Fuzzy Sets, Logic and Systems and Applications
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Lecture - 14
Properties of Fuzzy Sets

So, welcome to lecture number 14 of Fuzzy Sets, Logic and System and Applications. In this lecture today, we will discuss the distributivity property with respect to Fuzzy Sets. So, here is the list of all the properties that we have intended to cover.

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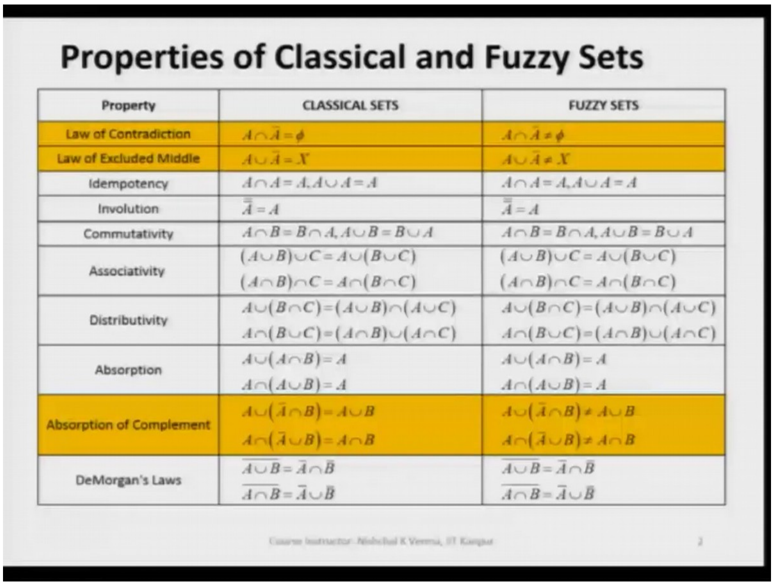

Properties of Classical and Fuzzy Sets

Property	CLASSICAL SETS	FUZZY SETS
Law of Contradiction	$A\cap\bar{A}=\phi$	$A\cap\bar{A}\neq\phi$
Law of Excluded Middle	$A\cup\bar{A}=X$	$A\cup\bar{A}\neq X$
Idempotency	$A\cap A=A, A\cup A=A$	$A\cap A=A, A\cup A=A$
Involution	$\bar{\bar{A}}=A$	$\bar{\bar{A}}=A$
Commutativity	$A\cap B=B\cap A, A\cup B=B\cup A$	$A\cap B=B\cap A, A\cup B=B\cup A$
Associativity	$(A\cup B)\cup C=A\cup(B\cup C)$ $(A\cap B)\cap C=A\cap(B\cap C)$	$(A\cup B)\cup C=A\cup(B\cup C)$ $(A\cap B)\cap C=A\cap(B\cap C)$
Distributivity	$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$	$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$
Absorption	$A\cup(A\cap B)=A$ $A\cap(A\cup B)=A$	$A\cup(A\cap B)=A$ $A\cap(A\cup B)=A$
Absorption of Complement	$A\cup(\bar{A}\cap B)=A\cup B$ $A\cap(\bar{A}\cup B)=A\cap B$	$A\cup(\bar{A}\cap B)\neq A\cup B$ $A\cap(\bar{A}\cup B)\neq A\cap B$
DeMorgan's Laws	$\overline{A\cup B}=\bar{A}\cap\bar{B}$ $\overline{A\cap B}=\bar{A}\cup\bar{B}$	$\overline{A\cup B}=\bar{A}\cap\bar{B}$ $\overline{A\cap B}=\bar{A}\cup\bar{B}$

So, so far we have covered the Law of Contradiction, Law of Excluded Middle, Idempotency, Involution, Commutativity, Associativity. All these properties with respect to fuzzy sets have been covered.

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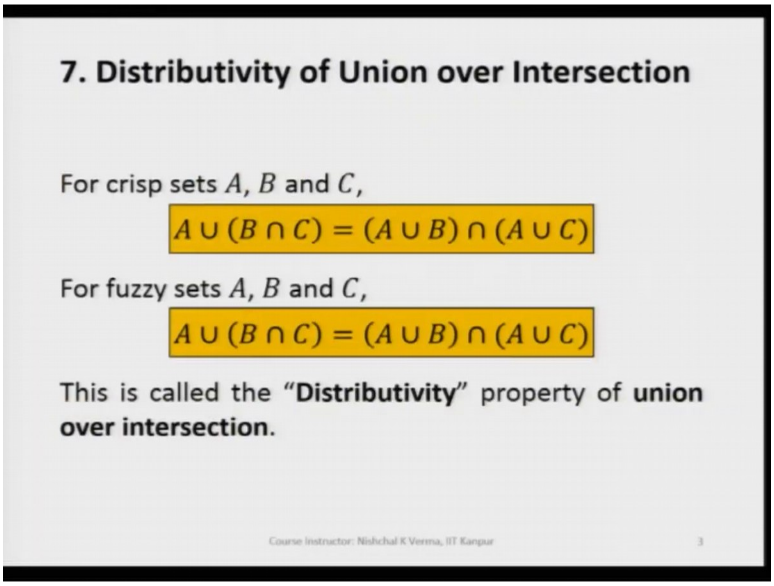

Now, today we will discuss distributivity property with respect to fuzzy sets and this, distributivity property is divided into two parts. The first part will be the distributivity of union over intersection and then the second part will be the distributivity of intersection over union. And the distributivity property will be discussed with respect to the fuzzy sets, the continuous fuzzy sets and the discrete fuzzy sets.

And we all know that for crisp sets A, B and C, this distributivity property of union over intersection is valid. This means that when we take the $A \cup (B \cap C)$, so if we take the union of these two entities, what we get is here we get the $(A \cup B) \cap (A \cup C)$. So, this is we know for the crisp sets.

And let's see what happens with the fuzzy sets. So, when we take fuzzy sets A, B and C, the same applies same holds good. This means that when we take $A \cup (B \cap C)$, this also is equal to the $(A \cup B) \cap (A \cup C)$, where in this case we have A, B, C, all these are the fuzzy sets. So, let us now understand by taking some examples.

