuzzy singleton. So, we see here the fuzzy singleton 5 which is a crisp number. So, this is a crisp number, right side we have the fuzzy number. Fuzzy number is always a fuzzy set, but this fuzzy set must satisfy the condition of normality and convexity.

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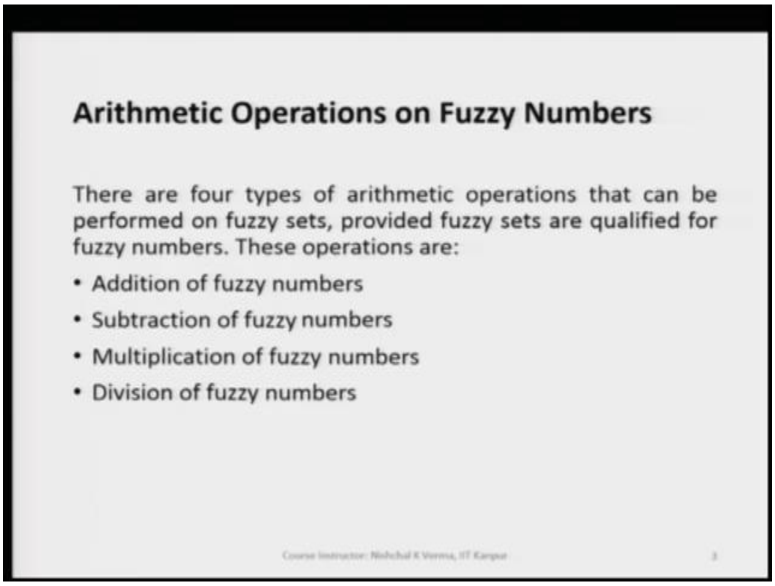

In this coming lectures, this lecture and the coming lectures we will learn the arithmetic operations on fuzzy number. So, by now we know what is a fuzzy number. So, similarly when we talk about fuzzy numbers when we say fuzzy numbers, it means we are we will be dealing with fuzzy sets. And, these fuzzy sets will be satisfying the criteria of normality and the convexity. And, when we talk of arithmetic operations here; so, arithmetic operations will be here arithmetic operations are addition of fuzzy numbers and then we have subtraction of fuzzy numbers, multiplication of fuzzy numbers, division of fuzzy numbers.

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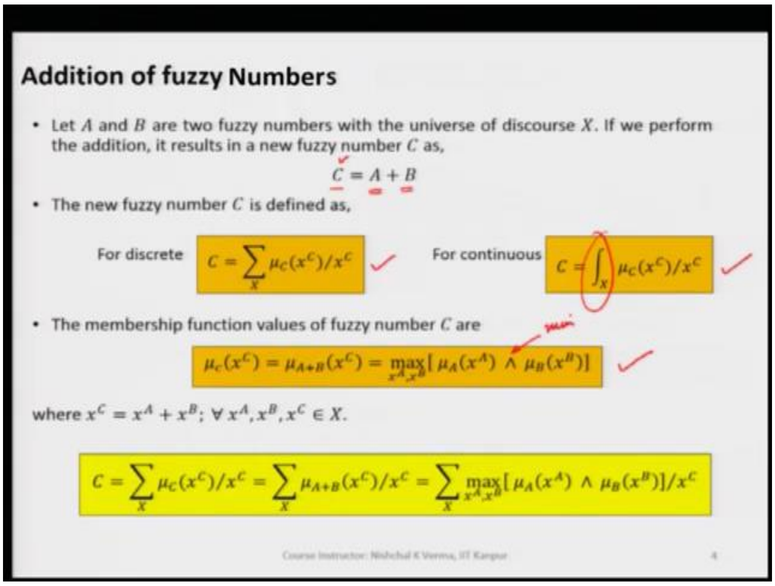

Now, let us see how we add two fuzzy numbers. So, when we say two fuzzy numbers; obviously, these fuzzy numbers will be a fuzzy sets and it is needless here in this lecture to say that these fuzzy sets again these should be with these conditions of convexity and normality satisfied. So, let us take two fuzzy numbers A and B and let us right here C which is the sum of these two fuzzy numbers A and B.

