Fuzzy Sets, Logic and Systems and Applications
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Lecture - 18
Distance between Fuzzy Sets

So, welcome to lecture number 18 of Fuzzy Sets, Logic and Systems and Applications. So, this lecture we will discuss the Distance between two Fuzzy Sets which is in continuation to over previous lecture, lecture number 17, where we have discussed how to find the distance between two fuzzy sets A and B.

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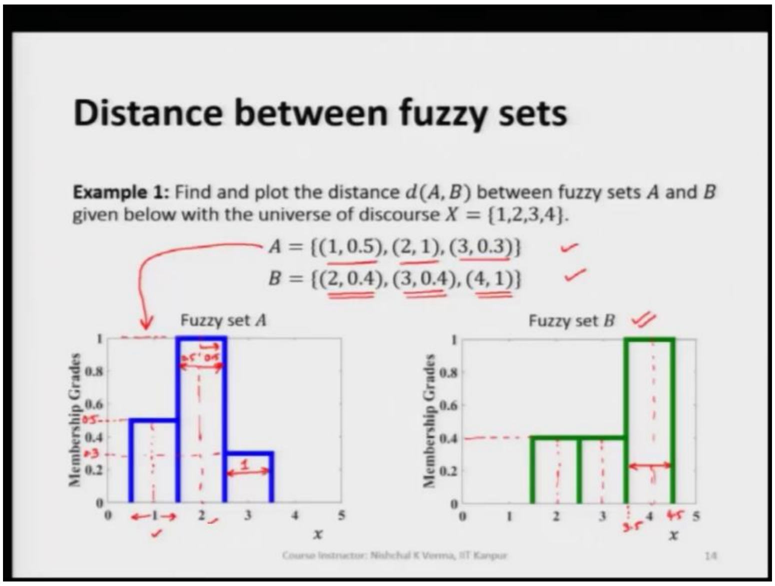

So, if we look at the fuzzy sets that we have taken these fuzzy sets were 2 discrete fuzzy sets, A and B and this is how we had plotted these 2 discrete fuzzy sets A and B.

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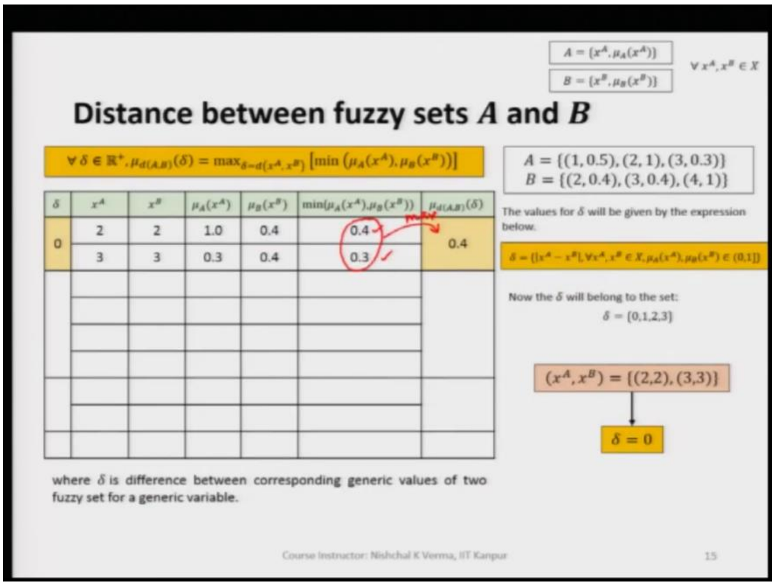

And in the previous lecture we also saw as to how we can tabulate the values of $\delta, x_A, x_B, \mu_A(x_A), \mu_B(x_B)$ and then we took minimum of the 2, that means, minimum of $\mu_A(x_A)$ and $\mu_B(x_B)$. So, when we have found the minimum of $\mu_A(x_A)$ and $\mu_B(x_B)$, we with respect to the previous example we found 2 values of minimum of $\mu_A(x_A), \mu_B(x_B)$ which were 0.4 and 0.3. So, in the last column of this table, we had to take the maximum of these values.

So, here we have 2 values $0.4, 0.3$, when we take the maximum of the these 2 values we get 0.4. So, this 0.4 is with respect to the delta value which is a 0 which was taken 0 and correspondingly we have taken the x_A, x_B values. Similarly, now we will move ahead and we will try to find if we have any such combinations which are leading to $\delta = 1$.

It may be possible that we may not find any such pairs for which $\delta = 1$ and if this is the case then we will move ahead and we will try to find the pairs for which we get $\delta = 2$. And this way we will keep moving ahead till the highest value for which the x_A, x_B pairs we are getting the highest value of δ.

