Fuzzy Sets, Logic and Systems and Applications
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Lecture - 13
Properties of Fuzzy Sets

So, welcome to lecture number 13 of Fuzzy Sets, Logic and Systems and Applications. In this lecture we will cover the remaining Properties of the Fuzzy Sets. So, we have already discussed some of the properties of classical and fuzzy sets in the previous lectures and these are the properties that are covered.

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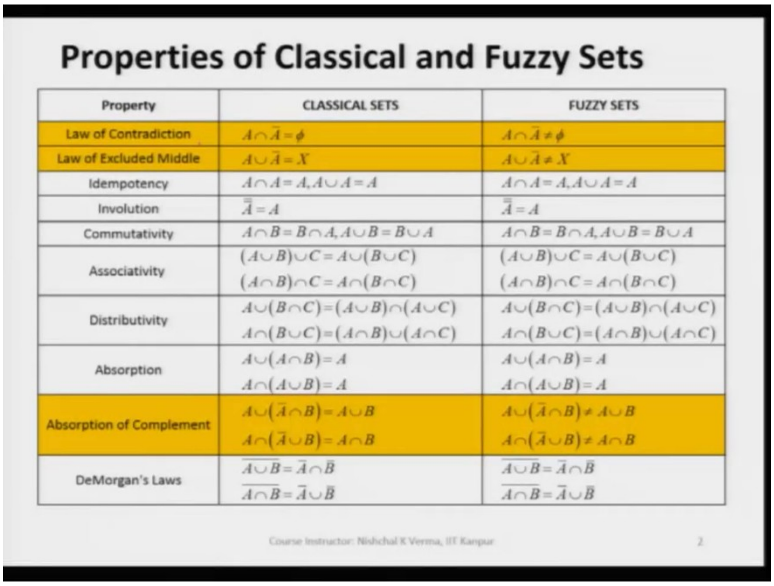

Property	CLASSICAL SETS	FUZZY SETS
Law of Contradiction	$A \cap \bar{A} = \phi$	$A \cap \bar{A} \neq \phi$
Law of Excluded Middle	$A \cup \bar{A} = X$	$A \cup \bar{A} \neq X$
Idempotency	$A \cap A = A, A \cup A = A$	$A \cap A = A, A \cup A = A$
Involution	$\bar{\bar{A}} = A$	$\bar{\bar{A}} = A$
Commutativity	$A \cap B = B \cap A, A \cup B = B \cup A$	$A \cap B = B \cap A, A \cup B = B \cup A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributivity	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Absorption	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Absorption of Complement	$A \cup (\bar{A} \cap B) = A \cup B$ $A \cap (\bar{A} \cup B) = A \cap B$	$A \cup (\bar{A} \cap B) \neq A \cup B$ $A \cap (\bar{A} \cup B) \neq A \cap B$
DeMorgan's Laws	$\overline{A \cup B} = \bar{A} \cap \bar{B}$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$	$\overline{A \cup B} = \bar{A} \cap \bar{B}$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$

So, I will just mention here law of contradiction for fuzzy set is discussed and then law of excluded middle is also discussed, idempotency property is discussed, involution is also discussed. So, we see that we have covered so far 4 properties as mentioned here with respect to fuzzy sets, now remaining properties will be discussed we will try to cover in this lecture. So, we will start with the commutativity property.

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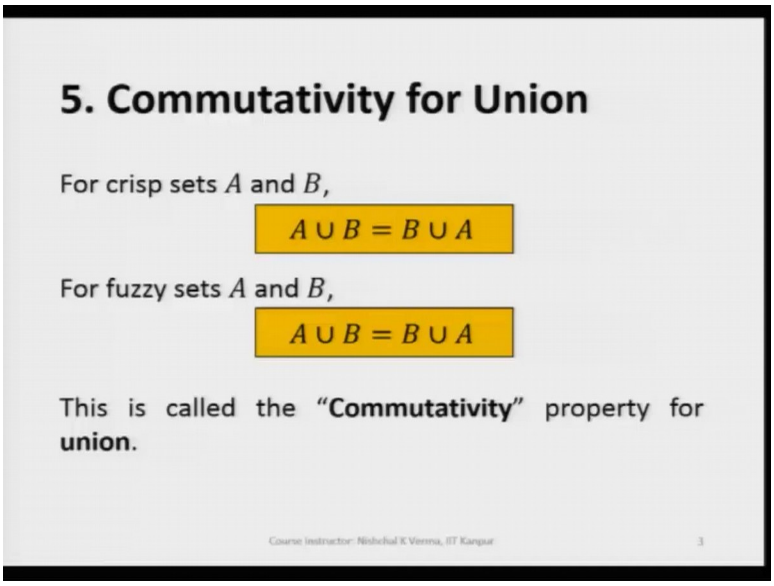

So, as we know for crisp sets A and B, $A \cup B = B \cup A$ and this is the commutativity property for union. So, let us now see what is happening when we take fuzzy sets instead of crisp sets. So, if we take two fuzzy sets A and B so, let's see whether we get $A \cup B = B \cup A$ or not, of course it is written so we will be getting these two equal. So, let us see what is happening and how are we getting this commutativity property for union satisfied.

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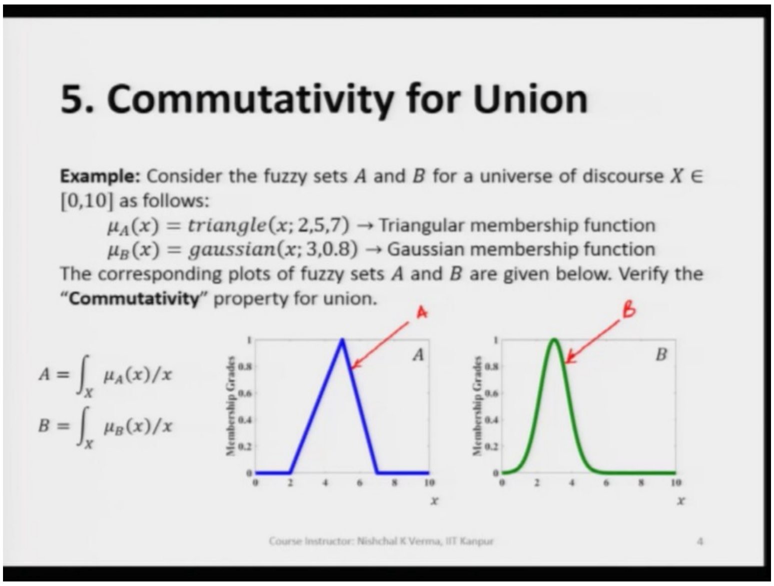

So, if you take an example here where we take a two continuous fuzzy sets A and B, here fuzzy sets A is a triangular fuzzy set, this fuzzy set A which triangular as we see and B fuzzy set is a Gaussian fuzzy set here as we see. So, we call this as the as B fuzzy set. So, now let's see whether $A \cup B = B \cup A$ or not.

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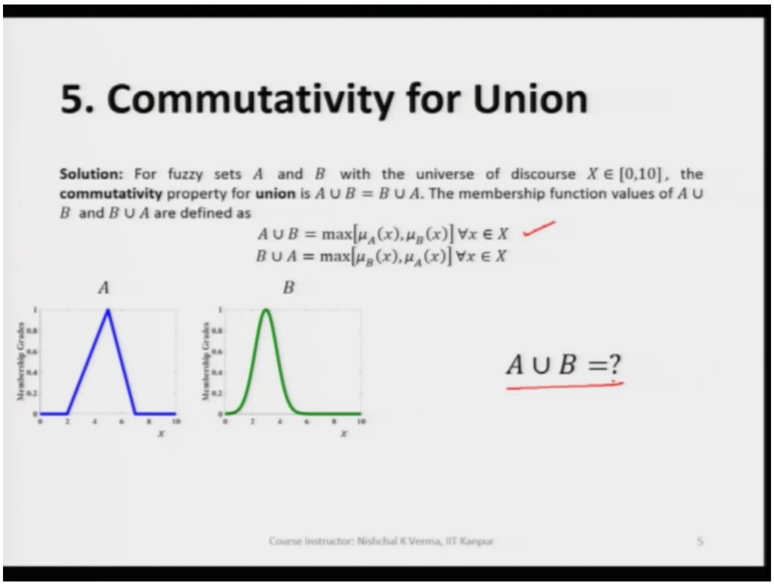

So, if we take $A \cup B$ here, so we apply this condition where we take the max of all the corresponding membership values from fuzzy set A and B with respect to their corresponding generic variable values.

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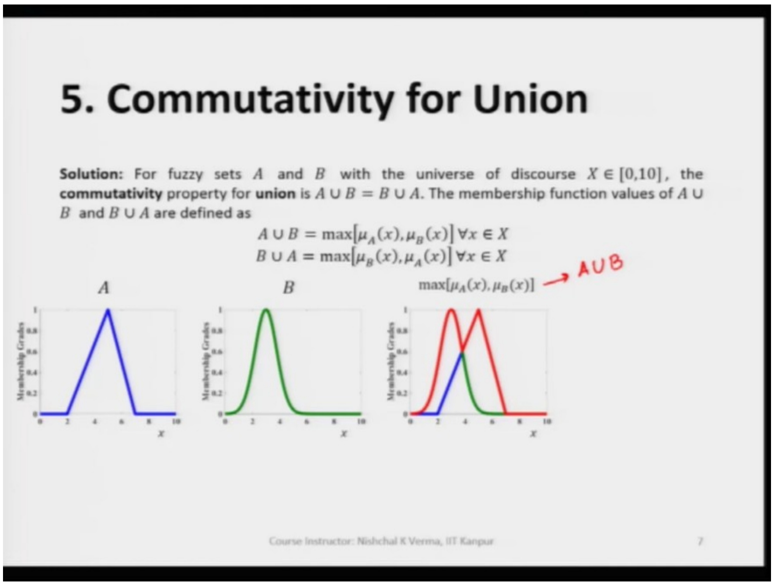

So, if we do that we find $A \cup B$ as this. So here, so this is nothing, but $A \cup B$. So, we have already discussed enough as to how we get the union of two fuzzy sets.

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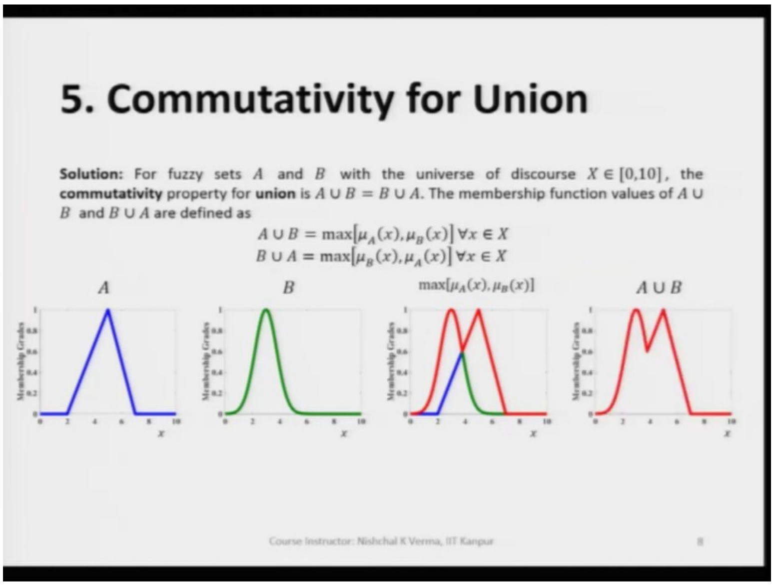

So, this way we get here after applying this condition we get $A \cup B$ as mentioned over here. Now, let us see what are we getting as the outcome when we take $B \cup A$.