So, we have the resultant fuzzy set which you C here and this C let us see how do we obtain ok. So, when we say C is a resultant fuzzy set obviously, C will have representation as we have already discussed. So, if it is a discreet fuzzy set if this resultant is a discrete fuzzy set, we will represent this fuzzy set C by

$$C = \sum_X \mu_C(x^C)/x^C$$

where this x_C is the generic variable values that will be coming from the fuzzy set the resultant fuzzy set C.

We can also say C as a fuzzy number because here when we are adding two fuzzy numbers A and B we will be getting another fuzzy number C. When the resultant fuzzy set here is a continuous fuzzy set C, we will replace the summation by the integration sign which is here.

$$C = \int_X \mu_C(x^C)/\, x^C$$

So, as we already know that in continuous in representation of the continuous fuzzy set, we represent the fuzzy set like this. Our job is to find the corresponding membership values of C when we add two fuzzy sets A and B.

So, we clearly see here the max criteria which gives us the addition of two fuzzy sets and this is with respect to the addition of two fuzzy sets A and B. So, $\mu_C(x^C) = \mu_{A+B}(x^C) = \max\limits_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$. So, it means what? It means that we will take min of all the corresponding membership values and then whatever values that we will be getting for all x_A x_B we will take max. And please note that the C is nothing, but A plus B.

So, we will see as to how we are going to get the complete fuzzy set we will take the example and then we will understand as to how we get the addition of two fuzzy numbers.

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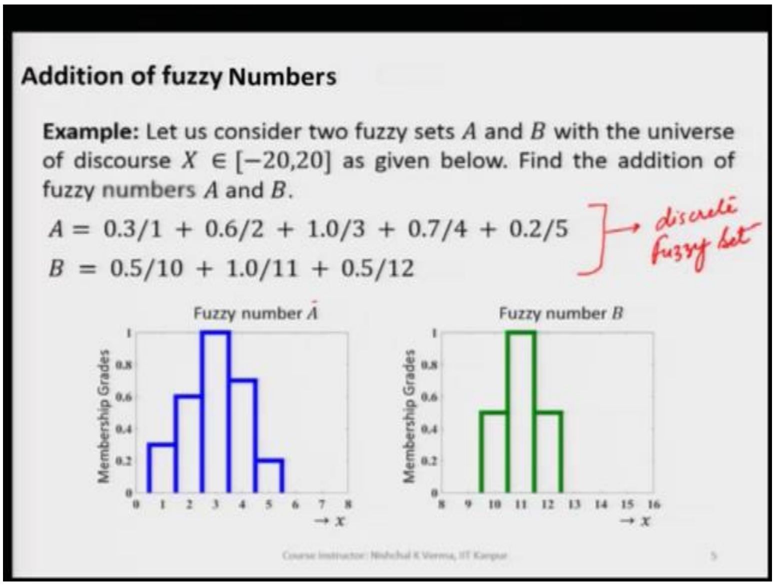

So, let us take this example here we have a fuzzy number A and then we have another fuzzy number B. If we see this is represented by these two discrete fuzzy expressions, these are the expressions. So, I will write here the discreet fuzzy sets because these fuzzy number, but finally, this is a fuzzy set discrete fuzzy set. So, these two are discrete fuzzy sets.

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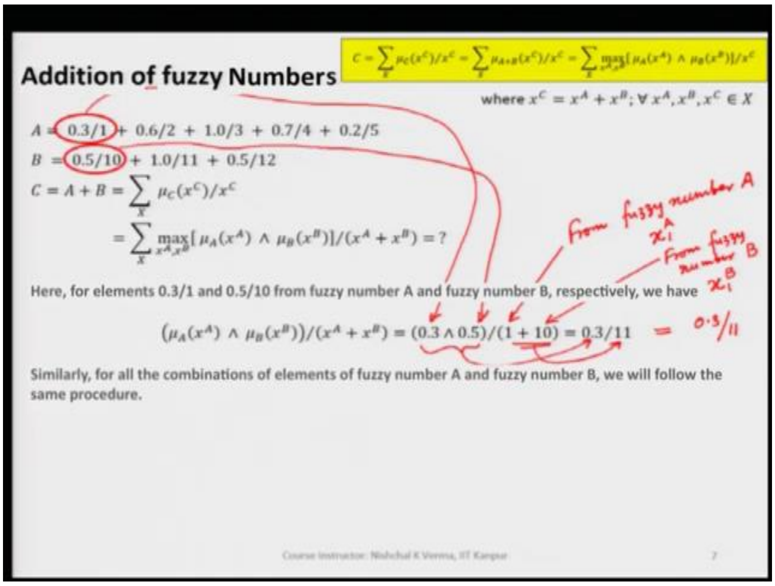