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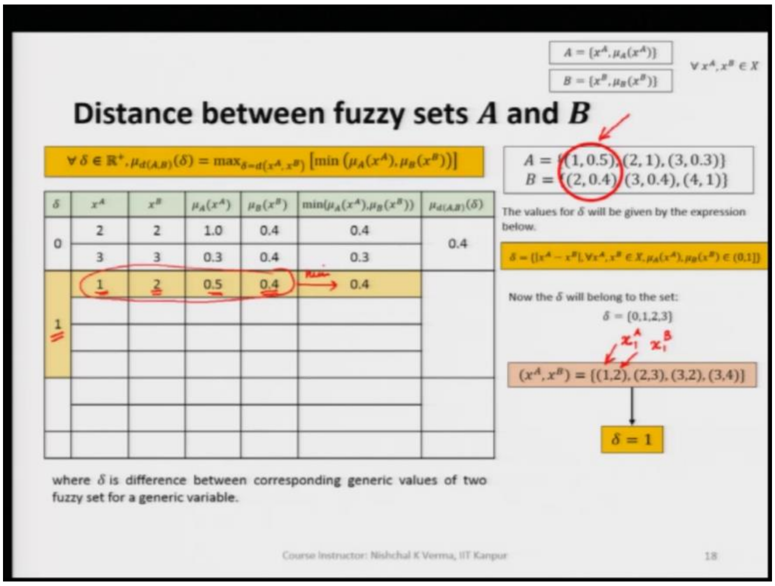

All right. So, let us now quickly try for $\delta = 1$ which is here. So, if we start finding the x_A, x_B values from these 2 discrete fuzzy sets A and B which are given, so we see that we have 4 such combinations. We have 1 from fuzzy set A which is here and 2 from fuzzy set B and then we have another element another pair $2, 3$ and this 2 is from fuzzy set A and here we have 3 which is from fuzzy set B.

Similarly, we have $3, 2$ as the third element 3 is from discrete fuzzy set A and 2 is from the discrete fuzzy set B and the 4th pair here is $3, 4$. So, all these 4 pairs are resulting $\delta = 1$, no other pair is going to result us $\delta = 1$. So, these pairs should be drawn from discrete fuzzy set A and B which is given in the example.

So, let us now use these pairs for $\delta = 1$ and tabulate the values of x_A, x_B,$\mu_A(x_A)$, $\mu_B(x_B)$ and then we take minimum of all these findings and then finally, we take max and let us see how does it go. So, for the first pair $1, 2$ which you see here it is encircled here and we write here $\delta = 1$.

So, this $\delta = 1$ which is in the first column, so, please look at the first column here first column of the table where we have entered in delta column which is first column of the table 1 and then let us tabulate all the corresponding entries with respect to $\delta = 1$.

So, first entry here will be for the pair $1, 2$. So, when we take $1, 2$ we write here as this is x_1^A and then we have 2 as x_1^B. This signifies that is the first generic variable of the discrete fuzzy set A and x_1^B signifies the generic variable value which is from fuzzy set B. Similarly, all other values can be understood.

Now, here enter in the second column which is x_A, we enter 1 and for x_B we enter 2 here and then let us find from the A discrete fuzzy set that is given we have 0.5 with respect to the generic variable value x_A here. So, we are getting 0.5 and then we enter here $\mu_B(x_B)$ which is 0.4. Now, in the 6th column here we take minimum of all these entries, minimum of all these entries and this gives us 0.4. Now let us take other pairs for which pair getting $\delta = 1$.

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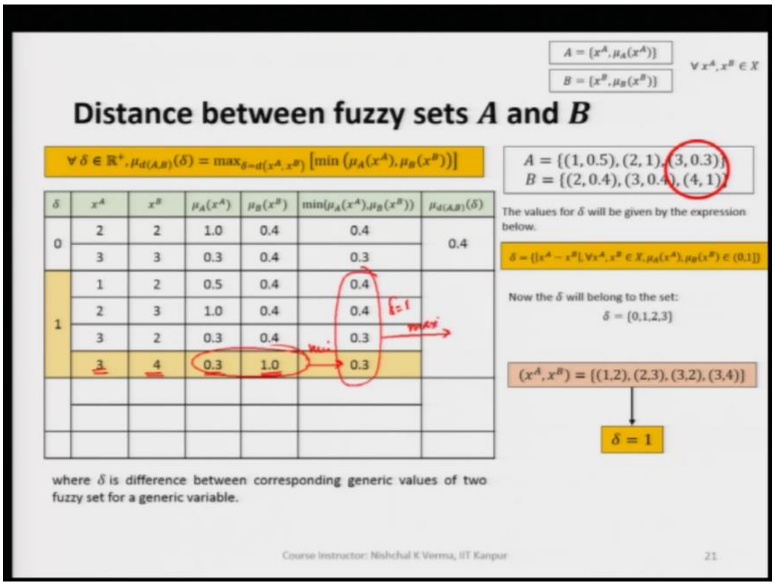

So, when we take the second pair which is $2, 3$, we are getting we are first of all we are entering these values 2, 3 x_A, x_B and then we take the $\mu_A(x_A)$, $\mu_B(x_B)$ and we take the minimum of these two which is coming out to be 0.4 and that is how we get here 0.4. Next, we take the third pair and third pair for which $\delta = 1$.

