Fundamentals of Electric Drives

Prof. Shyama Prasad Das

Department of Electrical Engineering

Indian Institute of Technology - Kanpur

Lecture – 09

Analysis of Single Phase Full Controlled Converter-Fed Separately Excited DC Motor

Welcome to this lecture on the fundamentals of electric drives! In our previous session, we explored the operation of a fully controlled rectifier feeding a separately excited DC motor. During that discussion, we identified two distinct categories of operation: continuous current operation and discontinuous current operation. We began our examination with discontinuous current operation, and now, let's pick up right where we left off in the last lecture.

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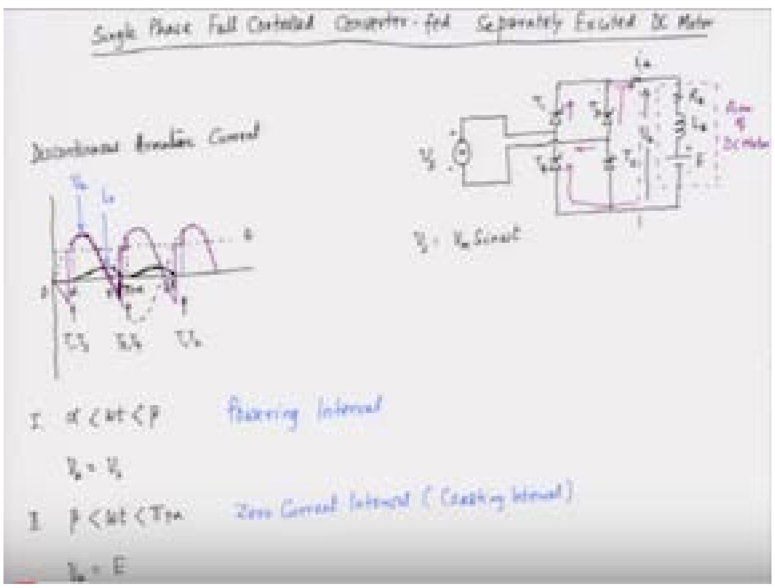

Now, let's delve into the concept of discontinuous armature current operation. In this scenario, the armature current is characterized by its discontinuous nature; it rises, falls to zero, and then starts rising again, only to fall back to zero once more. Essentially, the current reaches zero during every half cycle.

To analyze this circuit mathematically, we consider the single-phase converter alongside the load, which in this case is our DC motor. The DC motor is represented by its armature resistance, armature inductance, and back EMF. During the discontinuous current operation, we initiate the triggering of the SCRs, specifically T1 and T2, during the positive half cycle.

As we trigger these thyristors, T1 and T2, the current begins from zero, gradually building up until it reaches a peak and then declines back to zero at a specific angle known as the extinction angle, β. Following this point, the current remains at zero until we trigger the next pair of SCRs, which are T3 and T4, during the negative half cycle. At this moment, the output voltage reverses.

We can break down the output voltage and the operation of this converter into a few distinct modes, at least two modes to consider. The first mode is referred to as the powering interval, which occurs from α to β (i.e., $\alpha < \omega t < \beta$). This segment represents the powering interval, while the second mode, from β to $\pi + \alpha$, up to the instant when the next pair of SCRs is triggered, is known as the coasting interval or the zero current interval.

In this scenario, we have a phase where there is no current flowing, which we refer to as the zero current interval. Prior to this, we have the first zone, known as the powering interval. During this powering interval, we can observe the applied voltage, which is the source voltage, denoted as V_s. When we trigger the two SCRs, T1 and T2, the conduction path flows through T1, then through the load, and back to the source via T2. In this case, the armature voltage is identical to the source voltage, V_s, allowing the current to start from zero and gradually build up.

As we continue, once the voltage transitions to a negative value, this occurs even after $\omega t = \pi$, the SCRs still conduct. However, the current begins to decrease because the supply voltage is now negative. The current will reach zero at the angle $\omega t = \beta$, marking the end of the powering interval.

Following this, we enter the interval defined by ωt ranging from β to $\pi + \alpha$, this is the zero current interval. During this interval, there is no current flowing. At this moment, the back EMF aligns with the supply voltage, or V_a (the armature voltage), indicating that V_a is equal to the back EMF. This relationship is illustrated in this portion of the curve or region, where the supply voltage V_a corresponds to the back EMF. This phase is, therefore, referred to as the zero current interval. Now, let us analyze these two intervals separately.

