Fuzzy Sets, Logic and Systems and Applications
Prof. Nishchal K. Verma
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 19
Distance between fuzzy sets

Welcome to lecture number 19 of Fuzzy Sets, Logic and Systems and Applications. And here in this lecture we will have examples on a distance between two fuzzy sets. And this lecture is in continuation to lecture number 18 where we have already discussed the formulation for finding the membership values, corresponding to the generic variable value that was lambda for the distance between two fuzzy sets.

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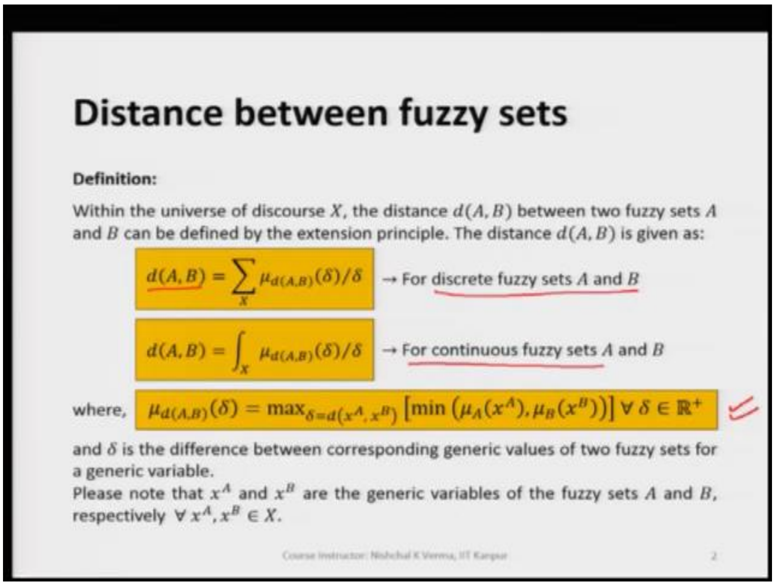

So, here before I move to the example, I would quickly go through the formulation. So, here as I mentioned earlier also that when we find the distance between two fuzzy sets let us say A and B. So, this distance is the fuzzy set this distance value is fuzzy and this is represented by a fuzzy set. So, this fuzzy set of course, will have the generic variable value delta as well as it is membership values. So, and of course here the delta is the difference between corresponding generic variable values up to fuzzy sets for a generic variable.

So, as it is written here that the distance between two fuzzy sets A and B that is represented by $d(A, B)$, which is equal to the summation of all the elements containing it's membership values over the universe of discourse. So, summation is used here for the discrete fuzzy set case and on the same lines we have another formulation, where we used integration sign integration symbol in place of a summation which is here for the continuous case.

And the membership value can be calculated by this formulation and these membership values are nothing but for respective delta values. So, since we have already explained enough in my previous lecture. So, let us now directly go to the example.

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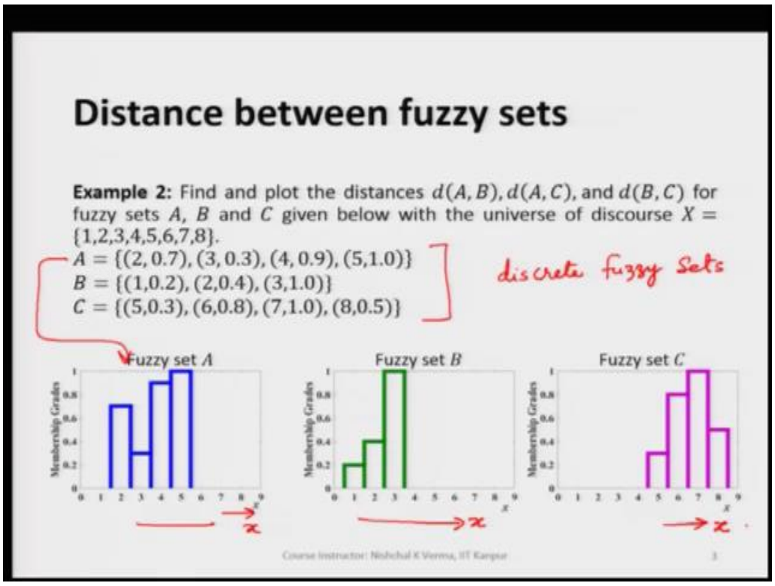

In this example we have three discrete sets A B and C and as I mentioned these are all discrete fuzzy sets. So, since these are the discrete fuzzy sets, we will apply the summation sign and we can you know accordingly minus to get all the values of the table that I already mentioned in the last lecture. So, we have here the fuzzy A this fuzzy which is given here is represented by a fuzzy set plot.

So, we have here a fuzzy set A and then we have fuzzy set B which is represented by the fuzzy set as shown here and then we have the third fuzzy set which is here. So, please note that these the generic variable for these fuzzy sets are nothing but the same the X. So, generic variable is X here in all the cases.

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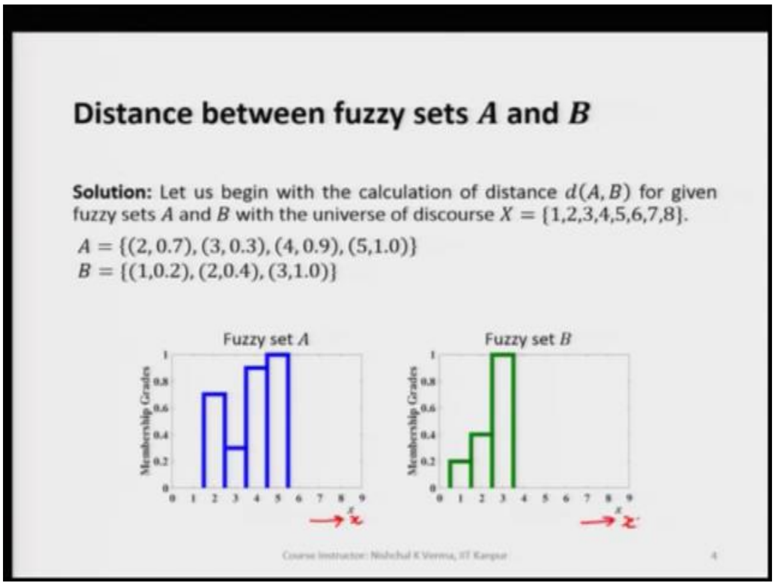

So, here let us now start with finding the distance between two discrete fuzzy sets, here also we have X here also we have X as generic variable in both the cases.

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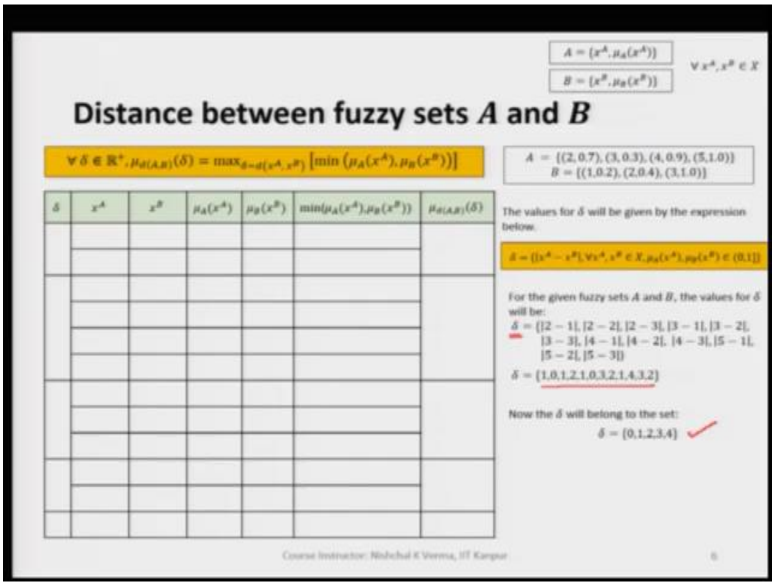

And if we see here as I mentioned in the last lecture that we have a table and this table will have the delta value which is here. And then we have x_A so x_A is nothing but the generic variable value which is present in the discrete set A similarly here is the generic variable value present in fuzzy set B. And here $\mu_{A}x_A$ and $\mu_{B}x_B$ both are the respective membership values and then we take min of these two $\mu_{A}x_A$ and $\mu_{B}x_B$.

So, that is how we fill these entries. So, let us now quickly go through and please note here that we have delta which is here this is always positive real number. So, that has to be noted because we are using here the mod of x A minus x B, so obviously delta is going to be the real positive value. So, let us quickly go through completing the this exercise of calculating the difference between two the generic variable value or in other words all the possible differences right from 0 and above.

So, when we find all combinations we get these values of δ. So, this δ will have 1 0, 1, 2, 1, 0, 3, 2, 1, 4, 3 ,2. So, like that when we rearrange this we will get $\delta(A)$ set which is 0, 1, 2, 3, 4. So, let us now start with delta is equal to 0.

