Fuzzy Sets, Logic and Systems and Applications
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Lecture - 15
Properties of Fuzzy Sets

So, welcome to lecture number 15 Fuzzy Sets, Logic and Systems and Applications. So, this lecture is in continuation to our discussion on Properties of Fuzzy Sets. Here we have these properties that we have intended to discuss or listed.

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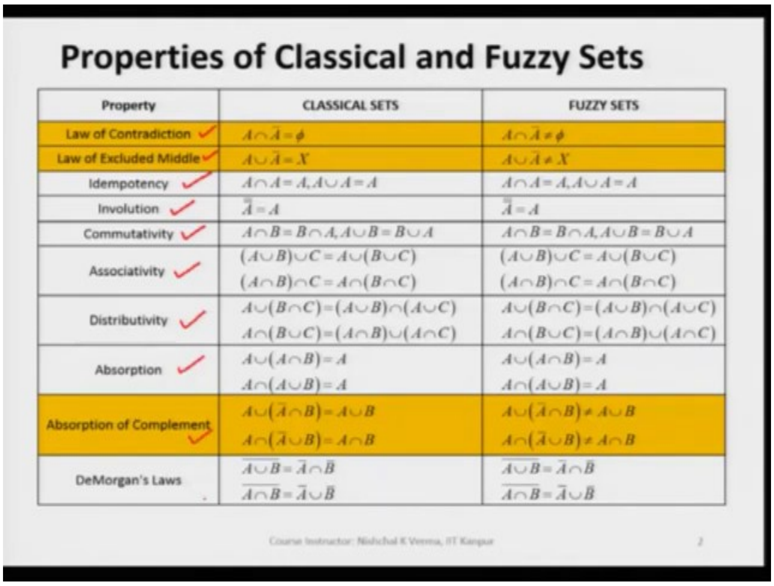
Properties of Classical and Fuzzy Sets

Property	CLASSICAL SETS	FUZZY SETS
Law of Contradiction	$A \cap \bar{A} = \phi$	$A \cap \bar{A} \neq \phi$
Law of Excluded Middle	$A \cup \bar{A} = X$	$A \cup \bar{A} \neq X$
Idempotency	$A \cap A = A, A \cup A = A$	$A \cap A = A, A \cup A = A$
Involution	$\bar{\bar{A}} = A$	$\bar{\bar{A}} = A$
Commutativity	$A \cap B = B \cap A, A \cup B = B \cup A$	$A \cap B = B \cap A, A \cup B = B \cup A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributivity	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Absorption	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Absorption of Complement	$A \cup (\bar{A} \cap B) = A \cup B$ $A \cap (\bar{A} \cup B) = A \cap B$	$A \cup (\bar{A} \cap B) \neq A \cup B$ $A \cap (\bar{A} \cup B) \neq A \cap B$
DeMorgan's Laws	$\overline{A \cup B} = \bar{A} \cap \bar{B}$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$	$\overline{A \cup B} = \bar{A} \cap \bar{B}$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$

So, far we have discussed a law of contradiction, law of excluded middle, idempotency, involution, commutativity, associativity, distributivity and these properties we have discussed with respect to fuzzy sets. Now, the rest of the properties we will try to discuss and in this continuation, we'll be discussing absorption, absorption of complement and De Morgan's laws.

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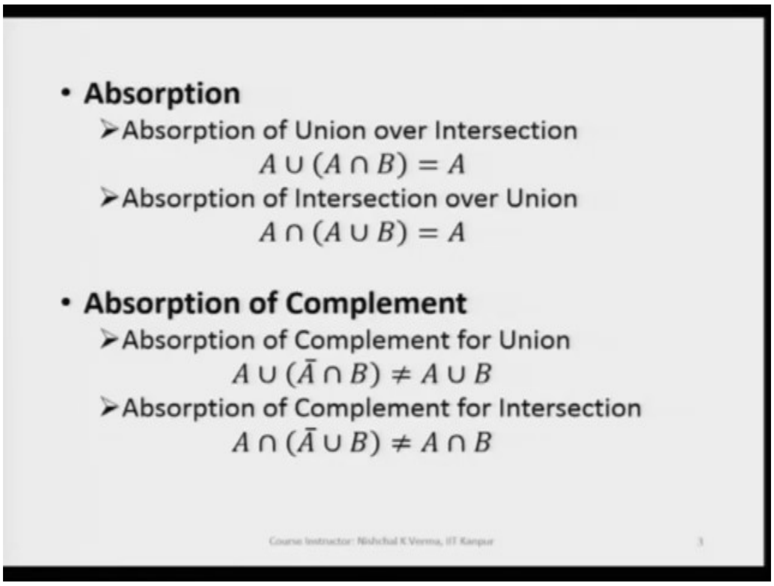

So, in today's lecture we will try to discuss absorption and this absorption will be the absorption of union over intersection and this is defined as $A \cup (A \cap B)$. So, this is regarded as the absorption of union over intersection and this is equal to A and then the absorption of intersection over union and this is nothing but the $A \cap (A \cup B) = A$.

So, let me tell you that when we discussed the absorption with respect to sets as we have already discussed the absorption of union over intersection for crisp sets and absorption of intersection over union for crisp sets. So, these hold good. Now, so let us we take fuzzy sets A and B. So, these absorption of union over intersection, absorption of intersection over union, both these absorptions here also hold good for fuzzy sets and then in this lecture we'll also discuss the absorption of complement.

So, when we say absorption of complement, again we have the absorption of complement for union absorption of complement for intersection. So, here we see that the absorption of complement for union hold good for the crisp set, but it doesn't hold for the fuzzy sets. Similarly, the absorption of complement for intersection hold good for crisp set, but this does not hold good for the fuzzy sets. So, we will discuss these absorptions for fuzzy sets and we'll take some examples also to see as to how it works.

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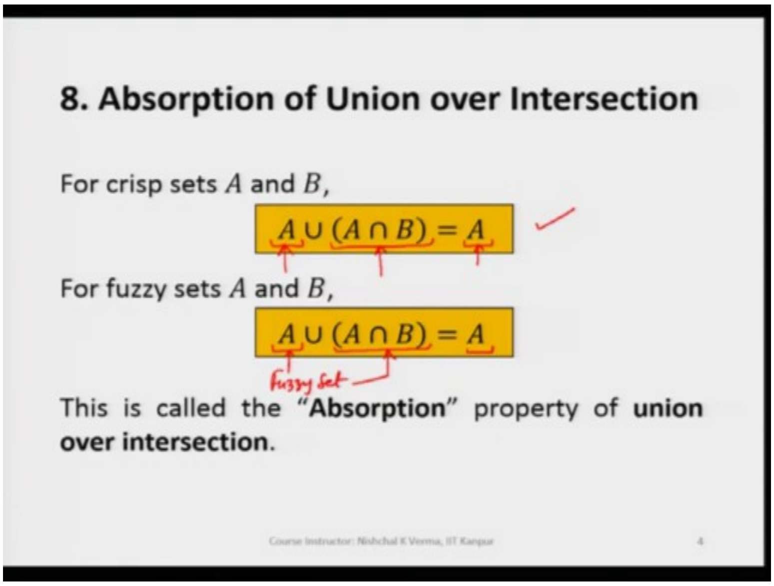

So, here is the absorption of union over intersection. So, as I have already mentioned that when we take crisp set A and B and the absorption of union over intersection. So, we have here the $A \cup (A \cap B)$. So, here we have two entities: 1 is the entity A and other is the entity $(A \cap B)$. So, when we take union of these two we get only A. So, this is for crisp set, same is true when we take fuzzy sets and this is similar to the crisp set.

So, here also we see that when we take union of A fuzzy set, this is A fuzzy set this A fuzzy set here and the $A \cap B$ fuzzy set. So, this is $A \cap B$ this is again A fuzzy set here. So, I will just right it like this show it like this. So, you see here that we have two fuzzy sets, one is A fuzzy set and other one is the fuzzy set which is the outcome of $A \cap B$. So, when we take the union of these two, we are getting the fuzzy set A. So, this means that the absorption property of union over intersection, we can say that is satisfied or that holds good.

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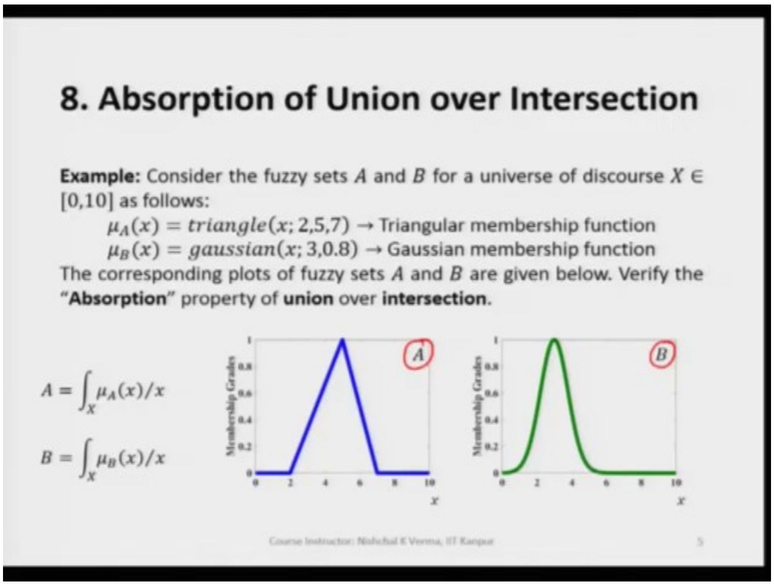

So, if we take an example here to understand this property better, we take two continuous fuzzy sets, one fuzzy set is A here and then the other one is B fuzzy set. So, we see here this two fuzzy sets, A and B they are defined within the universe of discourse right from 0 to 10. So, both the fuzzy sets are defined in this range and let us see when we have the absorption of union over intersection whether this hold good are not. Of course, this should hold good, but let us see by taking this example how or in what way this is holding good.

