Fuzzy Sets, Logic and Systems and Applications
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Lecture - 11
Set Theoretic Operations on Fuzzy Sets

Welcome to lecture number 11 of Fuzzy Sets, Logic and Systems and Applications. So, this lecture is in continuation to the lecture number 10. In this lecture, we will be continuing the discussions on Set Theoretic Operations on Fuzzy Sets.

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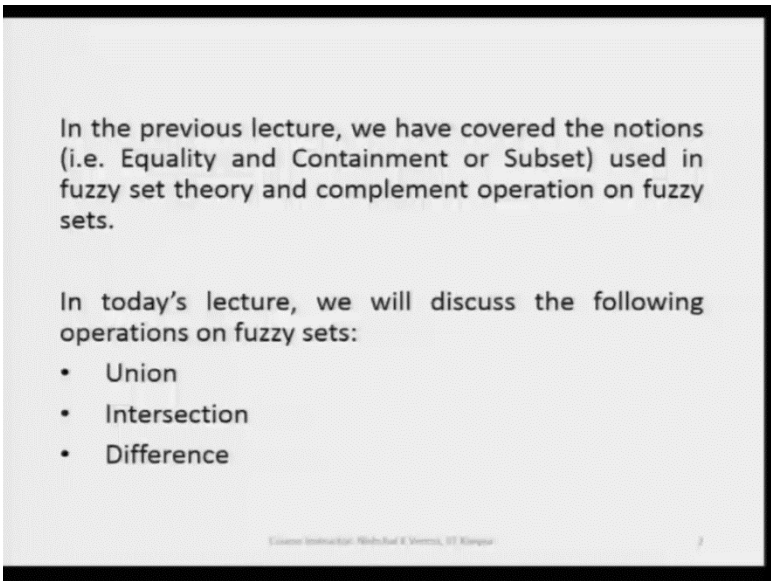

So, in the previous lecture, we have covered the notions; that means, the equality and containment or subset used in fuzzy set theory and complement operations on fuzzy sets. In today's lecture, we will be discussing the following operations on fuzzy sets. The first one is the union and then the second one is intersection and then the third one is the difference of fuzzy sets.

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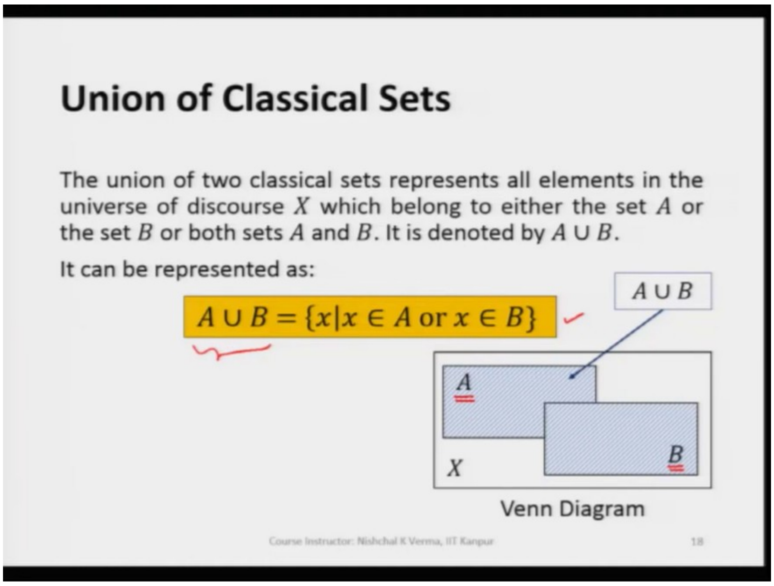

Now, let us talk about the union of classical sets. We all know the union of classical sets and we can clearly see here if we have two classical sets. So, let this be a classical set and then here B is also a classical set and we are interested in taking the union of these two classical sets in the universe of discourse.

So we take, we follow this condition and the union is represented by $A \cup B$ here,

$$A \cup B = \{x \vee x \in A \vee x \in B\}$$

So, this way we see here the shaded portion and shaded A and B and these two are together gives us the union of classical sets.