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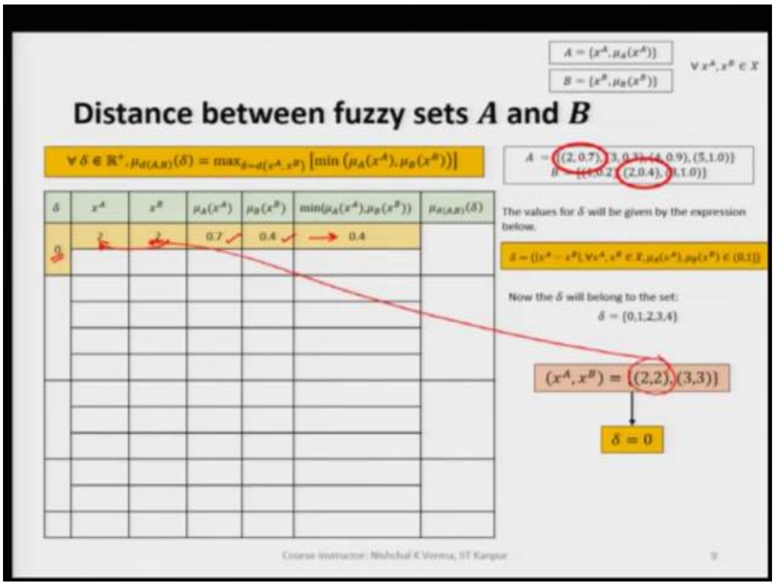

And here we will see that we have these two entries these two pairs for which we get $\delta = 0$. So, it is like this and these values are filled over here see here these values will go here and here so x_A, x_B and of course this is for $\delta = 0$. Similarly, we will find from the discrete fuzzy set given the respective membership values and then when we take min we are going to get 0.4. Now let us enter the values for the second pair.

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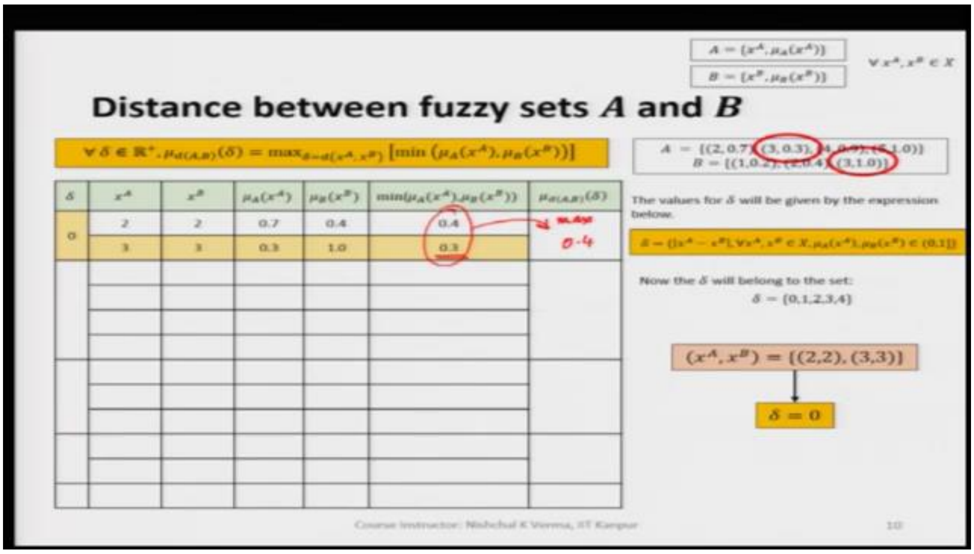

So, second pair is 3, 3 and we can see that these values are entered and the minimum we are getting here is 0.3. Now we have to take max of these two. So, this is here is used to compute the max, max is going to be 0.4 max of these two. So, as I already explained in my last lecture, lecture number 18 and 17 both that we first arrange all the values of lambda and then we find all the entries fill in the table and then we take the max values in the last column.

And then we use first column and the last column for constructing the fuzzy set and this fuzzy set is nothing but the distance between these two discrete fuzzy sets here. So now, let us move ahead and take $\delta = 1$.

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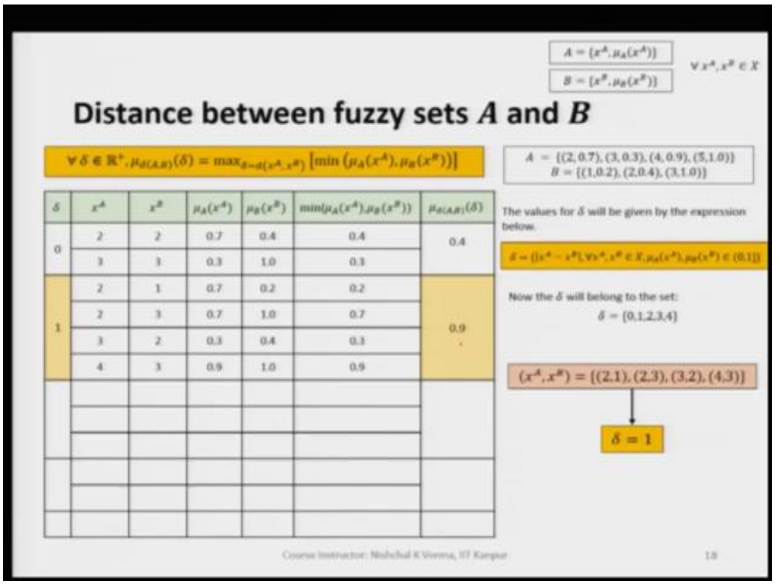

So when we take $\delta = 1$. So, we see that these are the combinations these are the pairs these four pairs are there and these four pairs will be entered in the table. You can see here I am quickly moving ahead and since you already know as to how we fill these entries, so we can quickly fill these entries here. Now for $\delta = 1$. So, we get the min values as listed here and when we take the max of these we are going to get 0.9, so this is written over here.

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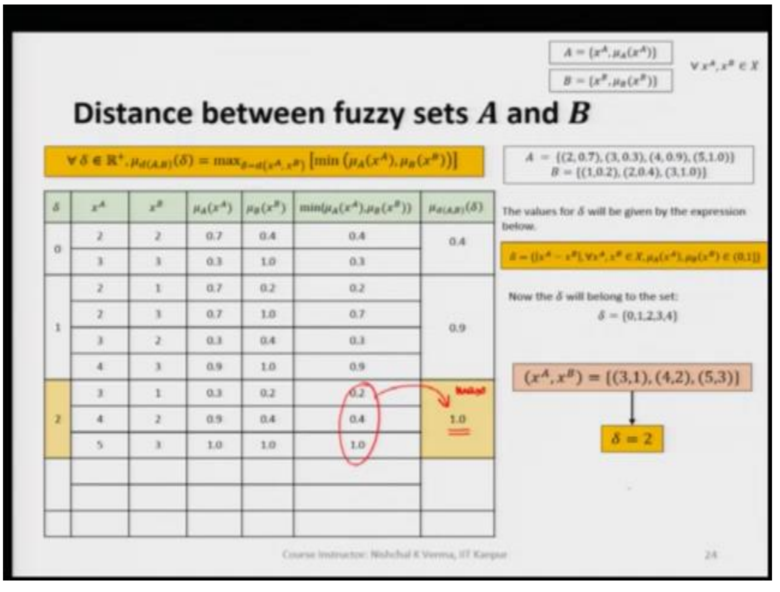

Similarly, let us go and have all these entries for $\delta = 2$. So, when we take $\delta = 2$ we get these three pairs and as I have already explained these pairs are coming from both the fuzzy sets that are given. The both the discrete fuzzy sets A and B, now let us fill these entries also here in the table.

So, when we fill these entries we are going to get here all the entries filled and we see here that we are getting the max of all these as 1. So, this way we are getting $\mu_{d(A,B)}(\delta) = 1$. So now, let us move ahead and find all the entries for $\delta = 3$.

