Fuzzy Sets, Logic and Systems and Applications

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Lecture – 10
Set Theoretic Operations on Fuzzy Sets

So, welcome to lecture number 10 of Fuzzy Sets, Logic and Systems and Applications. So, this lecture will cover Set Theoretic Operations on Fuzzy Sets. In today's lecture, we will discuss following operations on fuzzy sets. These operations are complement, union, intersection and difference.

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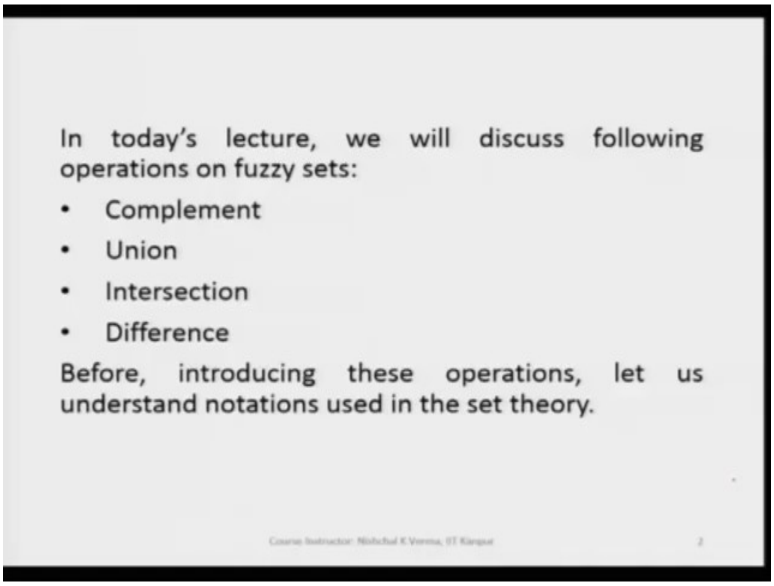

And before introducing these operations on fuzzy sets, let us first understand the notations that we use in set theory.

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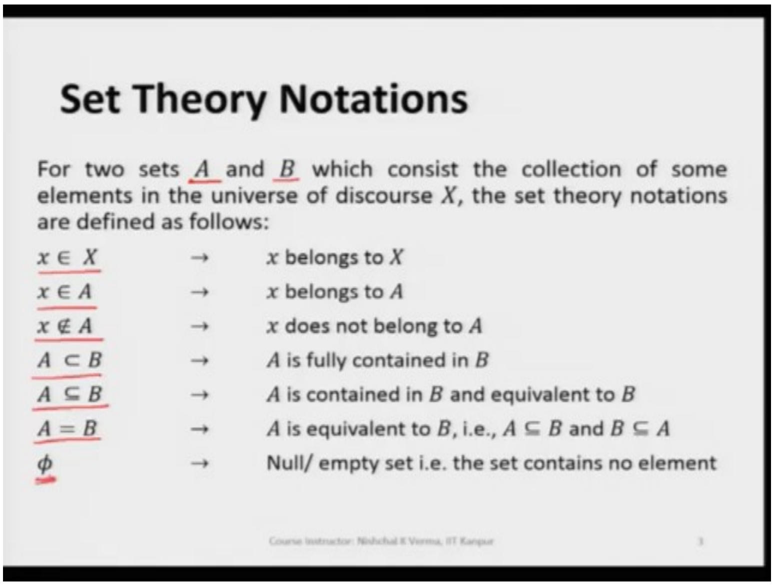

And if you see here, these are the notations that are used in set theory; like if we have two sets A and B here and which consists of the collection of some elements in the universe of discourse capital X, the set theory notations are as follows.

So, if we see here x belongs to capital X this means x belongs to X. Similarly, here this is for x belongs to A and here x does not belong to A. Here A is fully contained in B and here A is contained in B and equivalent to B. And of course, here A is equal to B. ϕ here represents the null or empty set.

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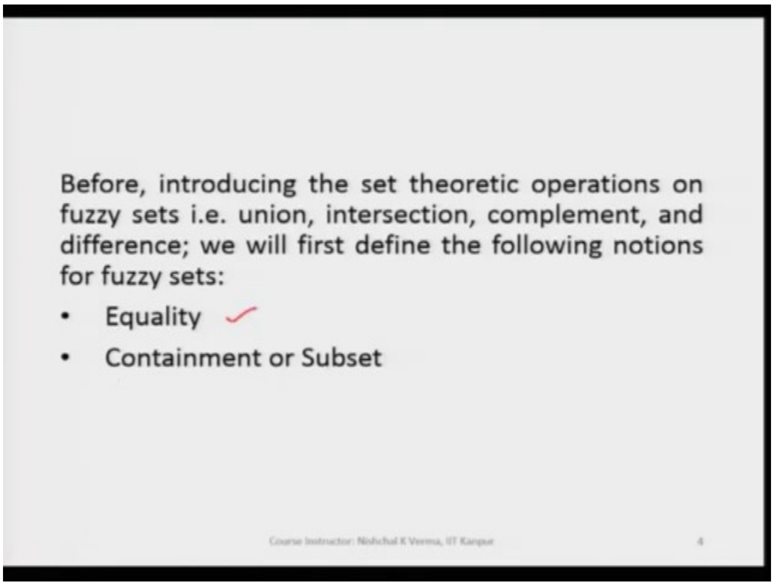