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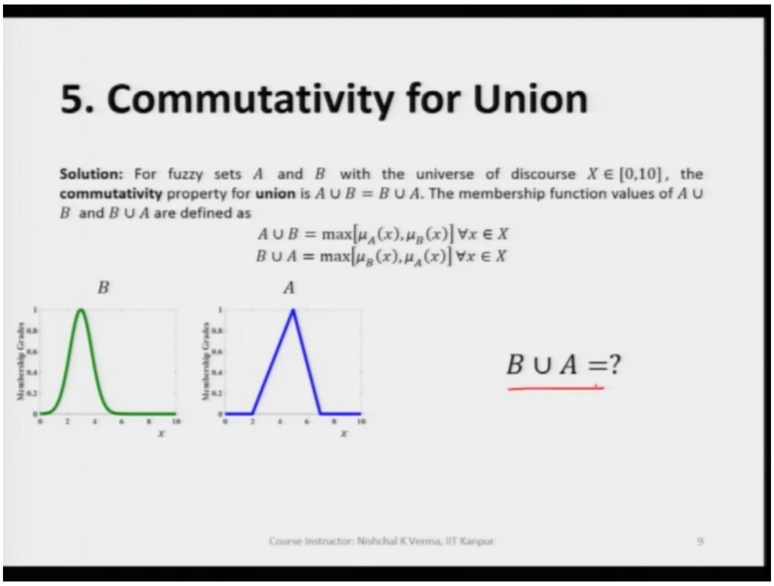

So, we if we take B fuzzy set here first and then we take A fuzzy set.

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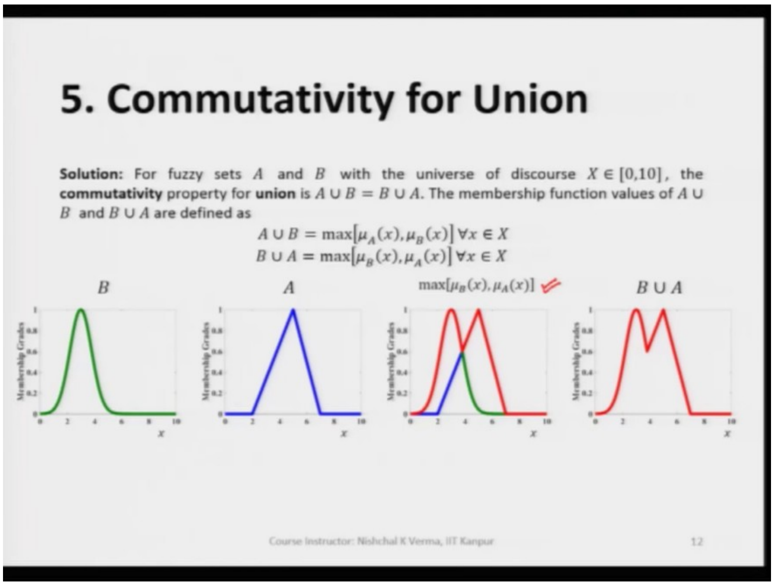

And, then we see here that if we take $B \cup A$ we are going to get this fuzzy set as $B \cup A = max\left[\mu_B(x), \mu_A(x)\right]$.

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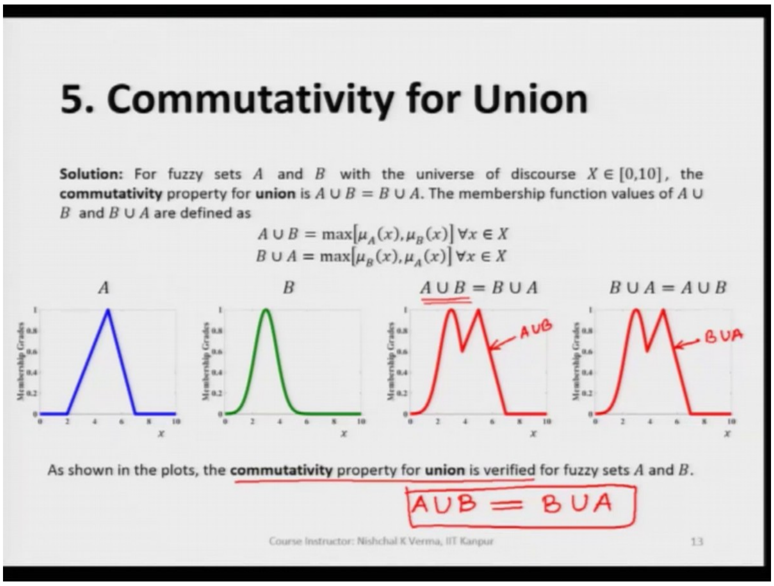

So, this way we see that what we are getting here is A union this $A \cup B$ and this $B \cup A$. And, we if we see here both the outcomes are same so, we can clearly say here that the commutativity property for union is verified or satisfied for fuzzy set A and B. And, this is written as $A \cup B = B \cup A$ and please note that this commutativity property for fuzzy sets A and B are satisfied.

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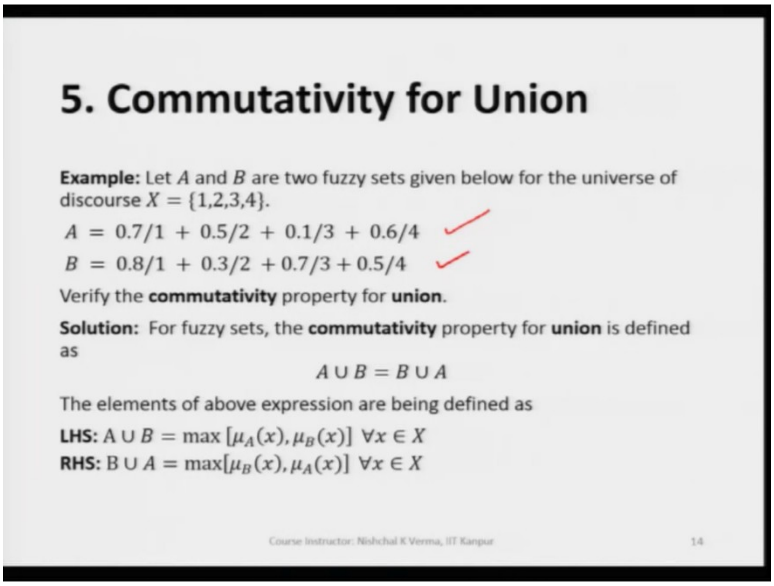

Now, let us take another example where we have two discrete fuzzy sets. So, if we take here A fuzzy set as a discrete fuzzy set and B also a fuzzy set which is discrete fuzzy set. So, let us know try to see whether $A \cup B = B \cup A$ or not; of course, this will this is going to be equal, but let us see how are we going to get this verified.

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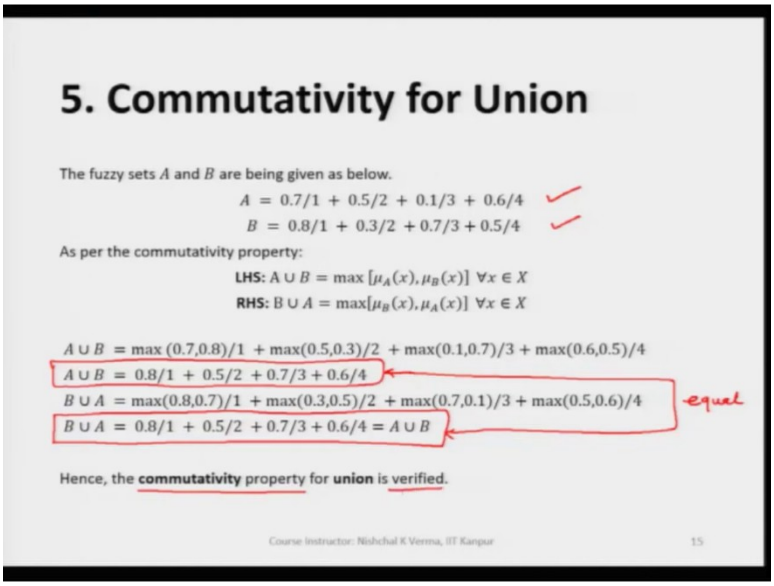

So, here we have fuzzy set A discrete fuzzy set A discreet fuzzy set B and see here if we find the $A \cup B$ of the two discrete fuzzy sets we are getting this and when we are taking $B \cup A$ we are getting this as the outcome. So, we can clearly see that all the elements of $A \cup B = B \cup A$

are same. So, we can clearly say here that these two sets are these two fuzzy sets are equal. So, when these two fuzzy sets are equal, we can very easily say or we can say that the commutativity property for union is verified.

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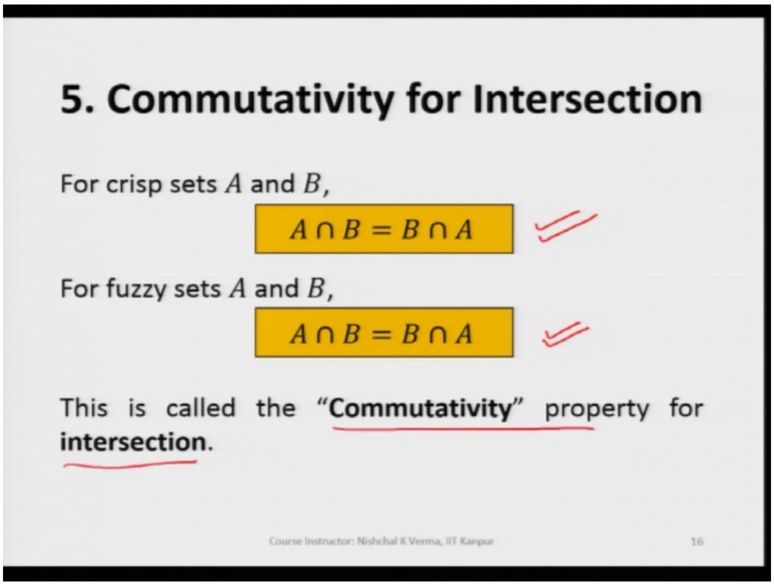

Now, on the same way we can define the commutativity property for intersection. So, here when we talk of intersection, so, let us first see what is this for crisp sets. So, if we take crisp sets A and B; so, when we take crisp sets A and B this property is satisfied means $A \cap B = B \cap A$ for crisp sets A and B. Now, let us see what is happening for fuzzy sets A and B. So, here also it is equal means when we take two fuzzy sets $A \cap B = B \cap A$. So, let us see how we are going to get this verified.