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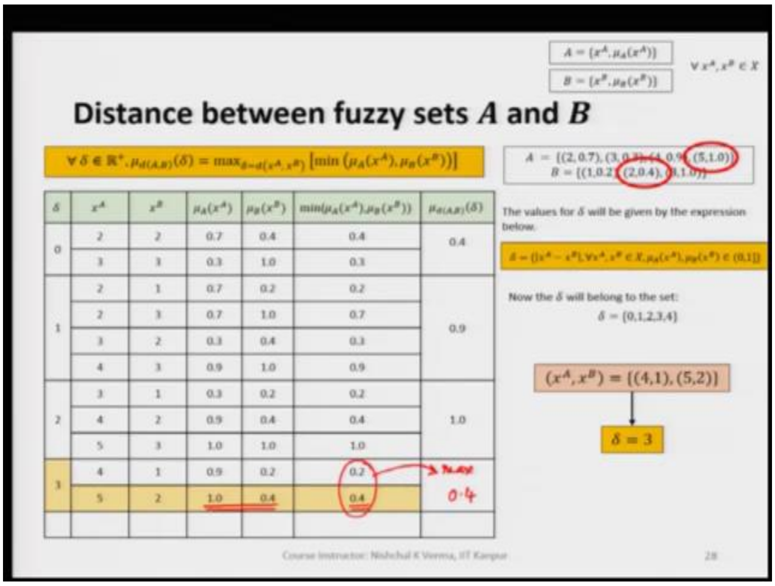

δ	x^A	x^B	$\mu_A(x^A)$	$\mu_B(x^B)$	$\min(\mu_A(x^A), \mu_B(x^B))$	$\mu_{d(A,B)}(\delta)$
0	2	2	0.7	0.4	0.4	0.4
	3	3	0.3	1.0	0.3	
1	2	1	0.7	0.2	0.2	0.9
	2	3	0.7	1.0	0.7	
	3	2	0.3	0.4	0.3	
	4	3	0.9	1.0	0.9	
2	3	1	0.3	0.2	0.2	1.0
	4	2	0.9	0.4	0.4	
	5	3	1.0	1.0	1.0	
3	4	1	0.9	0.2	0.2	
	5	2	1.0	0.4	0.4	

So, when we move ahead we see that we have two pairs which are resulting $\delta = 3$. So, let us fill these entries as well. So, when we fill these entries we find that the min is coming as 0.2 and then for the second pair when we see we are getting 0.4 as the min of these two. And as we have done in previous entries we take the max here finally and when we take max we are going to get 0.4, so that is how we have entered here 0.4. Now for the last value that is a $\delta = 4$

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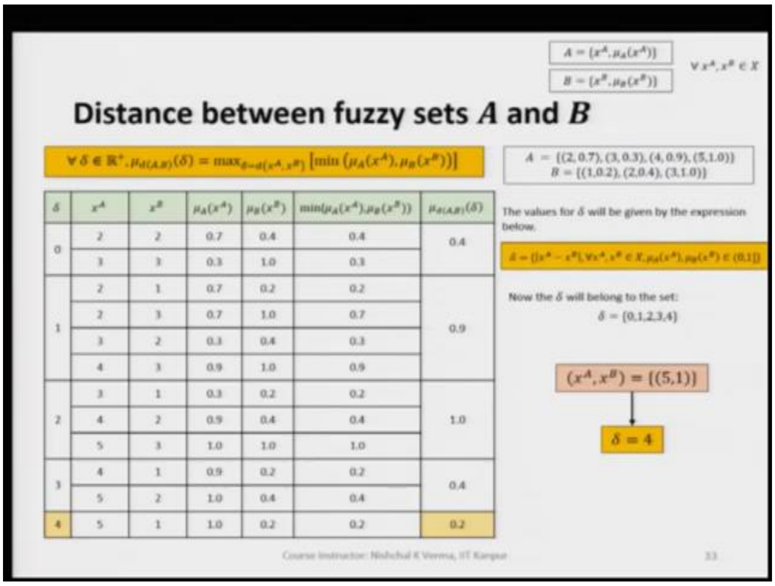

So, when we take $\delta = 4$ we are getting here a 5 and 1 only one pair for which we are getting the difference that is delta 4 and this we are entering here and of course since we have only one row here. So, max is going to be the same that is 0.2. So, when we do that we quickly get all the delta values which are mentioned in the first column and the corresponding membership values that are mentioned in the last column.

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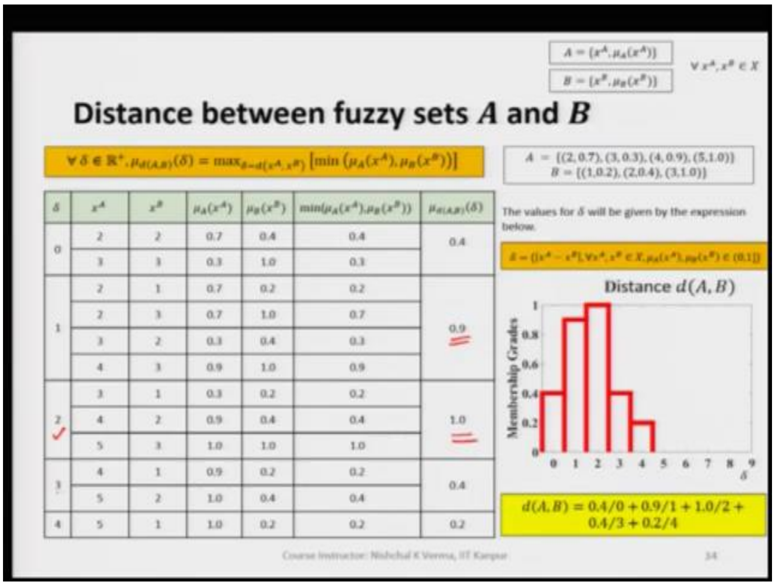

So, now may make use of these two columns like for $\delta = 0$, the corresponding membership value is 0.4 and then for $\delta = 1$ corresponding membership value is 0.9. And then for $\delta = 2$ corresponding membership value is 1 here and then for $\delta = 3$ the corresponding membership value is 4.

Similarly, for $\delta = 4$ we are getting corresponding membership value 0.2. Now let us make use of this to construct the fuzzy set which is the resultant fuzzy set which is nothing but the distance. So, here we have the distance between a two fuzzy sets A and B.

So, since we have written here we have made the use of the entries of column number one and the first column and last column and then we are getting here all these entries. And when we construct a fuzzy set this will look like this, this is nothing but the distance between two fuzzy sets A and B and this way we are able to get the distance between two fuzzy sets. Here in this case we have two discrete fuzzy sets and that is how we are able to calculate the distance and which is again coming out to be a discrete fuzzy set.

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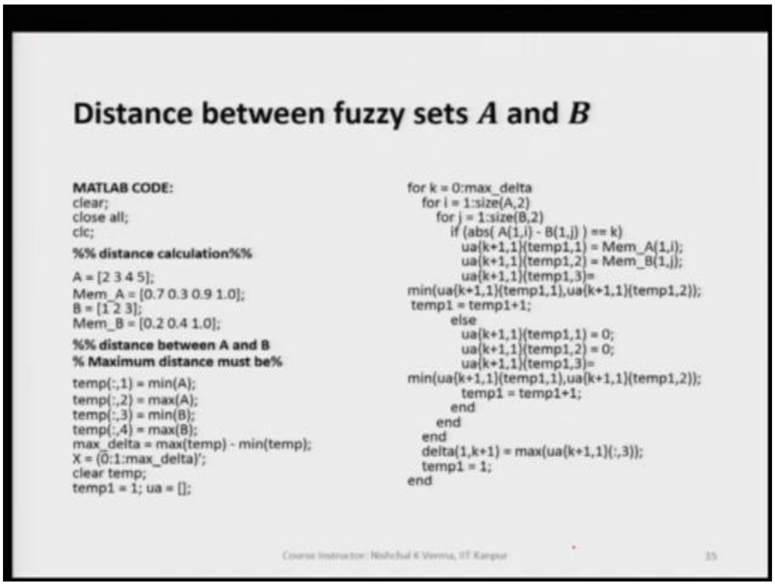

Now, we have a MATLAB code here. So, if you are interested you can make use of these MATALB codes, you can generate you can compute the distance between two fuzzy sets A and B and then you can generate the resultant fuzzy set. So, this is the same code that was used to generate all these figures plots and entries as well.

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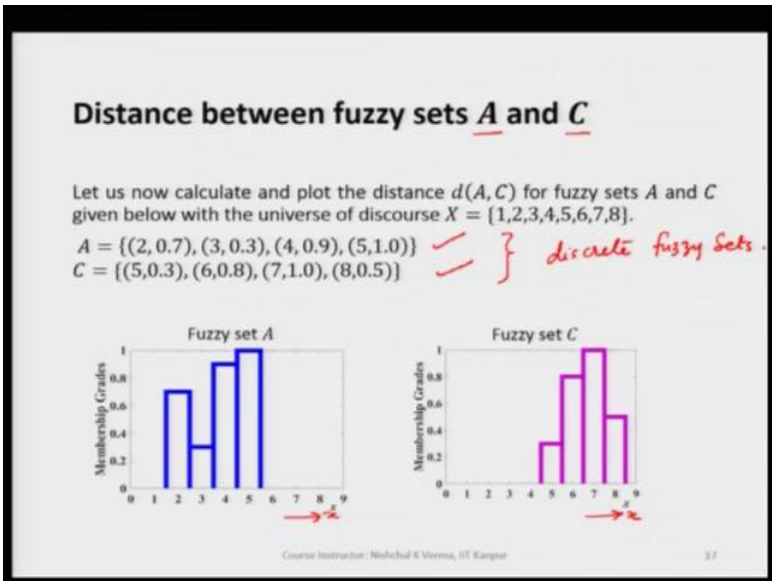

So, let us now take another example here, where we have to compute the distance between two discrete fuzzy sets A and C. In earlier case we computed the distance between fuzzy A and fuzzy B here we are computing the distance between fuzzy set A and fuzzy set C.