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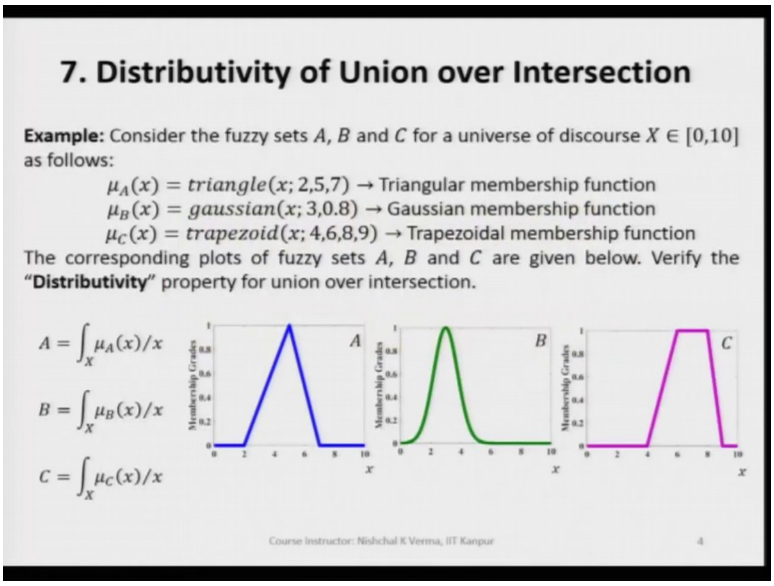

So, here we have an example which has three continuous fuzzy sets. So, now let us see whether for continuous fuzzy sets, the distributivity property of union over intersection is satisfied or not. Of course, this has to be satisfied, but let us see how is it satisfied.

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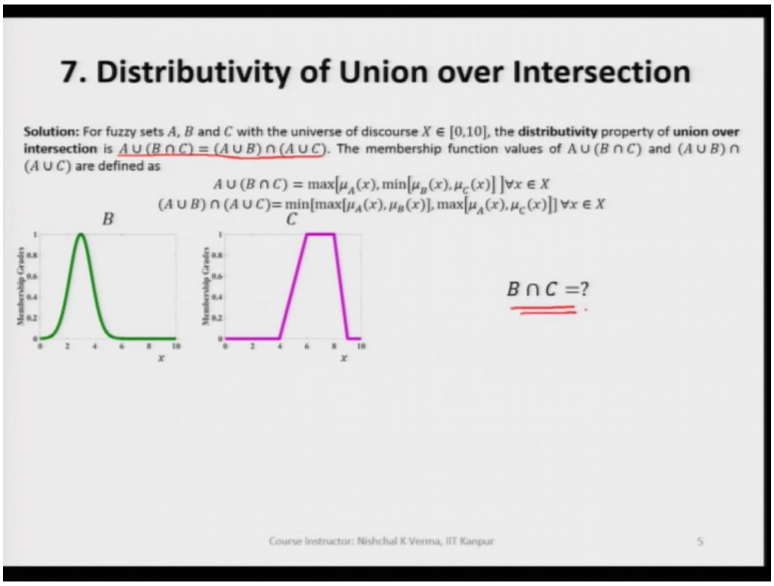

So, here we have to verify basically this thing.

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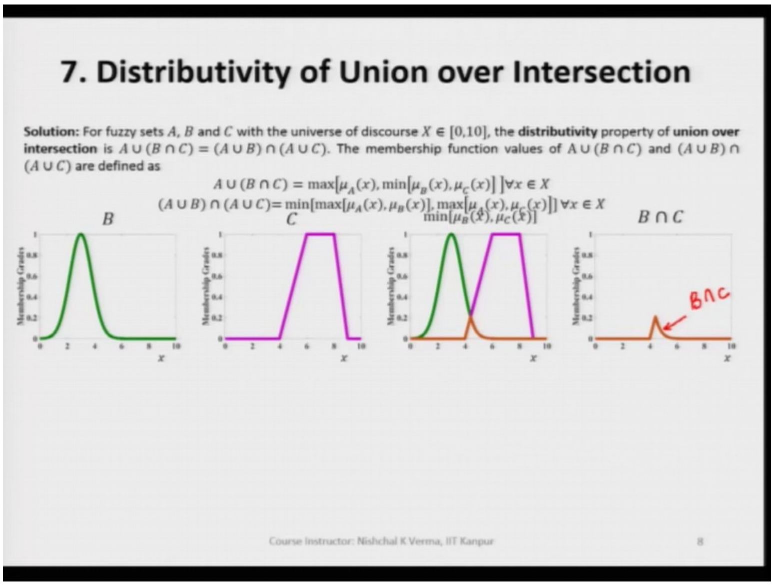

So, for this we are first taking the $B \cap C$ and the $B \cap C$, we can get these two fuzzy sets B and C and when we overlap these two fuzzy sets, we get this portion by applying the min criteria. So, this min criteria is going to give us a fuzzy set a sub normal fuzzy set as a result like this.

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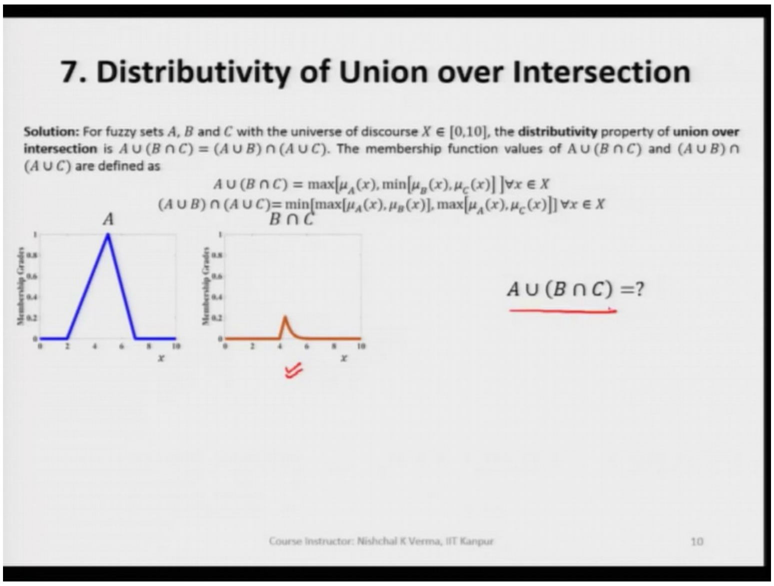

Now, what we have to get is what we have to obtain here is $A \cup (B \cap C)$. So, now we take the union of these two fuzzy sets, so this is what we have got as $B \cap C$. Now we take the union of these two, so let us see what are we going to get.

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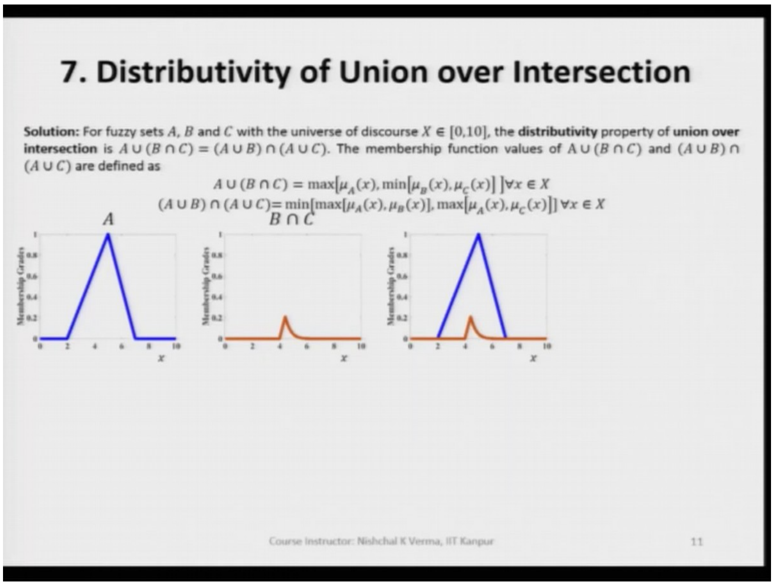

We overlap these two fuzzy sets here and when we take the union of these two fuzzy sets, of course by applying the max criteria we are going to get this as the $A \cup (B \cap C)$.