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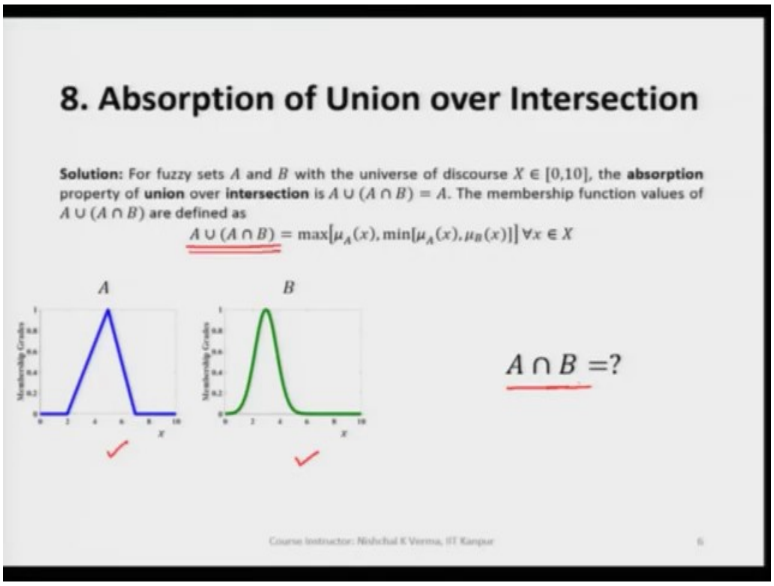

So, we have here the fuzzy set A and then we have fuzzy set B and since we are interested in this, the $A \cup (A \cap B)$ and so for this we need to first find $A \cap B$, $A \cap B$. So, we have A fuzzy set and B fuzzy set it's very easy to find $A \cap B$ we have done it many times.

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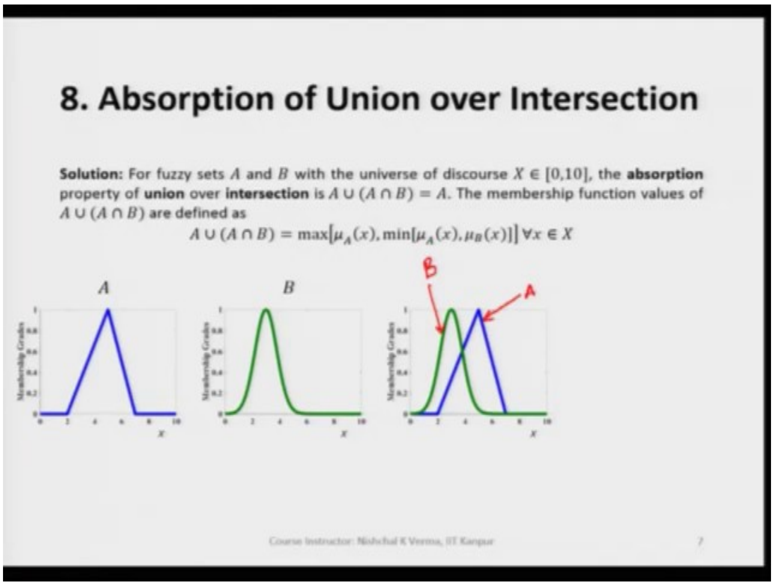

So, let us now find it. And to find this intersection we have to superimpose these two fuzzy sets on each other and we see here that we have this as A fuzzy set and this as the B fuzzy set and when we superimpose and then since we are finding the we are interested in getting the intersection. So, what we have to do here is to take we apply the min of these two corresponding membership values with respect to all the generic variable values within the universe of discourse.

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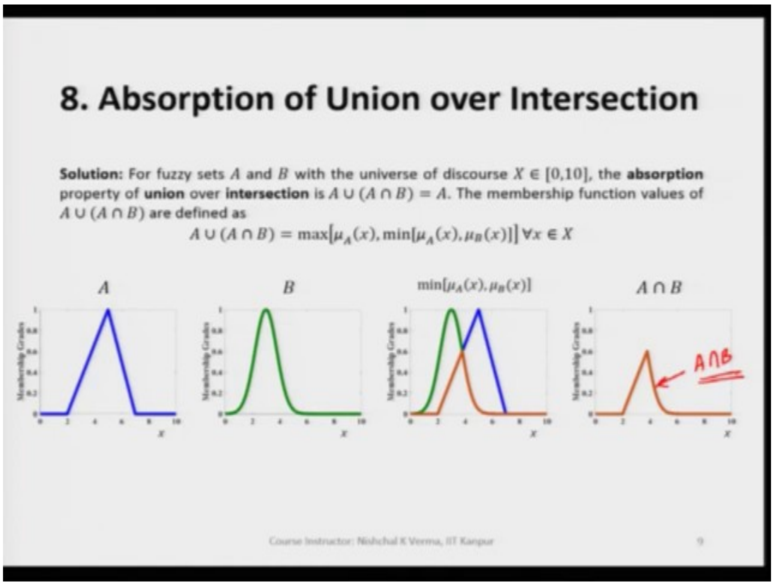

So, when we apply this min condition here, we find here this portion after applying the min portion we get here $A \cap B$. So, this is a fuzzy set, of course, this is a subnormal fuzzy set because none of membership value is approaching 1 or other words we can say the core of this fuzzy set is empty. So, that way here we were getting $A \cap B$ as the result and which is a subnormal fuzzy set. So, now this outcome $A \cap B$, let us take the union of this fuzzy set and A.

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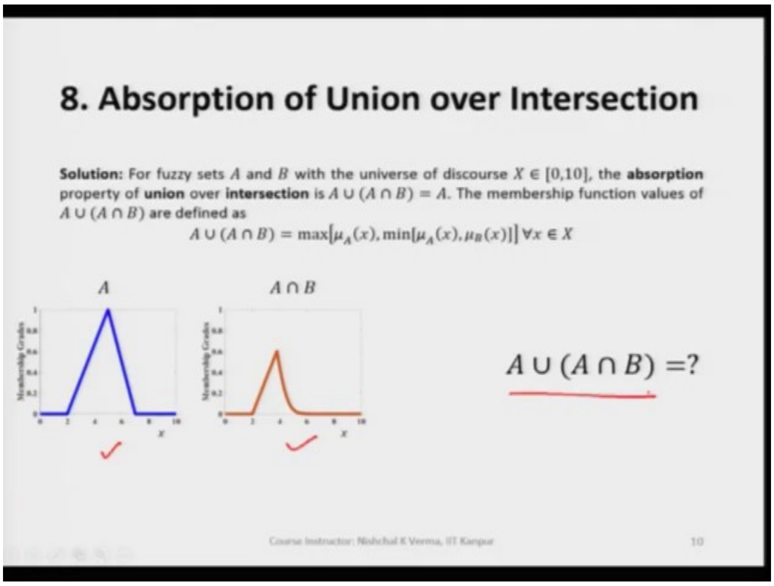

So, here we are taking A fuzzy set and the union of these two. So, here we are taking the union of these two fuzzy sets right and let us see what are we going to get here.

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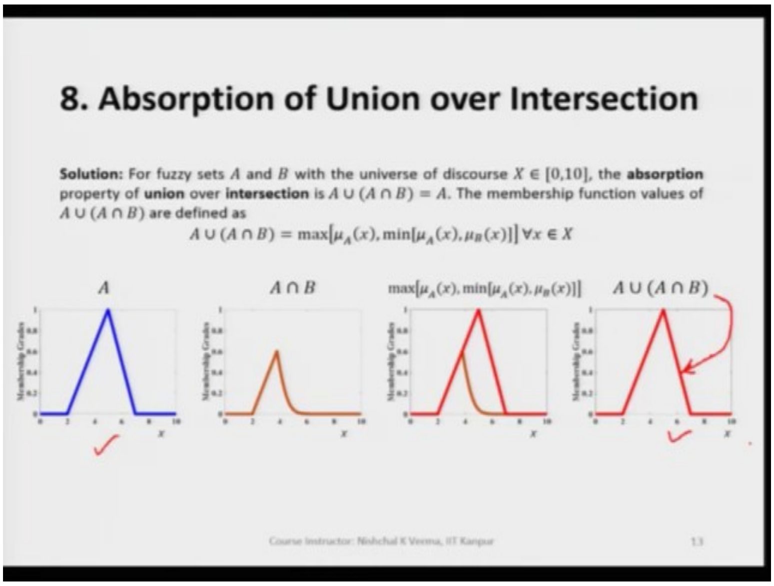

So, when we superimpose these two fuzzy sets on each other, we are getting here since we are trying to get are we are interested in the union, so we have to apply the max criteria and when we apply max criteria we are going to get a fuzzy set here which is like this, this is the fuzzy set. So, we clearly see that this is when we apply max criteria we are going to get this as a fuzzy set. Now, if we see the result and we compare we see that this same as A.