Now, during the powering interval, we focus on the region where ωt is between α and β, leading to the condition that $V_a = V_s$. This relationship allows us to establish a fundamental differential equation for the armature voltage, which mirrors the AC input voltage. The equation can be expressed as:

$$V_a = R_a I_a + L_a \frac{dI_a}{dt} + E$$

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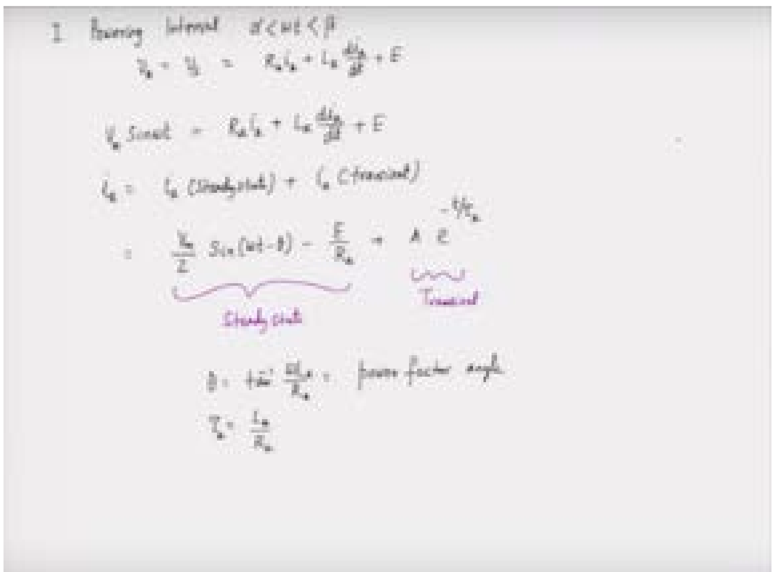

Here, E represents the back EMF, and the input voltage can be denoted as $V_m \sin(\omega t) = R_a I_a + L_a \frac{dI_a}{dt} + E$. This constitutes a first-order differential equation that we need to solve.

The solution to this differential equation is comprised of two distinct parts: a steady-state component and a transient component. The steady-state component includes terms such as

$$\frac{V_m}{Z} \sin(\omega t - \theta)$$

In this context, we have a circuit characterized by armature resistance, inductance, and back EMF. When we derive this equation for the RLE circuit, we find that the applied voltage V_a is equivalent to V_s.

Consequently, the steady-state current I_a exhibits two components: one induced by the input voltage V_a, which is an AC voltage, and the other stemming from the back EMF E. The back EMF opposes the flow of current, effectively driving it in the opposite direction.

In our analysis, the AC steady-state current is represented by

$$I_a = \frac{V_m}{Z}\sin(\omega t - \theta)$$

while the DC current, which opposes this AC current, is given by

$$-\frac{E}{R_a}.$$

This signifies the steady-state component, but we also introduce a transient component into the equation. The transient component can be expressed as

$$Ae^{-t/\tau_a},$$

where τ_a is the time constant of the circuit.

Now, you may wonder why the transient component is necessary. The reason lies in the variability of the triggering angle α. In this scenario, α is not fixed; it can be adjusted. However, when we trigger the SCRs, the current must initiate from zero to satisfy the boundary condition, which is crucial for the transient component. Thus, we need to determine the constant A.

To achieve this, we start from the condition at $t = \frac{\omega t}{\alpha}$, where the armature current $I_a = 0$. Substituting this condition into our established equation yields:

$$0 = \frac{V_m}{Z}\sin(\alpha - \theta) - \frac{E}{R_a} + Ae^{-\alpha/\tau_a}.$$

To clarify, we multiply the angular frequency ω by the angle α. Notably, $\omega\tau_a = \frac{\omega L_a}{R_a} = \tan(\theta)$, a relationship we defined previously. Hence, we can also express $\frac{\omega L_a}{R_a}$ as $\tan(\theta)$ and $\frac{R_a}{\omega L_a}$ as $\cot(\theta)$.

Now, substituting this back, we can rewrite the equation as:

$$0 = \frac{V_m}{Z}\sin(\alpha - \theta) - \frac{E}{R_a} + Ae^{-\alpha\cot(\theta)}.$$

At this point, our focus shifts to determining the value of A.