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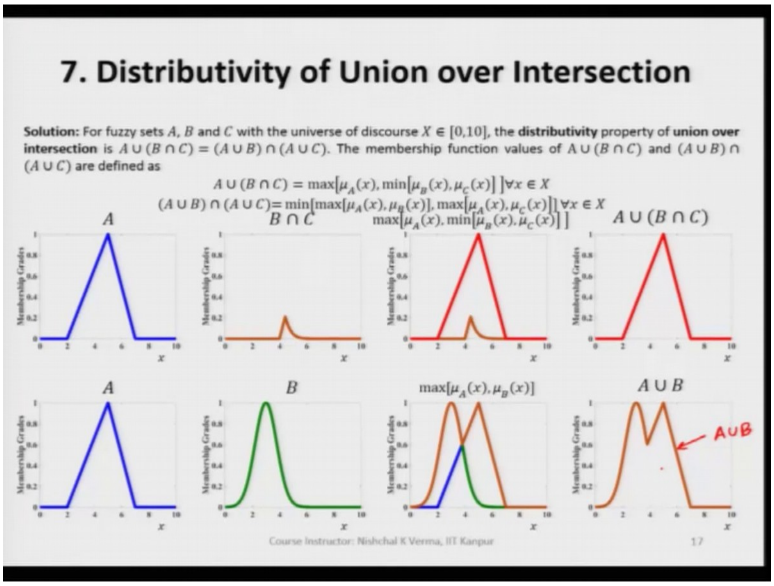

So, this way we get a triangular membership function which is a normal membership function. So, when we are trying to see whether it holds good or not, so for this we have to first find $A \cup B$. So, when we take $A \cup B$ set here, so after overlapping these two fuzzy sets A and B and when we apply the max criteria, when we take union of these two, we are going to get this as $A \cup B$, here this fuzzy set is a normal fuzzy set. So, we have got $A \cup B$ as shown here by the red color fuzzy set.

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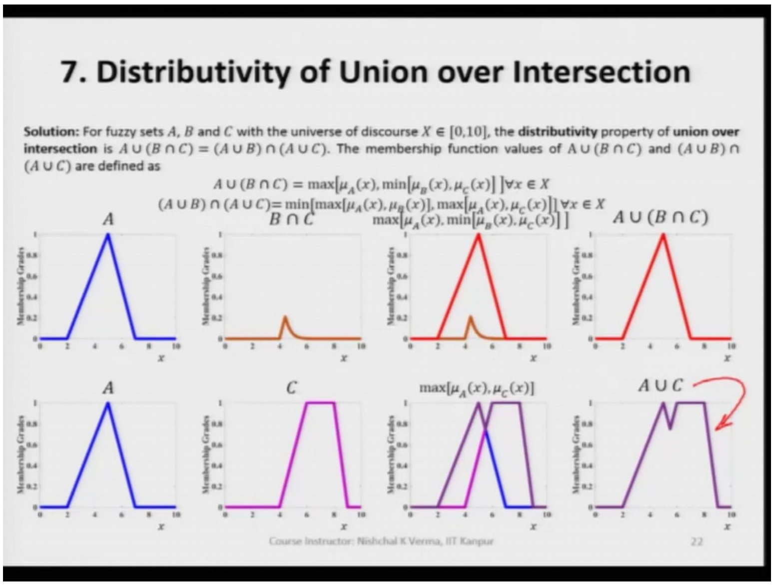

And now we have to obtain $A \cup C$. So, when we have A fuzzy set here and C fuzzy set here, so $A \cup C$ where when we overlap these two, we get after taking the max criteria applied, we are going to get $A \cup C$ here. So, this is $A \cup C$. Now, we have to take the intersection of $A \cup B$ and $A \cup C$. So, let us see what are we going to get out of these two.

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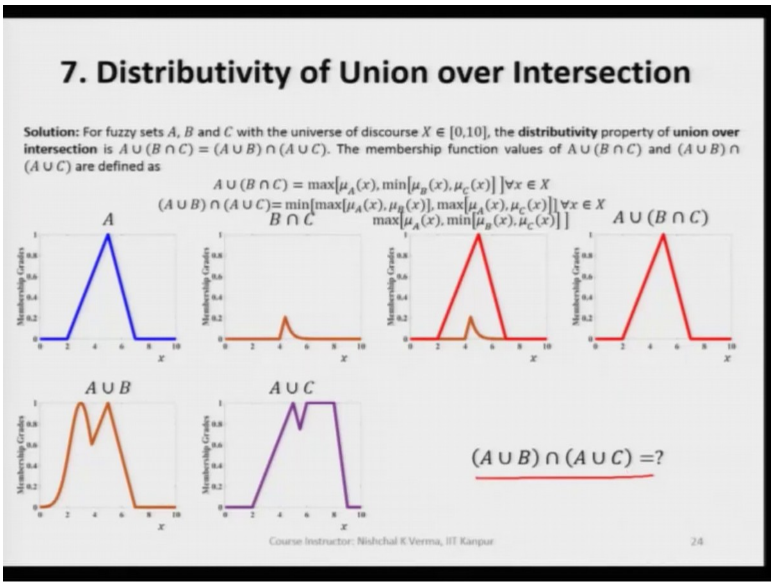

So, we have $A \cup B$ which we have already just got and then we have $A \cup C$. Now let us see what we are going to get as the intersection of these two fuzzy sets.

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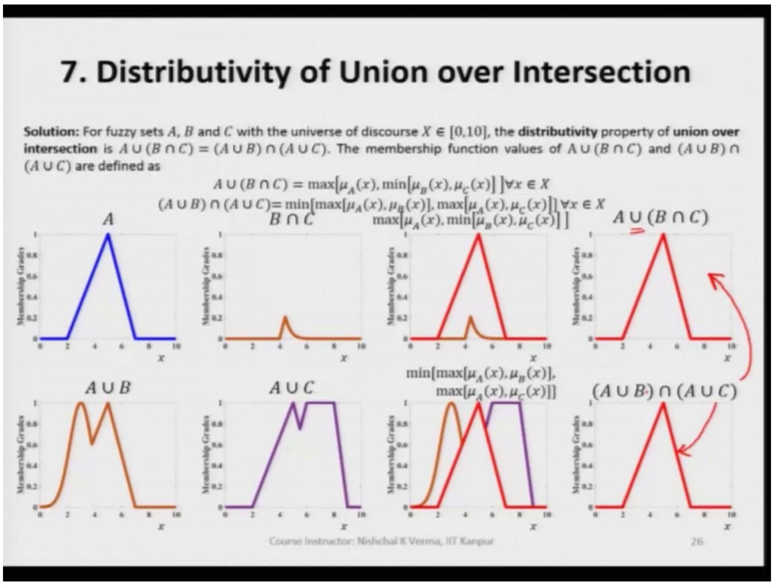

So, when we overlap these two fuzzy sets is the $A \cup B$ fuzzy set, this is $A \cup C$ fuzzy set. So, when we overlap these two fuzzy sets and since we are going to we are interested in the intersection of these two, so we have to apply the min criteria. So, when we apply the min criteria, min of the these two membership; min of these two fuzzy sets, so what we are going to get is here. So, this is what is the outcome.