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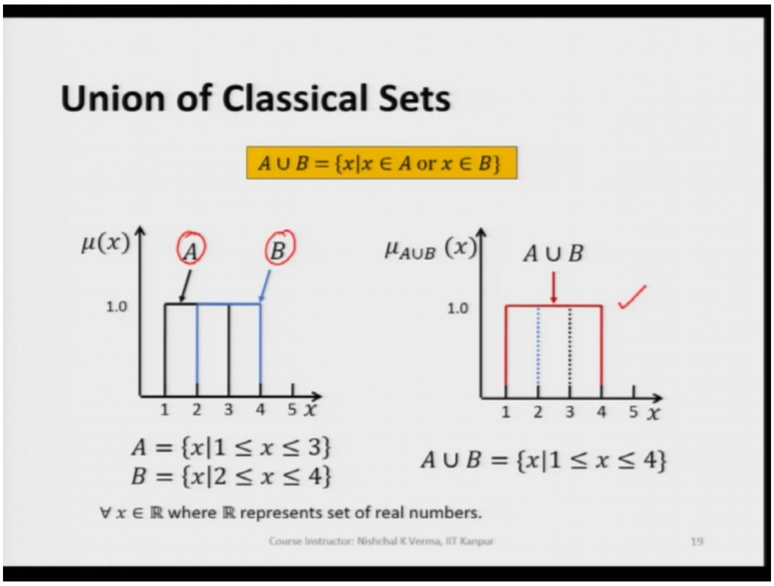

So, this can be clearly understood by this example and where we have two classical sets one is A and the other one is B and when we are interested in finding the A union B we can clearly see that all of the elements of A and B both are you know included in $A \cup B$ and this way we get the union of classical set classical sets A and B.

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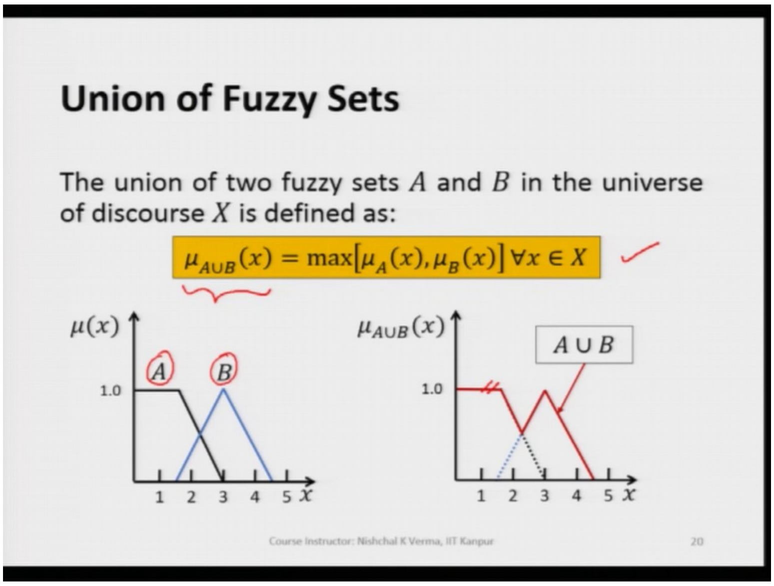

Let us now understand the union of fuzzy sets which are little bit different from the union of classical sets$A$ and B. So, if we have fuzzy sets A and B and we are interested in finding the union of these two, we must know that not only the elements which are present in these two sets, the membership values are also important and they place they play a very important role in managing in finding the union of fuzzy sets.

So, let us first look at the condition which we follow in finding the union of two fuzzy sets. So, as I mentioned that it is the membership values which are very important for the corresponding generic variable values and in case of a union of the two fuzzy sets

$$\mu_{A \cup B}(x) = max[\mu_A(x), \mu_B(x)] \forall x \in X$$

So this is the condition for union of fuzzy sets A and B. So, this can be clearly understood by the two continuous fuzzy sets. So, let this be fuzzy set A and this be another fuzzy set B. So, let us now find the union of these two fuzzy sets. So, if we apply this condition; in this condition says that at each and every generic variable value find, take the maximum of the corresponding membership values. So, if we do that we see that we are getting this as the $A \cup B$ means the union of these two fuzzy sets A and B. So, the maximum values if we plot, we'll be getting as this is shown by the red line the red color and this is going to be the $A \cup B$.