So, before we move to set theoretic operations, we have two notions of two notions that we will be discussing here. The first notion is equality and the second notion here is the containment of sets or subset.

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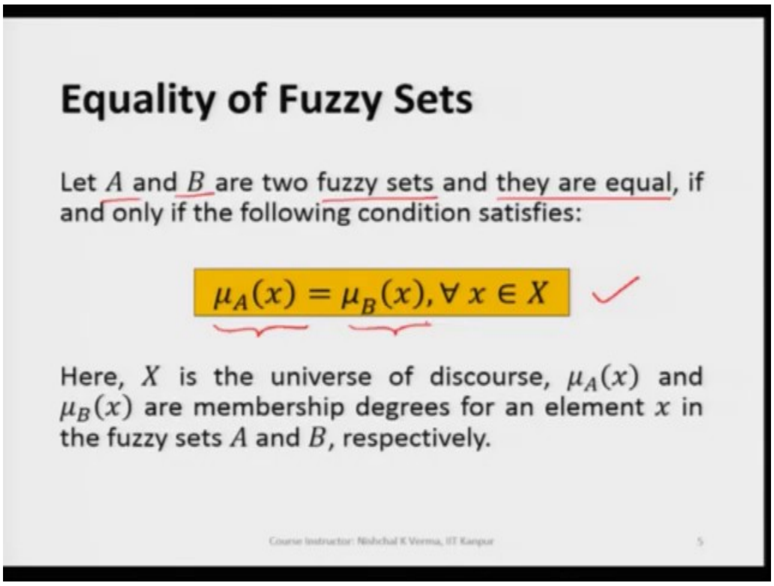

So, let us now first discuss the equality. So, equality of fuzzy sets is very important like when can we say two fuzzy sets are equal. So, here the point that needs to be noted is that even when we have the any fuzzy set the generic variable values remains the same, in both the sets may not be equal, because for equality their membership their corresponding membership values also need to be equal. So, that is how it is written here.

If A and B are two fuzzy sets and if we say these are equal A and B; so, if they we can say they are equal. So, this equality can be possible only when their corresponding membership values are also equal.

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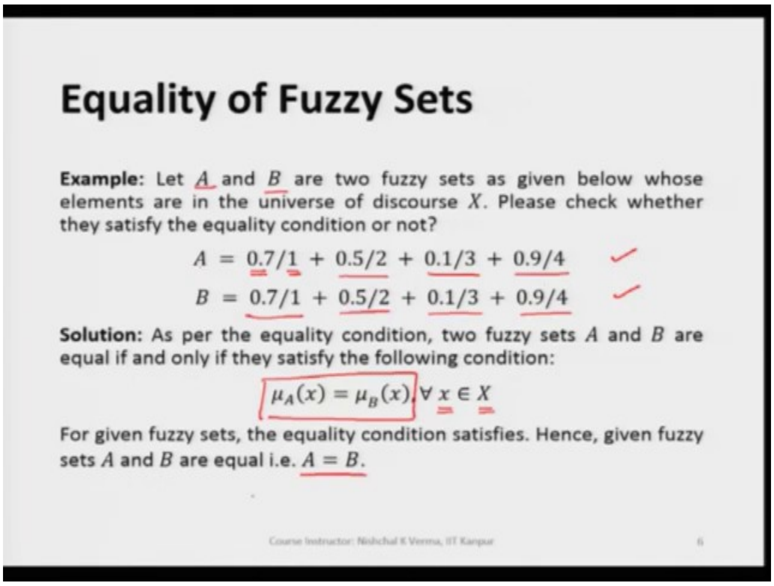

So, let us look at this example to better understand. So, if we have two fuzzy sets A and B and these fuzzy sets are discrete fuzzy sets. This A fuzzy set which is discrete fuzzy set, this B fuzzy set which is again a discrete fuzzy set. So, let us now look at these fuzzy sets and their you know the fuzzy elements. So, it means the generic variable values and the membership values; corresponding membership values. So, let us now look at these and see whether A and B both are equal or not.