So, when we see this outcome is same as we have got here, so these two fuzzy sets are equal. So, this means that if we have three fuzzy sets A, B and C, so if we take the union of this is the union. So, if we take the union of $A \cup (B \cap C)$, we are going to get this equal to the $(A \cup B) \cap (A \cup C)$.

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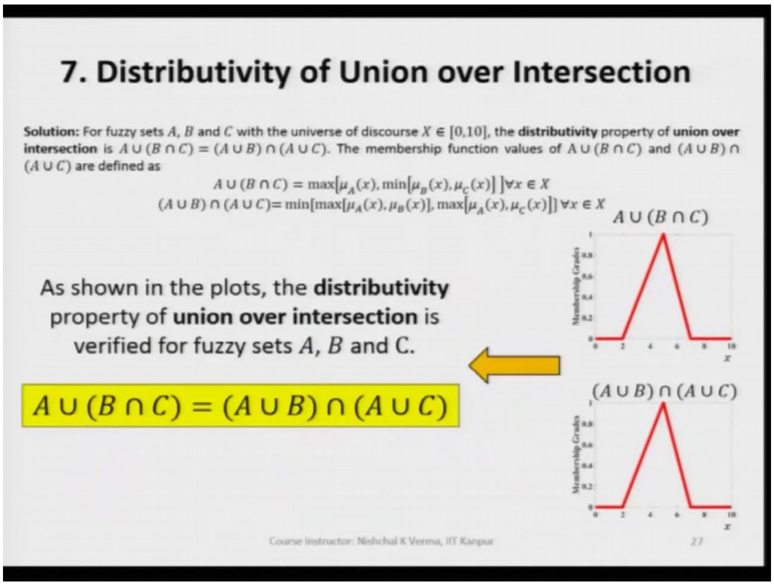

So, this is very clear, the results are same here; the outcomes are same. So, we can say here that the distributivity property of union over intersection is verified or in other word we can say distributivity property of union over intersection holds good for fuzzy sets as well.

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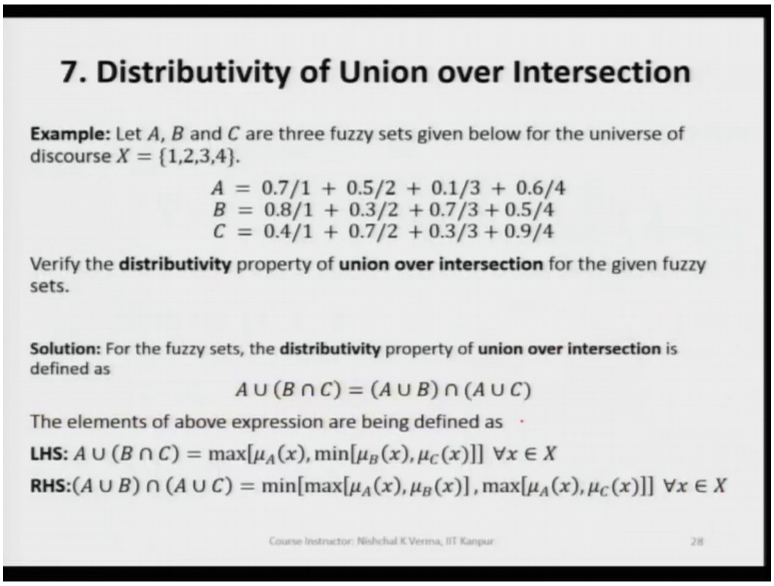

So, now the same can be checked by taking the discrete fuzzy sets A, B and C. So, here we have taken three discrete fuzzy sets and if we see here, we get $B \cap C$ as this discrete fuzzy set.

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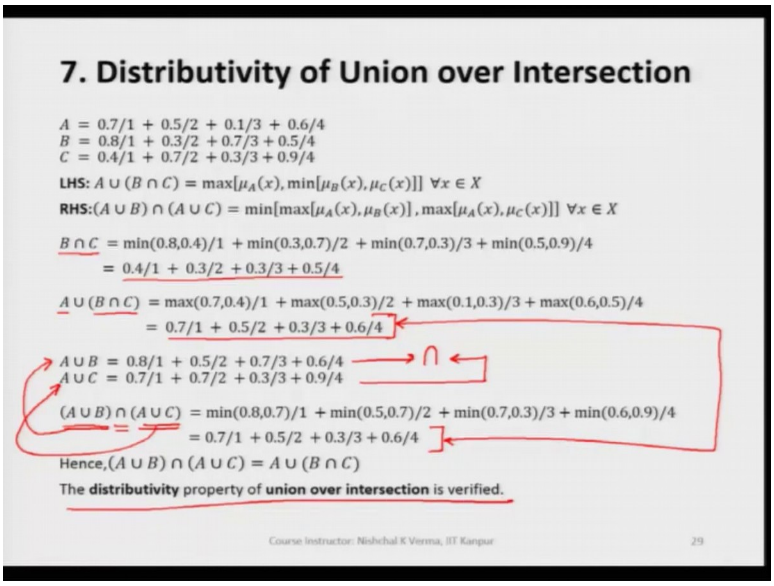

And then when we take the $A \cup (B \cap C) = 0.7/1 + 0.5/2 + 0.3/3 + 0.6/4$. So, this is what is the outcome that we get here. Now, if we find here the $(A \cup B) \cap (A \cup C)$ and this $A \cup B$ we get from here and $A \cup C$ we get from here.

So, if we take the intersection of these two, we take the intersection of these two like this, like we have this and we have this and we take the intersection. So, if we take this intersection, we are going to get discrete fuzzy set again as the outcome and this is $0.7/1 + 0.5/2 + 0.3/3 + 0.6/4$ and which is same as this fuzzy set. So, it is clearly visible that for discrete fuzzy sets also the distributivity property of union over intersection is verified.