Let us know quickly go ahead and see how to add these two fuzzy numbers. Alright, so we have A fuzzy number here and B fuzzy number here. As we have seen the first most operation is to get the minimum of x A $\mu_A(x_A)$ and $\mu_B(x_B)$. So, let us see as to how we are going to get this if we take the elements from A and B. So, the first element of A is $0.3/1$ and the first element of B is $0.5/10$. So, let us get this min operation and then we right by oblique and then $x_A + x_B$.

So, what do we do here is, we see that $\mu_A(x_A)$ here is 0.3 which is here and $\mu_B(x_B)$ is 0.5 which is here. When we take the min of these two we will get 0.3; as I mentioned this by oblique 1 plus 10. So, this is generic variable value which is from the first fuzzy set, fuzzy number A. So, this is from fuzzy number A and I can write it here this as x_A x_1^A.

Similarly this value is from fuzzy set or fuzzy number B, I can write here this as x_1^B. So, this way we have written here the min of 0.3 and 0.5 as 0.3 and then oblique 11 because 1 plus 10 here is 11. So, finally, we are getting $0.3/11$. So, this way we have got the first term and similarly we'll go through all the combinations of A and B elements.

Let us now go ahead and see what we are getting. And, then when we have got these combinations since we have the summation, so, we will sum these elements to get the final expression for fuzzy set C which is the resultant of fuzzy number A and B.

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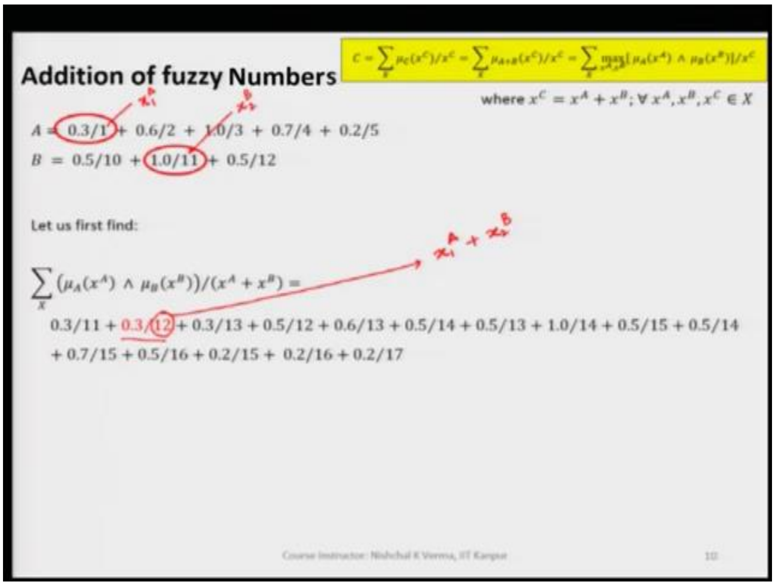

So, let us quickly go through this and here we have written the final expression, but I will explain as to how we are going to get these values as listed here. This we have already seen that how we are getting $0.3/11$. And, I can repeat here that this is the outcome when we have taken the first elements from both the sets, both the numbers of the elements from both the fuzzy numbers, next is the next combination.

So next combination will be we see here that, when we take $0.3/1$ and then from the fuzzy number A and then when we take the second element from fuzzy number B, we obtain $0.3/12$ and we know how are we getting this generic variable 12. This is because we have here 1 is 1 I can write again here $x_1^A$and here I can write here as x_2^B. So, this 12 is the outcome of $x_1^A + x_2^B$. So, this way the generic variable values are computed and now when we go ahead for other combinations let us say we are taking $0.3/1$ and $0.5/12$.