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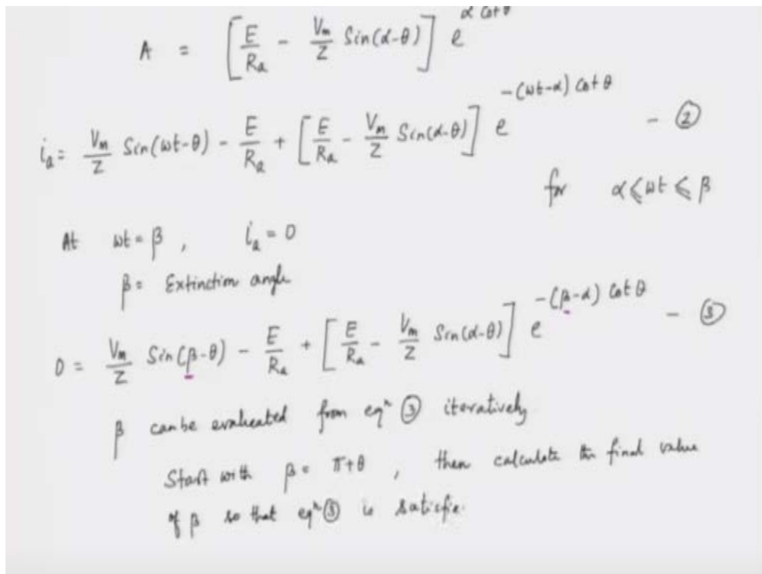

The value of the constant A is given by the expression:

$$A = \frac{E}{R_a} - \frac{V_m}{Z}\sin(\alpha - \theta)\,e^{\alpha\cot(\theta)}.$$

This expression has been derived from our previous equation. By determining the value of this constant A, we can substitute it back into our equation to find the value of the armature current I_a. Thus, the constant can indeed be calculated from this equation.

Now, let’s revisit and substitute this value into Equation 1. Recall that I_a is expressed as:

$$I_a = \frac{V_m}{Z}\sin(\omega t - \theta) - \frac{E}{R_a} + A.$$

Substituting our expression for A yields:

$$I_a = \frac{V_m}{Z}\sin(\omega t - \theta) - \frac{E}{R_a} + \left(\frac{E}{R_a} - \frac{V_m}{Z}\sin(\alpha - \theta)\, e^{-(\omega t - \alpha)\cot(\theta)}\right).$$

This equation represents the armature current I_a for the interval where ω t is greater than or equal to α but less than β.

Next, we need to determine the value of I_a when $\omega t = \beta$. At this point, we find that I_a equals zero:

$$I_a = 0 \text{ when } \omega t = \beta.$$

Here, β is known as the extinction angle of the converter. At ω t = β, the current drops to zero because the supply voltage becomes negative, leading to a cessation of current flow at this specific angle, β.

If we substitute this value into the previous equation, which we will refer to as Equation 2, we arrive at the following expression:

$$0 = \frac{V_m}{Z}\sin(\beta - \theta) - \frac{E}{R_a} + \frac{E}{R_a} - \frac{V_m}{Z}\sin(\alpha - \theta)\, e^{-(\beta - \alpha)\cot(\theta)}.$$

This results in a transcendental equation, not a simple algebraic one, because it incorporates both sine and exponential functions.

To evaluate the angle β, we must approach this iteratively. If we want to find the value of β, we can derive it from Equation 3. We start with an initial guess for β and then refine our estimate by exploring values in the vicinity of this initial guess.

For instance, we might choose $\beta = \pi + \theta$ as a reasonable initial value, where θ is the power factor angle we discussed earlier. From this starting point, we can calculate or evaluate β until we find a value that satisfies Equation 3. This iterative process will lead us to an approximate value for β.

Now, let's consider the boundary between continuous and discontinuous operation.

Now, let’s explore the boundary between continuous and discontinuous operation. We can establish that when $\beta = \pi + \alpha$, we reach this crucial boundary condition; beyond this point, the current becomes continuous.

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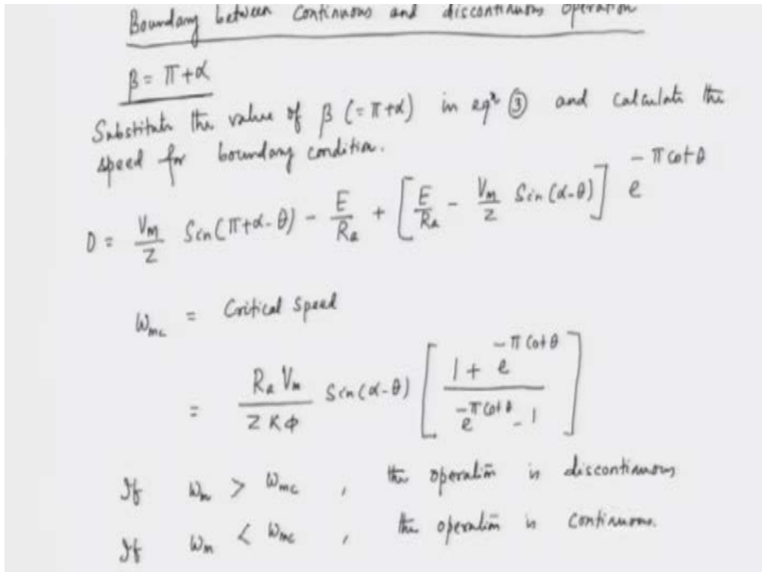

To understand what this entails, we substitute β = π + α into Equation 3 to identify the specific conditions that delineate the boundary between continuous and discontinuous operation.