So, we have $3, 2$ we have written here 3 and then 2 and then we have their corresponding membership values which we are getting from the discrete fuzzy sets A and B. Now, when we take minimum of these 2 we are obviously, getting 0.3.

So, that is how this row is enter and then we have a 4th element; $3, 4$, 4th pair for which we are getting $\delta = 1$. So, here also we enter $3, 4$ and then their corresponding membership values and when we take min of these min of these 2 entries, we are going to get 0.3. Now, when we have found all 4 minimum of the corresponding membership values which are these corresponding to $\delta = 1$, then we take the max of all these.

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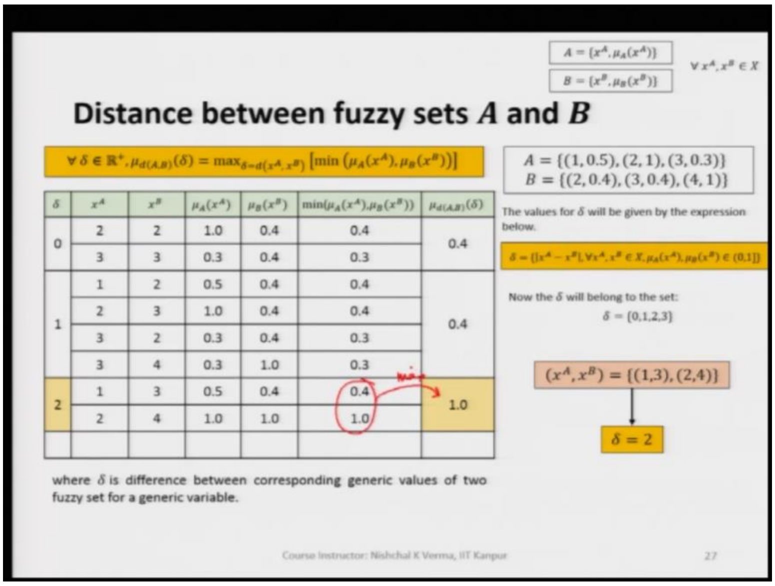

So, here we take max and then this max is going to give us 0.4. So, now, this way we are able to find all these values of x_A , x_B corresponding membership values and min of all these and then finally, max we are getting 0.4. So, that is how for $\delta = 1$, we have entered all the corresponding values. Now, we will check for $\delta = 2$.

So, the $\delta = 2$ let us see how many pairs how many such pairs we are getting for which we are getting $\delta = 2$. So, here if we see both the discrete fuzzy set, we are getting $1, 3$ and $3, 4$. So, these 2 pairs are giving us $\delta = 2$. So, now, let us on the same lines enter these values. So, when we take 1, 3 we are getting 0.5, 0.4 as the their corresponding membership values, when we take min we are getting 0.4 and similarly for the second pair which is 2, 4.

So, when we take 2, 4, we are getting the corresponding membership values 1 and 1. So, this way our all entries with respect to the $\delta = 2$ are complete and then when we take max we are getting one of course, this is the max of these 2 or 1.

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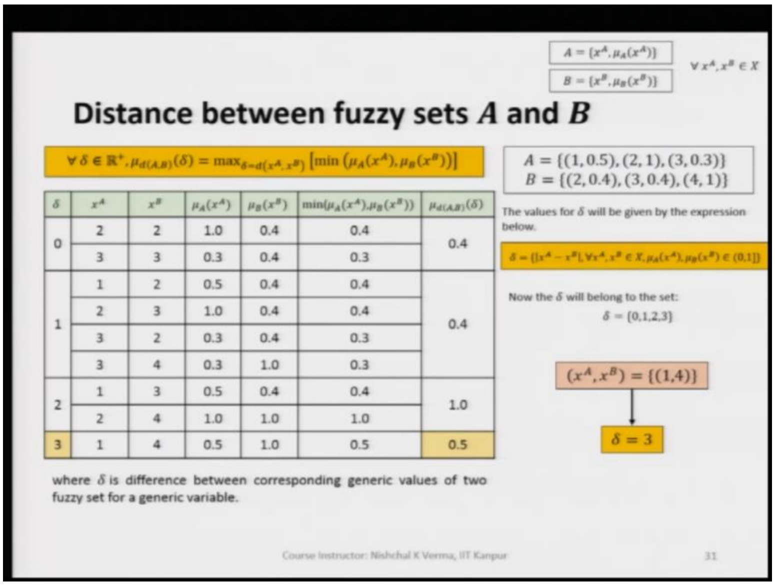

Next let us try to find all the pairs for which $\delta = 3$. So, this $\delta = 3$, we get only one such pair and when we use these values 1, 4 as the pair elements. So, 1, 4 for $\delta = 3$ and then the corresponding membership values are entered over here and when we take minimum of these we are getting 0.5 and since we have only one entry here, so, maximum of this is going to be 0.5. So, this will be 0.5.