So, if we see here for A; generic variable A, the first generate variable is 1 and the corresponding membership value is 0.7. So, now, if we see the first element here in B also this is same, means we have generic variable 1 and the corresponding membership value is 0.7. So, it means, we have the generic variable values; for the same generic variable values, we have the same membership values.

Similarly, if we look at all the elements of A and then we look at all the elements of B, so we see that we have exactly equal membership values for the generic variable values. So, this way; we find this relation is satisfied, means that you know $\mu_A(x)=\mu_B(x)$. And again this is needless to say that all x, all the generic variable values must belong to the universe of discourse within which we are working. So, this way fuzzy set A and fuzzy set B both can be said to be equal.

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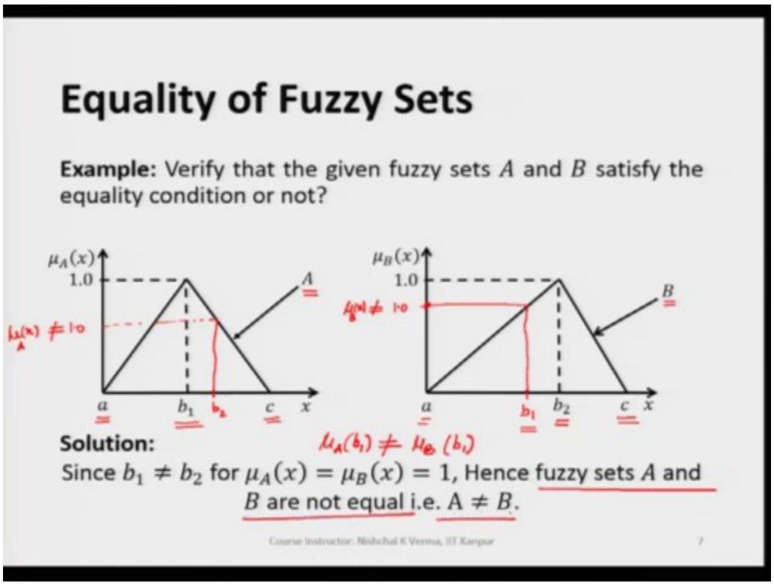

Now, if we take two continues fuzzy sets, and let us see whether these two fuzzy continues fuzzy sets are equal not. So, here we have this for you set A and the other one is B. So, we see that although, A and B; in between A and B, the generic variable values remain the same, but the corresponding, but their corresponding membership values are not same; not equal.

So, if we take any point if we take a point b_1 in between a, c and then if we take some point b_1 here in between a, c. So, we see that here the corresponding membership value corresponds; the membership value corresponding to b_1 in fuzzy set B is not equal to 1; is not equal to 1 means the there is the membership value corresponding the generic variable value b_1 whereas, here we can write this as $\mu_B(x)$. So, we can clearly see that for the same generic variable values b_1 in both the fuzzy sets the $\mu_A(x)$ is not equal to $\mu_B(x)$. So, instead of x here, because we are taking x is equal to b_1. So, I think we need to know we need to write here b_1.

So, this way we can clearly see that, the these two fuzzy sets are not equal. And if we take a point b_2 here in fuzzy set B and if we take the same fuzzy set here, we take a same point b_2 here in between a to c. So, we see here that in fuzzy set A, the corresponding membership value is less than is less than 1 or we can say is not equal to 1. So, this is $\mu_B(x)$.

So, this way we see for the same generic variable values in both the sets, their corresponding membership values are not equal. So, that way we can say a fuzzy set $A \neq B$.

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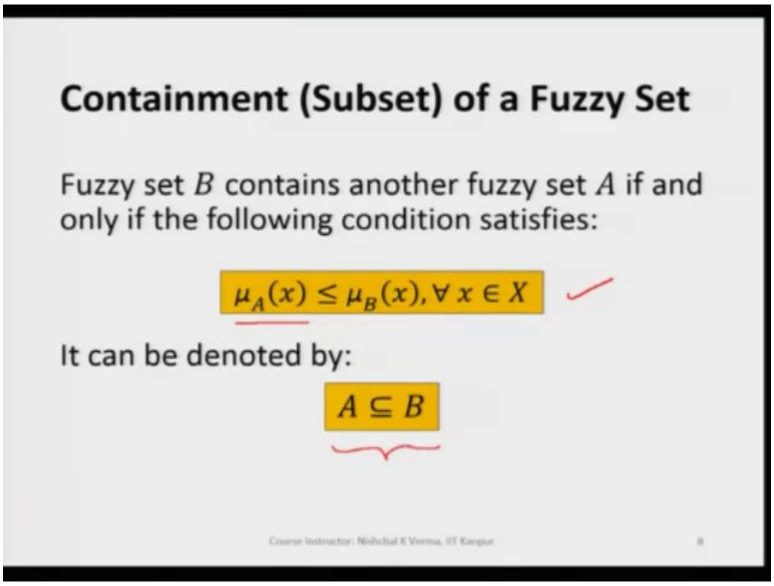