So, $0.3/1$ is from fuzzy number A and $0.5/12$ is from fuzzy number B. So, we are going to obtain here out of these two when we use these two, we are getting this as the. Similarly, when we go with other combinations you see the min of 0.6 and 0.5 we are getting 0.5 and then the generic variable values when we add, we add the addition we will get $2 + 10$ and this is nothing, but 12. So, $0.5/12$ we are getting and then here if we take this combination that is $0.6/2$ and $1/\ 11$ we are going to get the min value as 0.6 and the generic variable value here is 13.

So, finally, this element is here this element is $0.6/13$. I will quickly go ahead with other combinations here. So, with this combination we are going to get $0.5/14$, with this combination we are going to get $0.5/\ 13$ and then with this combination we are going to get $1/\ 14$. With this combination we are going to get $0.5/15$, with this combination we are going to get $0.5/14$ with this combination here we get $0.7/15$.

Similarly, we get here with this combination $0.5/\ 16$, with this combination we see here $0.2/15$, here with this combination we get $0.2/\ 16$. And finally, with this combination $0.2/5$ and $0.5/12$, we get $0.2/17$, 0.2 because here the min of 0.2 and 0.5 will be 0.2 and then the generic variable values will be straight away added to give us 17.

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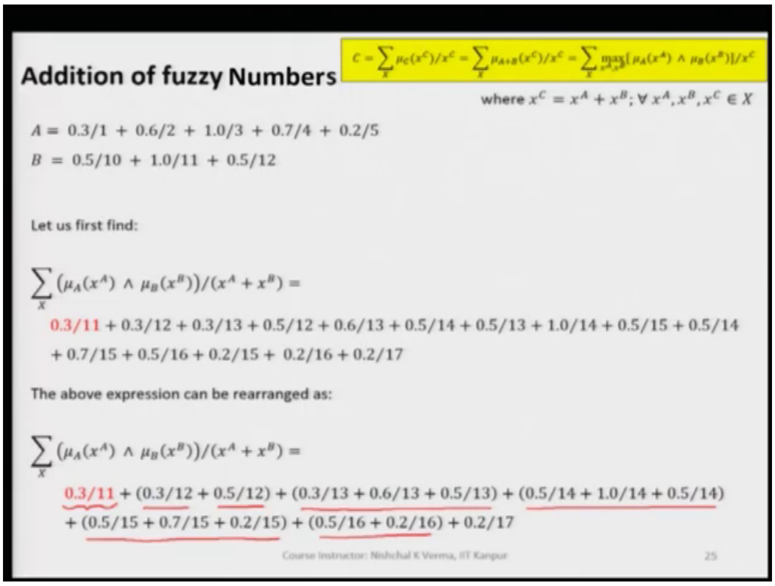

So, that is how we get all these values, now the next step is to rearrange these values that we have got these elements that we have got. So, let us now rearrange these values, these findings and when we rearrange we see that for the generic variable value 11, we have only one term that is with membership value 0.3. And, when we see for generic variable value 12 we have two terms here one is $0.3/12$ and the other one is $0.5/12$.

So, we keep writing all such terms for which we have the same generic variable values. So, for 12 for generic variable value 12 we are writing these two terms together, similarly for generic variable 13 we have three terms and we are writing these together for generic variable value 14 we have 1, 2, 3 three terms and here this also is being written together. Similarly, generic variable value 15, we have three terms and it is also written together and here also for generic variable value 16 we have two terms.

And finally, here as the last term we have for generic variable value 17 only one term that is with 0.2 as the membership value. So, this will be rearrange these terms and now what we do here is that, we take the max of these terms for which we have generic variable values same.