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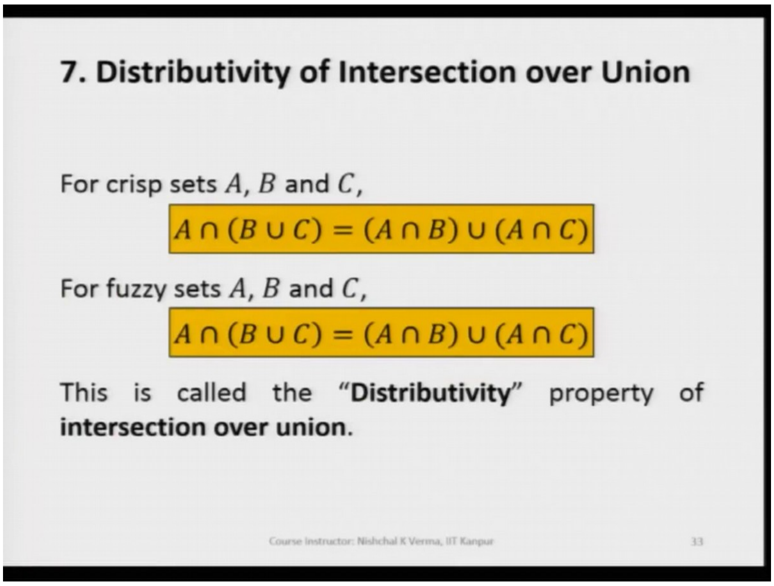

So, now we can go ahead and define that the distributivity of intersection over union for crisp sets, the distributivity property of intersection over union holds good and this we already know. So, this means that A intersection or other words we can say the $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

So, this holds good for crisp sets and if we change this by fuzzy sets A, B and C, so this distributivity property of intersection over union is here also satisfied. So, this mean that the distributivity of intersection over union for fuzzy sets is also holding good. So as it is written here, distributivity property of intersection over union and this holds good for fuzzy sets A, B and C same as crisp sets.

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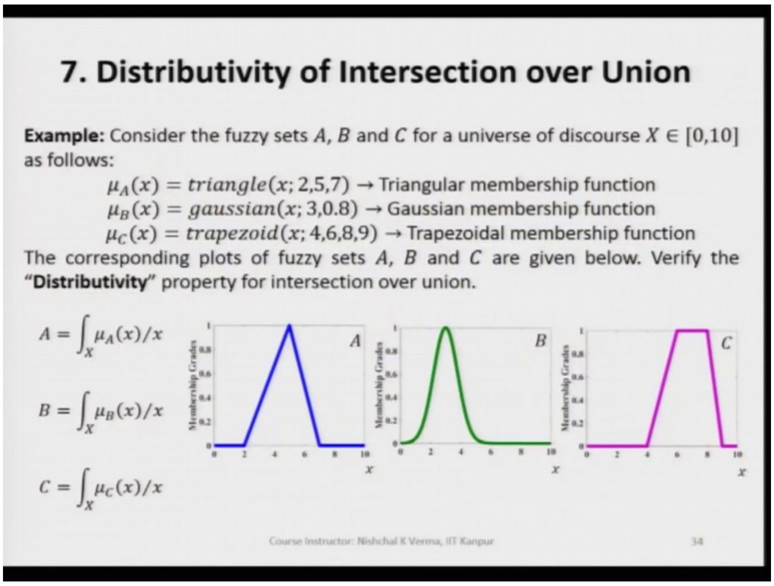

Now, let us try to see how this is holding good for fuzzy sets A, B and C. So, let us take an example here where we take three continuous fuzzy sets A, B and C. So, these are the fuzzy sets you can see here.

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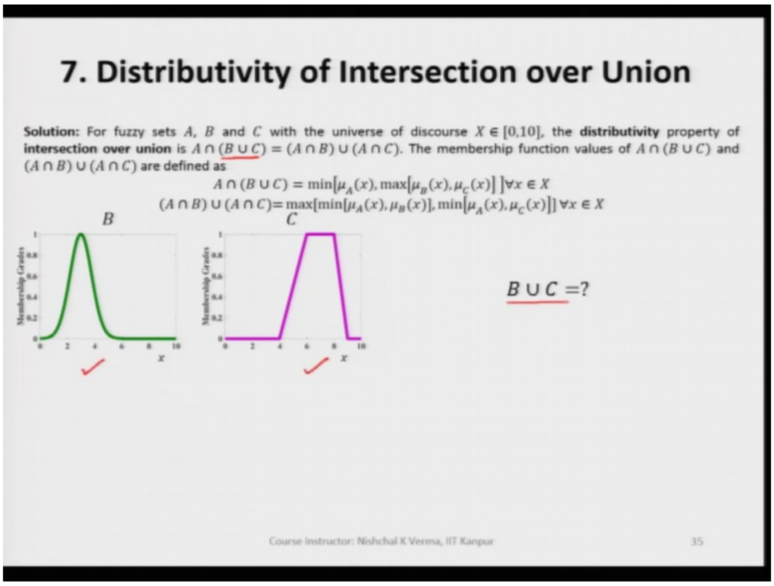

Now, for verifying to verify this distributivity of intersection over union, we have to first get the $B \cup C$.

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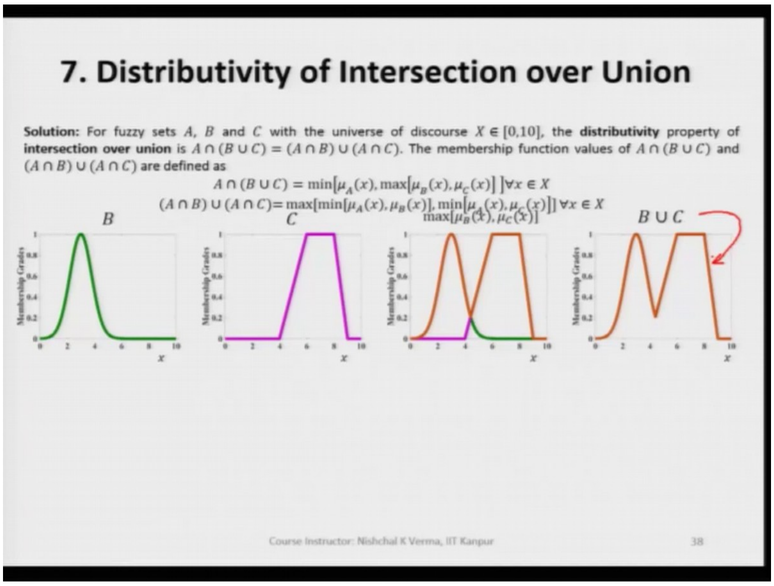

So $B \cup C$, we have we are taking B fuzzy set and C fuzzy set and $B \cup C$, we can get just by overlapping these two fuzzy sets and apply the max criteria, we are going to get this as the $B \cup C$.