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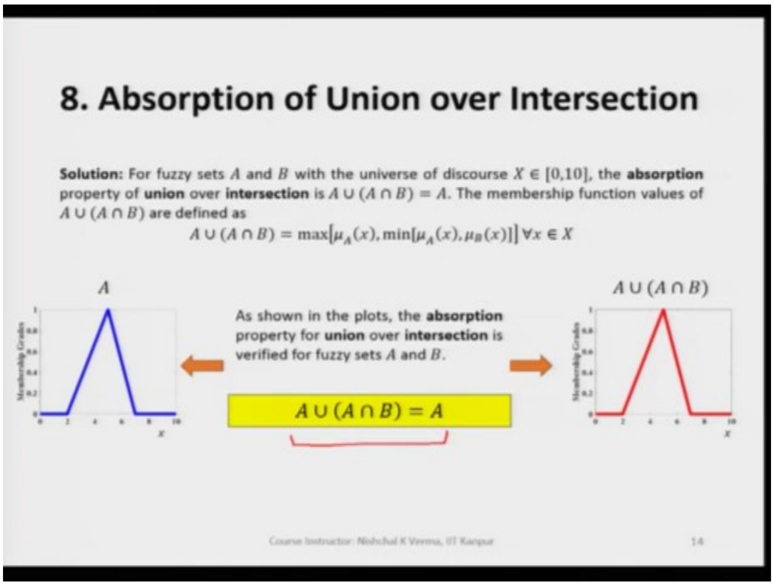

So, here, A is this and $A \cup (A \cap B)$ both the fuzzy sets are same. So, we can clearly say here that the outcome is A, and that is how we can say that the absorption property for union over intersection is verified and of course these sets are the fuzzy sets. So, here we have taken two fuzzy sets A and B and we applied absorption property of union over intersection and we found that these two are verified.

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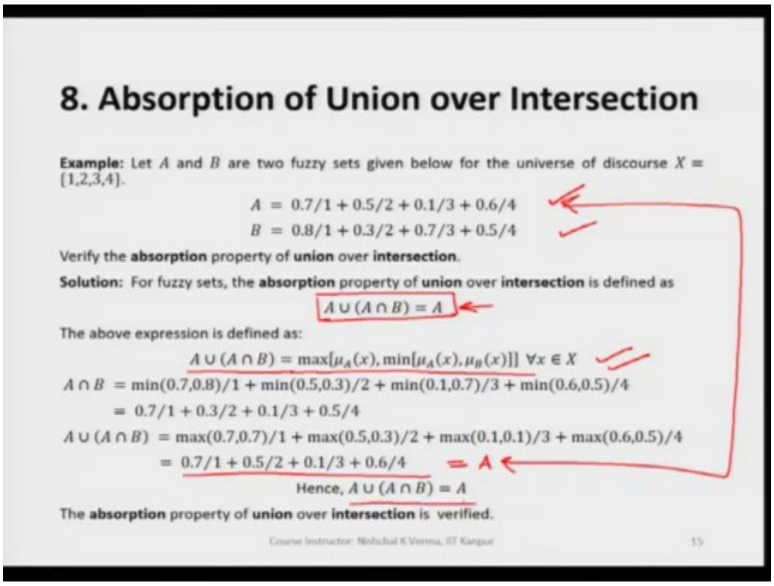

Now, let us take another example where we take two fuzzy sets, A and B and these fuzzy sets are discrete fuzzy sets. So, we have A fuzzy set here and then we have B fuzzy set here and what we have to check is this property, where you see that this absorption of union over intersection should come out like this. So, the above expression is defined as when, when we apply the max correct criteria we can write it like this and then when we apply this to these two discrete fuzzy sets, we are going to get as a result this after applying the max and min criteria which is mentioned over here and when we see this, this is same as A.

So, we can clearly see here that this is same as A is you can say, you can see here that A was $0.7/1+0.5/2+0.1/3+0.6/4$ and the result is also coming out to be the same. So, we can say here the result of $A \cup (A \cap B)$ and which is coming out to be the same as A. So, we can clearly say that this absorption of union over intersection is verified.

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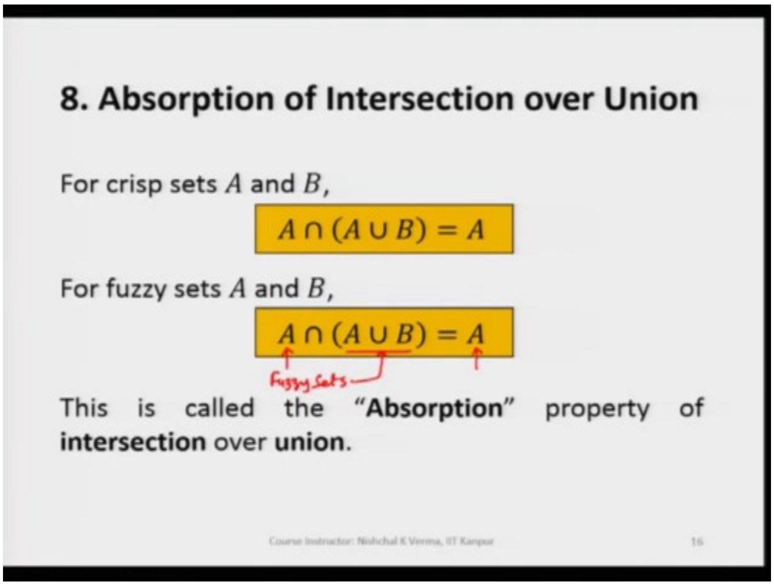

Now, let us discuss the absorption of intersection over union. So, you see earlier was absorption of union over intersection, now we are going to discuss the absorption of intersection over union. So, let us see this for crisp sets A and B. So, we have for crisp sets, we have you see the $A \cap (A \cup B)$ which is coming out to be A. Now when we take fuzzy sets instead of crisp sets, so, we see that this property holds good for the fuzzy sets as well.

So, if we take A as fuzzy set if this is fuzzy set and here B also is fuzzy sets, so, when we take union of these two fuzzy sets this will also be a fuzzy set. We can write it like this we can show it like this, that these are the fuzzy sets, this A is a fuzzy set and then $A \cup B$ is also fuzzy set and when we take the intersection of these two fuzzy sets we are going to A as a result which is again a fuzzy set. So, A fuzzy set we are going to get as a result. So, we can say that the absorption of intersection over union is also holding good. So, now let’s take some examples and understand this better.

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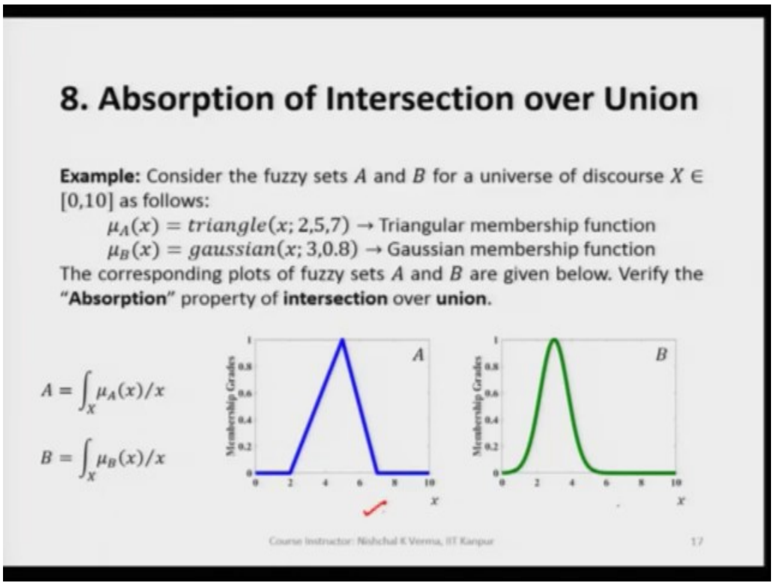

So, as we have seen previously by taking this by taking the example here we take three fuzzy sets, three continuous fuzzy sets and we see that here we are taking fuzzy sets A and B for this example. So, A is this and B is this.

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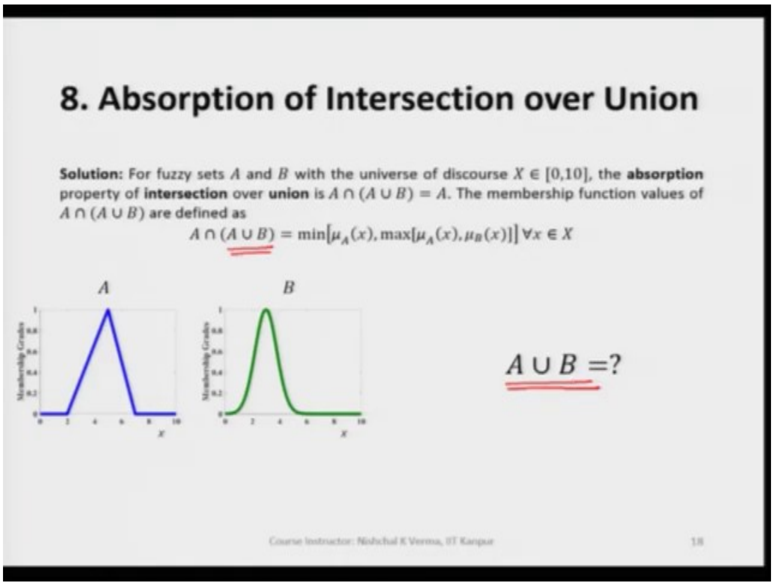

And then since we have to verify that the $A \cap (A \cup B)$, So, we have to first get the $A \cup B$. So, this is $A \cup B$ now let us find $A \cup B$ for A and B.

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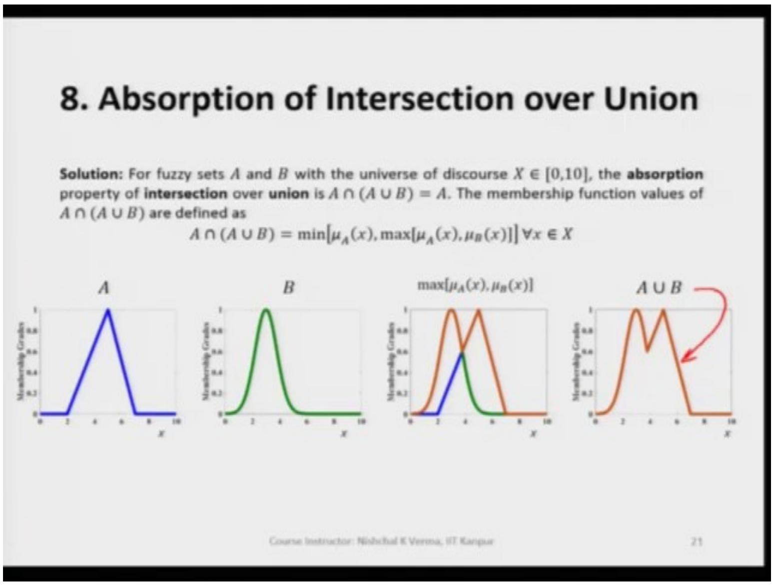

So, when we superimpose these two fuzzy sets on each other, we get to see after applying the max criteria we are going to get this as the $A \cup B$ which is a fuzzy set again and now if we take the intersection of this fuzzy set and A fuzzy set, here we have A fuzzy set and $A \cup B$, let us see what are we going to get.