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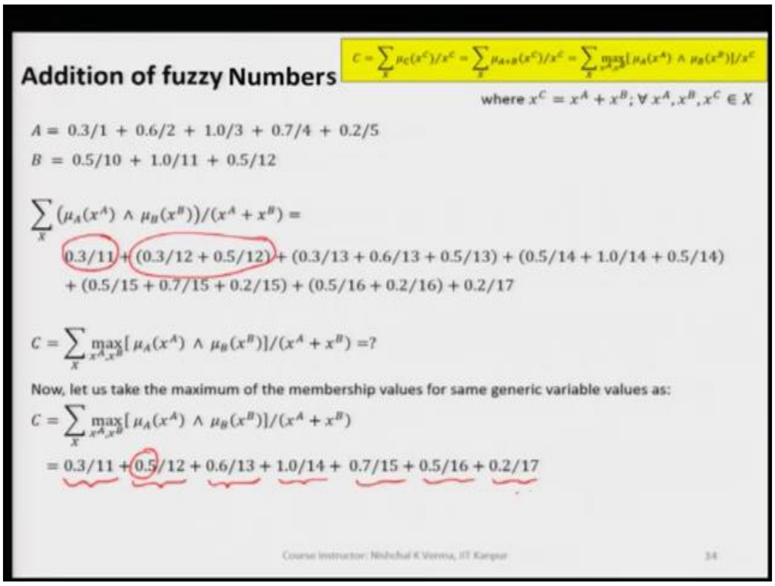

So, let us do that and when we do that here when we take the max as we see here. So, since we have $0.3/11$ as this is only one term. So, we just write this as it is, but here we if we have more number of terms for the same generic variable values. So, like for 12 we have two terms and then for these two terms we take the max of its corresponding membership values. So, max when we take max of 0.3 and 0.5 we get 0.5 here. So, we write $0.5/12$.

Similarly for generic variable value 13 when we take max we get only one membership value which is 0.6. So, we have this term as $0.6/13$. So, for generic variable value 14 we have 1 as the membership value. So, $1/14$ will remain here and then similarly for 15 we have 0.7, for 16 we have 0.5, for 17 we have 0.2. So, all these are the corresponding max values with respect to the generic variable values. So, this way we have the expression here of the fuzzy set the resultant fuzzy C and which is nothing but a fuzzy number here and this is, this C as I have already mentioned the C is the addition of two fuzzy numbers A and B.

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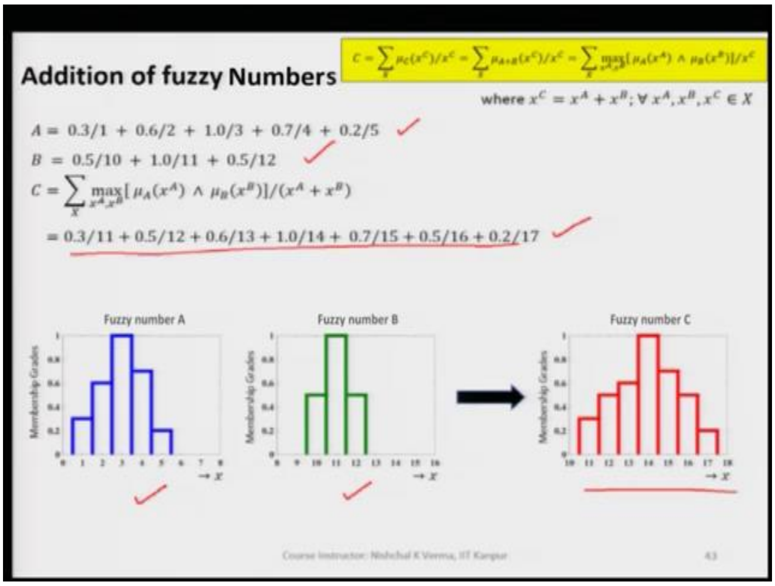

So, this is how we obtain an addition of two fuzzy numbers A and B. So, let's now see as to what was given to us. So, A was given to us as the discrete fuzzy set, B was also given to us as the discrete fuzzy set and then what we are obtaining here is this. So, when we add these two fuzzy numbers we have C fuzzy number. So, this is the resultant fuzzy number, let us know represent these three fuzzy numbers A, B and C and let us see how these three look like.

So, if we see here fuzzy number A, fuzzy number B and then when we compute the addition, when we find the addition of these two fuzzy numbers we get another fuzzy number which is C which is outcome of the addition. So, we see that the spread is increased the spread of the resultant fuzzy number is increased. It is probably because of the uncertainty level that is getting increased in the addition of two fuzzy sets A and B.

So, the uncertainty level here in fuzzy number A was some uncertainty and then the uncertain level in fuzzy number B, but whatever uncertain levels in the both fuzzy numbers were there we see that the uncertainty level in fuzzy number C is more than the uncertainty level of fuzzy number A and fuzzy number B. So, this way we see that we how can we add fuzzy number A and fuzzy number B to get another fuzzy number C.