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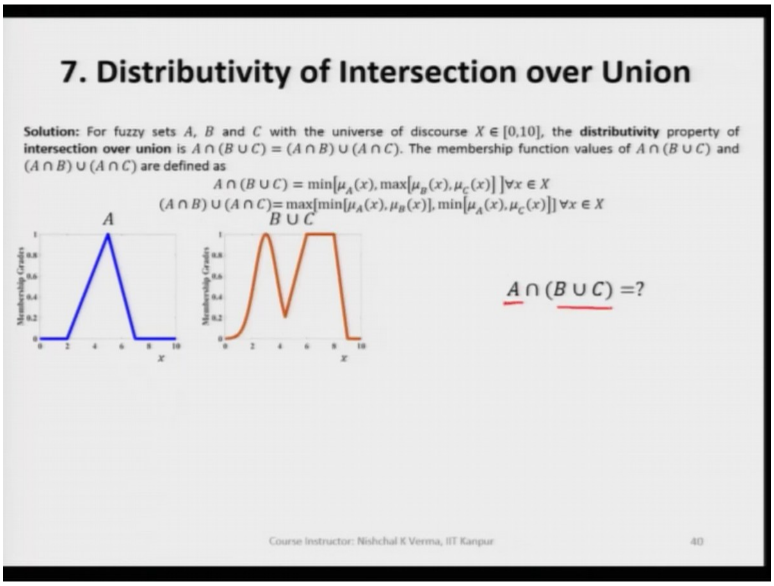

And now let us take a fuzzy set and then we take the $A \cap (B \cup C)$ which we have just got.

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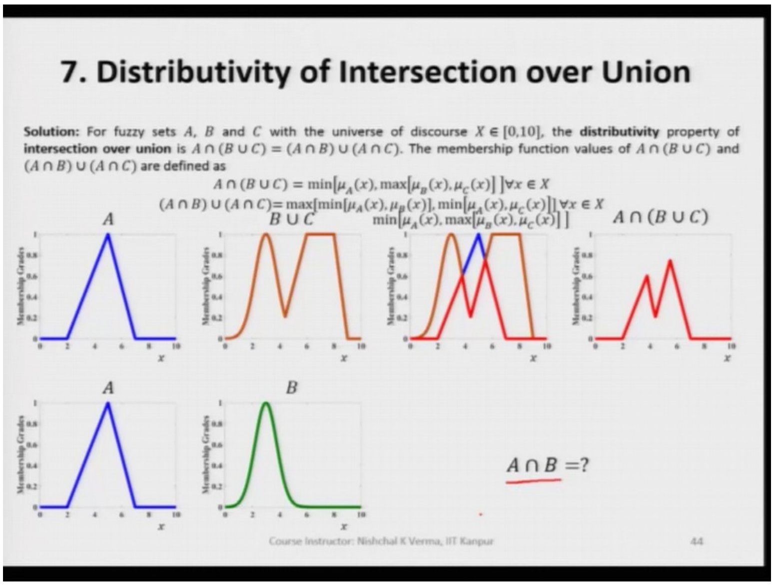

So, $A \cap (B \cup C)$ here. So, we can again overlap these two fuzzy sets and since we are taking here intersection, we apply the min criteria and then on applying the min criteria, we are going to get this as the result. So, this is represented by the red color fuzzy set. So, this is the outcome of the $A \cap (B \cup C)$. So, this will keep this here this outcome here and then now we will try to find the $(A \cap B) \cup (A \cap C)$ and let us see if this is coming out to be same as this or not.

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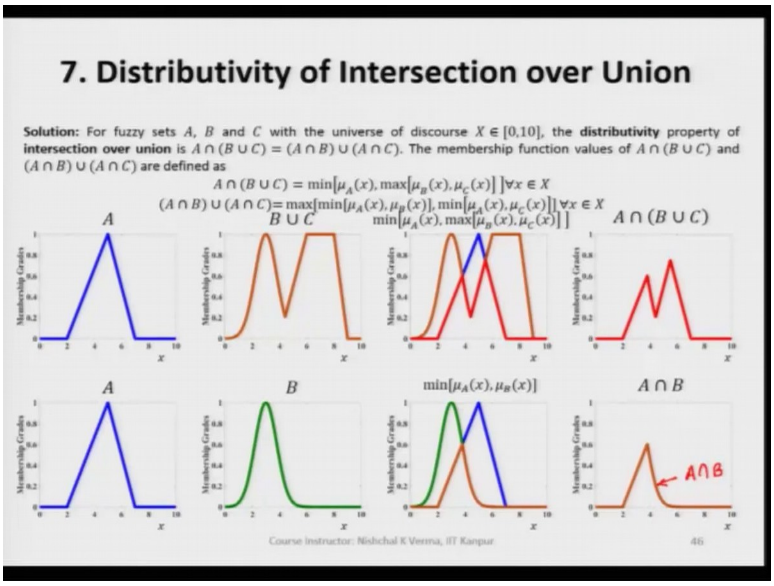

So, let us now try to find the $A \cap B$. So, $A \cap B$ again we take fuzzy set and fuzzy set B and we try to we first overlap these two fuzzy sets and since we are taking the intersection, so we have to apply the min criteria. So when we apply a min criteria, we get a fuzzy set here, a sub normal fuzzy set and we can and this is $A \cup B$, $A \cap B$. Now, after this we have to find $A \cap C$.

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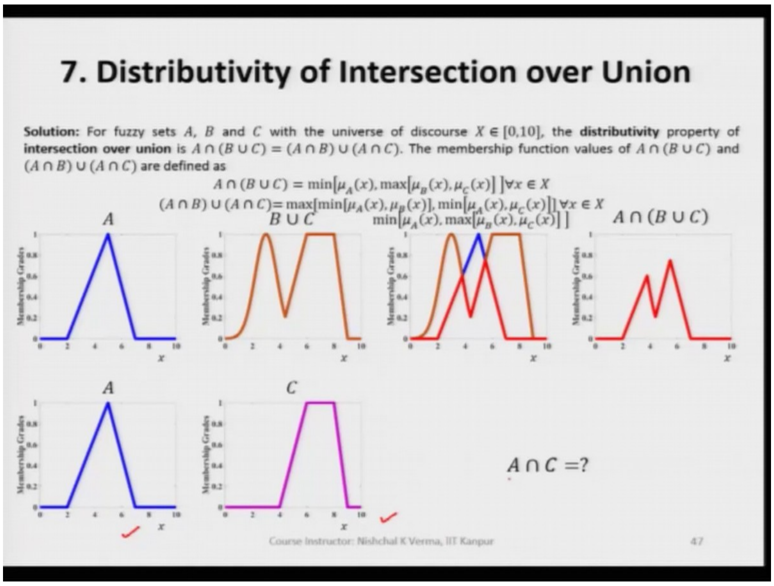

So, when we take A fuzzy set and B, C fuzzy set, we take the intersection, we find the $A \cap C$.