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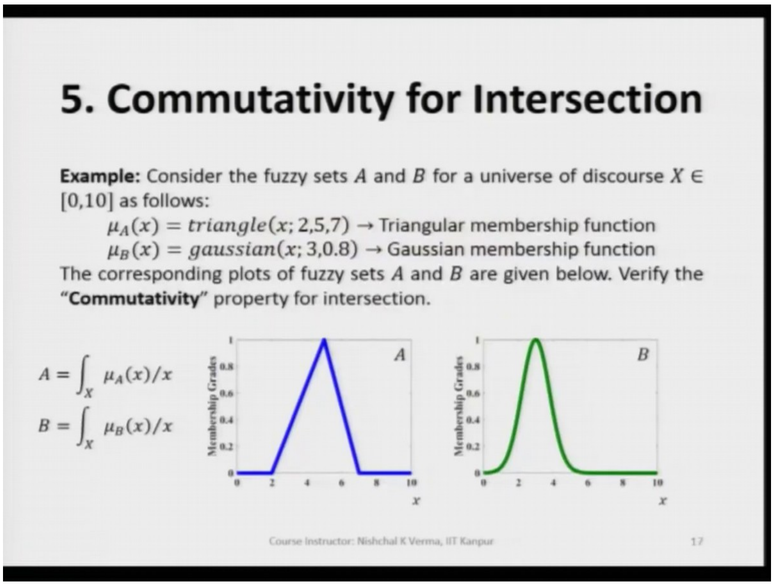

So, if you take an example here, in this example we have two fuzzy sets as we have taken in the previous example.

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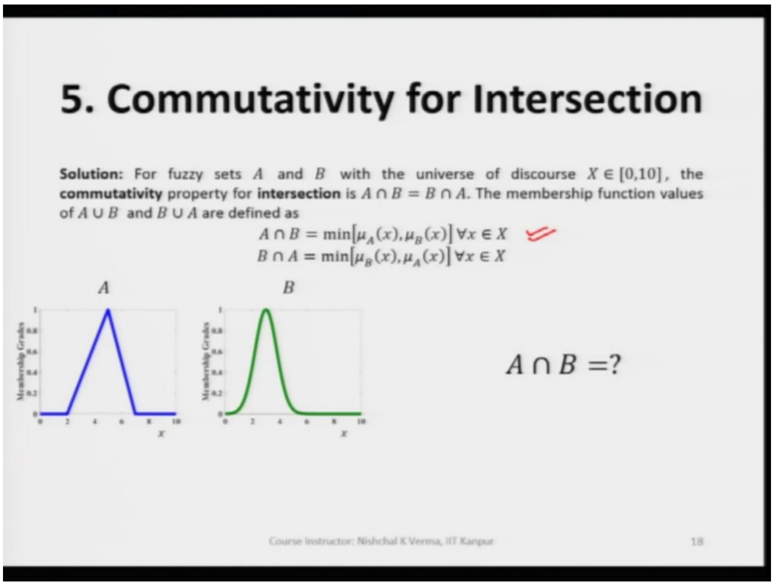

So, we have taken the same example here also. So, we see that when we take $A \cap B$ using this condition we take min of all membership values corresponding to the generic variable values for A fuzzy set and B fuzzy set.

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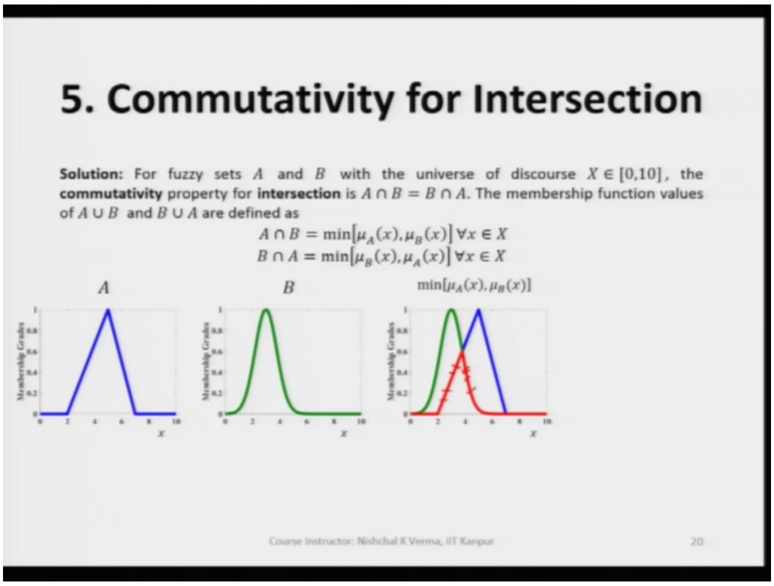

So, $A \cap B$ we are going to get like this. So, this is what is the $A \cap B$ which is represented by the red color fuzzy set. So, this is what is the outcome and let me also make it very clear how do we get that here.

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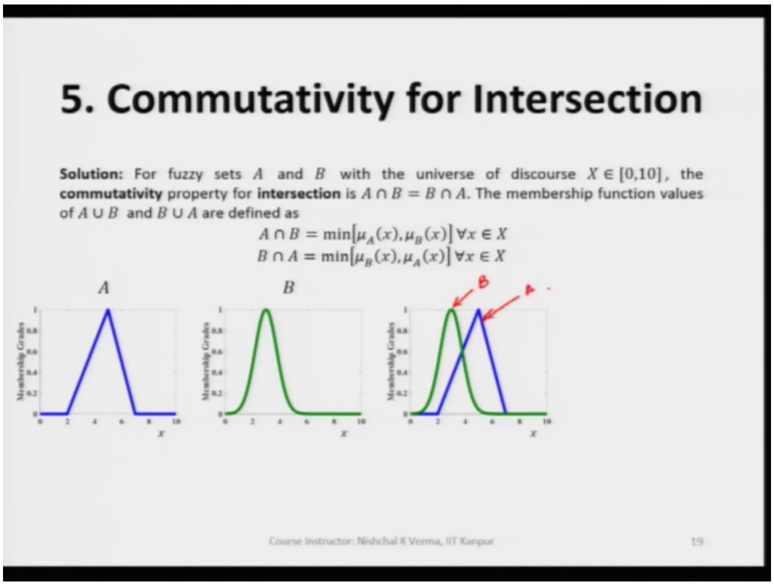

So, you see here that we have this as B fuzzy set, this as A fuzzy set and when we take min of the respective membership values for corresponding generic variable values we get here, the portion which is represented by a red color plot.

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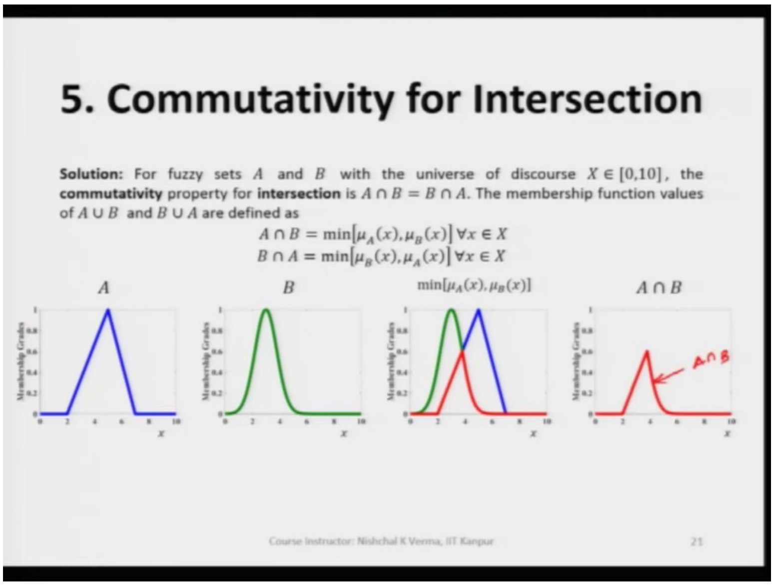

So, we see that this is what is we are going to get as $A \cap B$.

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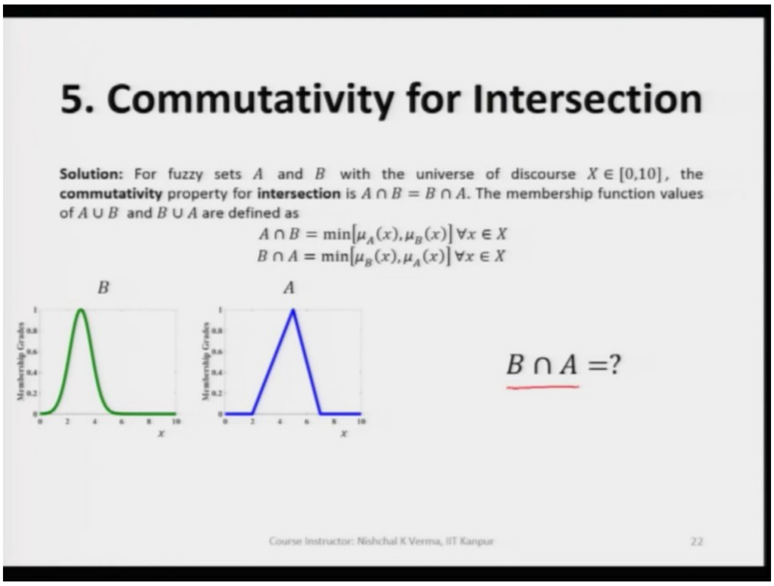

Now, when we take B first when we take B first and then A, of course this B and A are two fuzzy sets. So, if we do that let us see what is happening.

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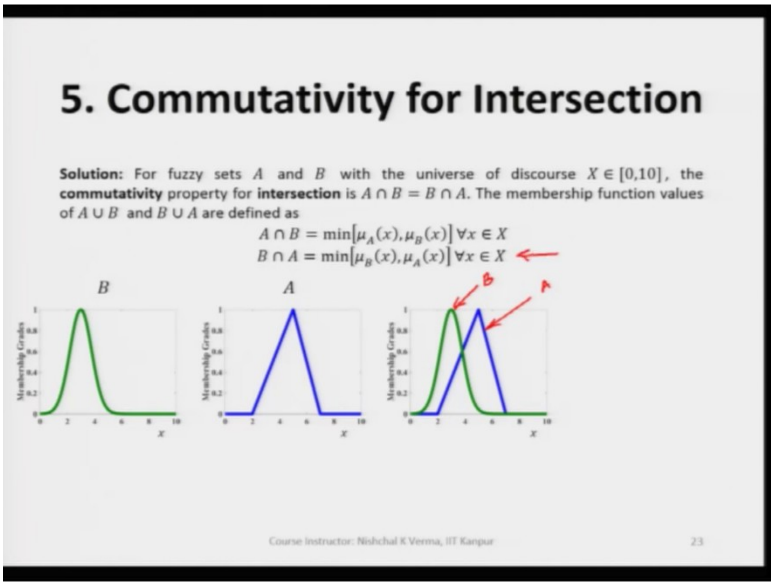

So, here also we have B fuzzy set and then A fuzzy set and then when we apply this condition the condition for intersection.