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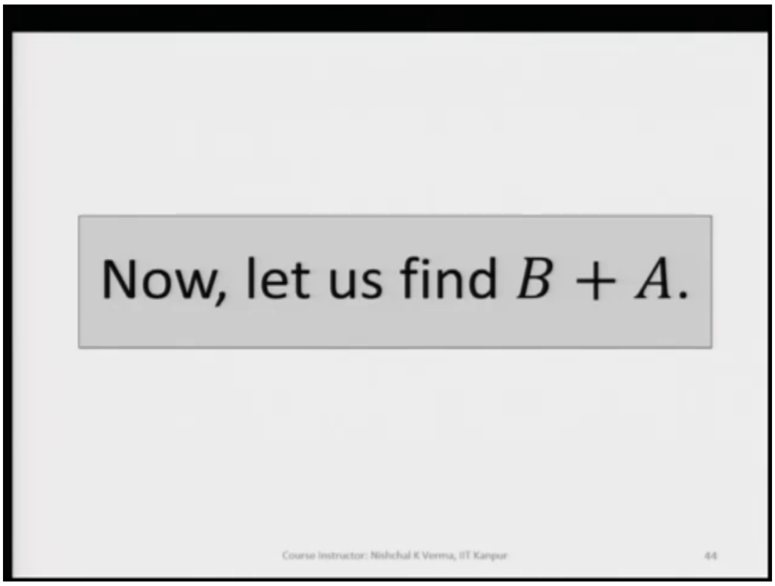

On the same lines now we can find we can compute the addition of the fuzzy number B and A. So, here we are just changing the order, we are just taking B first and then A. So, let us see what are we getting.

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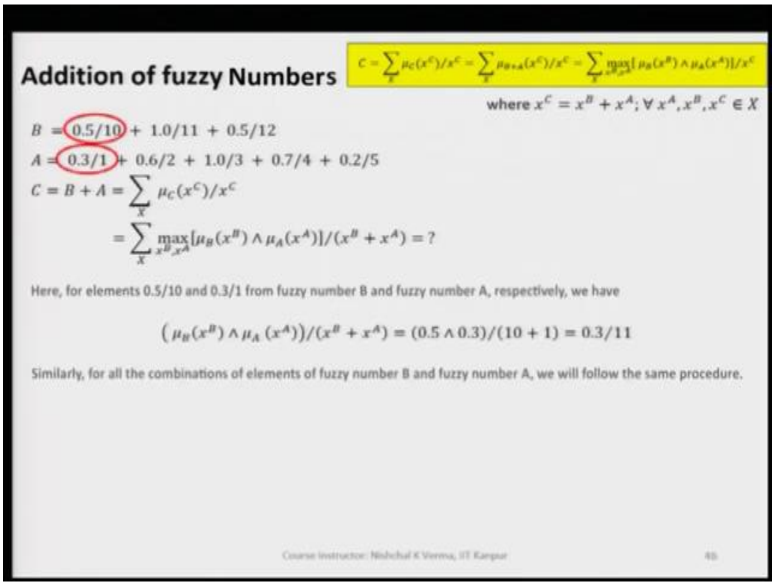

So, when we take B first and we do the same exercise.

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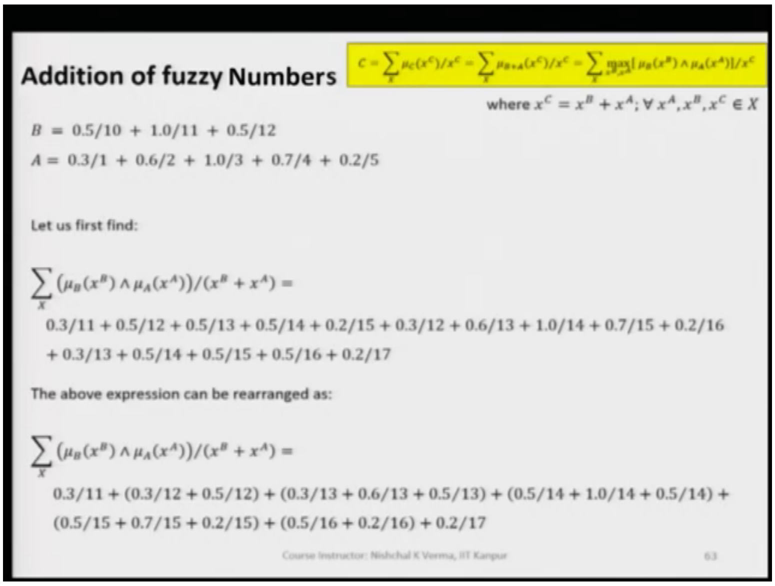