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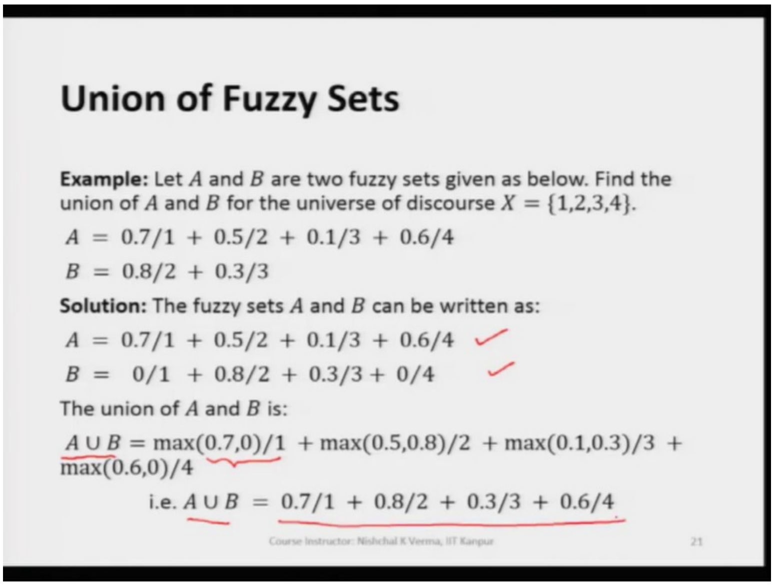

Similarly, if we are taking two discrete fuzzy sets A and B and we can use the same condition to find out A intersection, $A \cup B$ and this way here if we have A as the discrete fuzzy set, B here as another discrete set. So if we apply this condition as $A \cup B$, the we take the union of these two fuzzy sets. We take the respective, maximum of the respective membership values.

So, here it is very clear that we have the $A \cup B$ which is coming out to be like this. So, this way we can quickly find the union of two fuzzy sets; two discrete fuzzy sets as well.

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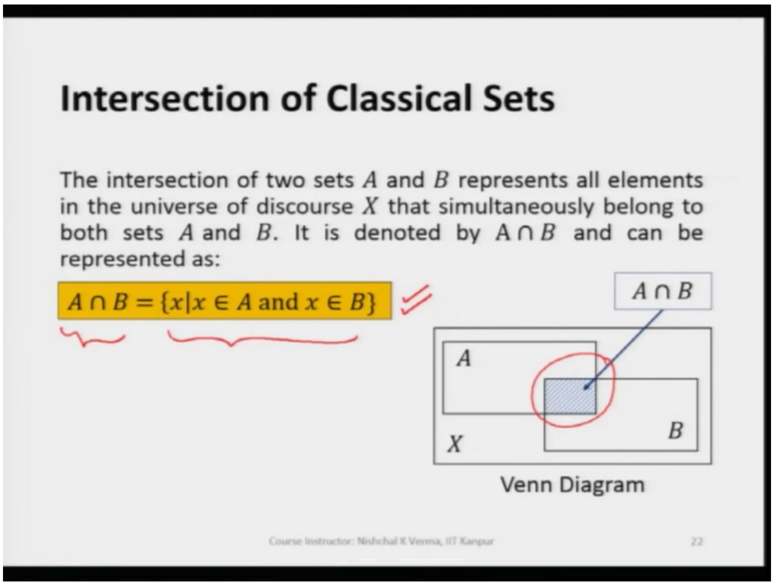

Now let us discuss the intersection of two fuzzy sets. So, before we move to intersection of two fuzzy sets or I would say intersection of fuzzy sets, let us first understand what happens in the intersection of classical sets. So, in the intersection of classical sets we follow this condition and this is represented by $A \cap B$. So, $A \cap B$ basically contains all the elements which are present in both the sets A and B. So, if we apply this condition A intersection will be represented by this section. So, it means the elements which are common in both the classical sets are regarded as $A \cap B$.