Now, the containment or sub or subset of a fuzzy set. So, fuzzy set B; if we take the same example, if we take two fuzzy sets A and B, so the containment can be defined as the; if we have in fuzzy sets A and B, we follows the membership values follow this condition; we can say the fuzzy set B contains fuzzy set A. So, if we have any two fuzzy set A and B and their corresponding membership values follow this condition; means, $\mu_A(x)$ is less than or equal to $\mu_B(x)$ for every x that belongs to the universe of discourse capital X.

So, then if this case is if this is the case if this condition is followed; we can say that, A is contained in B. So, the fuzzy set A is contained in B.

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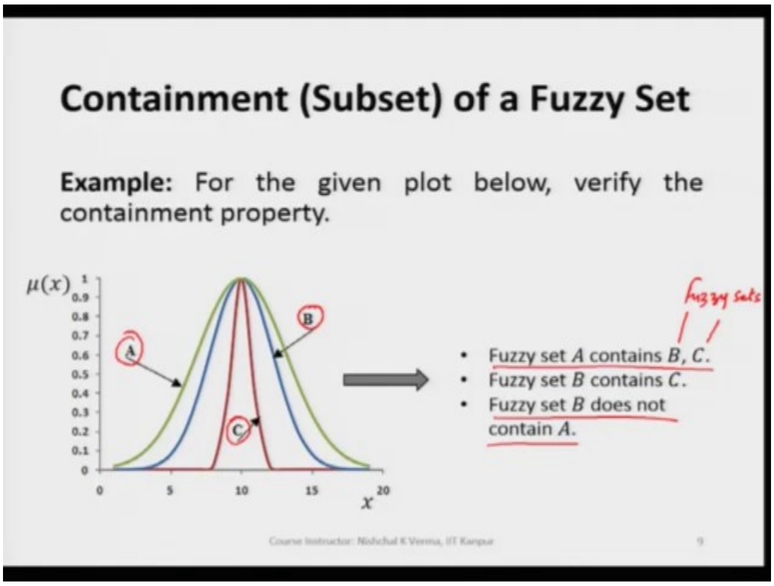

Now, let us understand this a little bit more clearly with this example. So, we have here a diagram and in this diagram, we have three fuzzy sets. So, here we have the fuzzy set A, which is shown by the green line I mean in green colour. The B fuzzy set is shown in blue colour and C fuzzy set is shown in the red colour.

So, by just looking at it, we can clearly say that all the membership values corresponding to the generic variables in A, in C is less than their corresponding membership values less than the membership values corresponding to the generic variable values of B and A. So, this way, we can say that fuzzy set A contains BC, B and C fuzzy set this is also, this is fuzzy set this is for set these two are fuzzy sets.

And, we can also say that here that; fuzzy set B does not contain A. So, if we compare fuzzy set B and fuzzy set A, we can clearly see here that the membership values corresponding to their generic variable values in both the cases. If we compare, then the membership values of B fuzzy set are less than membership values of A. So, that way we can say fuzzy set B does not contain A.

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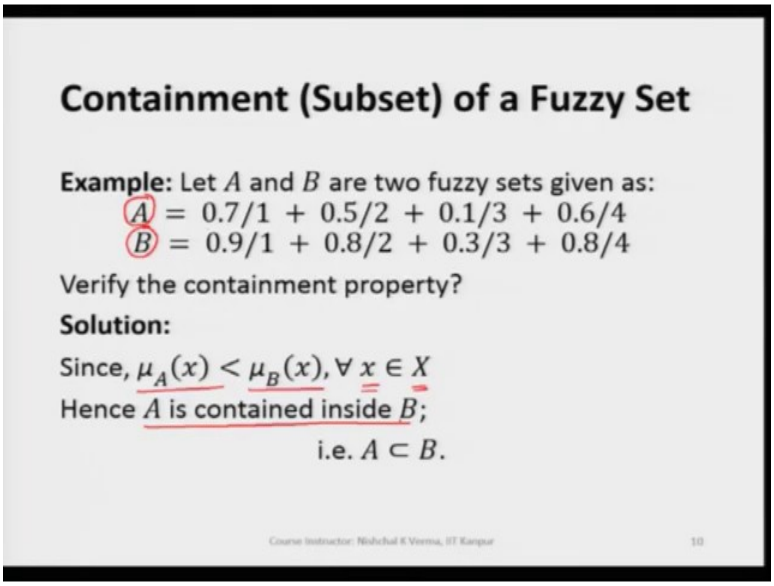

So, let us now take an example here to understand the containment or the subset of a fuzzy set better. So, in this example, here we have two fuzzy sets. The first fuzzy set is the two discrete fuzzy sets basically. And the first fuzzy set is A and the second fuzzy set is B.