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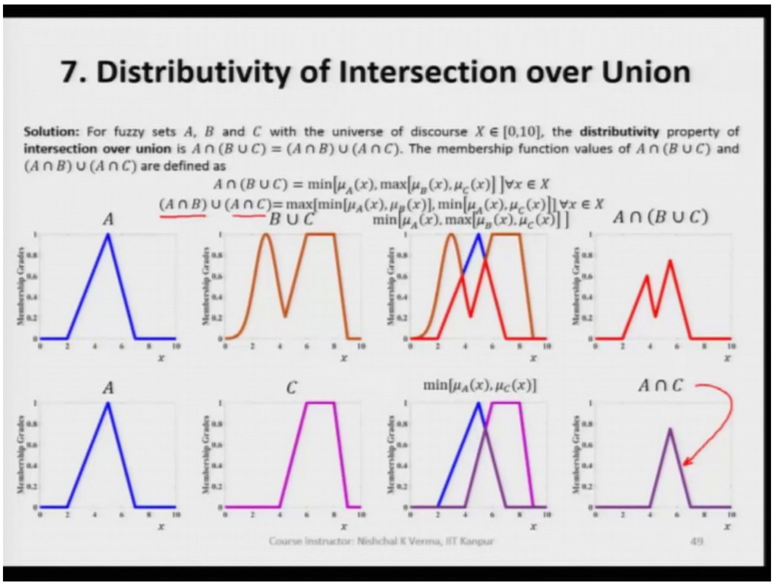

We again do the same and we overlap these two fuzzy sets A and C together. We apply the min criteria, after applying the min criteria we get the sub normal fuzzy set which is $A \cap C$ which is the outcome here.

So, after this what we have to do is we have to find the union of these two fuzzy sets. So, we have now $A \cap C$ and earlier we found $A \cap B$. So, now we take the union of these two.

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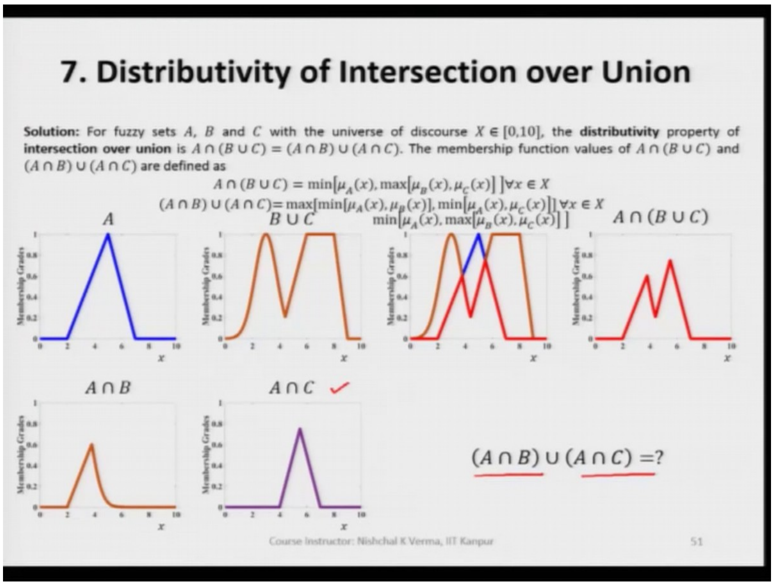

So, we had here $A \cap B$ and then we have $A \cap C$. Now, we have to find the union of these two fuzzy sets. Let us see how does it look like.

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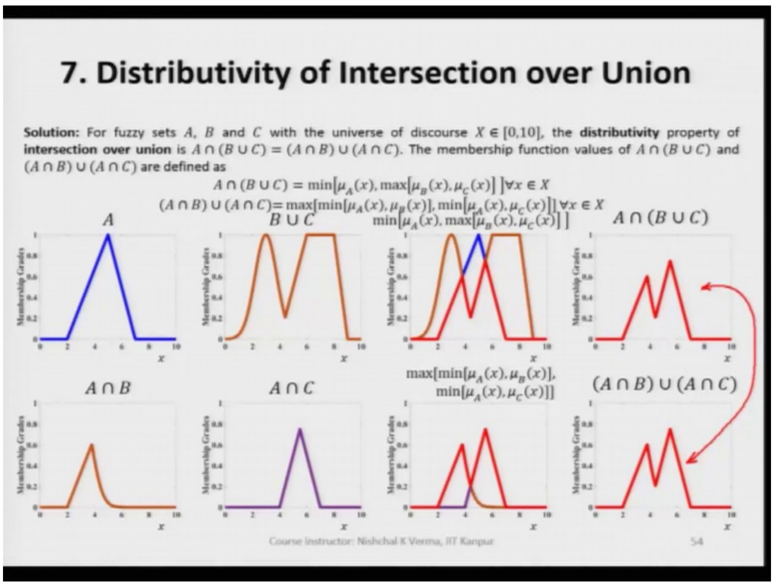

So, we see here that we overlap these two fuzzy sets here. So, this fuzzy set and this fuzzy set. So, what is this is $A \cap B$ and here we have $A \cap C$.

We have to take the union of these two fuzzy sets, it means that we have to apply the max criteria and when we apply max criteria, we are going to get this as the result. So, we are going to get here this as the union of $A \cap B$ and $A \cap C$. So, if we see these outcome is same as this outcome is same as the previous outcome which is which we have got on taking the $A \cap (B \cup C)$.

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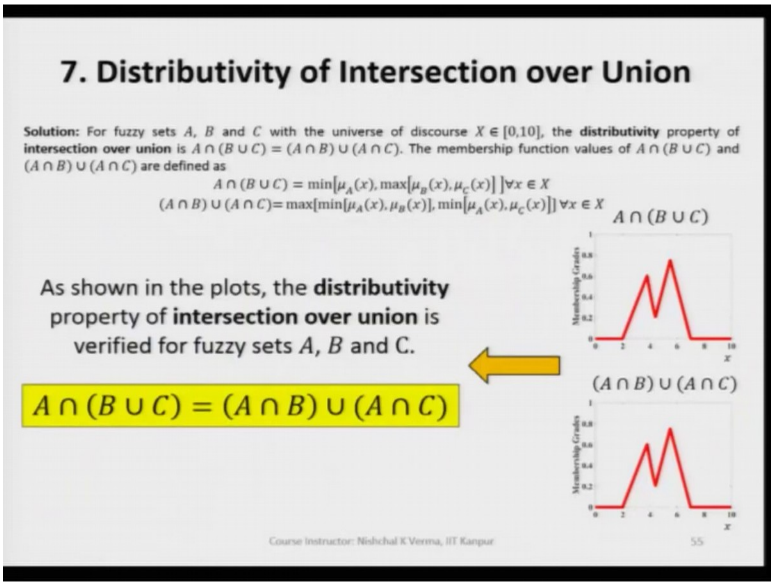

So, this way we can say here; this way we can see here say here that the distributivity property of intersection over union is verified for fuzzy sets A, B and C which is here.