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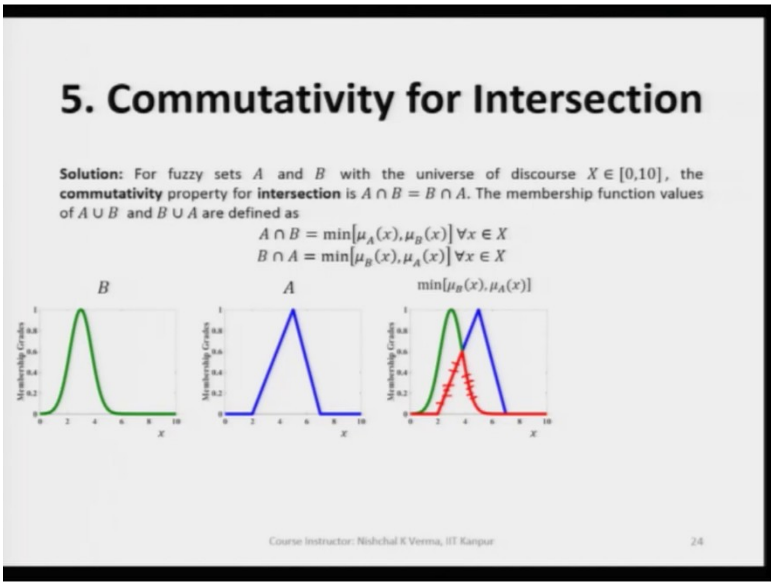

So, when we find this we are also we get again the same portion of the fuzzy sets.

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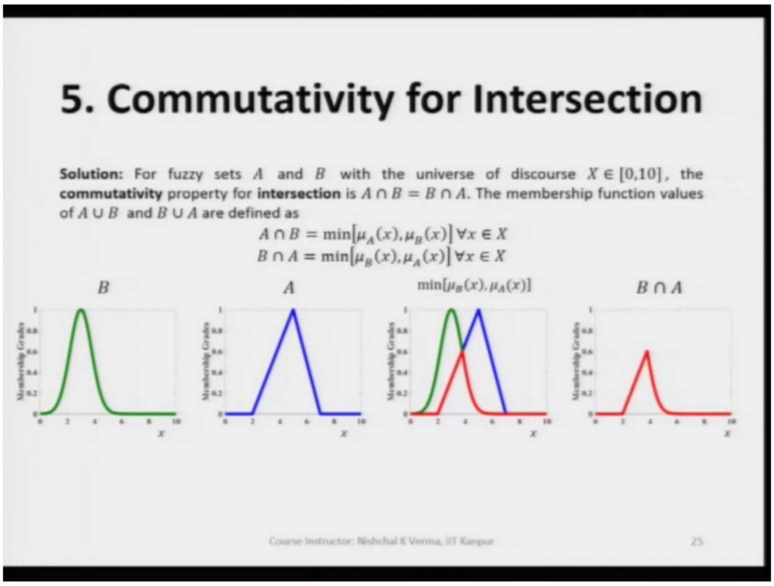

So, $B \cap A = A \cap B$.

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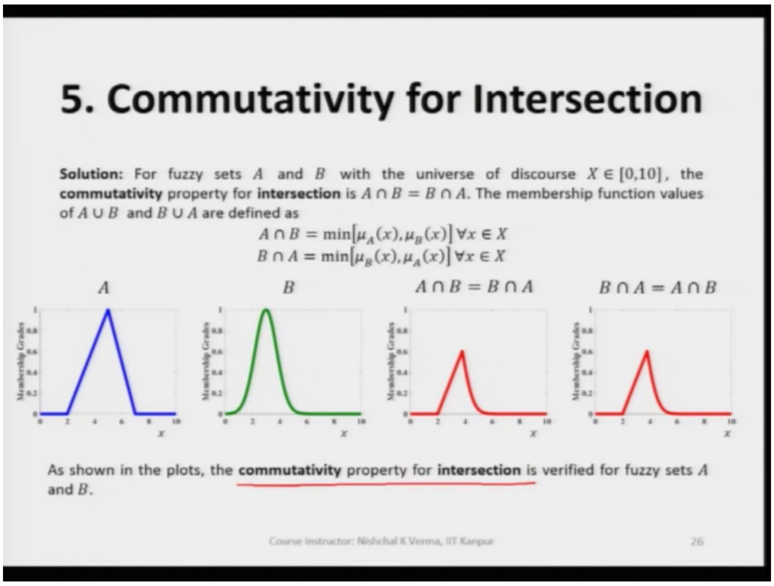

So, this way we can say that commutativity property for intersection is verified for fuzzy sets A and B.

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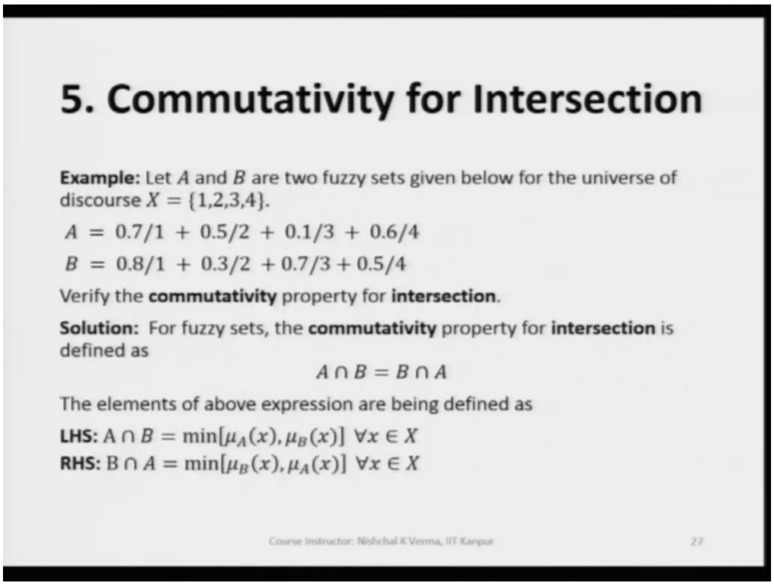

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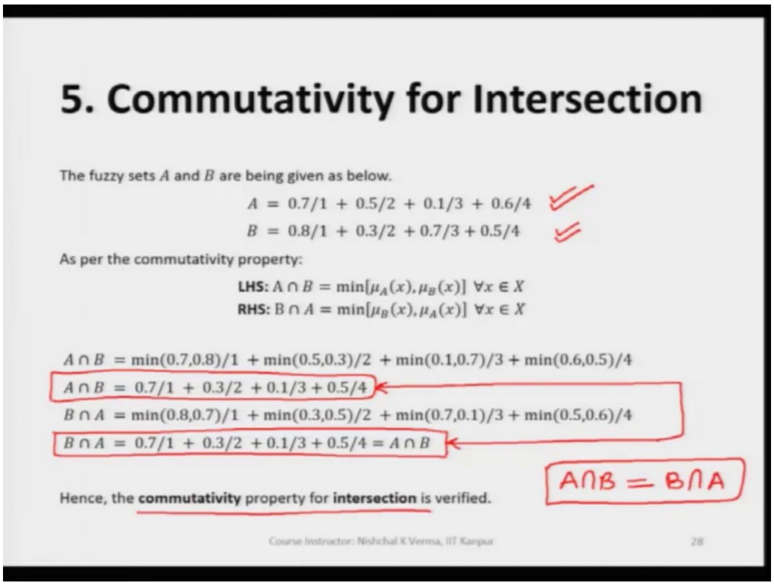

Now, again if you take the discrete fuzzy sets A and B, here also we see that if we take $A \cap B$, here for fuzzy set for discrete fuzzy set A and discreet fuzzy set B. So, we see that we have $A \cap B$ as the outcome. So, this $A \cap B = 0.7/1 + 0.3/2 + 0.1/3 + 0.5/4$ as the outcome. Now, let us see what are we getting when we take $B \cap A$. So, this way we see that we are getting the same outcome as we have gotten for $A \cup B$.

So, all the elements remain same and then and hence we can always say that $A \cap B = B \cap A$ for discrete fuzzy sets also. And, hence we can say since we have already checked this condition for continuous fuzzy set A and B and here in this case also for discrete fuzzy set we have checked. So, we can say that commutativity property for intersection is verified.

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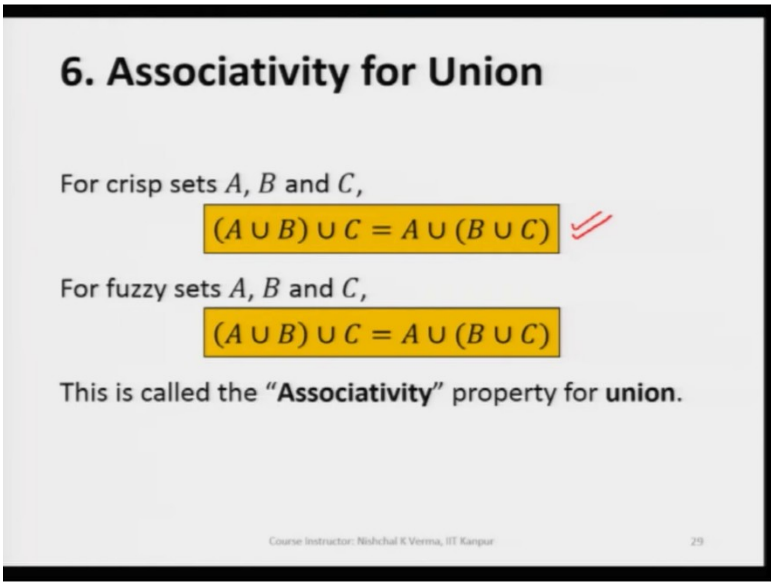

Now, let us go to the associativity property for union. So, as we already know that for crisp sets A, B and C we have this equality as the associativity for union, means $(A \cup B) \cup C = A \cup (B \cup C)$. So, for fuzzy sets A, B and C this condition also is satisfied. So, let us now see how is it happening.