So, now, we understood as to how we fill this table for which is going to help us in finding or making the fuzzy set which is giving us the distance between 2 discrete fuzzy sets A and B.

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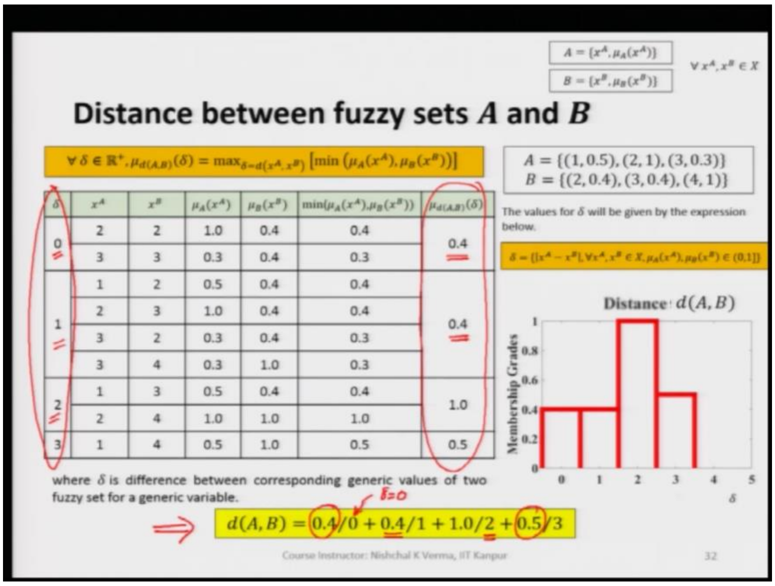

So, now, we can use this delta, when this table is complete. We use this table with the pick all the delta values and we pick all the $\mu_{d(A,B)}$ values $\mu_{d(A,B)}(\delta)$ values. So, when we take these 2 entries the δ entries and corresponding membership values, so these 2 is the first column and the last column. So, we see that when we take 0 as δ, so, $\delta = 0$ is is going to give us $\mu_{d(A,B)}(\delta) = 0.4$.

Similarly, if we take $\delta = 0$ we are getting $\mu_{d(A,B)}(\delta) = 0.4$. Similarly for 2, $\delta = 2$ $\delta = 3$ we get the its corresponding $\mu_{d(A,B)}(\delta)$. And when we use this these entries because one is the generic variable value which is delta which is nothing but the difference as we have already discussed was δ is the difference between the corresponding generic variable values of the 2 fuzzy sets A and B and then the last column gives us the it is corresponding membership values.

So, we can make use of these 2 entries and we can construct here a fuzzy set which is mentioned by $d(A, B) = 0.4/0$ means at generic variable value 0 which is the difference that is a $\delta = 0$ this is $\delta = 0$ and for this $\delta = 0$, we are getting here 0.4 as its corresponding membership value. Similarly here, we are getting another element as $0.4/1$.

So, we see that $\delta = 1$, we are getting 0.4 as its corresponding membership value. And then for $\delta = 2$, we are getting 1 and similarly for $\delta = 3$ we are getting its corresponding membership 0.5. So, this is how we are making use of this table and we are constructing fuzzy set which is nothing but the distance between the fuzzy sets A and B which is represented by $d(A, B)$ which is this.

Now, let us plot this fuzzy set and if we make use of these values here the its elements we can very easily plot or find the fuzzy set which is nothing but the distance between the 2 fuzzy sets. So, here we can clearly see that this is delta this x axis is here is delta and the y axis is the ordinate here is the membership grade that is mu of delta.

So, at 0 we have its corresponding membership as 0.4 which is here, which is here and at 1 also we have 0.4 which is here this level and then at 2 we have 1 which is here and at 3 we have 0.5, see here 0.5 there is 0.5. So, this way we are able to find the fuzzy distance between 2 discrete fuzzy sets.

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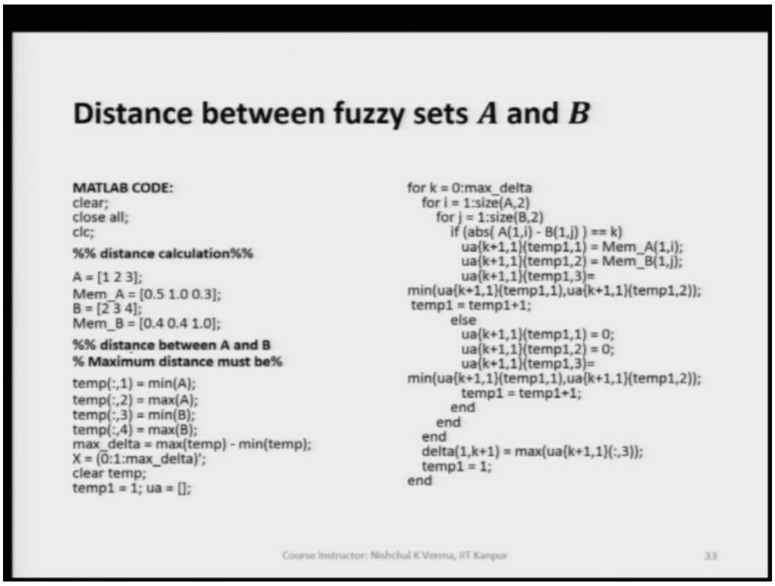

So, here we have just to help you we have included the MATLAB code for finding the distance between to discrete fuzzy sets A and B. So, you can make use of this MATLAB code to find the distance between any 2 discrete fuzzy sets.