So, these two fuzzy sets we see, we have the elements here and the generic variables values that generic variable values $1,2,3,4$ and their corresponding membership values. So, in both the fuzzy sets, we have same generic variable values $1,2,3,4$. But, the membership values are different.

So, if we see clearly the membership values, so membership values of A fuzzy sets are lesser than that of B for respective membership generic variable values. So, that is how we can say $\mu_A(x) \le \mu_B(x)$. So, this means the the membership values in fuzzy set A are less than the membership values in fuzzy set B for corresponding generic variable values. And of course, this is needless to say that all these x, the generic variable values they all belong to the universe of discourse.

So, this way we can say that A is contained inside B or we can say A is contained in B.

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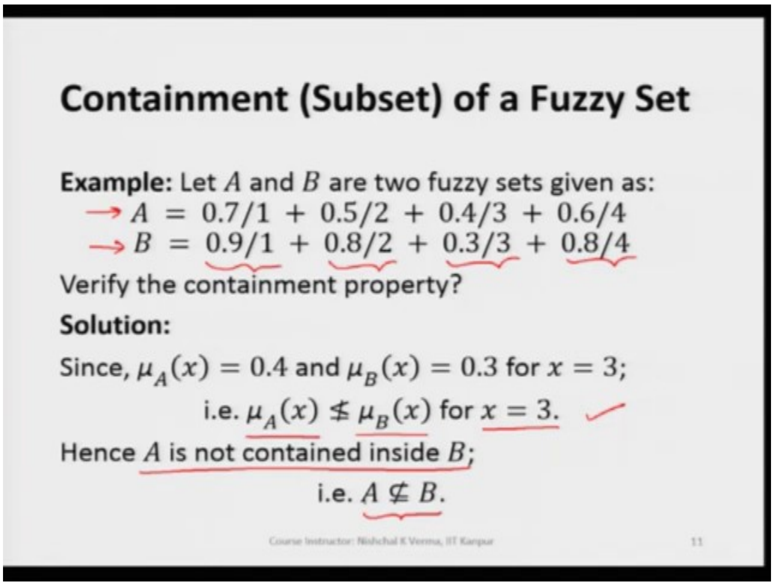

Here, we have another example and we see again two discrete fuzzy sets A and B. And just by looking at the generic variable values and their corresponding membership values, we see that, $\mu_A(x) \ntrianglelefteq \mu_B(x)$ for the generic variable value x is equal to 3. So, if we look at B here; B member; B fuzzy set and A fuzzy set, so if we look at these elements, we compare these elements generic variable values remain again the same $1, 2, 3, 4$ in both the fuzzy sets, but their corresponding membership function values are membership values are different.

So here, for x is equal to 3 for the membership for the generic variable value 3, the membership value in the corresponding membership value in fuzzy set A is more than that of the membership value in B. So, that that is why, we can say that; this condition is not satisfied means, $\mu_A(x) \ntrianglelefteq \mu_B(x)$ for all x's and this is one of the generic variable value x is equal to 3 which violates. Otherwise, if for x is equal to 3, these value the corresponding value of membership would be equal are the less in A then B, we would have set that AB is containing A or A is contained in B.

So, since at x is equal to 3, it is violated. We can clearly say that, A is not contained in B. And mathematically, we can represent this by A is equal to A is not contained in B.

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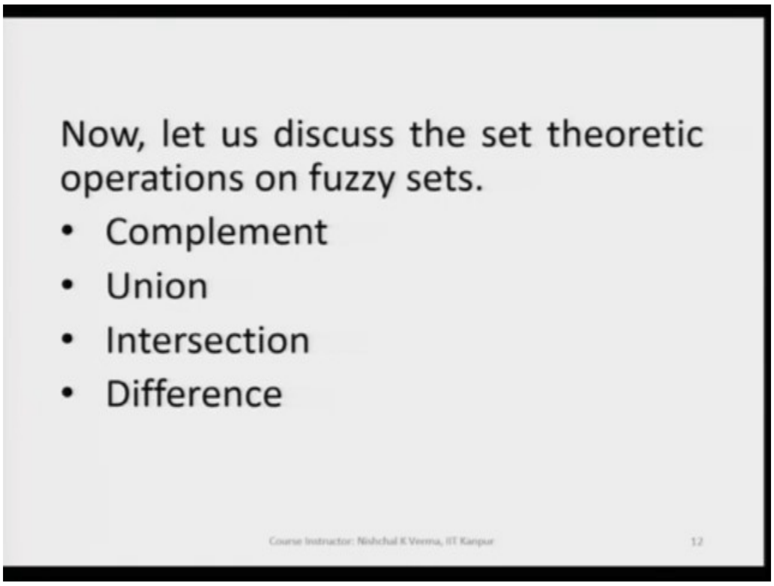