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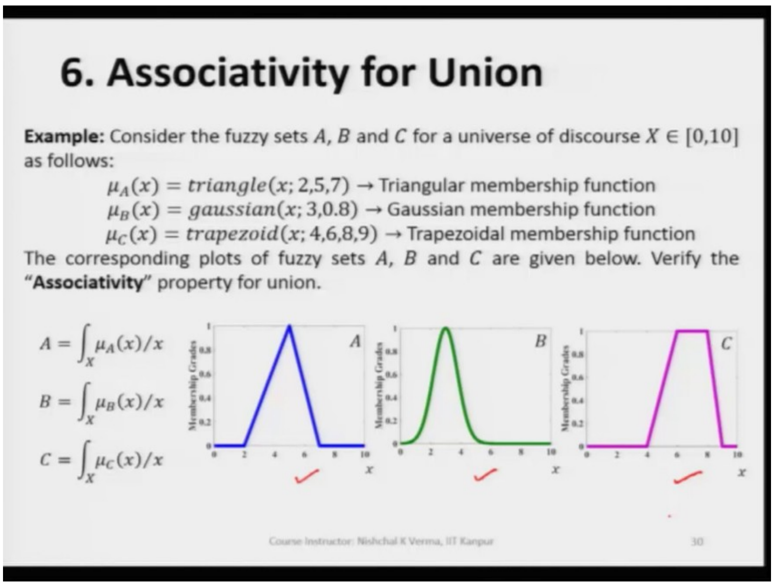

So, if you take an example here again for the associativity property for union we take fuzzy set A, continuous fuzzy set A, continuous fuzzy set B and then the third fuzzy set continues fuzzy set C.

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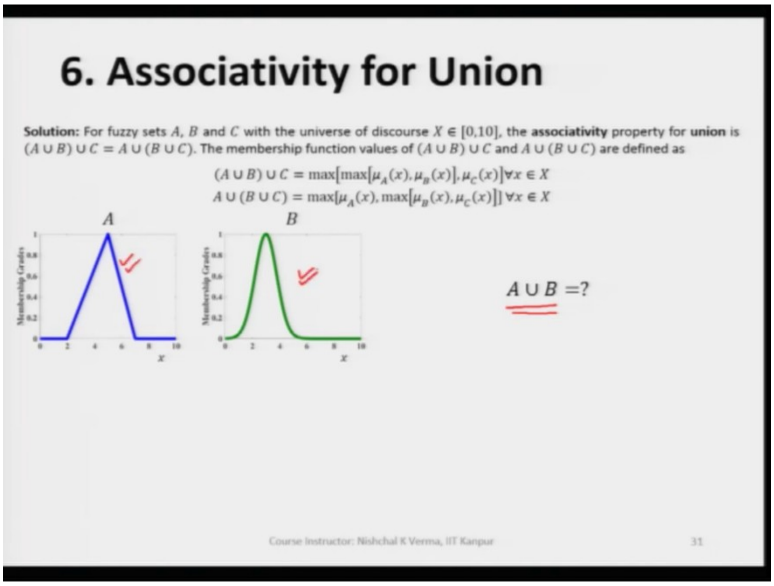

So, we see that if we take the $A \cup B$ here where A is this fuzzy set and B is this fuzzy set.

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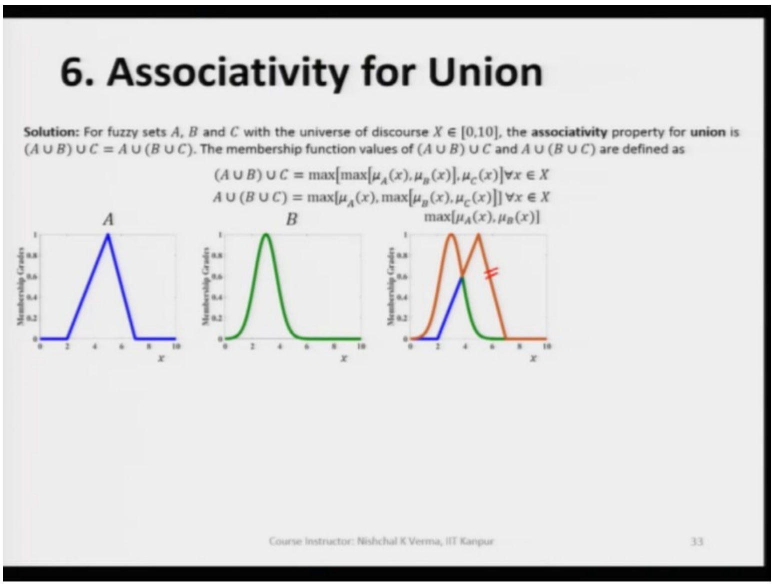

So, we are going to get $A \cup B$ like this. So, this is what we are going to get, means we are going to get the max of all the respective membership values corresponding to the generic variable values of both the fuzzy sets.

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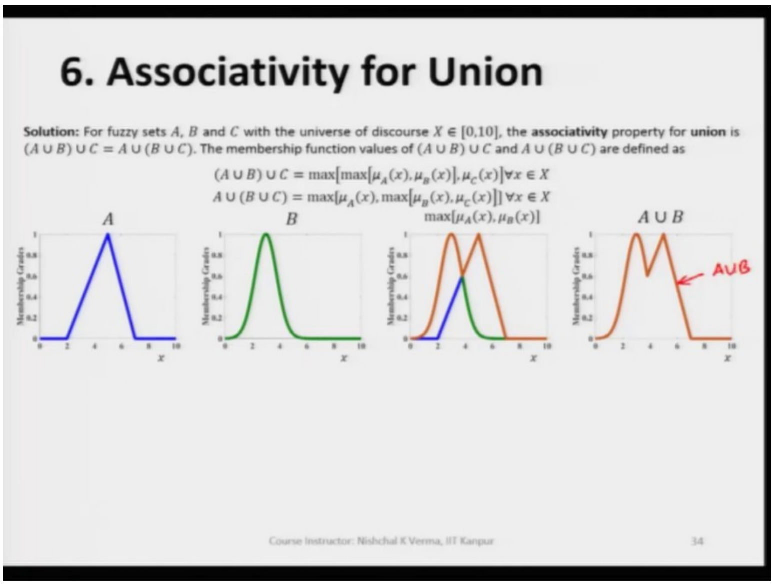

So, this way we can say here that we are getting $A \cup B$ like this.

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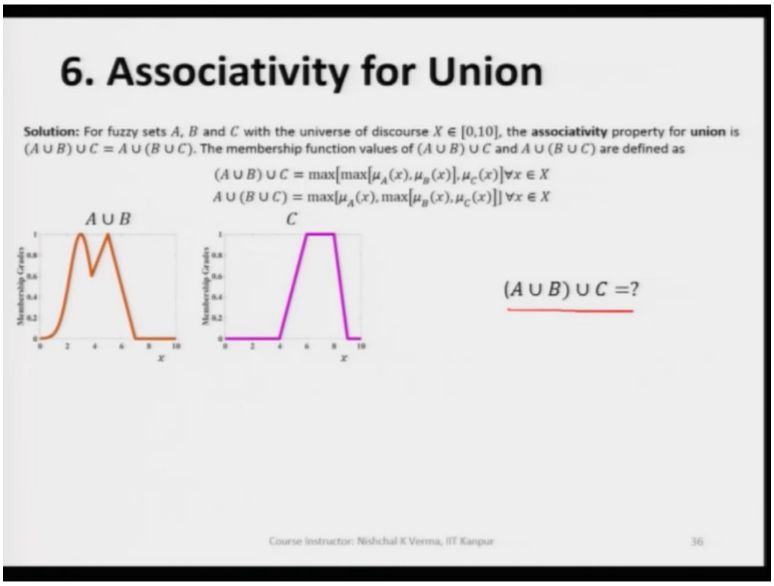

Now, since we already have $A \cup B$, now take the we use this and take the $(A \cup B) \cup C$. So, let's see what is happening. So, we have C fuzzy set here; so, $(A \cup B) \cup C$ means $(A \cup B) \cup C$ we are going to get it like this.

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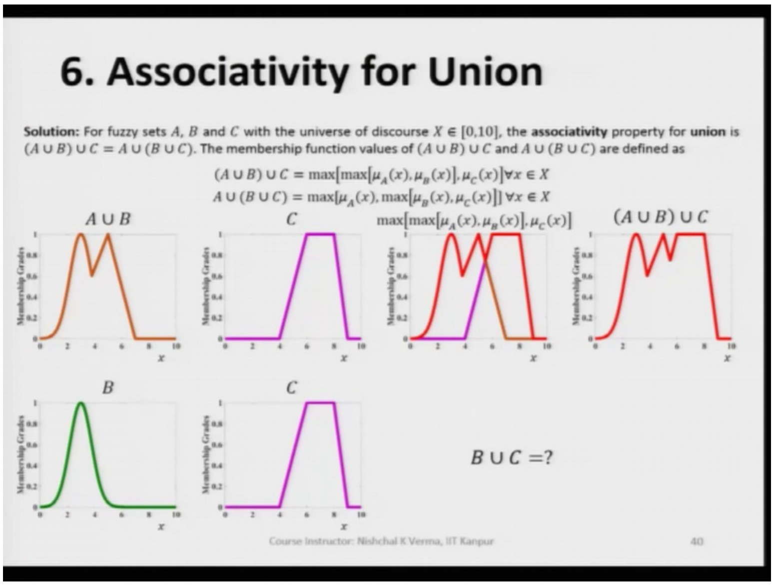

So, if we apply the same criteria, we are going to get this portion as the $(A \cup B) \cup C$ by applying the same max criteria.

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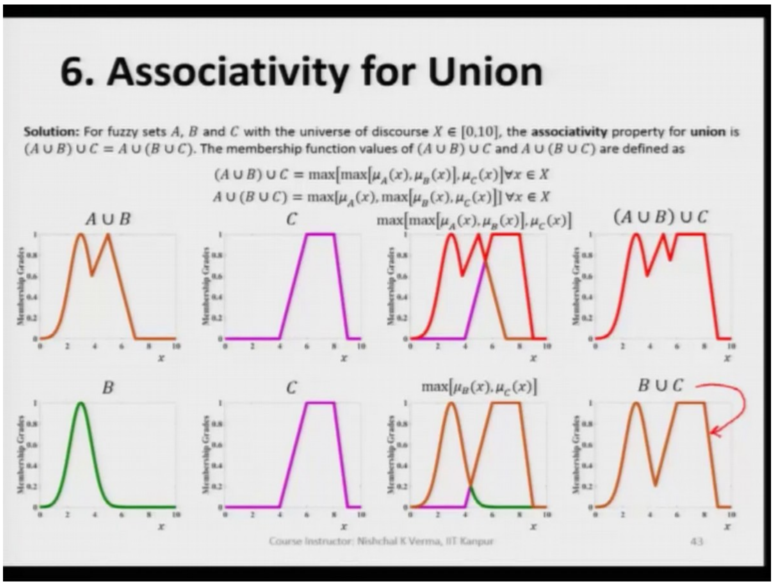

Now, if we take B the union of fuzzy set B and C, we are going to get we are going to get this as we are going to get this as the outcome means $B \cup C$ is this outcome.