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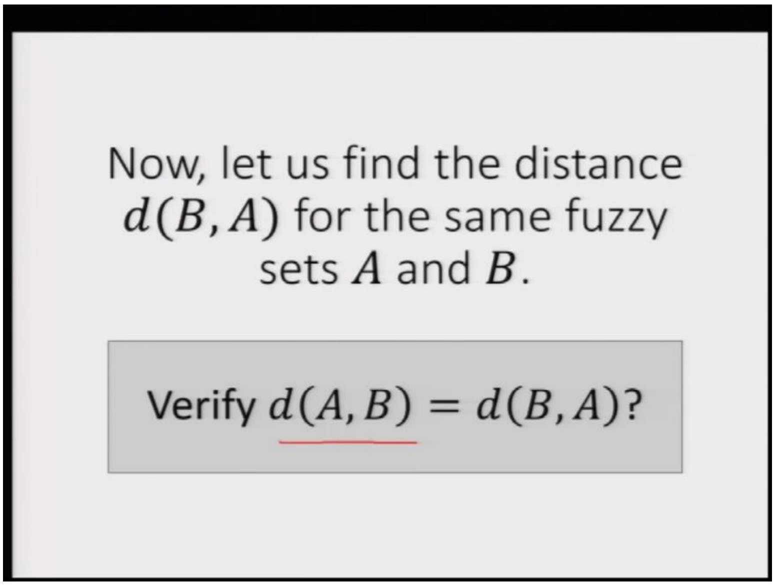

Now, let us find the distance between B and A and this way we will try to verify whether distance between 2 fuzzy sets A and B whether this is equal to the distance between 2 fuzzy sets B and A. So, this is very interesting example which will show us that these 2 distances these 2 the distance either we find the distance between A and B are B and A and the where these A and B are two fuzzy sets these two are always equal. So, let us verify this. So, we have already we have fuzzy set A and B. So, let's now find the distance between B and A.

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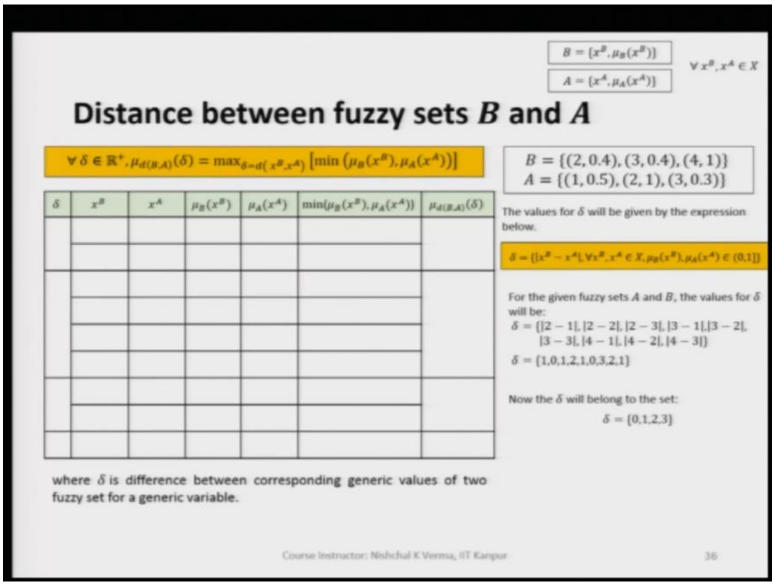

So, let us now for the same example that we have already taken let us find the distance between fuzzy set B and A. Please note that here we will take B first and then A and let us now enter all these values in the table which we have already discussed as to how we can fill up these values. So, here also we'll have because we are taking the same sets same discrete fuzzy sets A and B. So, obviously, here we are going to have the delta the difference as 0, 1, 2 and 3.