We see that here we are getting when we take this

$$\sum_X \big(\mu_B(x^B) \wedge \mu_A(x^A)\big)/(x^B + x^A)$$

So, we see that for this combination we are getting $0.3/11, 0.5/12, 0.5/13, 0.5/14, 0.2/15, 0.3/12, 0.6/13$. So, $1.0/14$ and then here we are getting $0.7/15, 0.2/16, 0.3/13, 0.5/14, 0.5/15, 0.5/16, 0.2/17$.

Now, when we rearrange these terms as I mentioned earlier, so we see that we are getting the expression like this and when we take the maximum or the max of all the membership values corresponding to the same generic variable value, we are going to get the final expression like this.

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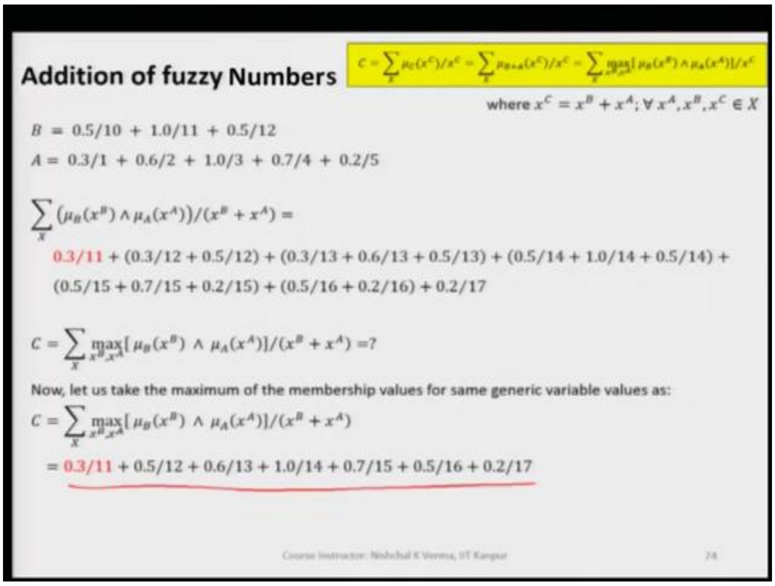

So, this is after taking the max. So, this is coming out to be $0.3/11 + 0.5/12 + 0.6/13 + 1.0/14 + 0.7/15 + 0.5/16 + 0.2/17$. So, this is how we are getting the result the resultant fuzzy set.

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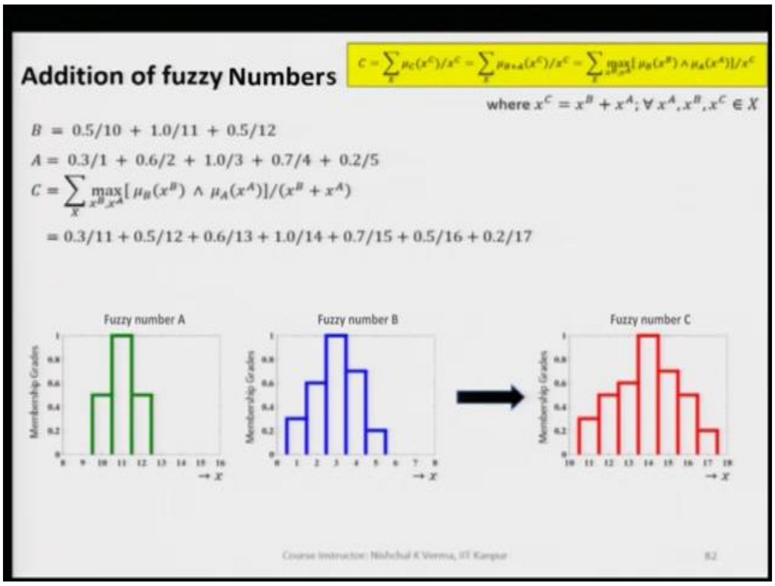

So, if we see here that we are obtain fuzzy set C which is the outcome of we obtain fuzzy number C, which is the outcome of fuzzy number A fuzzy number B and fuzzy number A.