When we substitute β into the equation, we have:

$$0 = \frac{V_m}{Z} \sin(\pi + \alpha - \theta) - \frac{E}{R_a} + \frac{E}{R_a} - \frac{V_m}{Z} \sin(\alpha - \theta)\, e^{-(\pi + \alpha)}.$$

After substituting, we simplify the equation to find the boundary condition. Notably, since $\sin(\pi + x) = -\sin(x)$, we can rewrite the equation accordingly:

$$0 = -\frac{V_m}{Z} \sin(\alpha - \theta) + \frac{E}{R_a} - \frac{V_m}{Z} \sin(\alpha - \theta)\, e^{-\pi \cot(\theta)}.$$

Here, the terms involving α will cancel each other out, leading us to focus on the condition that results in:

$$-\pi \cot(\theta)\, e^{-\pi \cot(\theta)}.$$

This establishes the critical condition that separates continuous operation from discontinuous operation.

From this simplification, we can derive the value of the critical speed, denoted as ω_{mc}. The expression for this critical speed is given by:

$$\Omega_{mc} = \frac{R_a V_m}{Z\phi \sin(\alpha - \theta)} \left(\frac{1 + e^{-\pi \cot(\theta)}}{e^{-\pi \cot(\theta)} - 1} \right).$$

This critical speed implies that if the speed of the motor exceeds ω_{mc}, the operation will be discontinuous. Conversely, if the speed is less than ω_{mc}, the operation remains continuous.

Now, let us proceed to draw the torque-speed characteristic under discontinuous current operation.

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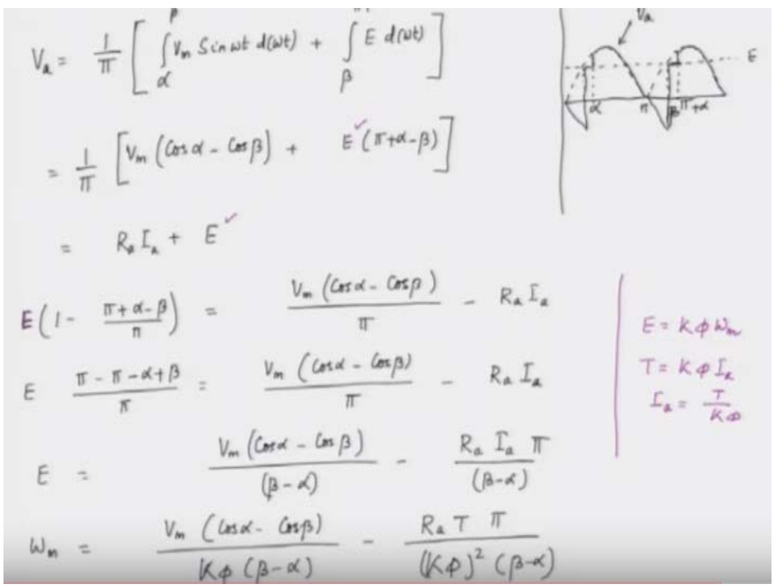

Now, when the current is discontinuous, we observe several key characteristics. Let me illustrate the speed characteristic of the output voltage once again. We have the back EMF represented as E, and the SCRs are triggered at an angle α. This triggering occurs at π as well, and we have the extinction angle denoted as β. The output voltage waveform appears as follows: it rises up to the angle β, then descends, creating a characteristic shape. To clarify, the points of interest are α, π, β, and again at π + α.

If we examine this periodic waveform, we can denote the armature voltage as V_a. Our goal now is to determine the average armature voltage applied to the DC motor. To find the average armature voltage, we will integrate over one half cycle, specifically from α to β and from β to π + α.