So, since these sets are already given A has been given like this and C fuzzy set has been given like this, both of these fuzzy sets are discrete fuzzy sets again I am writing this discrete fuzzy sets. So of course, as we have seen in the previous examples that the resultant fuzzy set the distance is also going to be the discrete fuzzy set. So, here also we see that we have the generic variable x in both the cases.

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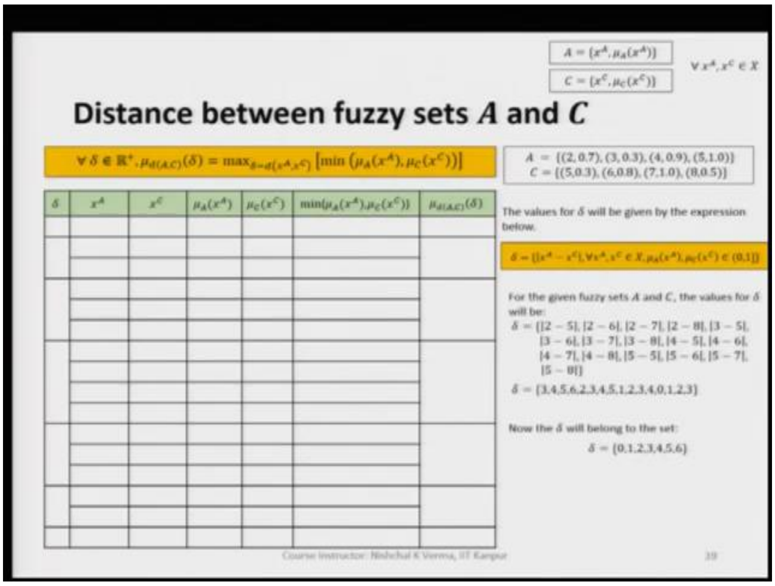

And when we take these two fuzzy sets again on the same lines as we have done in the previous example, if we try to find the delta values. So, as I had already explained and I already suggested that we should start with delta is equal to 0.

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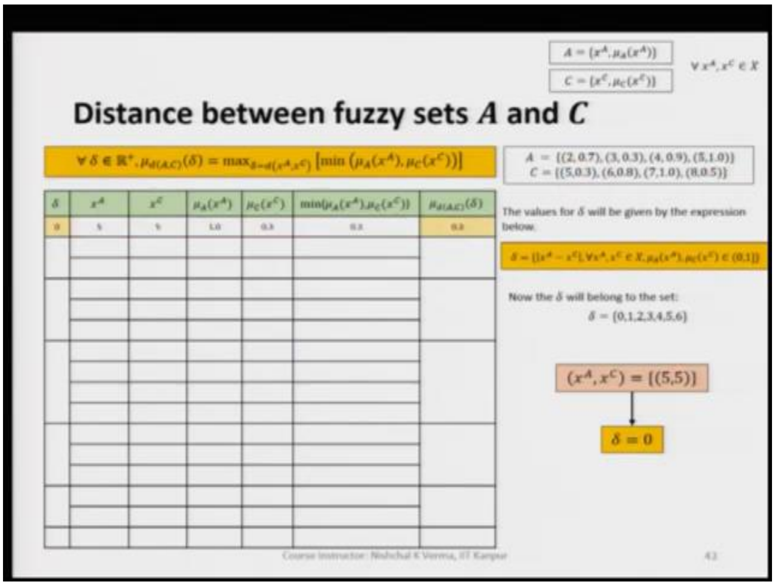

So, for $\delta = 0$ here we get these pairs from x_A and x_C, the generic variables of both the fuzzy sets and in this case $\delta = 0$ we are getting only one pair that is 5, 5. So, let us quickly enter this value in the table. So, as we have already seen in the previous examples we have a delta column which is the first column. So, let us enter the delta value and this is nothing but 0. So, we enter here in the first row we are entering here $\delta = 0$. Now let us quickly take these values of x_A and x_C which are like this 5 and 5 for which delta is coming out to be 0.

So, we can fill these entries and now we will go ahead and find the corresponding membership values. So, let us now see what are the corresponding membership values. So, here if you see 5 is from here so corresponding membership value here is 1 and then in the fuzzy set C we see that we have corresponding to 5 we have membership value here as 0.3. So, when we take minimum of these two we are getting 0.3 and then similarly see here we have a $\delta = 0$ we have only one row.

So, the max of 0.3 since we have only one element so we can straight away right 0.3. So, this way we are able to enter the first row values for $\delta = 0$.

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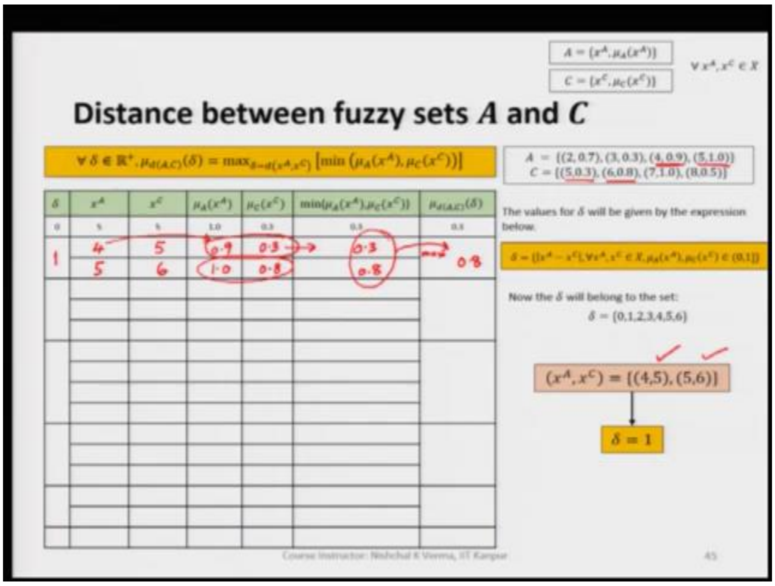

Now, let us quickly go ahead and take delta is equal to 1. So, we have delta is equal to 1 here and let us find all these pairs which are responsible for creating delta is equal to 1. So, we have two pairs here first pair and the second pair first pair is 4, 5, second pair is 5, 6. So, let us now enter these values. So, first pair here is a for delta is equal to 1, we have x_A 4 and x_C 5 and here we have 5 the other pair values 6 like this and then we'll find the corresponding membership values. So, corresponding membership value here will be 0.9 corresponding to 4 so 0.9 and then corresponding to 5 we have 0.3.

Similarly for 5, 6 we have here corresponding to 5 we have membership value 1 and then we have here corresponding to 6 we have 0.8. So now, here if you take minimum of these two we are getting 0.3 and minimum of these two we are getting 0.8. When we take maximum of these two we are getting 0.8. So, this is how we are able to enter the values corresponding to $\delta = 1$.

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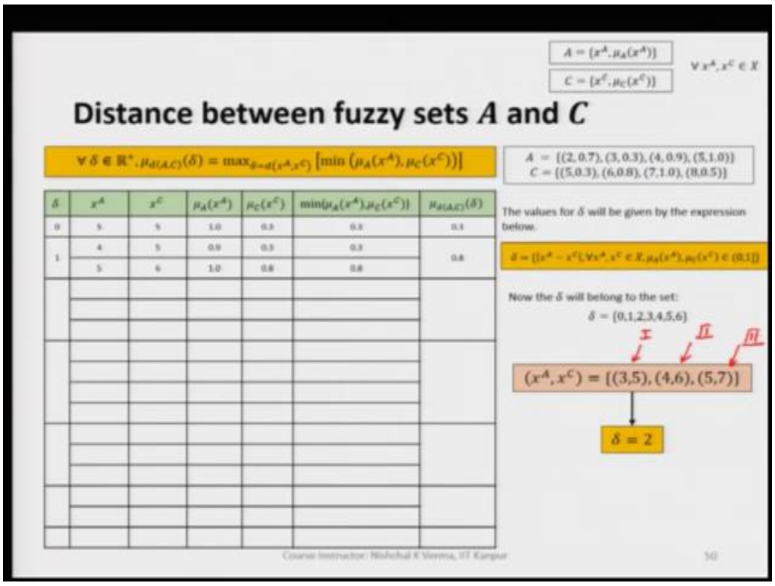

Now, let us enter the values of $\delta = 2$. So, when we take $\delta = 2$. So, let us see how many pairs we get which are giving us the $\delta = 2$ the difference 2. So, we get three pairs so first pair, second pair and third pair all these pairs are responsible for generating the difference of two.