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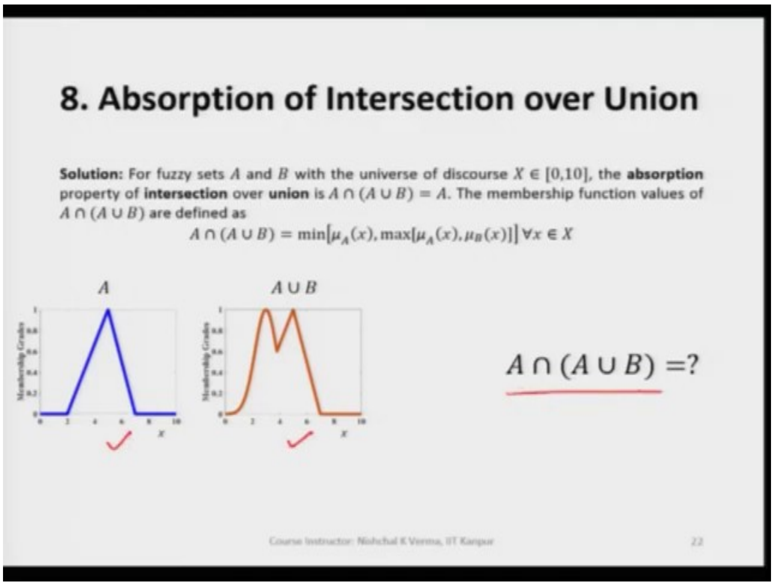

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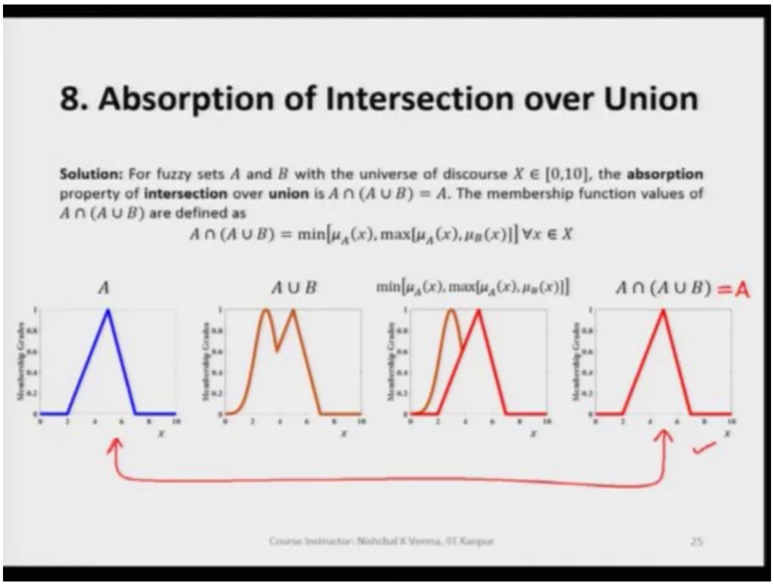

So, when we superimpose again these two fuzzy sets on each other, we are going to get this as the outcome which is again we see nothing but the same fuzzy set as A or we can say these two fuzzy sets are equal. So, we can say that we are going to get here the outcome which is a fuzzy set A. So, this way we can say absorption of intersection over union is satisfied for the fuzzy sets A and B.

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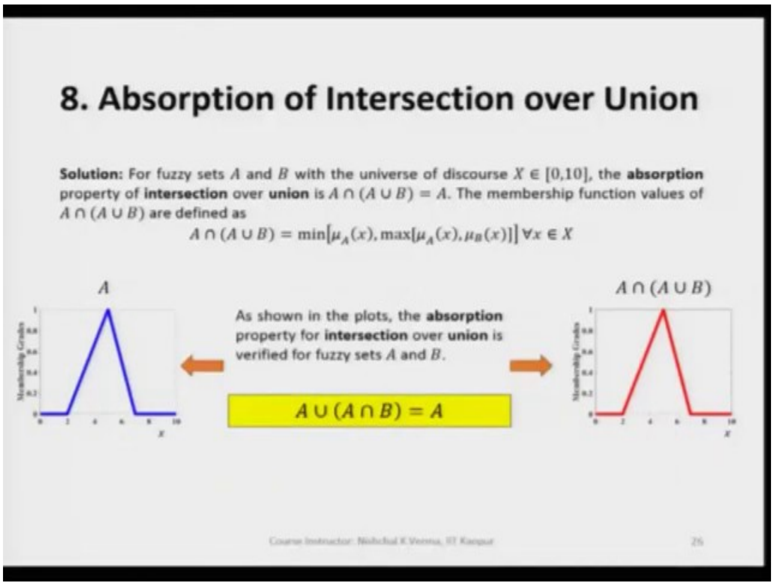

Please note that here in this example we have taken the continuous fuzzy sets A and B.

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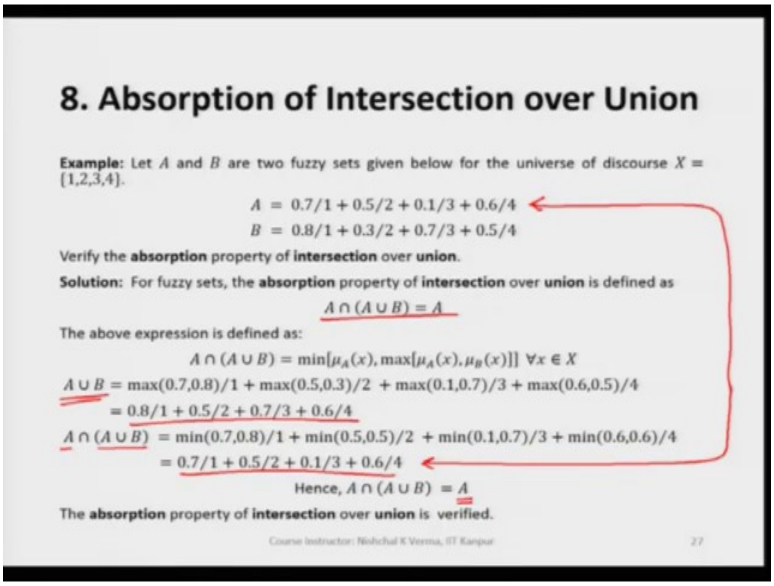

Now, we take another example were we take to discrete fuzzy sets A and B and let us see how this is satisfying this is verifying the absorption of intersection over union. So, as we have done in the previous example, we will find out first the $A \cup B$ and $A \cup B$ is here. So, when we find $A \cup B$, we get here this as the fuzzy set discrete fuzzy set and then when we taken $A \cap (A \cup B)$ we are going to get this thing which is nothing but same as A.

You can verify here this fuzzy set with this, this two fuzzy sets are equal here and this way we can clearly say that the absorption property of intersection over union is verified. So, we have seen that whether we take the discrete fuzzy set or the continuous fuzzy sets, for the both the kinds of fuzzy sets the absorption of intersection over union is also verified.

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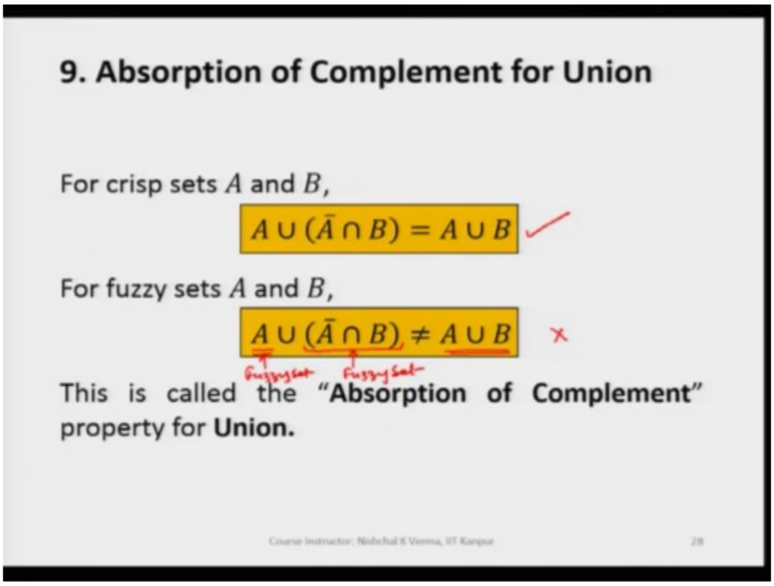

Now, we come to the absorption of complement for union. So, absorption of complement for union is here this is represented by the$A \cup (\acute{A} \cap B)$. So, here we have the A and its complement and we see that when we take the $A \cup (\acute{A} \cap B)$ we see that the $\acute{A}$ is, $\acute{A}$ is not there in the outcome.

So, this holds good for crisp sets A and B we all know this, let us now see what is happening when we take fuzzy sets A and B. So, when we take fuzzy sets A and B here, we see that we take when we take $A \cup (\acute{A} \cap B)$. So, if we take you see here when we take $\acute{A} \cap B$, this going to be a fuzzy set this going to the result is a fuzzy set here, this also is a fuzzy set. So, if we take the union of these two fuzzy sets and the result here is not going to be the$A \cap B$ as we were getting this in the crisp set case.