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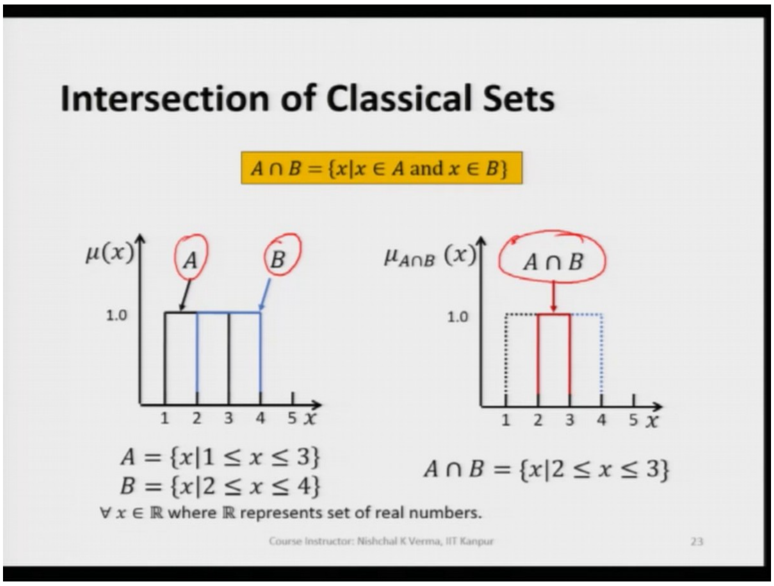

So, now this can also be understood by this example where we have two classical sets here A and B and $A \cap B$ will be the common portion of this two continuous classical sets.

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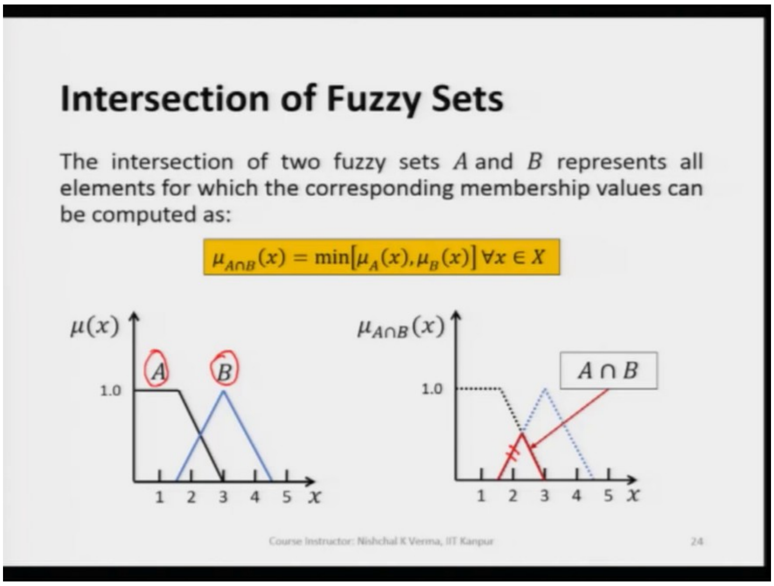

Now let us understand the intersection of fuzzy sets and the intersection of two fuzzy sets A and B represents all the elements for which the corresponding membership values can be computed as

$$\mu_{A \cap B}(x) = min[\mu_A(x), \mu_B(x)] \forall x \in X$$

So, let us take an example here to better understand the intersection of two fuzzy sets. So, let's let this be a fuzzy set A and this be another fuzzy set B and when we apply this condition and take the min of the corresponding membership values. We well get this red portion as the $A \cap B$ and this will also be a fuzzy set. So, if we take the intersection of fuzzy set A and fuzzy set B, we will get another fuzzy set which is shown by a red line a red color and this comes here the out of the intersection of two continuous fuzzy sets.