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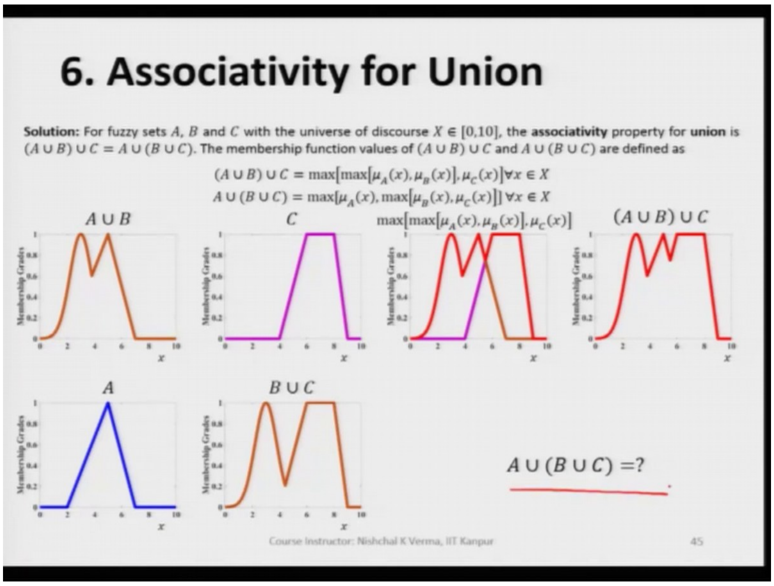

And, if you take the $(A \cup B) \cup C$ we are going to get here this as the outcome by applying the same max criteria.

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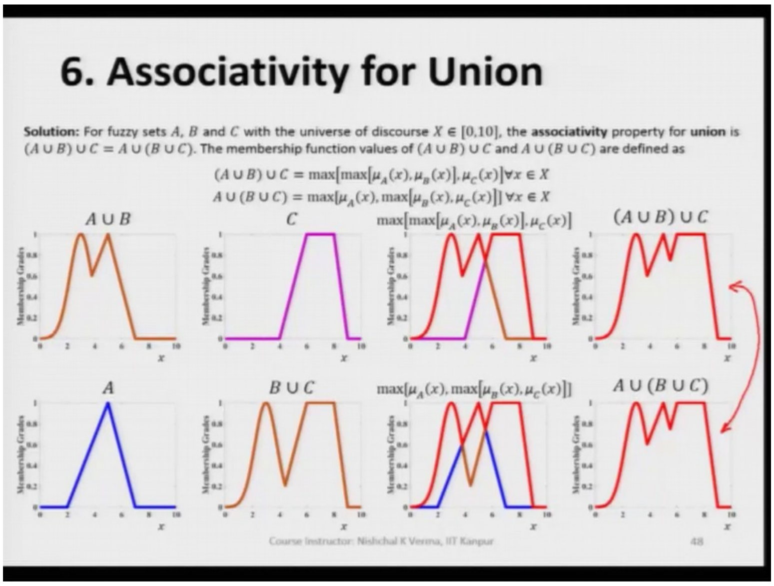

So, we can clearly see here that these two outcomes are same as same in both the cases, means either we take $A \cup B \cup C$ or we take $(A \cup B) \cup C$.

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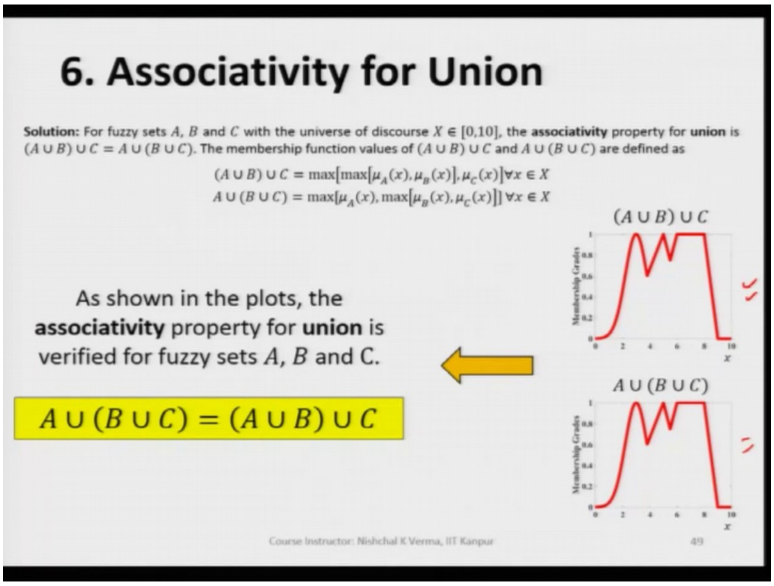

So, we can clearly say that the associativity property for union is satisfied or verified for fuzzy set for fuzzy sets A, B and C.

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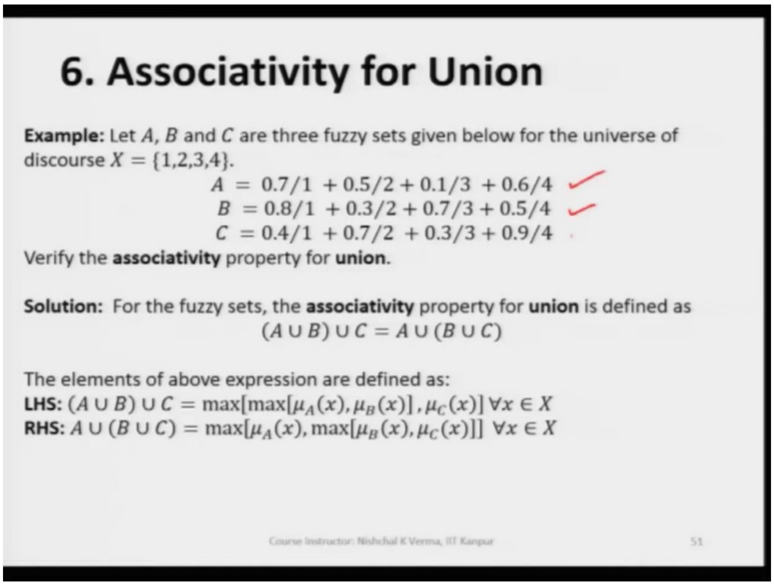

Now, the same can be tested, same can be checked by taking the discrete fuzzy sets A, B and C.

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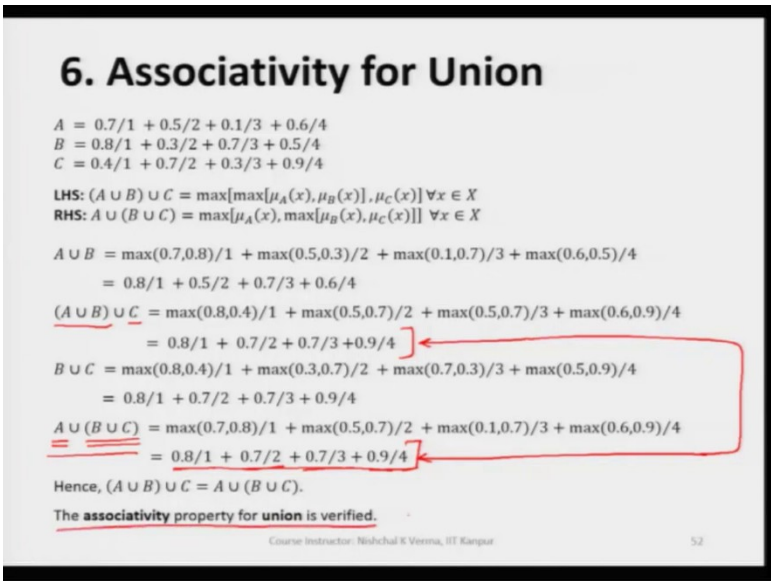

So, if we take these three fuzzy sets A, B and C so, we find here if we directly go to the outcome of $(A \cup B) \cup C$ we are going to get this as the outcome. So, this is what is the outcome, this is a fuzzy set a discrete fuzzy set. So, let me read the element of this fuzzy set $(A \cup B) \cup C = 0.8/1 + 0.7/2 + 0.7/3 + 0.9/4$.

And, when we find the A union the fuzzy set which we have got as a result of $B \cup C$, if we take this thing we get the outcome here as $0.8/1, 0.7/2, 0.7/3, 0.9/4$. So, if we look at these two fuzzy sets these two outcomes, we see that these two are equal these two are same. So, then it is very clear that $(A \cup B) \cup C = A \cup (B \cup C)$ means the associativity property for union is satisfied or verified. So, this way you have checked the associativity property for union.

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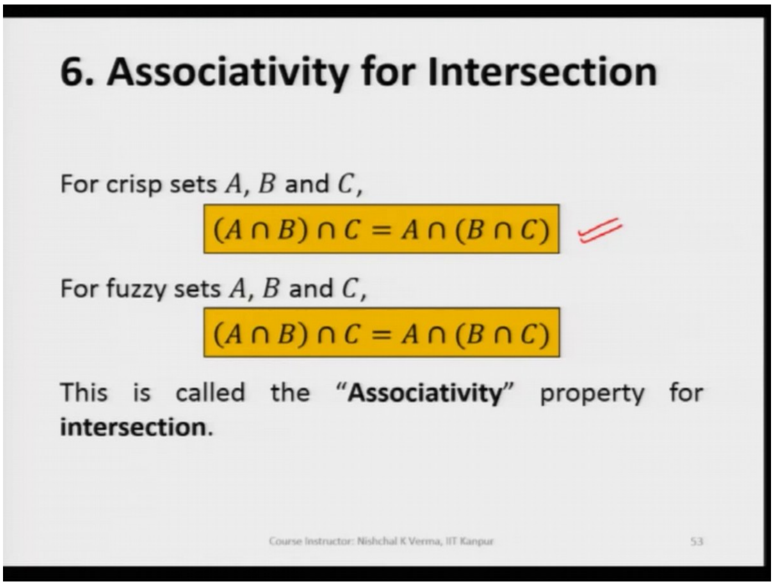

Now, let us do the same thing, let us now check the associativity property for intersection. So, for crisp sets A, B and C this property, the associativity property for intersection is satisfied and for fuzzy sets A, B and C are also satisfied, but we need to verify by taking some examples.