Now, let us discuss the set theory op set theoretic operations on fuzzy sets and these set theoretic operations are complement, union, intersection and difference. So, since these operations we are interested in applying on fuzzy sets. So, before we move to these operations on fuzzy sets, we would be discussing these operations on crisp sets or classical sets and then we will transition from classical sets to the fuzzy sets with respect to these operations, set theoretic operations to understand the set theoretic operations and fuzzy sets better.

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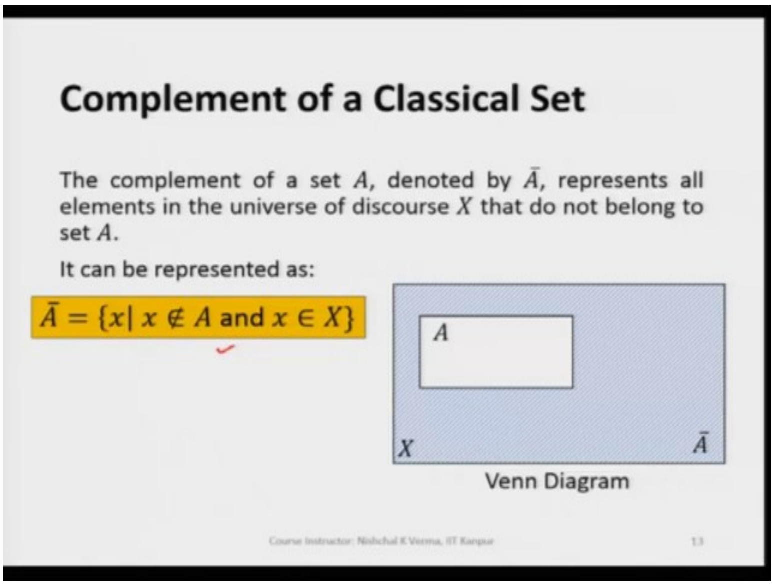

So, let us now first understand; what do we mean by a complement of classical set. So, if we have a classical set it is represented by A, we represent a complement of this classical set by $\acute{A}$. And this can be represented here mathematically as $\acute{A}=\{x \vee x \notin A \wedge x \in X$.

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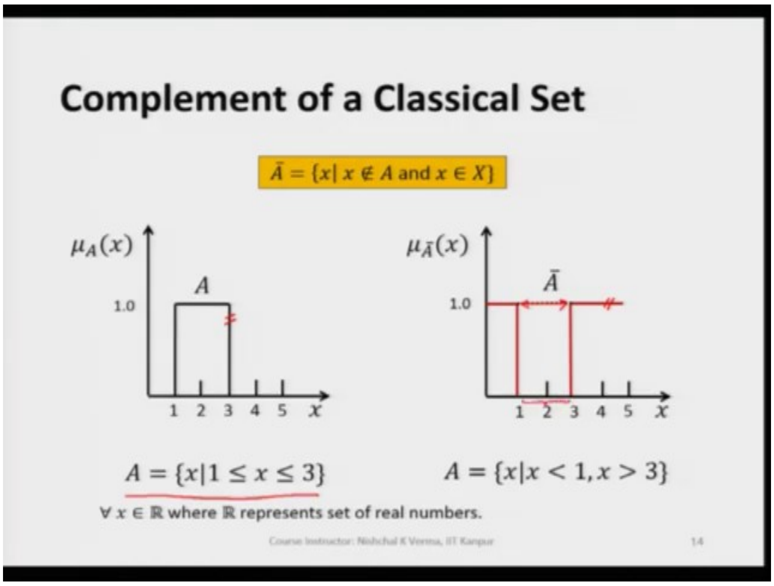

So, let us now understand the complement of a classical set. So, in this diagram here, we see a classical set which is continuous and this is right from 1 to 3. So, as it was just defined if we are interested in finding the complement of a classical set A, we represent this by A bar and this $\acute{A}$ will be the collection of all the points that are in the universe of discourse excluding the points that are coming into excluding the elements that are there in set A.