The integration for the first half cycle is as follows:

$$\text{Average } V_a = \frac{1}{\pi}\left(\int_{\alpha}^{\beta} V_m \sin(\omega t)\ d(\omega t) + \int_{\beta}^{\pi+\alpha} E\ d(\omega t)\right).$$

Evaluating the first integral, we find:

$$\int_{\alpha}^{\beta} V_m \sin(\omega t)\ d(\omega t) = V_m[-\cos(\omega t)]_{\alpha}^{\beta} = V_m(-\cos(\beta) + \cos(\alpha)).$$

For the second integral, since the voltage remains constant at E over that interval, we have:

$$\int_{\beta}^{\pi+\alpha} E\ d(\omega t) = E(\pi + \alpha - \beta).$$

Combining these results, we express the average armature voltage as:

$$V_a = \frac{1}{\pi}[V_m(\cos(\alpha) - \cos(\beta)) + E(\pi + \alpha - \beta)].$$

This average armature voltage must equal the sum of the resistance drop and the back EMF. Thus, we can write:

$$V_a = E + I_a R_a,$$

where I_a is the armature current. Rearranging gives us:

$$E = V_a - I_a R_a.$$

Now, substituting our expression for V_a into this equation, we have:

$$E = \frac{V_m(\cos(\alpha) - \cos(\beta)) + E(\pi + \alpha - \beta)}{\pi} - I_a R_a.$$

We can simplify this to isolate E:

$$E\left(1-\frac{\pi+\alpha-\beta}{\pi}\right)=\frac{V_m(\cos(\alpha)-\cos(\beta))}{\pi}-I_aR_a.$$

This gives us:

$$E=\frac{V_m(\cos(\alpha)-\cos(\beta))}{\beta-\alpha}-\frac{R_aI_a(\beta-\alpha)}{\pi}.$$

Next, we need to establish the relationship between torque and speed. We already know from previous lectures that the back EMF is given by:

$$E=K\phi\omega_m,$$

and the torque can be expressed as:

$$T=K\pi I_a.$$

From this, we can substitute for the armature current I_a in terms of torque:

$$I_a=\frac{T}{K\pi}.$$

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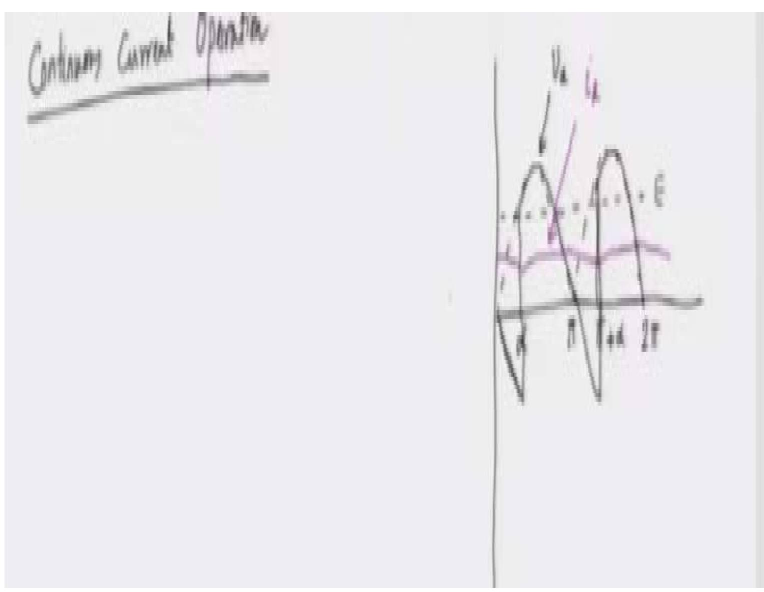

Substituting this into our equation allows us to derive an expression for the motor speed:

$$\omega_m = \frac{V_m(\cos(\alpha) - \cos(\beta))}{K\phi(\beta - \alpha)} - \frac{R_a T(\beta - \alpha)}{K\phi\pi}.$$

Thus, the torque-speed characteristic for a separately excited DC motor operating under discontinuous conditions is established.

Now, let us examine the torque-speed characteristic for continuous operation.

In the case of continuous current, we observe the following characteristics: we have our voltage and back EMF, with the SCRs triggered at an angle α. This occurs at π and continues on to π + α. If we were to plot the current in this scenario, we would see that the current remains continuous, never dropping to zero.

The voltage waveform is shaped accordingly, showing that we are triggering at angle α and maintaining this continuity all the way to π + α. To visualize this, the voltage waveform looks something like this. We mark 2π, π, and observe that the current waveform, denoted as I_a, remains continuous, never reaching zero.

This represents a continuous current operation where there is no interruption in the current flow. The current is consistently maintained throughout both half-cycles, which simplifies our analysis. As a result, we don't need to tackle the more complex equations associated with discontinuous current operation.

In our next lecture, we will focus on deriving the average voltage equation and the torque-speed relationship specifically for continuous current operation.