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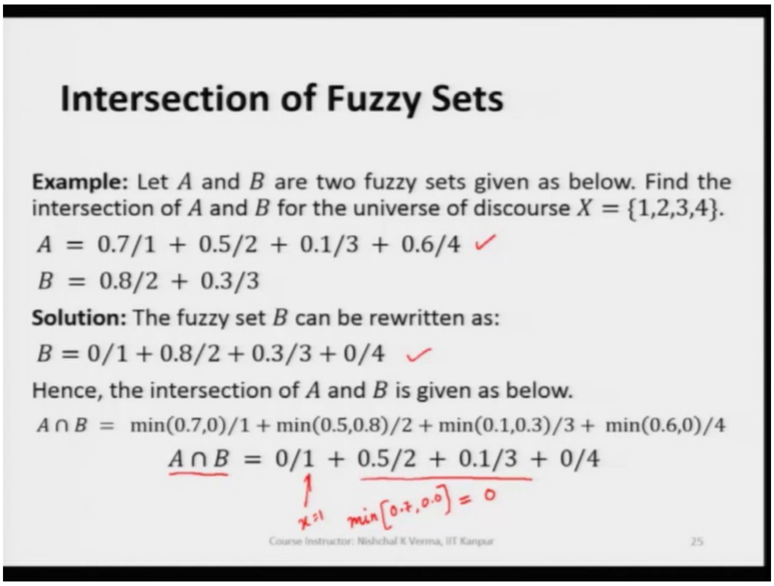

So, if we are interested in knowing what is happening to the intersection of two fuzzy sets when we take two discrete fuzzy sets. So, here we have two discrete fuzzy sets; the fuzzy set A and fuzzy set B. Here we have two fuzzy sets, fuzzy set A and then fuzzy set B. So, if we apply the same condition, we get $A \cap B$ which is you know for corresponding generic variable value this is $x=1$ and then for this we have if we take min of the corresponding membership values like in one case for a fuzzy set A we have 0.7 and then we have for the other fuzzy set we have 0.

So, the minimum of this will come out to be 0. So that is why 0 has been mentioned here although 0 is never written when we express when we represent a fuzzy set. So, this can be neglected here. So, the outcome will be $A \cap B = 0/1 + 0.5/2 + 0.1/3 + 0/4$. So, this 0 has been written here this term the first and last term is included just to make you understand as to what is happening and why this generic variable values is not included in this outcome.

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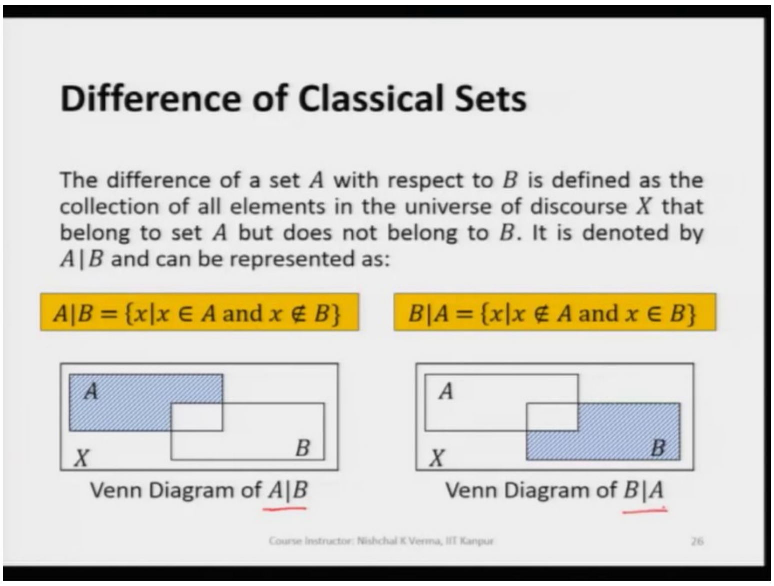

Now, let us discuss a difference of classical fuzzy set. So, the difference of a set A with respect to B is defined as $A \vee B = \{x \vee x \in A \wedge x \notin B\}$

So, this is with respect to the classical set, it is denoted by A, A straight line B so, $A \vee B$ and this is a set which includes all the elements which are present in A, but not in B. So, we can see here in the Venn diagram that here we have the difference of A and B and here we see the difference of B and A.