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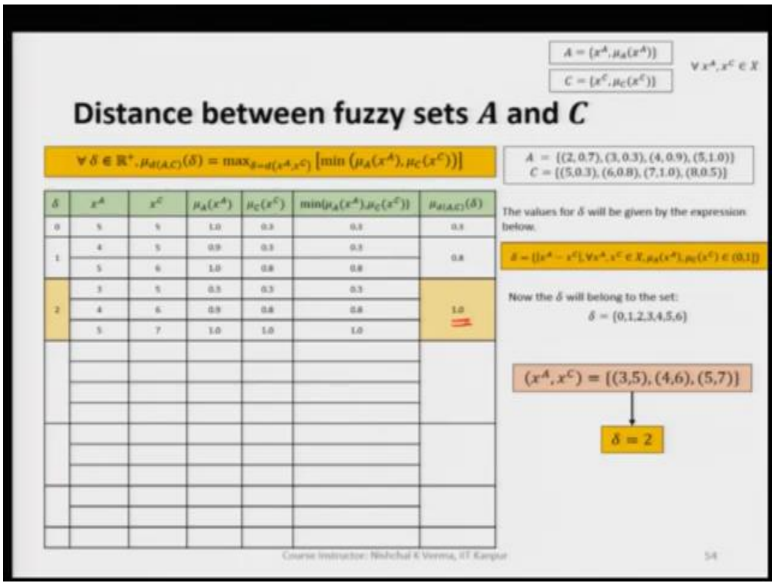

So, let us now quickly enter these values here and we know as to how we enter these values and then when we take maximum of these we are going to get 1. So, this is how we are getting the corresponding to $\delta = 2$, we are getting it's corresponding membership value as 1.

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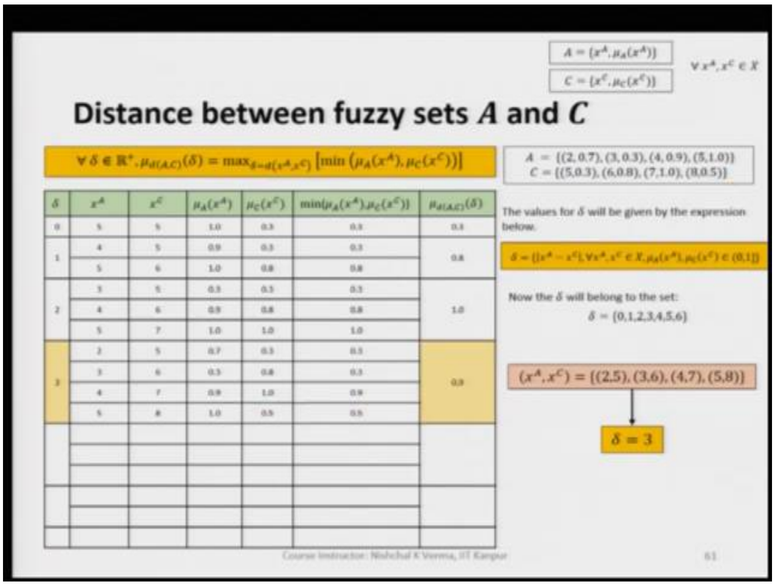

So, now let us find the number of pairs for $\delta = 3$. So, here we see that there are 4 pairs which are coming from fuzzy set A and C, as x_A and x_C values and the generic variable values. So, these four pairs let us enter quickly here in the table. So, here we have entered all these values and then when we take maximum we are getting here as 0.9.

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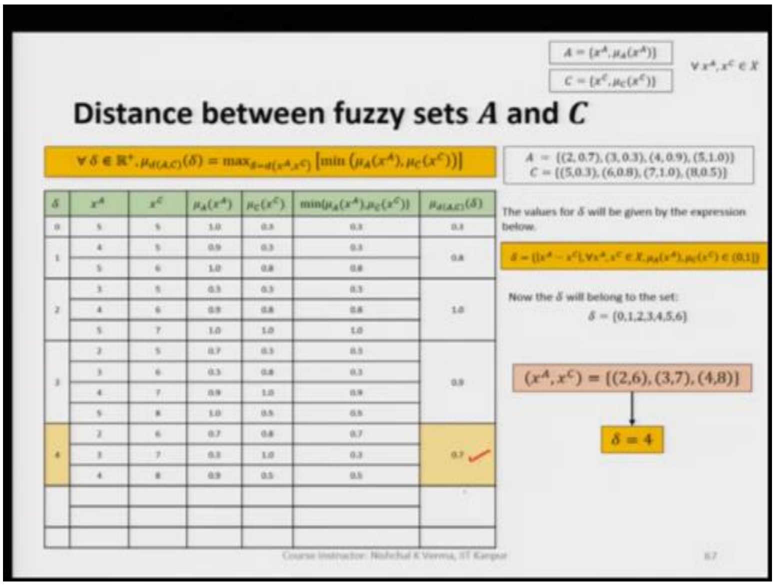

Similarly, now let us go ahead and find all those pairs which are responsible for generating the difference that is $\delta = 4$. So, $\delta = 4$ let us see how many pairs are responsible for generating. So, there are three pairs so first pair here 2, 6 and then 3, 7 and 4, 8 these three pairs are responsible for generating $\delta = 4$. So, $\delta = 4$ as I already mentioned is nothing but the difference between the x_A and x_C and these values are basically the generic variable values and these values are coming from the fuzzy sets that are given to us.

Here in this case these fuzzy sets are A and C and these both the fuzzy sets are discrete fuzzy sets. So, this way we are able to quickly manage to enter all the values in the table corresponding to $\delta = 4$ and the maximum value that we are getting here is the max of all these entries we are getting here as 0.7.

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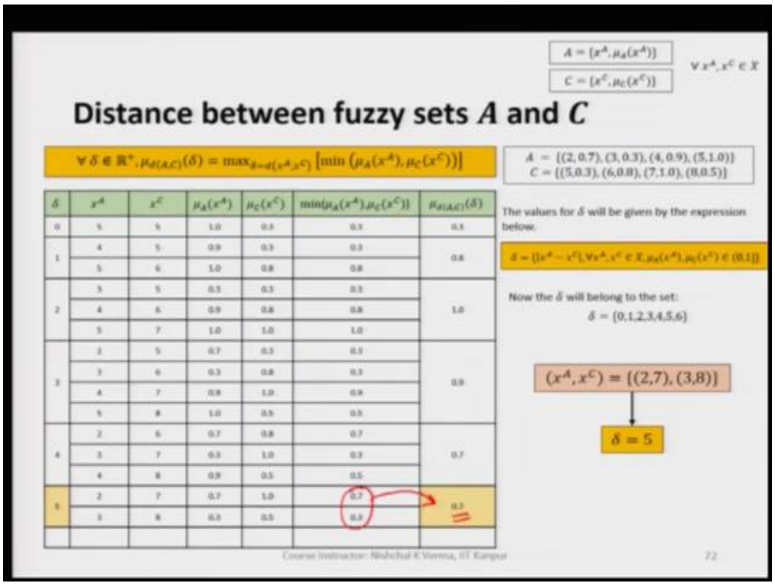

Now, let us do the same exercise for $\delta = 5$ and when we see we are getting two pairs 2, 7 and 3, 8 and both are responsible for $\delta = 5$. So, let us enter these values here and here we are getting 0.7 as the maximum of 0.7 and 0.3 so this way this is also done.

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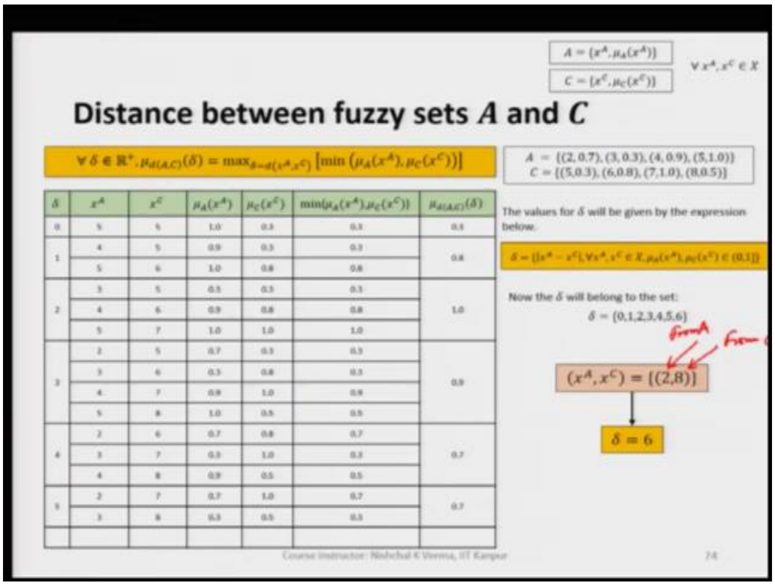

And now let us go ahead and try to find all the entries corresponding to $\delta = 6$. So, when we want to have the difference of 6, let’s see how many pairs that we are having which are responsible for creating the difference of 6. So, this is pair of 2 and 8 which is responsible for creating $\delta = 6$ that is the difference. And of course as I already mentioned that this is coming from the A set and this is coming from the 8 is coming from C fuzzy set.