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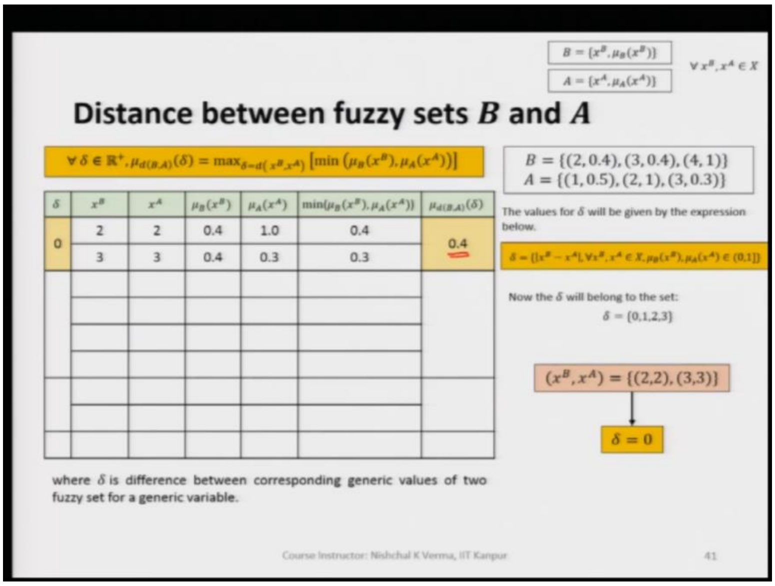

So, for all these delta values, let us now enter these values in this table. So, for 0 for $\delta = 0$, again we have 2 pairs here we have 2, 2 and 3, 3 for which we are going to get delta is equal to 0. So, let us now enter this value, this value of $\delta = 0$ and for this 0 for this value of $\delta = 0$, we are now entering corresponding values of x_B and x_A.

So, when we do that we are going to get 2, 2 ,0.4, 1 and then if we take min of these 2 we are going to get 0.4. Similarly, for 3, 3 we are going to get 0.3. Now, as we have already seen in the previous example $\mu_{d(B,A)}(\delta)$ we are going to take the max of these two values. So, when we take max of these two values we are going to get 0.4. Now, here we have completed all the entries for $\delta = 0$. Now, let us do the same for $\delta = 1$.

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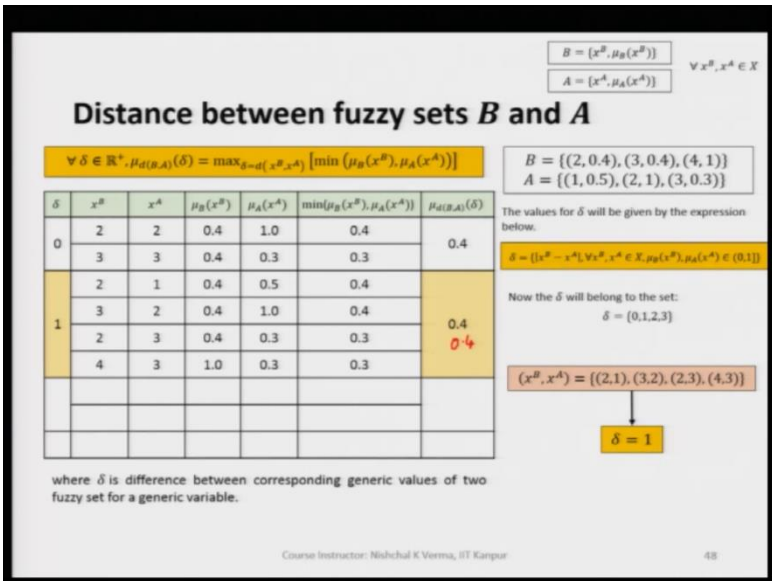

So, when we take $\delta = 1$, we have 4 entries as we have seen in the previous example also, here also we have 4 entries 4 pairs. So, let us now put all these in the table. So, here we will write $\delta = 1$ and the first pair we write 2 1 and then 0.4, 0.5 and then we take minimum of these two, we are going to get 0.4 and this way we are able to complete the entries for the first pair for which $\delta = 1$. Now the next pair we have 3, 2.

Then the next pair the third pair we have 2, 3 and then the last pair which is the 4th a pair we have 4, 3 and then we at this is stage we know how are we getting these values these corresponding values of membership and then we have taken the min of the corresponding membership values of A and B. And then when we take max of these values we are going to get here 0.4. So, this is 0.4.

So, if we look at this, if you look at this we have completed all the entries corresponding to $\delta = 1$. Now, let us move ahead and fill all the entries corresponding to $\delta = 2$.

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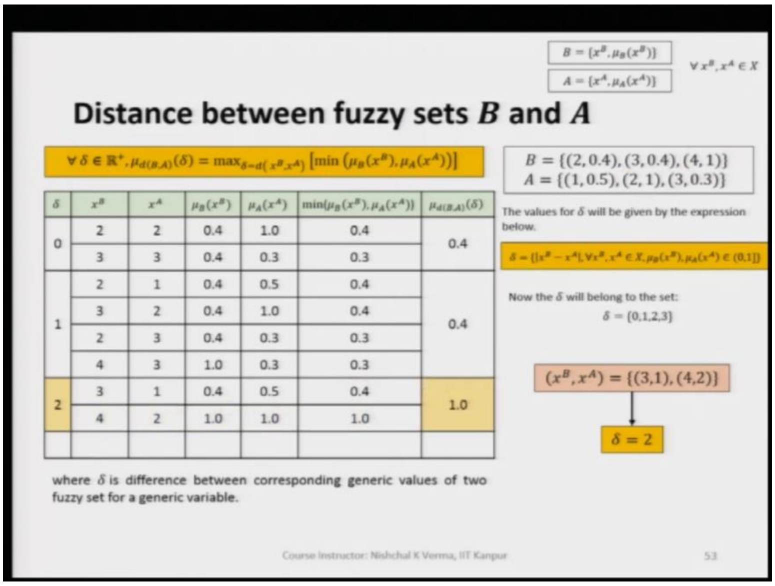