So, here this the result of these $\acute{A} \cap B$ and if we take the $A \cup (\acute{A} \cap B)$, we are not going to get $A \cup B$. So, this is to be noted here. So, it means that for crisp case, for when we have crisp sets A and B, the absorption of complement for union holds good. If we take the fuzzy sets, so, this absorption of complement for union doesn’t hold good. So, let us now take some example and we understand little better.

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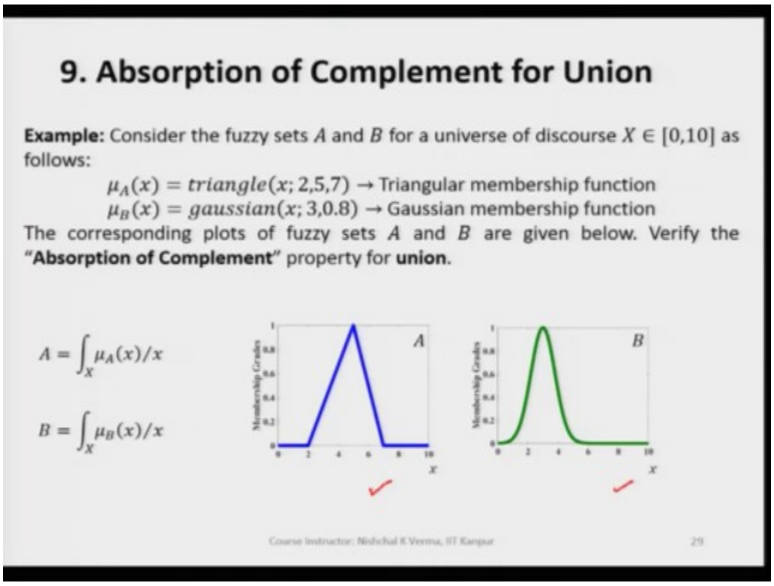

And in this example, we take again two fuzzy sets A and B. So, first fuzzy set is A and second fuzzy set is B let's now check whether there is absorption of complement for union is same as what we have discussed or not.

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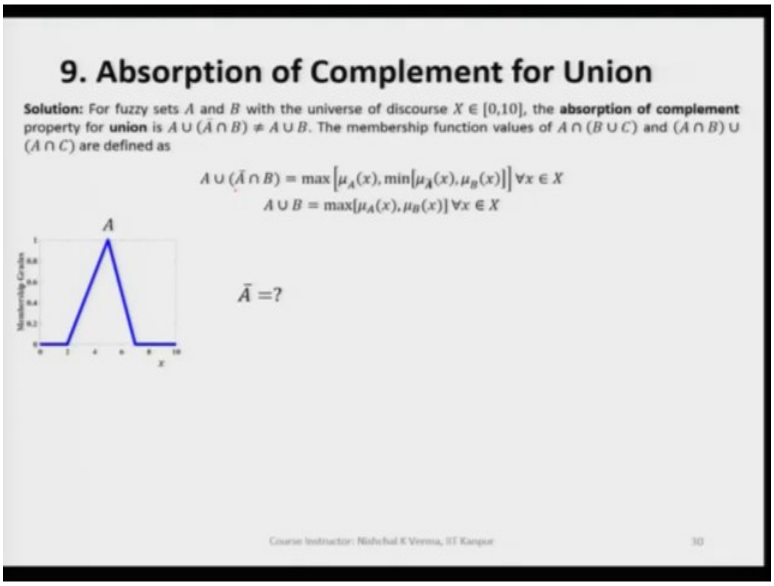

So, since we have to find here we have to use $\acute{A}$. So, we have A here.

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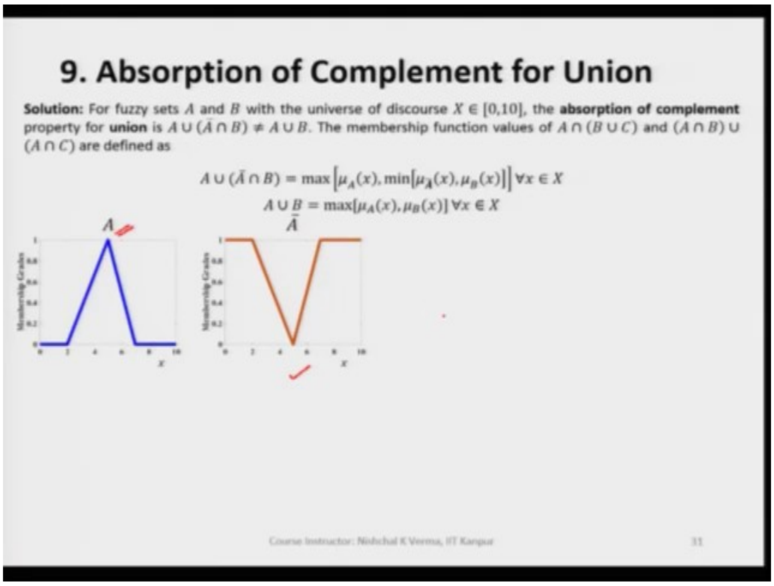

Let us find $\acute{A}$. So, $\acute{A}$ is this here, this is fuzzy set A and if we find $\acute{A}$ will look like this and then let us now take the $\acute{A} \cap B$.

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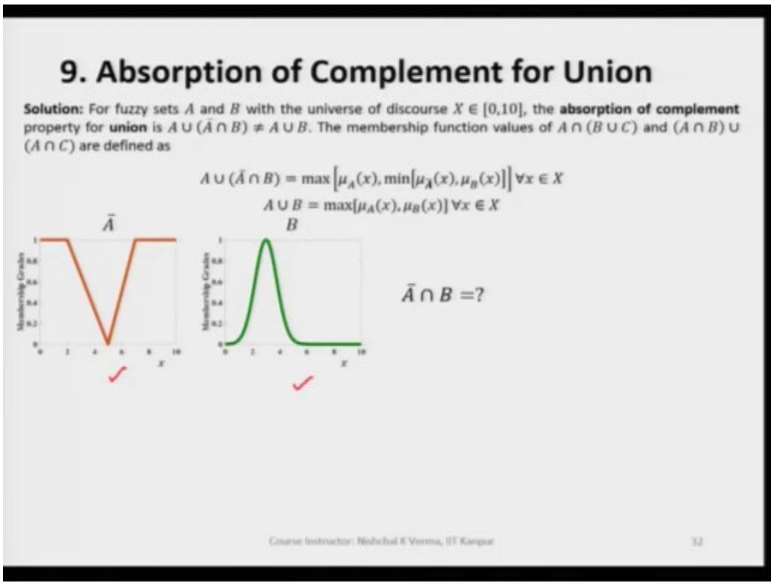

So, $\acute{A}$ and then B fuzzy set, these two fuzzy sets if we take the intersection, we have to go for superimposing this two sets on each other.

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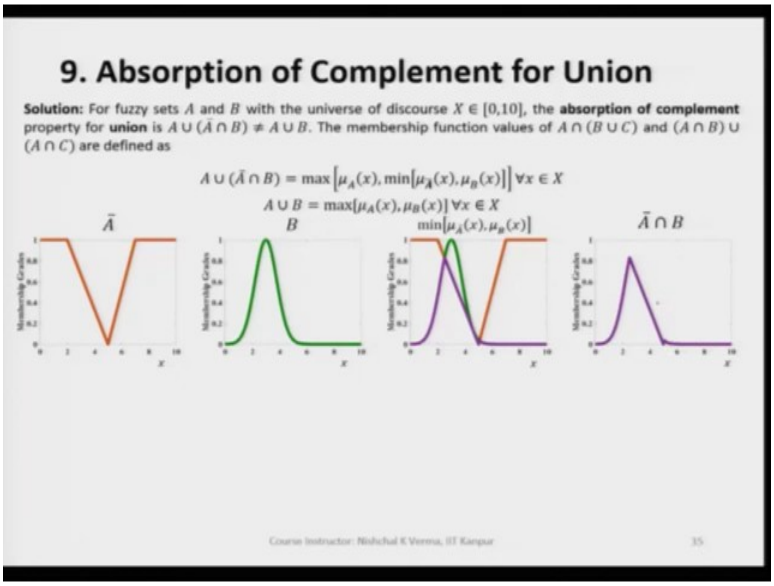

And which we are doing here and then since we are taking the intersection we have to apply the min criteria. So, when we apply min criteria, we find a fuzzy set here we find a fuzzy set as a result which is again you know A, the $\acute{A} \cap B$ which is by looking at it we can say a subnormal fuzzy set. So, we have got this now we have to take the union of this fuzzy set with A.

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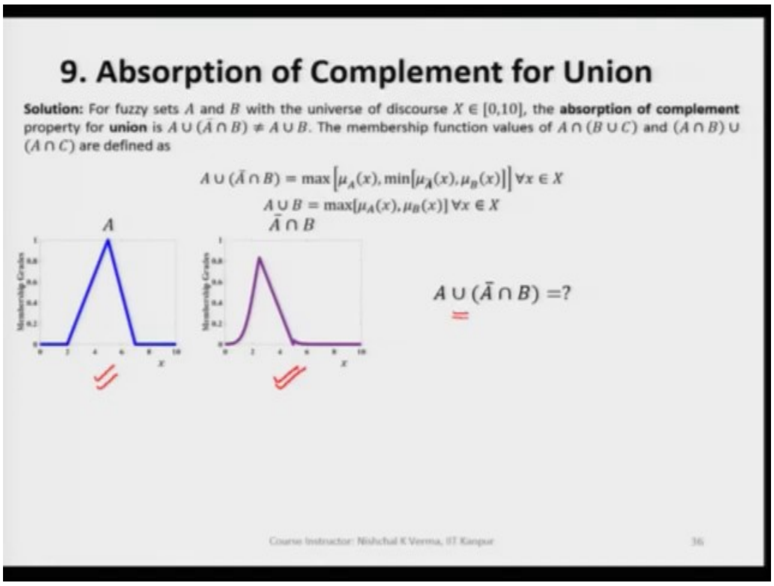

So, A is here and the $\acute{A} \cap B$ is also here, now let us take the union of these two fuzzy set.