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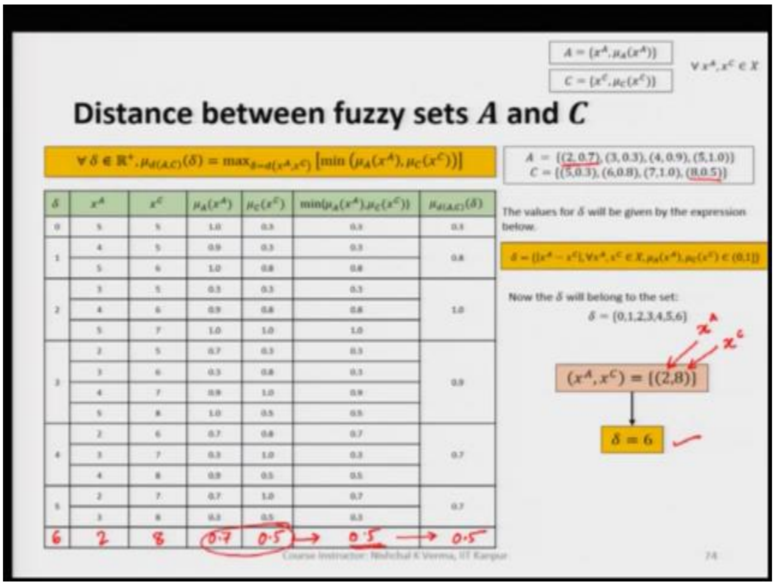

So, are in the other words we can say this is the generic variable value from A set and this is the generic variable value from C set. So, here we have only one pair and this pair is responsible for creating the distance of 6, now let us enter this quickly. So, we will right here at 6 and then we will right here two we will right here 8 and then let us now look for the corresponding membership values, we will directly get these values from the fuzzy set that has been given to us.

So, corresponding to 2 we are getting it's membership value 0.7 this is coming from here from fuzzy set A. And now corresponding to 8 which is here now let us quickly find and enter it's membership value which is nothing but 0.5. When we take min of these we have getting 0.5 and since we have only one row, so when it comes to taking max so max will remain the same.

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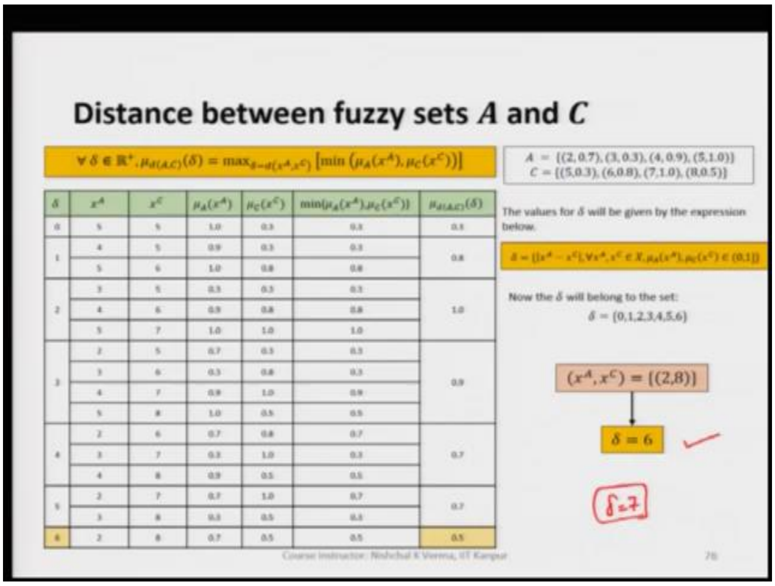

So, this way we got all the entries that are needed to compute the distance between two fuzzy sets. And now if we see here at the table we have the first row as the delta row that is all the possible values or delta are there. And I would like to mention one thing here that see delta is equal to 6 we have computed we have found all the values we have entries in the table. So, can we have any entry for $\delta = 7$ also. So, the answer is no because we cannot find any pair here which are responsible for creating the difference of that is $\delta = 7$. So, no pair is responsible for creating $\delta = 7$.

So, that is how or that is why we are not including the $\delta = 7$, because no pair here is present which can create a difference $\delta = 7$. So, that is why we stop here and in the table we are having only entries of $\delta = 6$.

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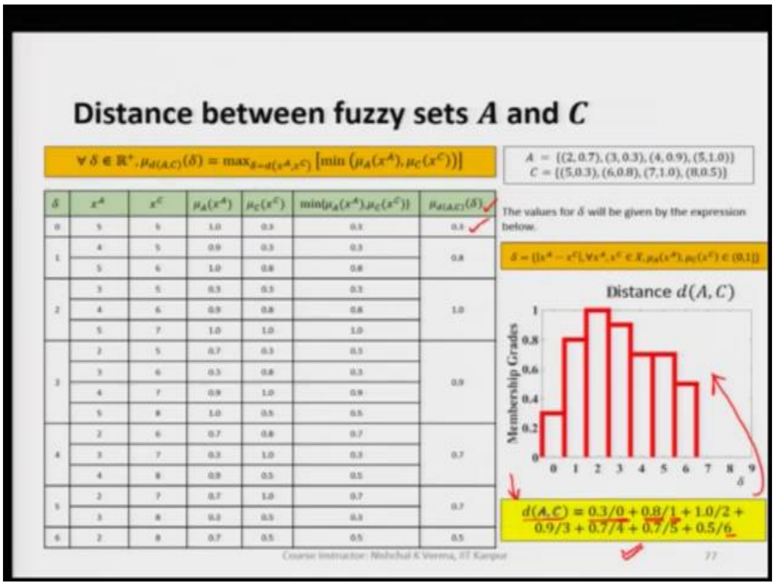

So, now we are using these delta values and it is corresponding membership values which are in the last column and we see that we have corresponding to $\delta = 0$. So, for 0 we are getting 0.3 as the membership value. So, we have written over here and then we have 1 for delta value 1 we have 0.8. So, this is also written and similarly for all the values of delta up to 6 we have written all the elements all the membership values and this $d(A, C)$ this $d(A, C)$, means the distance between two fuzzy sets A and C we are getting A fuzzy set which is nothing but the distance between two discrete fuzzy sets A and B.

So, this way we are able to find the distance in between two fuzzy sets and it is needless to say that here both the fuzzy sets are discrete and hence they are getting the distance also a discrete fuzzy set and this can be plotted here like this. So, I in the last lecture I mentioned as to how we can plot the resultant fuzzy set which is here you can see.

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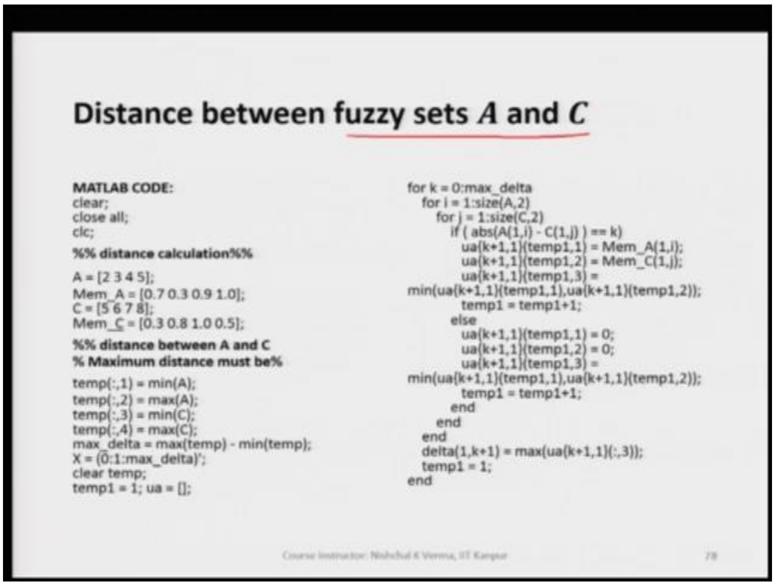

So, here also for the distance between two fuzzy sets A and B we have the MATALB code and you can use these MATALB code for computing the distance between two fuzzy sets and this MATALB code can also be used for plotting the resultant fuzzy sets.