So, when we apply the same logic, we have two pairs for here for $\delta = 2$; one is the 3, 1 and the second one is the 4, 2. So, here we will write all the entries in the table and when we again take the maximum of this, we are going to get 1 which is mentioned over here. Now, next when we move to $\delta = 3$, so, we see that we have only one pair only one pair for which we are getting $\delta = 3$.

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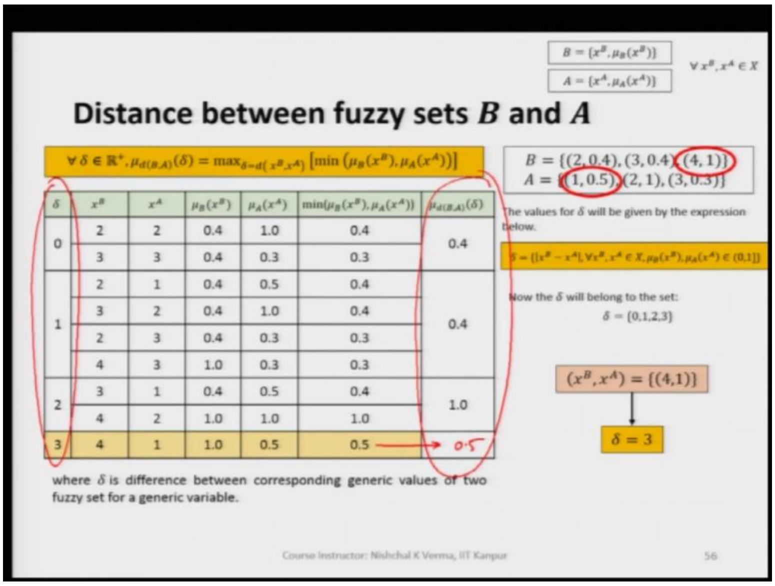

So, we fill the table with $\delta = 3$ and its corresponding values. So, here since we have only one value if we take max we get only 0.5. Now, please note that we do not have any pair for which we get $\delta = 4$.

So, we will stop here. And then again as we have done in the previous example, we take the first column which is delta column and the second column which is the$\mu_{d(B,A)}(\delta)$. So, here we will take these 2 columns, we will make use of these 2 entries and we will write the fuzzy set which is which will be representing the distance between 2 fuzzy sets B and A.

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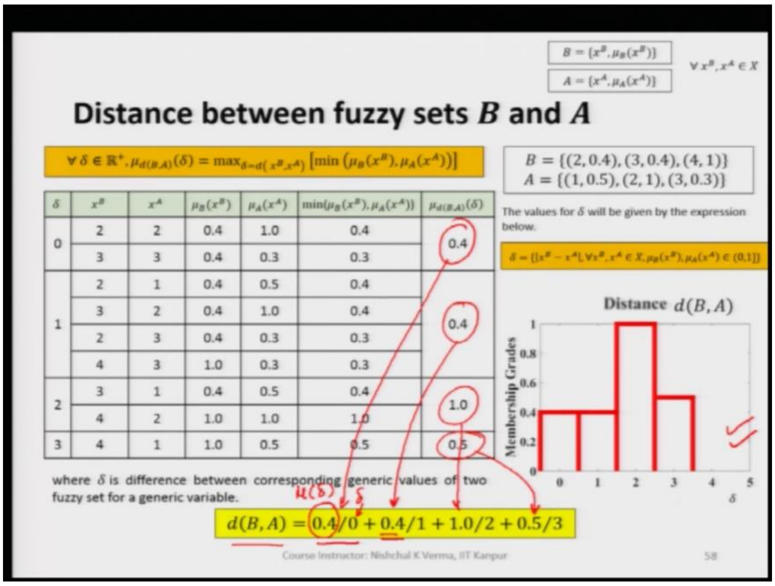

So, when we make use of these entries here we are going to write $d(B, A)$ this is nothing but the distance between the fuzzy set discrete fuzzy set A, B and A. In earlier case we had the distance between the fuzzy set A and B. So, the difference here is that we have change the sequence. So, here we are taking B first the fuzzy set B first and then A. So, $d(B, A)$ represents the distance between fuzzy set B and A and which is nothing but the $0.4/1$.

So, 0.4 is the μ corresponding to δ and this $\delta = 0$, this delta here is 0. So, 4 delta is equal to 0 which is which we can see in the very first column we have all the delta values mentioned. We have delta is equal to 0, 1, 2, 3 and then we when we take the corresponding membership values which is which are mentioned in the last column, so, we have for $\delta = 1$, we are getting 0.4.