So, if we look at this set which is a classical set. So here, we see that they in this is a continuous set. So, here there are winds which are right from starting right from 1, 2, 3 are covered and if we have some universe of discourse, so we exclude these points and we take all the points which are contained which are there in the universe of discourse.

So, if we do that, we will get the $\acute{A}$ like this and we see that this is the points which are there in the A are excluded. So, this is it is this part is excluded. So, this way we find the complement of classical set A.

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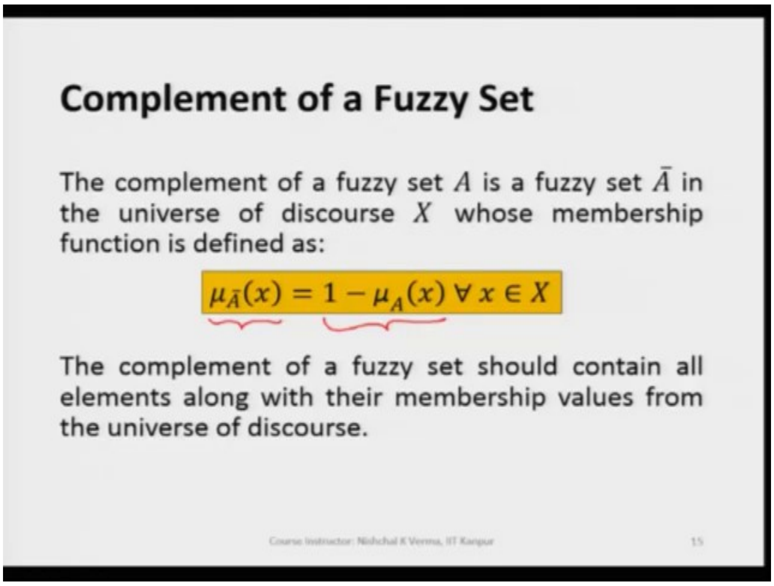

Now, if you are interested in finding a complement of fuzzy set, so the complement of a set A is fuzzy set, that is also represented by $\acute{A}$. And of course, here that we have a universe of discourse X and in fuzzy set also we take all the points all the elements you know, but with this condition which is mentioned here. So, fuzzy set, if we have a fuzzy set A and then the complement of fuzzy set A will have its membership values corresponding to the generic variable values which will be defined as $\mu_{\acute{A}}(x)$.

So, you see here $\mu_{\acute{A}}(x)$ and all the corresponding membership values will be subtracted from 1. So, all the corresponding membership values of A fuzzy set will be subtracted from 1. So, this way if we follow, we get the membership values corresponding to complement fuzzy set and we can say that complement of a fuzzy set should contain all the elements along with their membership values from the universe of discourse.

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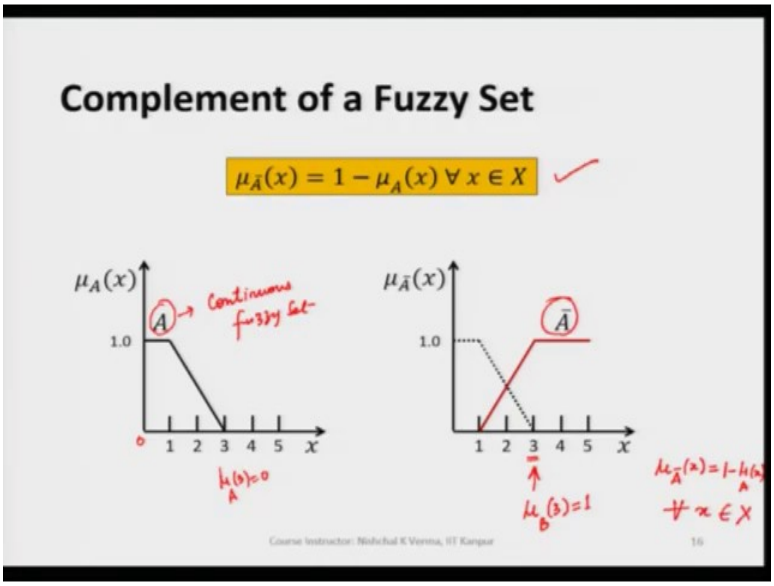

So, to understand the complement of a fuzzy set better, let us take this example. Here, we have a fuzzy set A and please note that this fuzzy set here, this fuzzy set is a continuous fuzzy set, is a continuous fuzzy set. So, we have here this 0. So, right from 0 to 3, this fuzzy set is spreading. And we have varied membership values also like at 0, we have membership value 1 and at 3, we have membership value 0.