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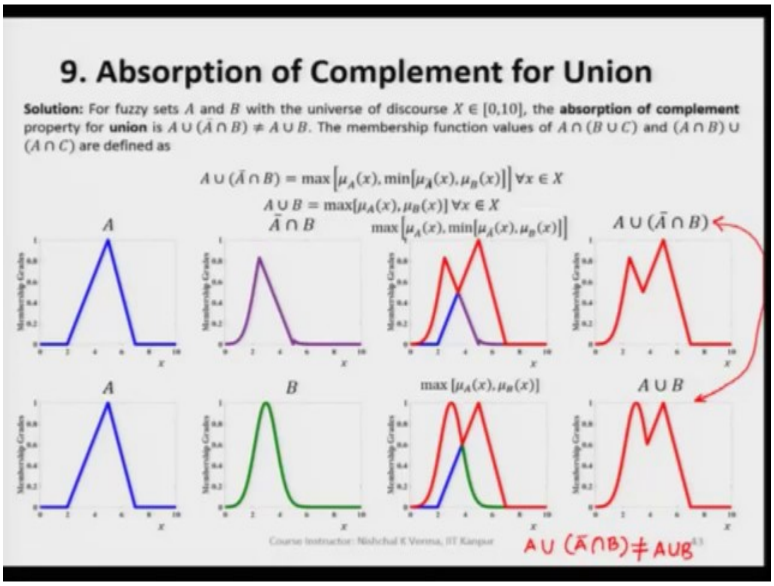

Since this is the union we have to apply the max criteria. So, when we apply max criteria, we are going to get this fuzzy set as a result of this thing and then if we take $A \cup B$, $A \cup B$ is here and we can clearly say that this is not same as this fuzzy set. So, here $A \cup (\acute{A} \cap B) \neq A \cup B$.

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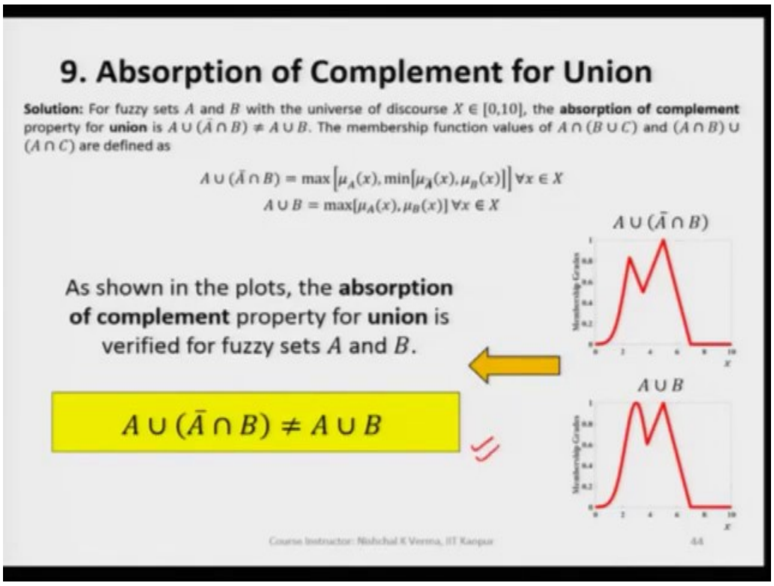

So, since these two outcomes are not equal for fuzzy sets A and B, we can clearly say that the absorption of complement for union is not holding good when we take fuzzy sets A and B.

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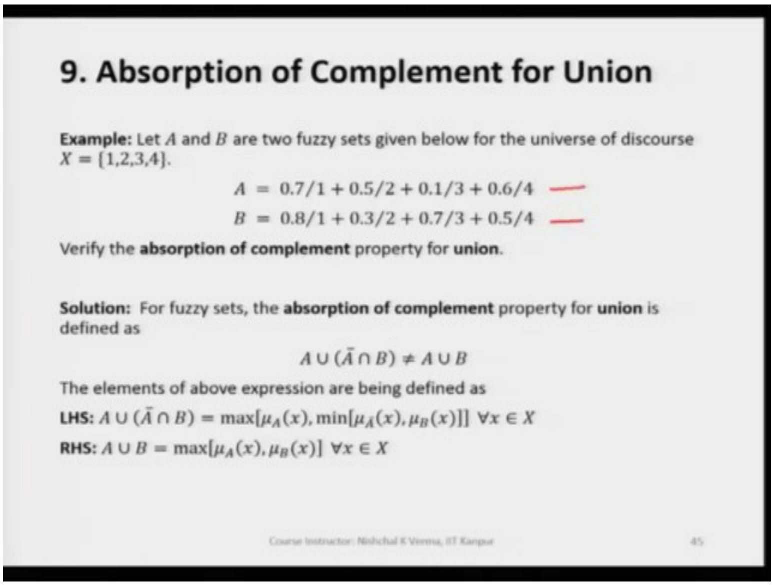

Now, when we take discrete fuzzy sets and see whether for discrete fuzzy sets also this absorption of complement for union is satisfied or not. So, we take two fuzzy sets two discrete fuzzy sets A and B here.

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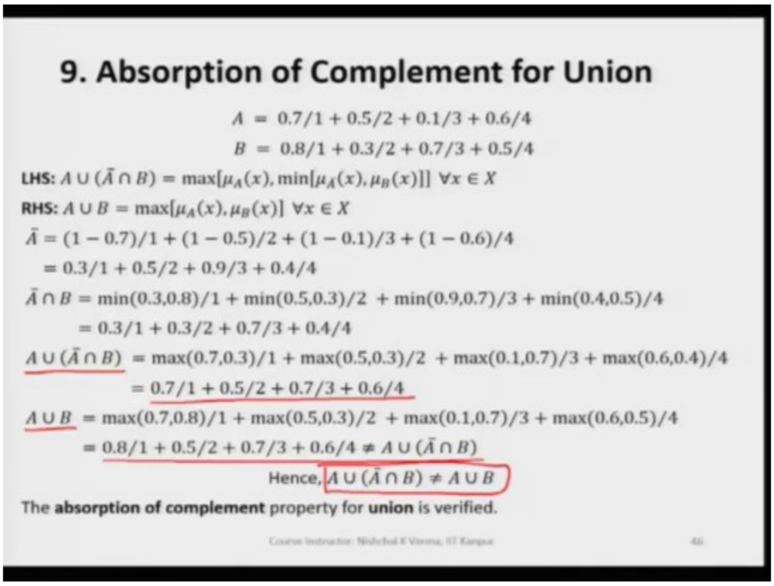

And see here we find A, the $A \cup (\acute{A} \cap B)$ and we find that we are getting here $0.7/1+0.5/2+0.7/3+0.6/4$ and when we take the $A \cup B$ we see that we are finding we are getting $0.8/1+0.5/2+0.7/3+0.6/4$ and we see that this is not equal to the when we take the $A \cup (\acute{A} \cap B)$.

So, we can clearly say here also here also this absorption of complement for union does not hold good. So, in both the cases we clearly see that the absorption of complement for union is not satisfied or does not hold good.

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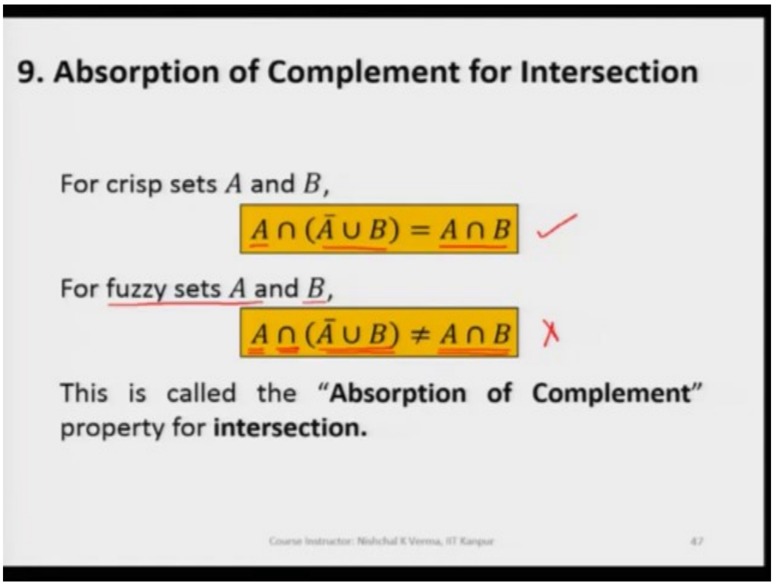

Now, the absorption of complement for intersection: so, let us now see what is happening for absorption of complement for intersection. So, of course, when we take crisp sets A and B, we take the $A \cap (\acute{A} \cup B)$, we see that we are getting $A \cap B$. When we take fuzzy sets A and B, this does not hold good and hence we write here the $A \cap$¿. So, we can clearly say that the absorption of complement for intersection for fuzzy sets A and B does not hold good.