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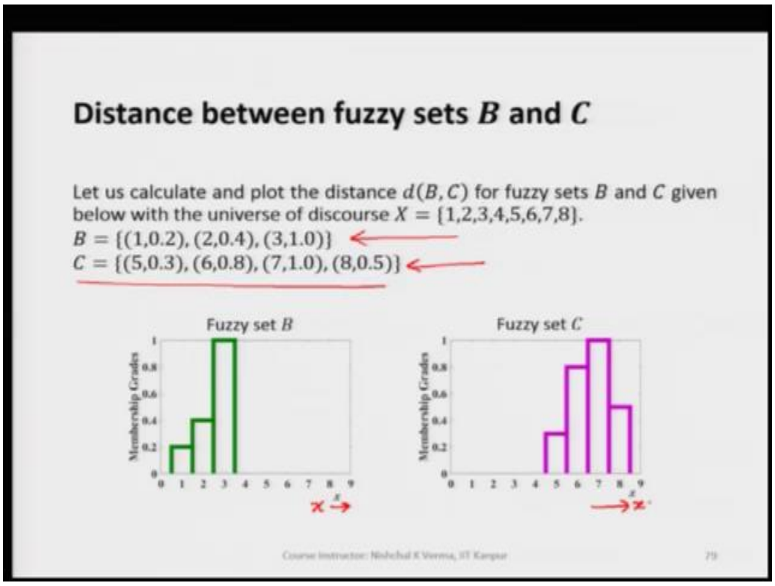

Now, let us find the distance between fuzzy sets B and C. Where we have B as this discrete fuzzy set which is given and C here is another fuzzy set which is given by this expression. So, we have again all these two fuzzy sets B and C both are on the same generic variables.

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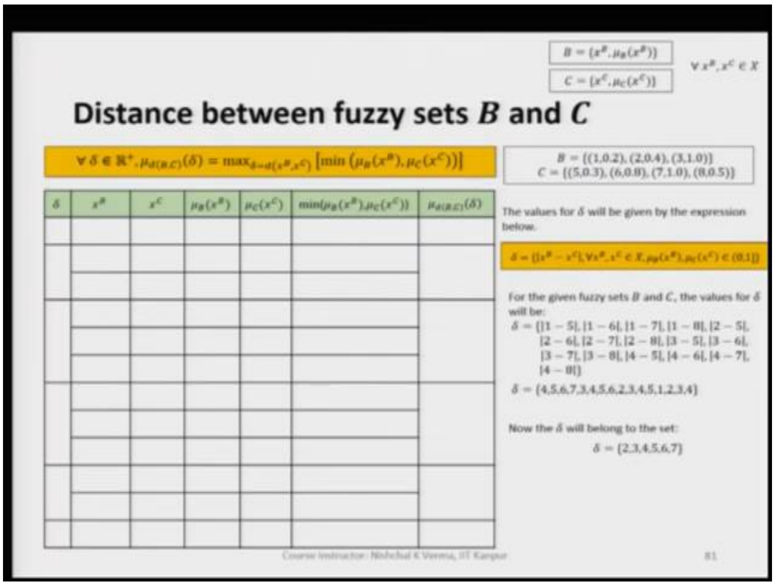

So, let us quickly now go ahead and find out the distance between these two fuzzy sets. So, we remember that we first need to find a set of delta values and this delta values will start from 0. So, we will quickly see that first we will try to find the difference of 0 and for this difference of 0, we try to use the generic variable values of both the sets.

So, we see that we do not find any pair any such pair which creates $\delta = 0$. Similarly, we do not find any such pair which create the difference of one that is $\delta = 1$. So, here a delta is equal to 0 and 1 we are not including in the set and then when we move ahead we find that for $\delta = 2$, means the difference of two can easily be created and from the entries that are given in the fuzzy sets B and C. So, 2, 3, 4, 5, 6, 7 all these delta values are possible, we will see as to how we are getting these differences.

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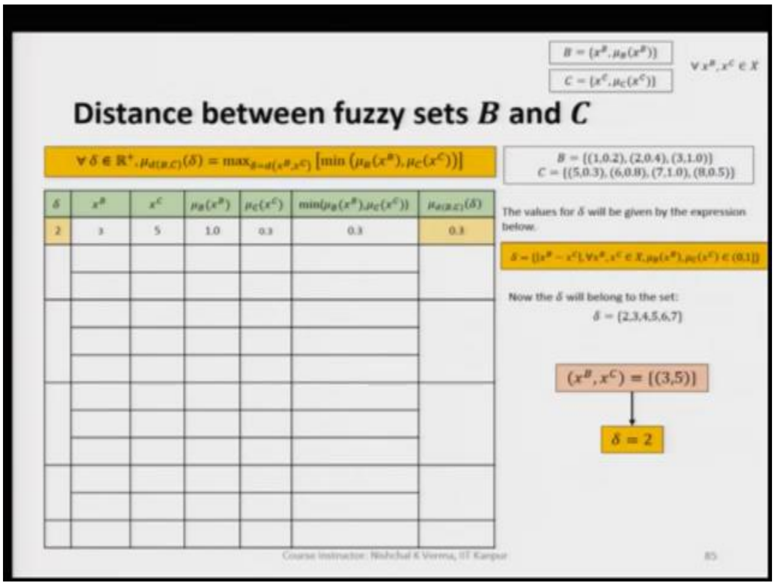

So, let us take first the $\delta = 2$. So, $\delta = 2$ we have one pair these entries we have. So, 3 and 5 so this is one pair and this pair is responsible for creating the difference of 2 that is $\delta = 2$. So, let us quickly enter all these entries in the table and that is how we get the max here also 0.3 in the last column. And now let us quickly go ahead and find all these entries for $\delta = 3$.

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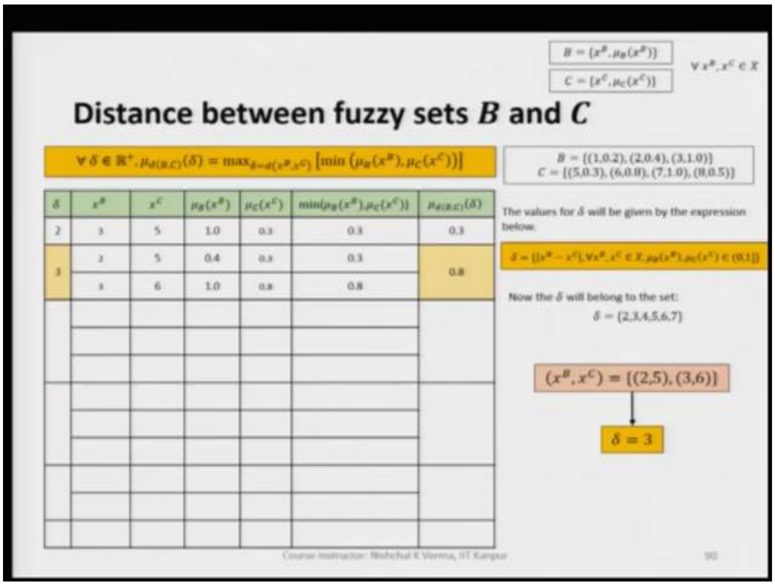

So, for $\delta = 3$ we see all these entries and you can follow the same procedure for finding all these entries. And you know when you take max of these min values here then you get 0.8 for $\delta = 3$ and let us now see what we are getting for delta is to 4.

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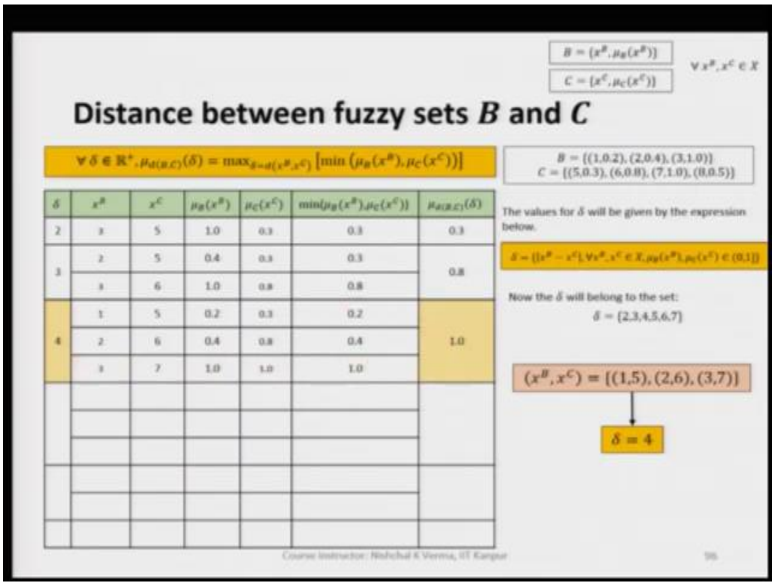

So, for $\delta = 4$ we are getting three pairs, here first pair second pair and then we have third pair. So, this way we get three pairs and all these three pairs we are entering in three rows and then when we take the max of these min entries. So, we are getting 1.0 means 1. So, we quickly get the corresponding to 4 means $\delta = 4$ we are getting it’s membership value 1.