So, if we follow this criteria like complement fuzzy set $\acute{A}$, its membership values will be computed as 1 minus $\mu_A(x)$, for every x belonging into the universe of discourse. So, if we do that we are going to get $\acute{A}$ like this. So, we can clearly see here that at 3 at the generic generic variable value 3 in A, we have $\mu(x)=0$.

So, instead of $\mu(x)$, I will be writing $\mu(3)$, $\mu(3)$. And since this belongs to the fuzzy set A we'll write $\mu_A(3)=0$. Now, if we look at the fuzzy set $\acute{A}$ which is complement of a fuzzy set. So, we will look at this general variable value 3 so, this is $\mu_B(3)=1$. So, likewise, if we check at all the points, all the generic variable values we'll get to see that this follows, the membership value follows membership values of $\acute{A}$ follow this condition, $1-\mu_A(x)$. And this is for all the for every x belonging to the universe of discourse.

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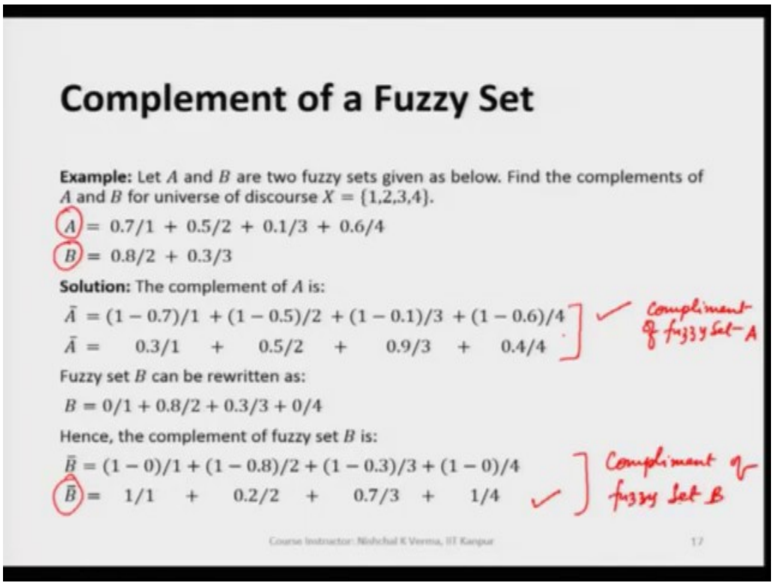

Similarly, we take an example with two discrete fuzzy sets. So, if we have a fuzzy set A, which is here and if we are interested in finding the complement of this fuzzy set. We can clearly use the criteria as I just mentioned and we get the complement of a fuzzy set. So, this is complement of A. This is complement of fuzzy set A.

Similarly, this is complement of fuzzy set A, and similarly if we take this fuzzy set B, we can get the complement of fuzzy set here. And please note that the universe of discourse here is given and this is this contains $1,2,3,4$. So, we are excluding all other values except the other than $1,2,3,4$. So, this way we find complement of fuzzy set B. So, this will be the complement of fuzzy set B.

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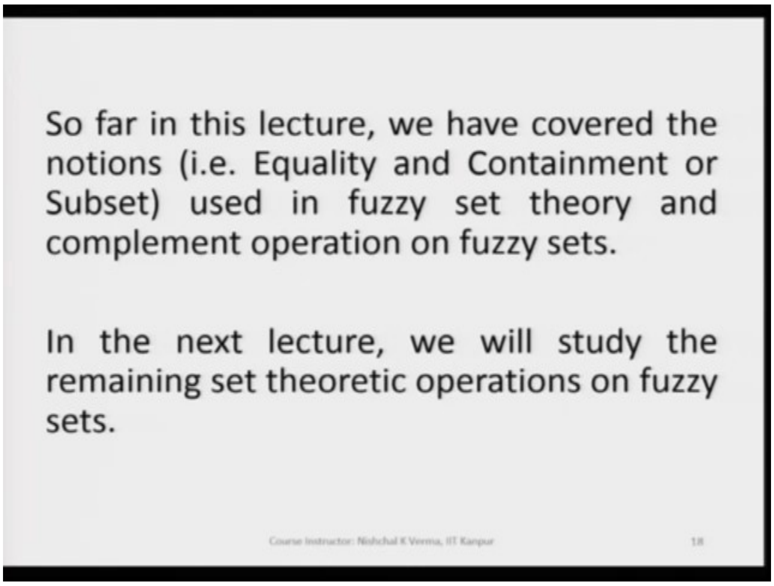

So, far in this lecture, we have covered the notions that means the equality and containment or subsets used in the fuzzy set theory and complement operations on fuzzy sets. So, in the remaining fuzzy set theoretic operations will be discussed in the next lecture.