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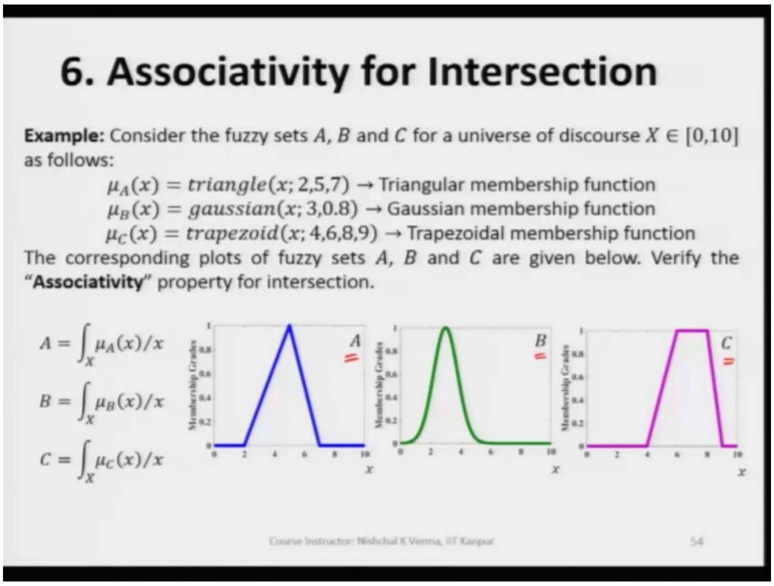

So, let us here also take few examples, two examples: first example is with the continuous fuzzy sets and the second example is with discrete fuzzy sets. So, the first example is here and here we take three fuzzy sets A, $B$and C.

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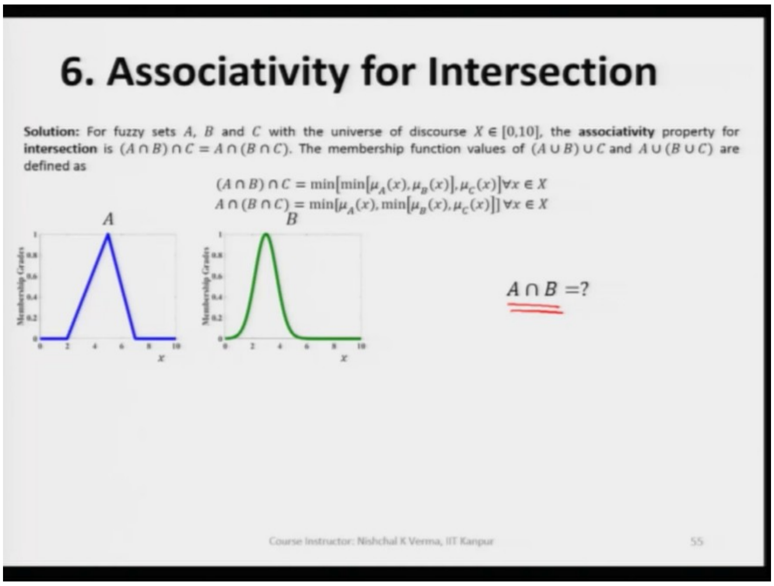

And, let's now try to see what are we getting when we do $A \cap B \cap C$. So, what does this mean when we say $A \cap B \cap C$? It means that we are going to get a fuzzy set as the outcome of $A \cap B$ and then whatever we are getting as the outcome, we take the intersection of this fuzzy set with the third fuzzy set C.

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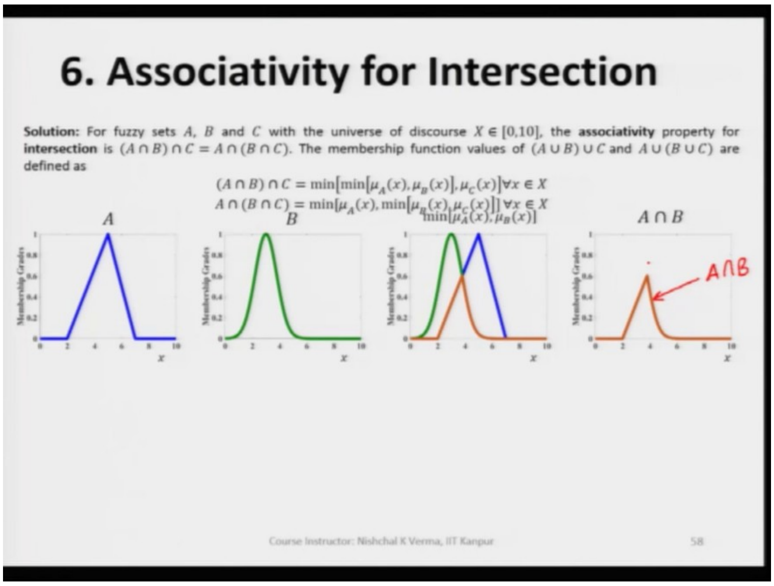

So, $A \cap B$ is here; so, this is what we are getting as a result by applying this condition, the min condition. So, this is $A \cap B$; so, this is a fuzzy set basically which is by just looking at it we can say that this fuzzy set is a sub normal fuzzy set.

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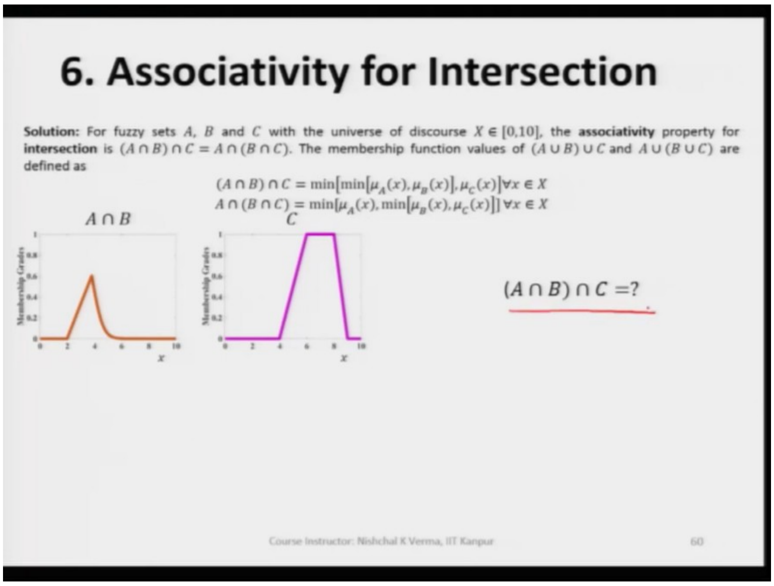

And, when we take the intersection of this fuzzy set with C fuzzy set, the third one the third continuous fuzzy set which is C, so it is here and if we do that let us see what are we getting.

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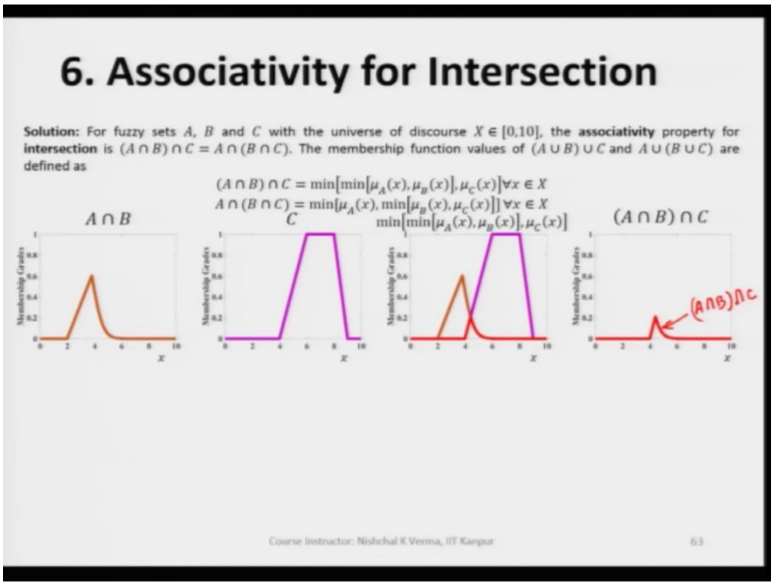

So, if we apply this the min condition we are getting this fuzzy set which is again a sub normal fuzzy set and this is the outcome of $A \cap B \cap C$. So, this is how we are getting the outcome of $A \cap B \cap C$.

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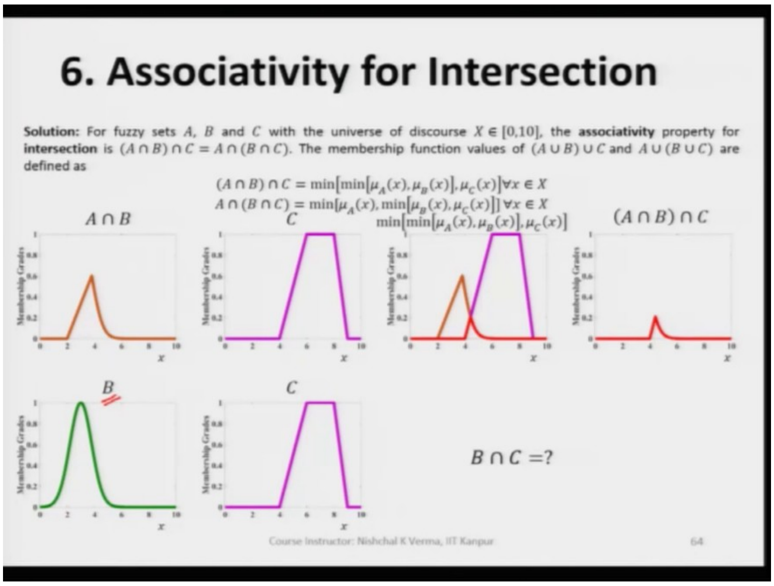

Now, let’s try to verify let us try to get $B \cap C$ first. So, B fuzzy set is here and please remember that this is please note that this is a continuous fuzzy set, all these fuzzy set A, B , C are continuous fuzzy sets. So, $B \cap C$ we are going to get like this.

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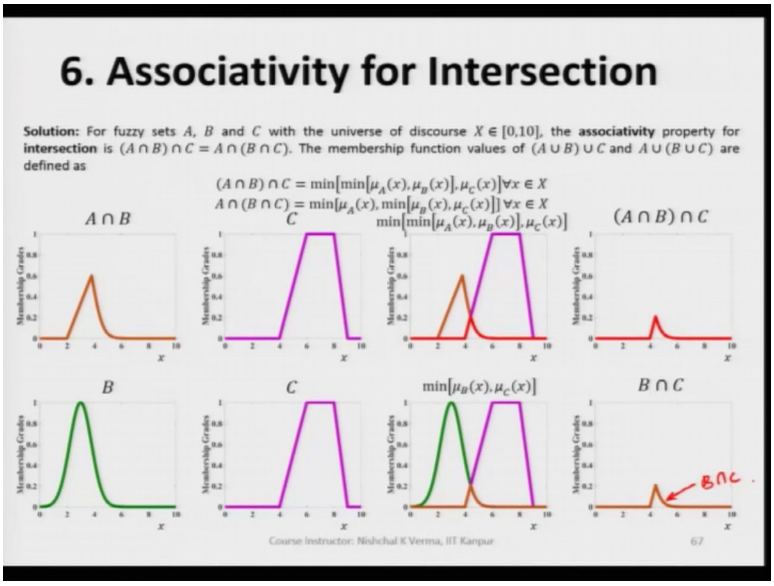

So, this is our$B \cap C$, now since this is a fuzzy set again by just looking at it we can say this is sub normal fuzzy set. So, if we take this fuzzy set or we see if we take the intersection of this fuzzy set and A, so let us see what are we going to get.