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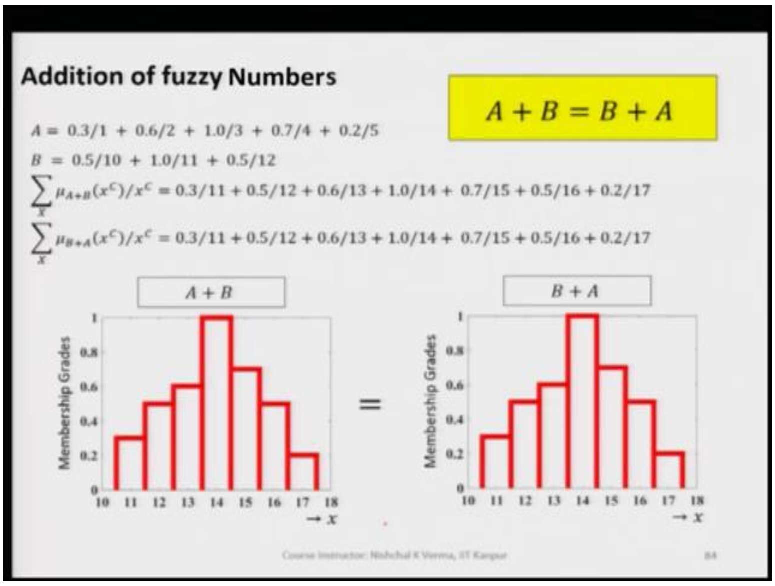

So, we see the same fuzzy number the same fuzzy set that we have obtained here and this way we can say that either we take A first or B first both the outcomes remain same. So, we can say the addition is commutative here the fuzzy addition is commutative.

So, this means the $A + B = B + A$ for fuzzy numbers. So, this way we have been able to know as to how we can add two fuzzy numbers and any two fuzzy numbers can be added this way. And, please note once again that these fuzzy numbers when we when we say fuzzy numbers means these fuzzy numbers are the fuzzy sets that satisfy the criteria of normality and convexity. It means these fuzzy numbers must be a normal fuzzy set first and then these fuzzy numbers should be satisfying the criteria of convexity.

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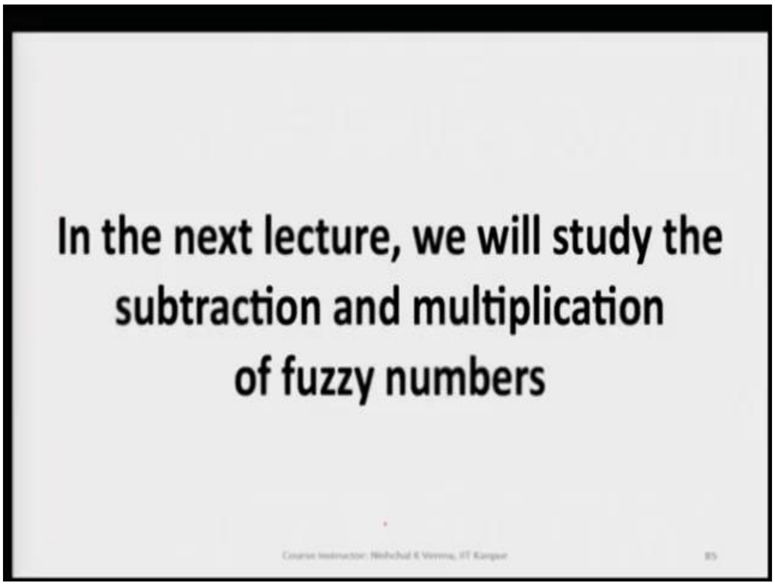

So, this way we understood as to how we can add any two fuzzy numbers. And, in the next lecture, we will study the other operations other arithmetic operations like subtraction, multiplication, divisions of fuzzy numbers.

Thank you.