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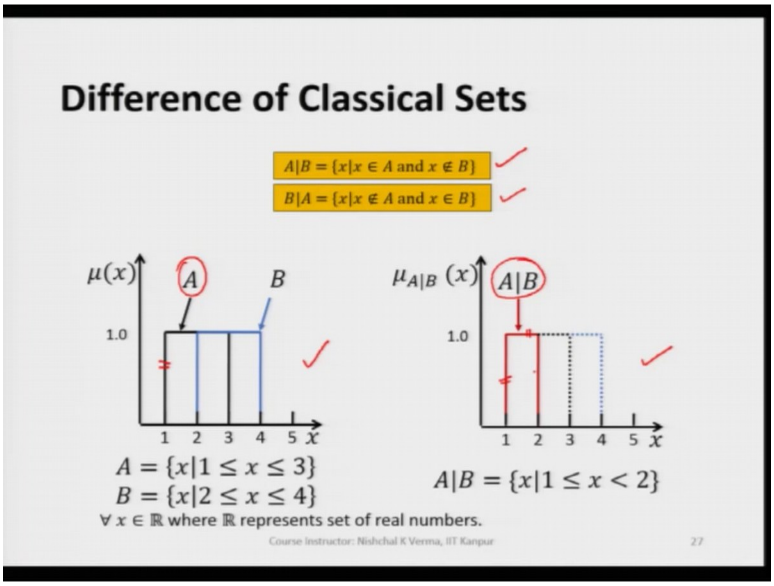

Now, here we have another example to understand the difference of classical sets. So, we are interested in the difference in A and B we follow this criteria, we are interested in finding the difference in B and A we follow this criteria and these diagrams will show us will give us the difference in A and B. So, A is this fuzzy set which is shown by the black line, the black color and B is the another classical set which is shown by the blue color. So, if we are interested in finding the A difference B, we represent first of all by this A oblique B. And then if we apply the condition that was mentioned we get only this portion this portion and this way we get A the difference of A and B and these sets are the classical sets.

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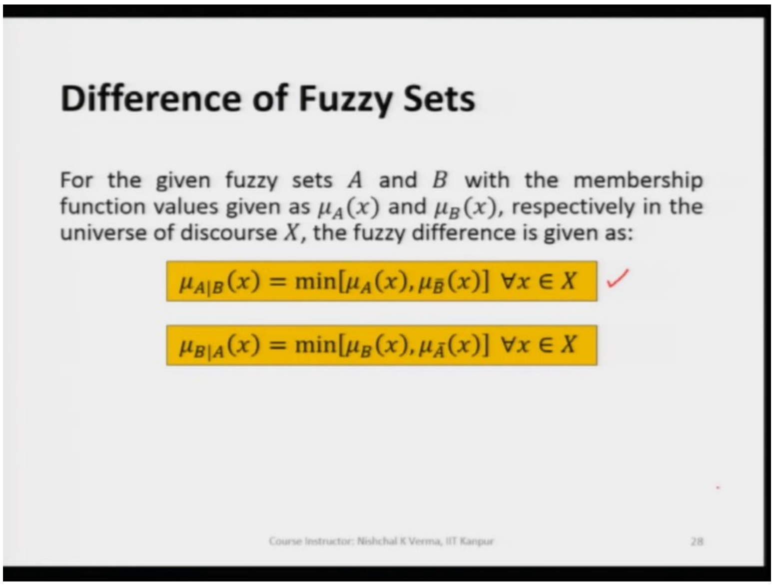

Now let us understand the difference of fuzzy sets, on the same lines as we have already done the other set theoretic operations where we have seen that the it's the membership values which are playing an important role in the set theoretic operations. So, here also we see that when we are interested in the difference of fuzzy sets let us say difference of fuzzy sets A and B, we try to first you know compute the their membership values respective membership values and the these conditions are the membership values are represented by

$$\mu_{A \vee B}(x) = min[\mu_A(x), \mu_B(x)] \forall x \in X$$

And then if we are interested in the difference in B and A fuzzy set B and A, we find the respective membership values as $\mu_{B \vee A}(x) = min[\mu_B(x), \mu_A(x)] \forall x \in X$