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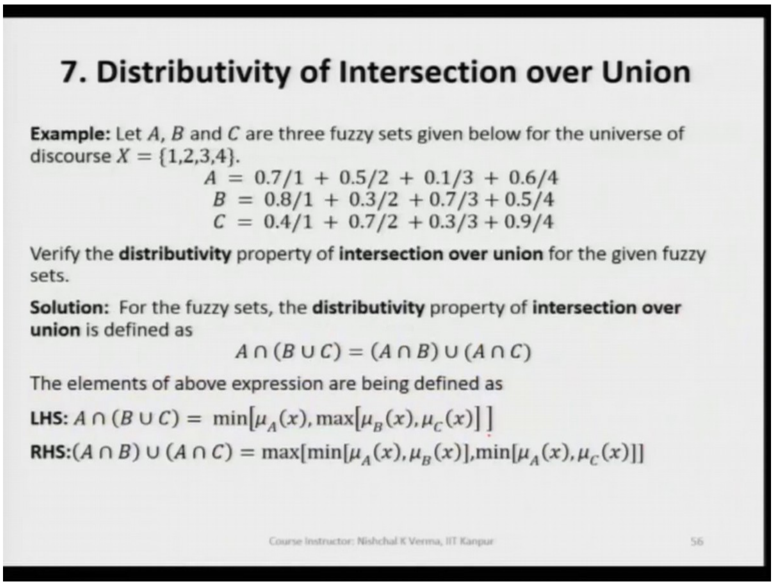

Now, the same can be verified by taking three discrete fuzzy sets. So, these three discrete fuzzy sets we have taken here. So, we have A fuzzy set as $0.7/1+0.5/2+0.1/3+0.6/4$ and then we have B as the discrete fuzzy set here as $0.8/1+0.3/2+0.7/3+0.5/4$. And the 3rd one, 3rd fuzzy set 3rd discrete fuzzy set here is $0.4/1, 0.7/2, 0.3/3, 0.9/4$. Let us now try to verify the distributivity property of intersection over union.

So, for this we have to first find $B \cup C$ and then we take the if we find the $A \cap (B \cup C)$. So, let us now go ahead and do that.

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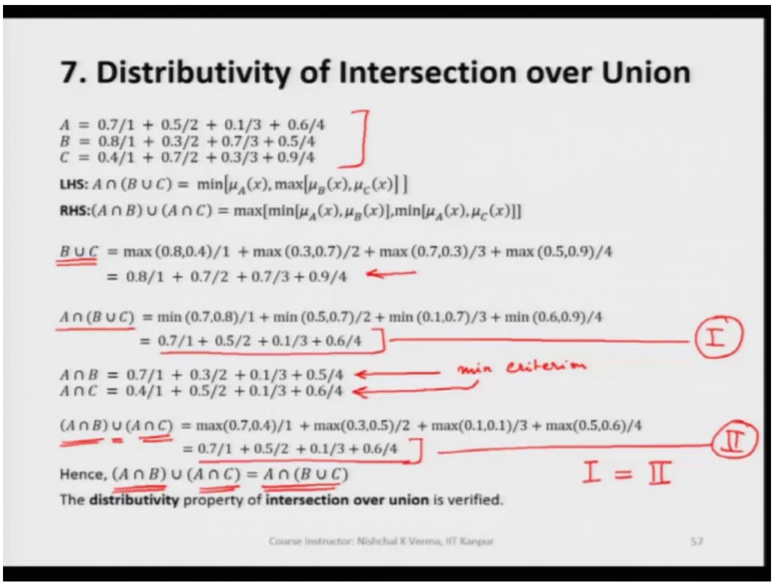

So, $B \cup C$. is here and this is coming out to be after applying the max criteria, we are going to get $0.8/1+0.7/2+0.7/3+0.9/4$ and this is nothing, but is again a discrete fuzzy set. So, all these A, B, C are also discrete fuzzy sets.

Now, let us take the intersection here $A \cap (B \cup C)$ here and then when we apply since we are taking the intersection, we have to follow the min criteria here, we have to apply min criteria here. So, when we do that we are getting $0.7/1+0.5/2+0.1/3+0.6/4$ as a discrete fuzzy set. So, this way we have the outcome and now we need to find the $A \cap B$ and $A \cap C$. So if we find$A \cap B$, we are getting $0.7/1+0.3/2+0.1/3+0.5/4$ by after applying the min criteria; min criteria.

So here also $A \cap C$, we are getting after applying the min criteria $0.4/1+0.5/2+0.1/3+0.6/4$. So now when we have found the intersections, we are getting two discrete fuzzy sets as $A \cap B$, $A \cap C$, now we have to take the union of these two fuzzy, discrete fuzzy sets. So, when we are taking the union of these two fuzzy sets; so since we are taking union now what we have to do is, we have to apply a max criteria. So when we apply max criteria, let us see what we are going to get.

So when we apply max criteria, we are getting $0.7/1+0.5/2+0.1/3+0.6/4$. So, this way when we compare the these two outcome, this was the first outcome, if I say here and this is the second outcome here, so we see that the first outcome is equal to the second outcome or in other words, we can say that the $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$. So, this way we can clearly see that the distributivity property of intersection over union is verified.

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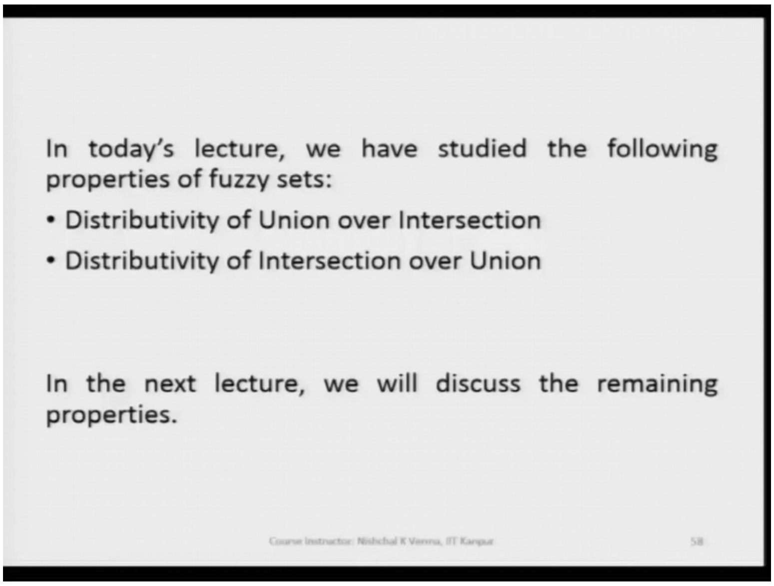

So, after verifying this we clearly we can say that the distributivity property of union over intersection and distributivity property of intersection over union both are satisfied for fuzzy sets. Here in our examples we have taken two fuzzy sets discrete as well as continuous and we have seen that these two distributivity properties are verified. So, at this point we will stop and in the next lecture we will discuss the remaining properties.

Thank you.