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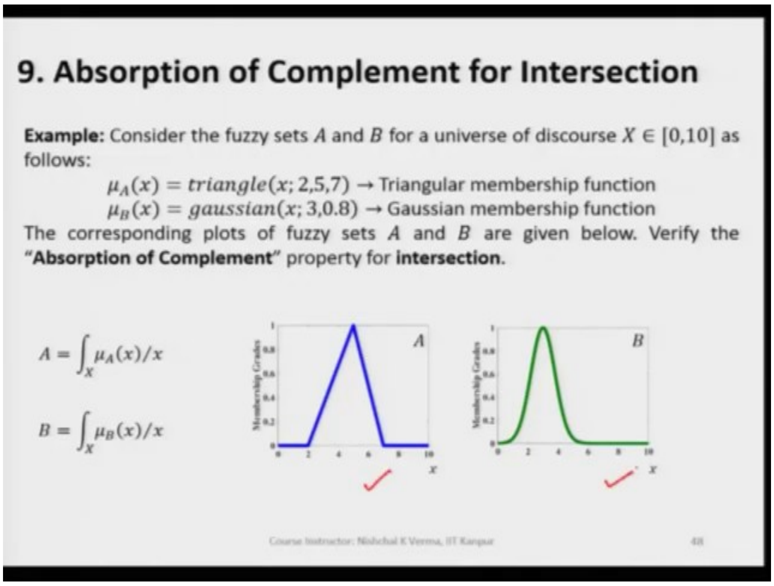

Let us now understand this better by taking few examples. So, here we take an example which contains two continuous fuzzy sets A and B. So, A is here and B is here.

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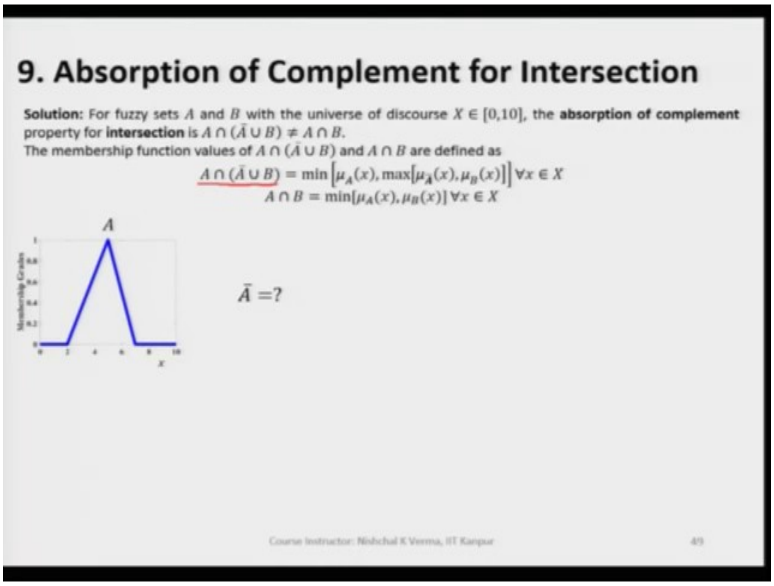

And first we are interested in finding $A \cap (\acute{A} \cup B)$. So, for this we need to find the $\acute{A} \cup B$. So, $\acute{A}$ is this.

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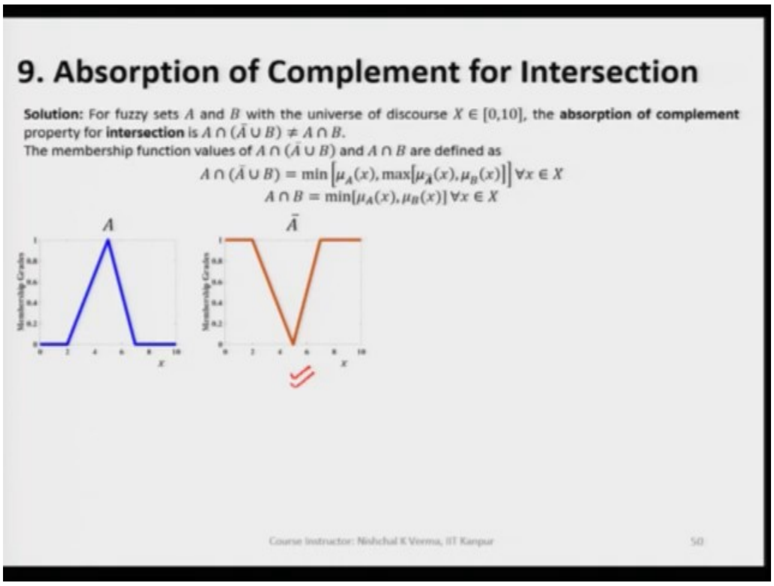

This is $\acute{A}$ and then if we take the $\acute{A} \cup B$, where B is this fuzzy set.

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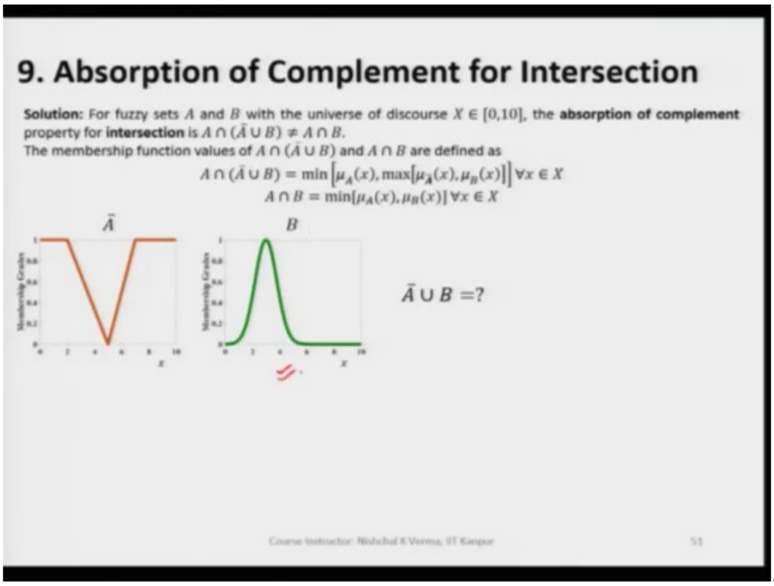

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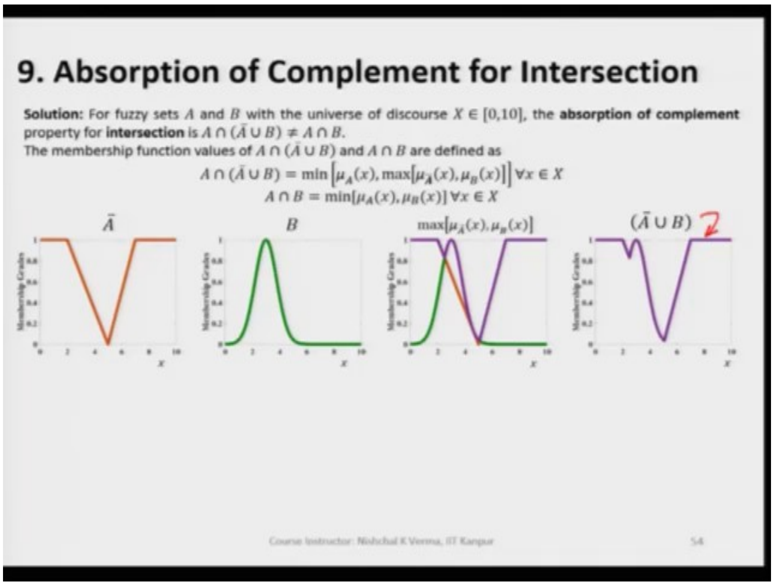

We see that after superimposing these two fuzzy sets on each other and since we are interested in union, we have to apply the max criteria. When we apply max criteria what are we going to get is here. So, this is the $\acute{A} \cup B$ which is again a fuzzy set. So, this is the outcome which we are getting when we take the $\acute{A} \cup B$. Now, we take the intersection of this fuzzy set with A.

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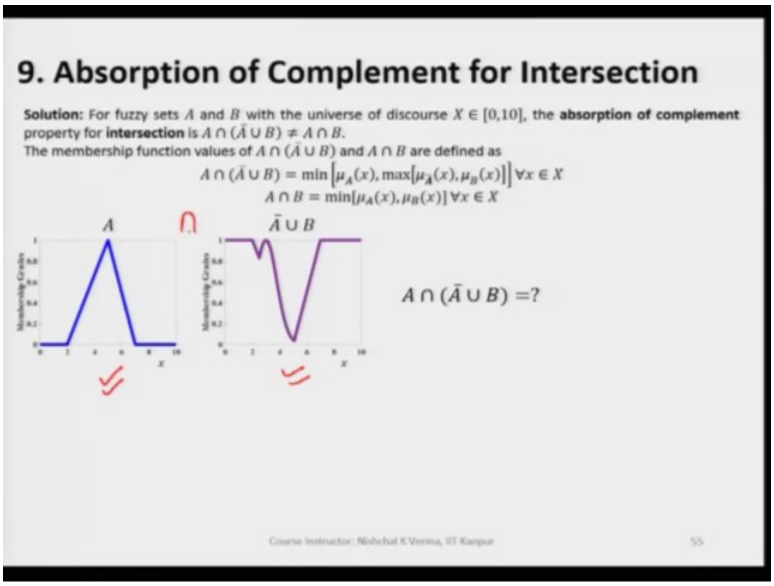

So, for this we have taken A here and we have taken a fuzzy set which is the union of $\acute{A} \cup B$.