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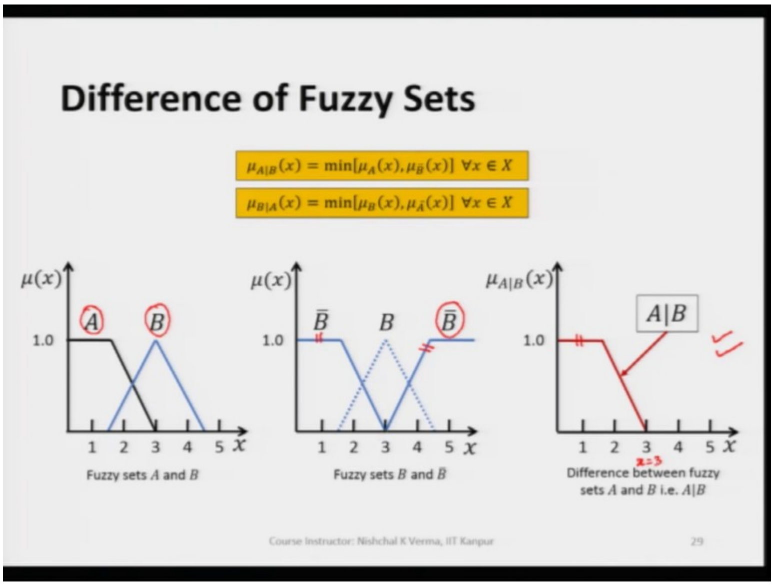

So, let us now understand the difference of fuzzy sets here and if we apply these two conditions to the fuzzy sets that are given here is fuzzy set A fuzzy set B. And if we apply the condition here as the we are interested in finding the difference between fuzzy sets A and B. So if we are interested in the difference of fuzzy sets A and B, we have to find the mu B bar. So, for finding this we have to first find the compliment of B. So, this is what is the compliment of B fuzzy set and when we have this, we can compute the respective we can find the respective membership values.

So, if we have B fuzzy set here we can say this is the compliment of B. So, this is compliment of B fuzzy set and then when we apply this criteria this condition we find fuzzy set the difference between fuzzy set A and B which is coming out to be represented in red color. So, we take all the minimum values so, if we plot here the minimum value everywhere wherever we see these conditions are satisfied in this figure and if we take at 0.3 at generic variable $x=3$. So, we see that x has for A fuzzy set we have a 0 membership value and B bar also has membership value 0. So, if we take min we are getting 0 here as well, all right.

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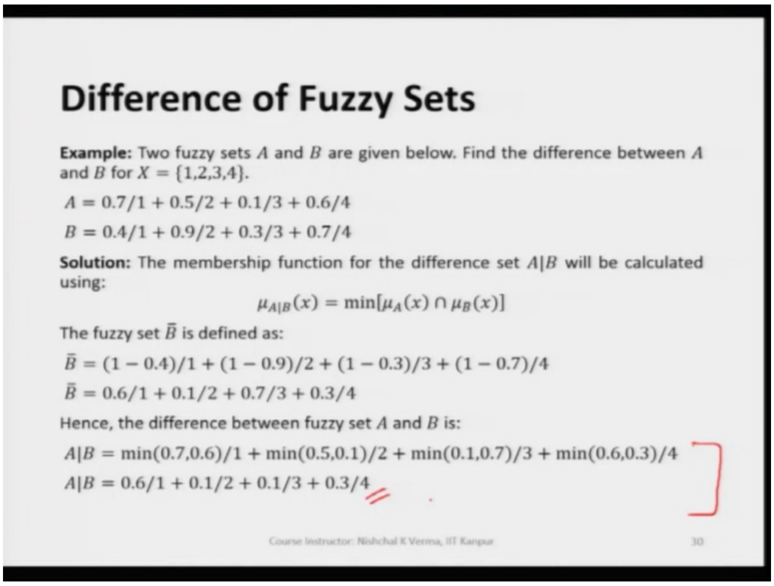

So, we take some more examples here to make to understand the difference of fuzzy sets better and here we have two discrete fuzzy sets A and B. And if you apply these conditions, we are we can get this as the outcome of the computations and this is fuzzy this is the difference between two fuzzy sets A and B. So, we have in this lecture we have covered, these set theoretic operations and in the next lecture we will study the properties of fuzzy sets.

Thank you.