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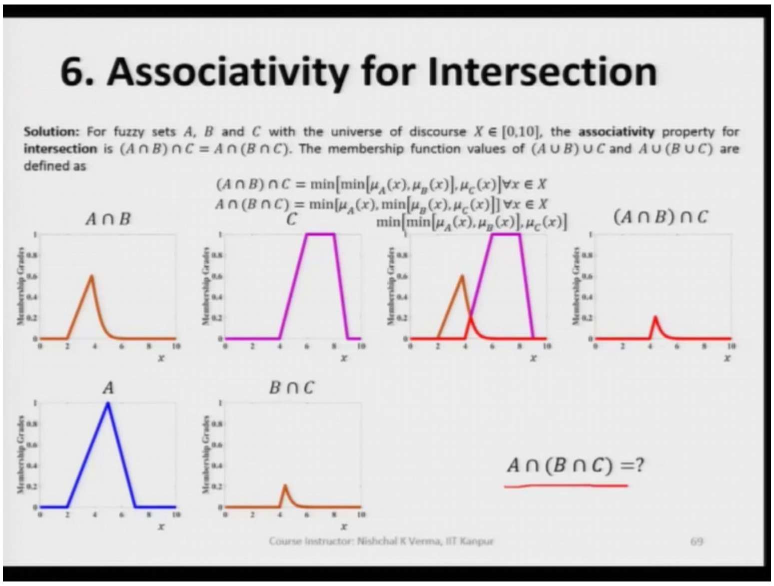

So, here we have A so, here we have to follow the sequence. So, we have A fuzzy set first, we take the sub normal fuzzy set $B \cap C$ and when we find $A \cap B \cap C$, let's see what are we going to get.

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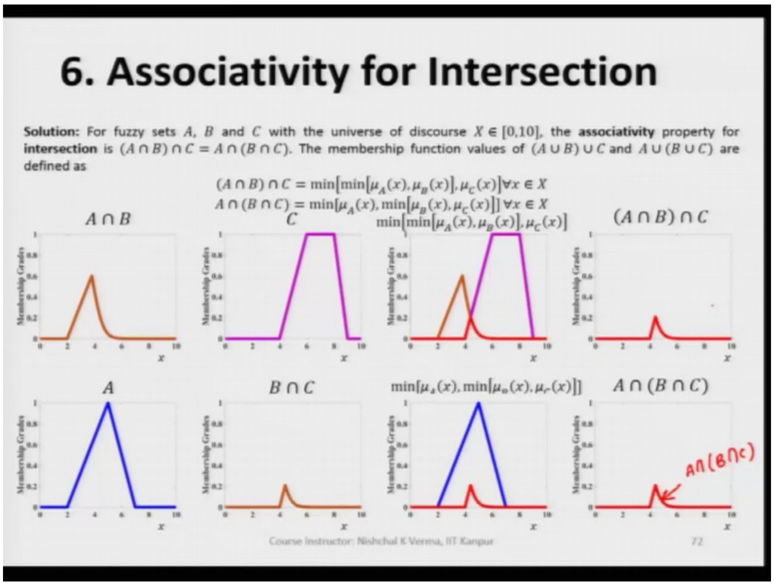

So, when we apply the min condition we are going to get this as this as $A \cap B \cap C$. So, just by looking at this we can clearly say that these two fuzzy sets are equal.

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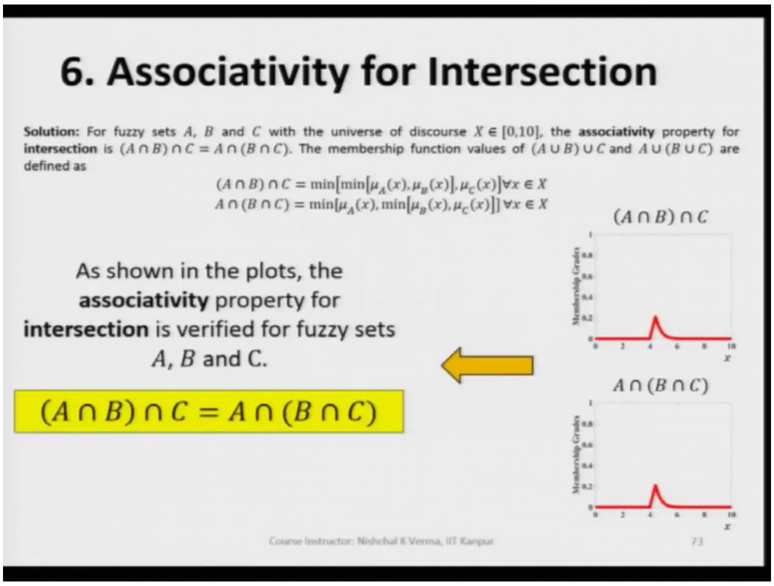

So, this way we can say the associativity property for intersection of three fuzzy sets A, B and C are satisfied.

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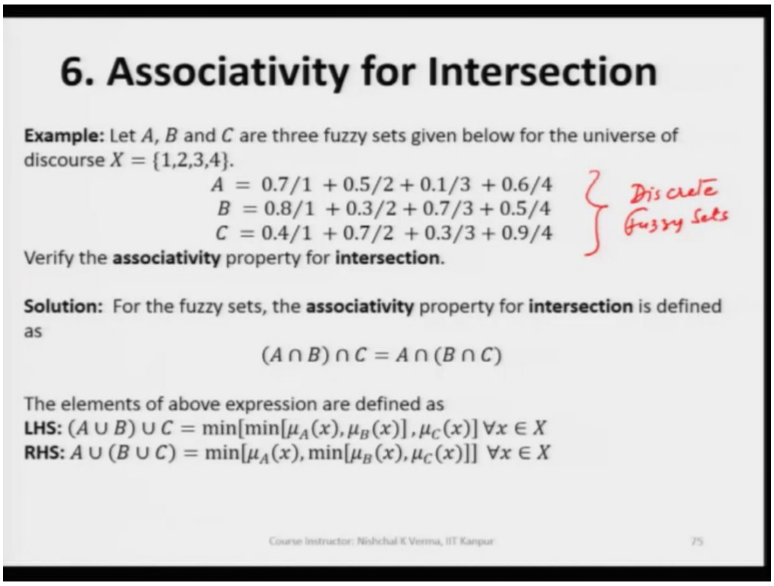

Now, let us see what is happening when we take the discreet fuzzy sets. So, here we have three discrete fuzzy sets A and A, B and C is three are discrete fuzzy sets.

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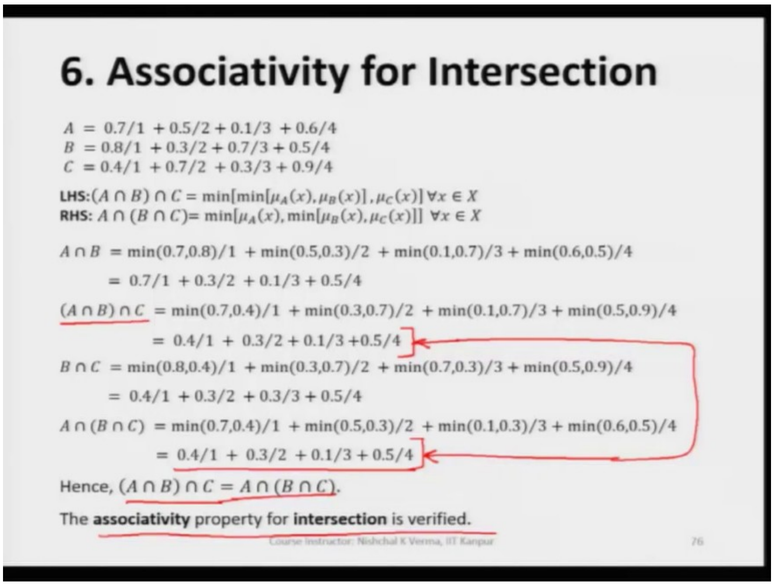

And, if we try to find here, I am going directly to the outcome of $(A \cap B) \cap C$. So, we are going to get a fuzzy set a discrete fuzzy set which is $(A \cap B) \cap C = 0.4/1 + 0.3/2 + 0.1/3 + 0.5/4$. So, let me repeat it. So, we are going to get the outcome as $0.4/1 + 0.3/2 + 0.1/3 + 0.5/4$. So, this is what is the outcome that we are getting as the $A \cap (B \cap C)$.

So now, when we compute the $A \cap (B \cap C)$, so this is what we are getting when we apply the min criteria. So, if we look at all the elements of these two fuzzy sets, we find $A \cap (B \cap C) = 0.4/1 + 0.3/2 + 0.1/3 + 0.5/4$. So, we see that all the elements are same as we have here. So, this way we can say that if we take $A \cap (B \cap C)$ and if we take the $(A \cap B) \cap C$ both are equal.

So, this way we can say the associativity property for intersection is verified for all the three discrete fuzzy sets also. So, this way we understand that the associativity property for intersection are all satisfied for fuzzy sets also.

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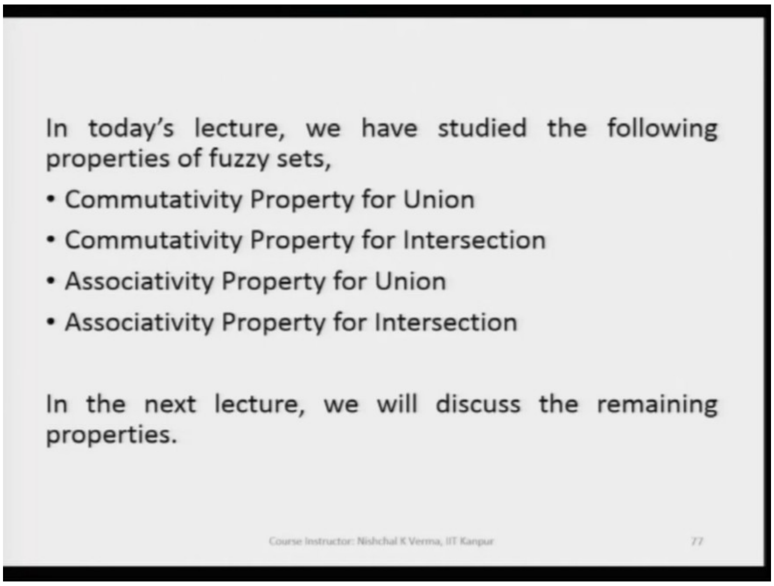

So, in today's lecture we have so far covered the following properties of fuzzy sets: the commutativity property for union, commutativity property for intersection, associativity property for union, associativity property for intersection. So, we have covered all these four properties for the fuzzy sets and we will stop here for this lecture. And, in the next lecture we will cover the remaining properties of fuzzy sets that we have already seen listed.

Thank you.