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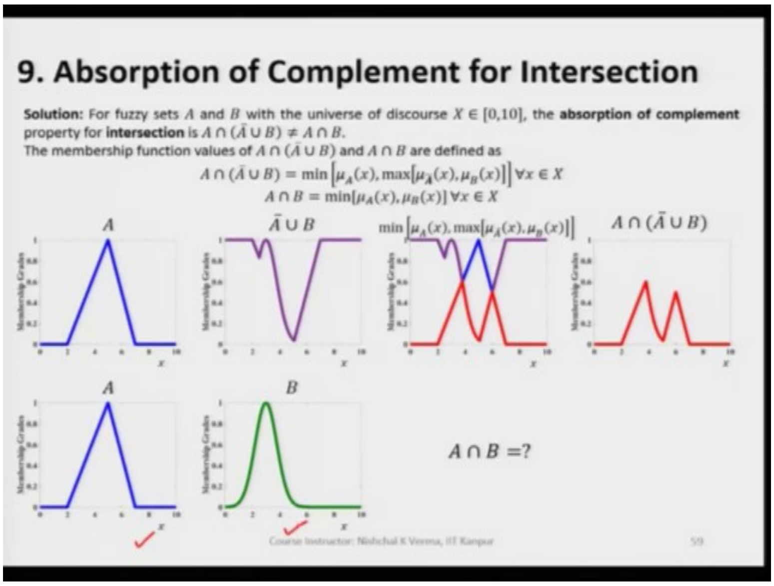

So, when we take the intersection of these two fuzzy sets when we take intersection of these two fuzzy set what are we going to get is here. We superimpose these two fuzzy sets on each other and then when we do that since we are taking the intersection we have to apply the min criteria. When we apply min criteria when we apply min criteria we are going to get this outcome as a fuzzy set. So, A, so, this is intersection of these two fuzzy sets and then let see whether this is equal to $A \cap B$ or not.

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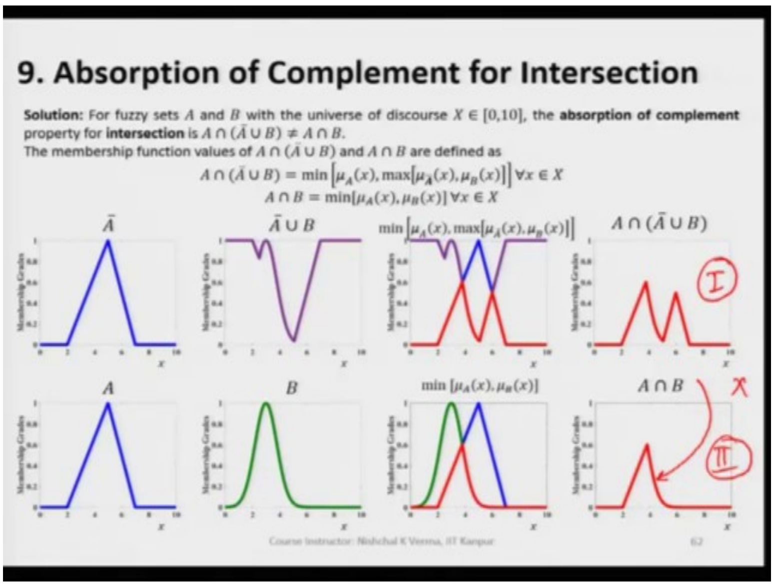

So, when we take A fuzzy set here and B fuzzy set here and we take the intersection of these two fuzzy sets, we apply the min criteria because again we are taking the intersection. So, when we apply min criteria we are going to get this fuzzy set as the outcome of $A \cap B$. And when we compare these two when we compare this fuzzy set and this fuzzy set, so, these two are not equal. So, this way we can say that the absorption of complement for intersection does not hold good for fuzzy sets A and B.

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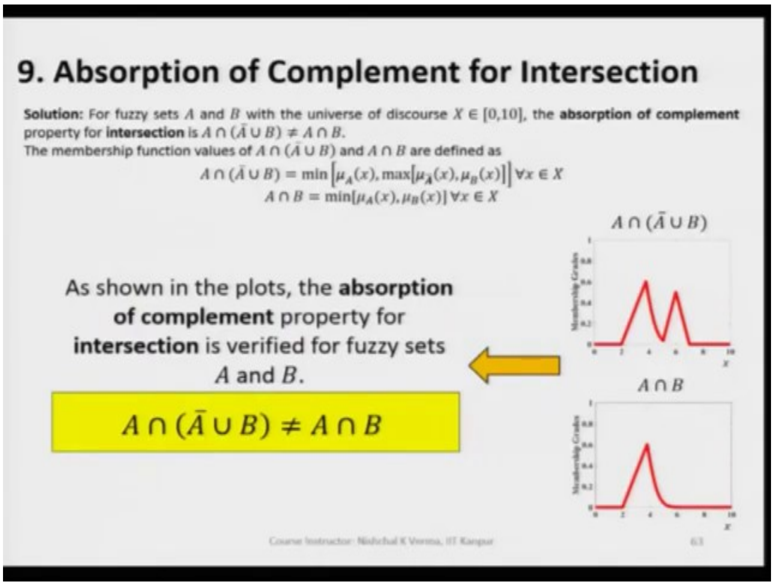

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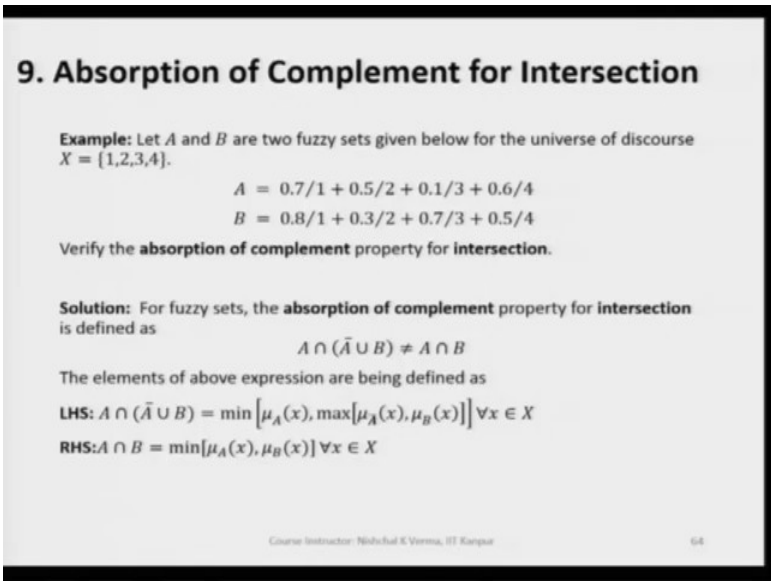

Now, if we take the discrete fuzzy sets instead of continuous fuzzy sets and we see that here also which is very clear from this example.

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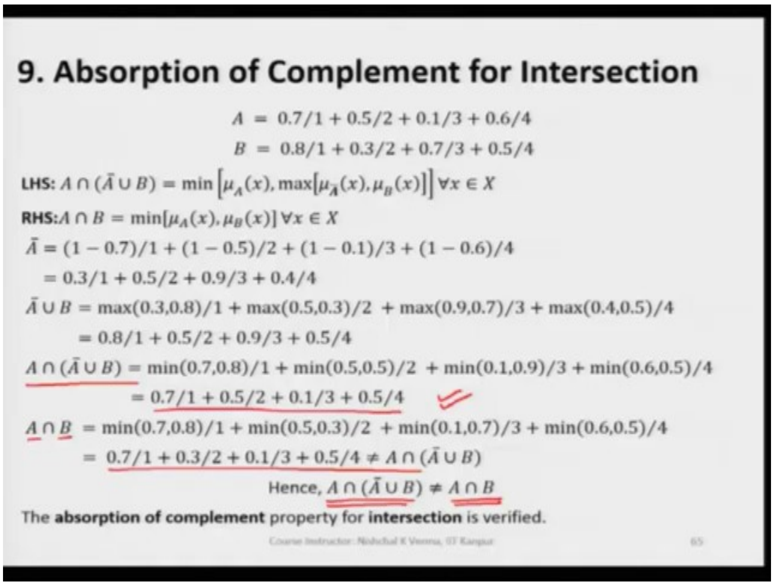

Here also when we try to find the $A \cap (\acute{A} \cup B)$, we see here that we are getting $0.7/1+0.5/2+0.1/3+0.5/4$. So, this is a discrete fuzzy set as the outcome and then when we take the intersection of A and B here which is coming out to be $0.7/1+0.5/2+0.1/3+0.5/4$ which is clearly not equal to this case.

So, this way we can say this is not equal to this. It means what? It means the absorption of complement for intersection does not hold good for discrete fuzzy sets A and$B$.

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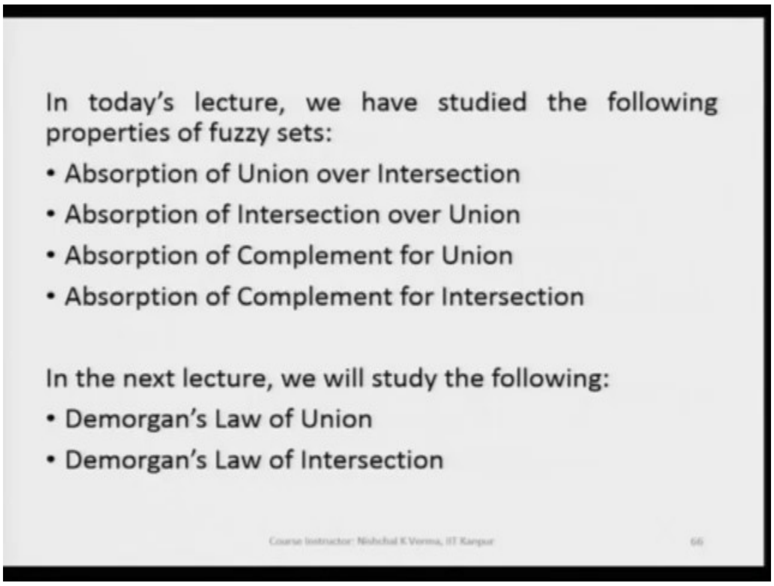

So, this way we have checked, we have verified, the absorption of union over intersection, absorption of intersection over union, absorption of complement for union, absorption of complement for intersection and these have been verified these have been checked for fuzzy sets which are continuous and discrete.

So, we will stop here in this lecture and in the next lecture will study will try to cover the following. So, we will be discussing the De Morgan's law of union and De Morgan's law of intersection for fuzzy sets.

Thank you.