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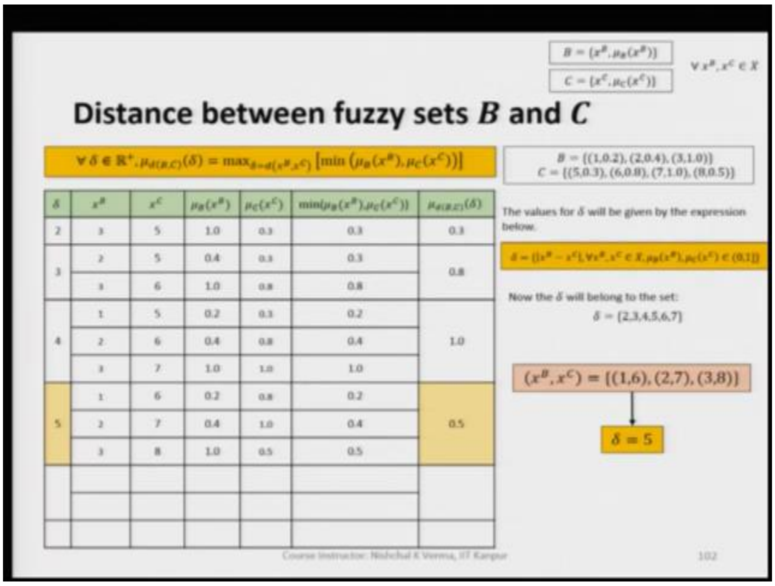

Now, let us take $\delta = 5$ and again we have a three pairs of x_B and x_C. So, let us now enter these values on the same manners, you see here so we get for $\delta = 5$, we get it is membership value as 0.5.

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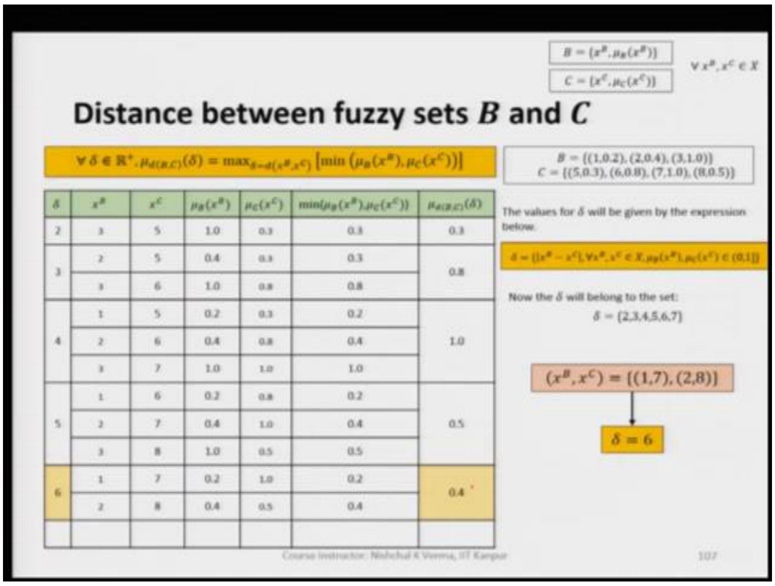

And when we take $\delta = 6$ we get all these entries filled and since we have two pairs only which are responsible for creating the difference of a 6, that is delta is equal to 6 we have two rows here and when we take max of this we are getting 0.4. So, we are having it's membership value 0.4.

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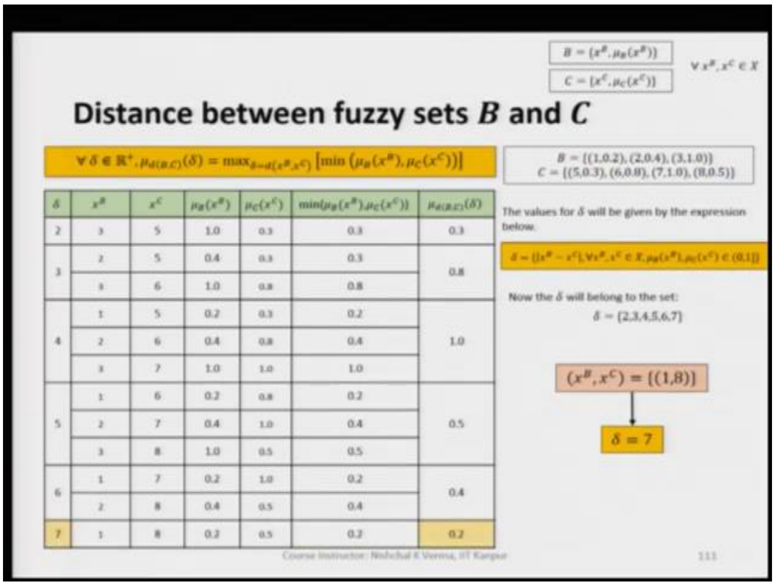

Next is a $\delta = 7$. So, for $\delta = 7$ we are having only one pair and this pair here for delta is to 7. So, we enter the values of x_B and x_C which are responsible for creating the difference between these as 7. So, we just enter these values here so x_B here is 1 and x_C here is 8 and this way we are getting it's corresponding membership values from the fuzzy sets B and C and that is how we fill these entries.

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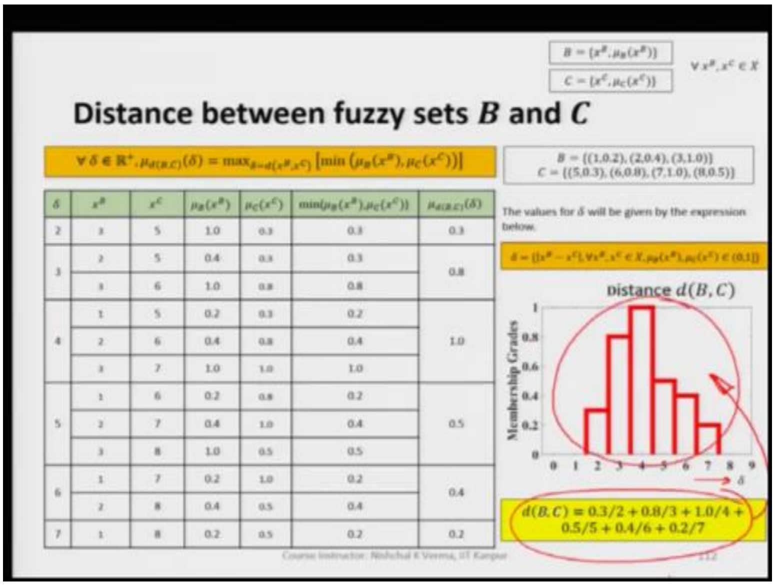

So, we see here that when we pick these membership values and the corresponding delta values from the last column and first column respectively. We see that we are able to get a discrete fuzzy set and this is nothing but the distance between two fuzzy sets B and C over the generic variable x. So, when we are interested in plotting this we are getting this plot, so this plot represents the distance between B and C.

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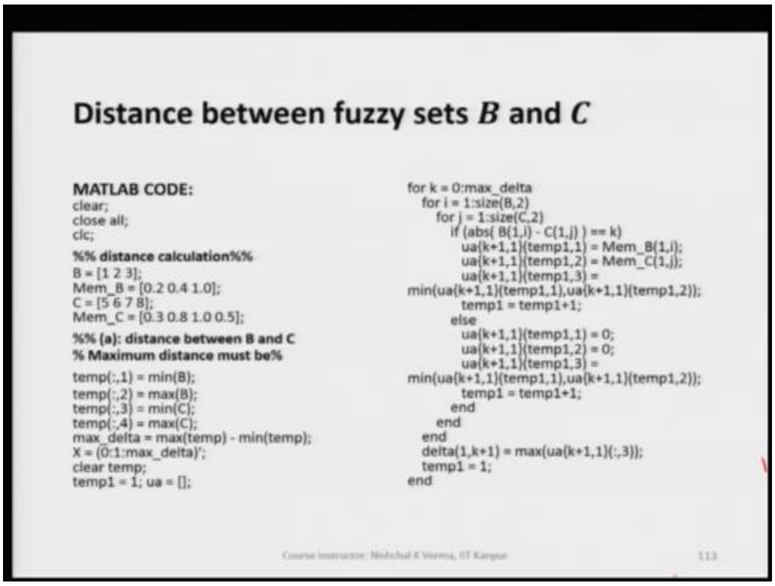

And here we have the MATALB code for finding the distance between two fuzzy sets B and C. So, if you are interested this MATALB codes you can use for finding the distance between two fuzzy sets and generating the plot for the resultant discrete fuzzy set. So, this way we have seen that the distance between two fuzzy set can be found.

So, if the fuzzy sets are discrete fuzzy sets, the resultant distance between these two fuzzy sets will be a discrete fuzzy set and if these fuzzy sets are continuous fuzzy sets, ofcourse, the distance between these two fuzzy sets are going to be the continuous fuzzy sets. So, this way we understood as to how we can manage to find the distance between two fuzzy sets. So, here we will stop and then in the next lecture we will study the fuzzy arithmetic.

Thank you very much.