So, this 0.4 is from here and then for $\delta = 1$ we are getting the corresponding membership value from here and then similarly for $\delta = 2$ we are getting the corresponding membership value here, similarly for $\delta = 3$ we are getting the corresponding membership. So, this way we have got a fuzzy set which is nothing, but the distance between discrete fuzzy set B and discrete fuzzy set A.

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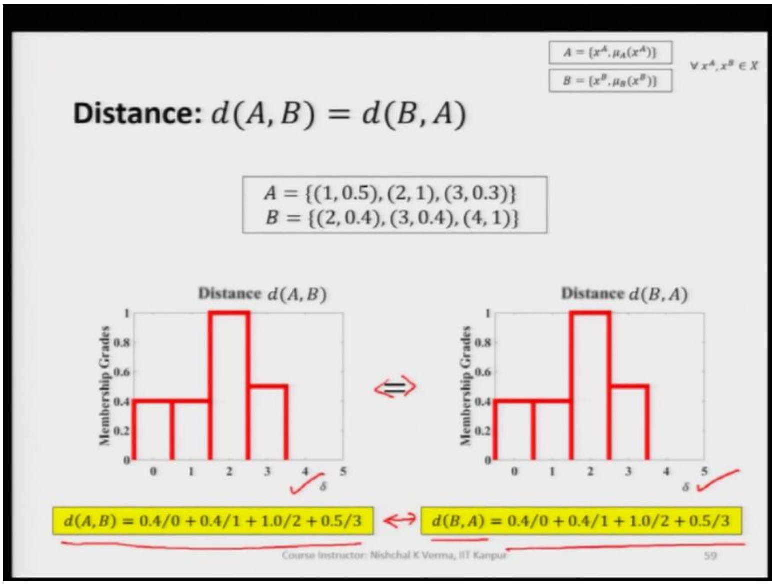

And when we plot these fuzzy set we are going to get here a fuzzy set which is exactly same as the fuzzy set as the distance between fuzzy set A and B we have got in the previous example. So, we can clearly see here that whether we calculate the distance between A and B fuzzy sets or B and A both are same. So, we can clearly see here if you plot or even the values also we see that $d(A, B)$ is same as $d(B, A)$.

So, this way, we can say that we have verified that $d(A, B) = d(B, A)$. So, this means that the distance between fuzzy sets A and B is exactly same as the distance between fuzzy sets B and A. So, by now we have understood very clearly as to how we can find the distance between any two fuzzy sets. So, if we can manage to find these entries here first the delta value, so difference.

So, we will first point all the possible values of the delta from the given fuzzy set like we have seen in the, this example and then we find all such pairs for which we are getting the different delta values. So, these pairs will be used to find their corresponding membership values and this these values then we will fill in the table as x_B, x_A,$\mu_B(x_B)$, $\mu_A(x_A)$.

And then we take the minimum of these values these membership values and then when we have done for all such pairs all the minimums are all the minimum values are found out then we take max of these values to get the final value which is $\mu_{d(B,A)}(\delta)$. So, this way we can manage to get all the entries all the values of delta and its corresponding membership values and these 2 values delta and mu of delta will help us in constructing the fuzzy set the discrete fuzzy set here in this case because we have taken discrete fuzzy set only.

And so, the distance between the two discrete fuzzy set is of course, of obviously, going to return us the discrete fuzzy set as the distance between the two discrete fuzzy sets.

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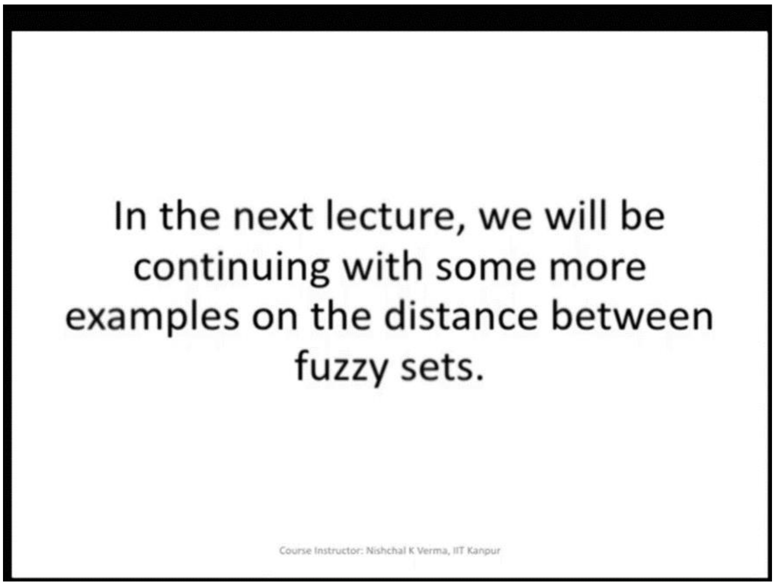

So, with this I would like to stop here and in the next lecture I will try to cover a few more examples on distance between fuzzy sets.

Thank you.