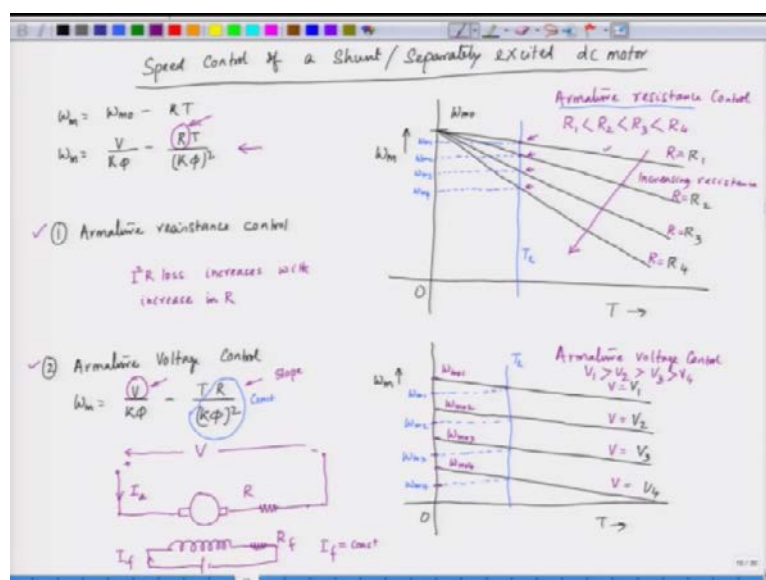


**Fundamentals of Electric Drives**  
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**Lecture - 06**

**Speed Torque Characteristics of Separately Excited DC Motor and Series DC Motor**

Hello, and welcome to this lecture on the fundamentals of electric drives. In our previous session, we explored the speed control methods for separately excited and shunt excited DC motors. Today, we will briefly review those concepts and then move forward with our discussion.

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This is the speed-torque characteristic of a separately excited DC motor. The speed, denoted by  $\omega_m$ , is given by the equation:

$$\omega_m = \frac{V}{k\phi} - \frac{RT}{k\phi^2}$$

where  $V$  is the terminal voltage,  $\phi$  is the field flux,  $R$  is the armature resistance, and  $T$  is the torque. This characteristic forms a straight line, as we discussed in the last lecture.

We explored various methods for controlling speed, starting with armature resistance control, where we adjust the value of  $R$ . When we change  $R$ , the slope of the speed-torque characteristic

shifts, allowing us to control the speed of the motor.

Initially, the characteristic corresponds to a resistance value  $R_1$ , and the intersection of this characteristic with the load torque  $T_l$  determines the operating point. This intersection gives us the first operating speed  $\omega_{m1}$ .

When we increase the resistance to  $R_2$ , the characteristic shifts, and the new intersection point corresponds to a reduced speed  $\omega_{m2}$ . Increasing the resistance further to  $R_3$  moves the operating point again, yielding an even lower speed  $\omega_{m3}$ , and similarly for  $R_4$ , where the speed decreases to  $\omega_{m4}$ .

This method effectively reduces the speed, but it comes at a cost. Since the armature carries the full current, inserting resistance leads to significant  $I^2 R$  losses, making this method less efficient. While it is simple to implement, requiring only a rheostat in the armature circuit to vary the resistance and control the speed, the energy loss makes it an inefficient approach.

The issue with the armature resistance control method is that as the resistance  $R$  increases, the  $I^2 R$  losses also increase, leading to inefficiency. To address this, we turn to a more efficient method: armature voltage control.

In this second method, the speed-torque characteristic remains the same, but instead of varying the resistance, we control the applied voltage  $V$ . The armature resistance is kept constant at its minimum value, and we do not change it.

Here's how it works: we change the applied voltage  $V$  while keeping the armature resistance fixed. Recall that in a separately excited motor, the field is excited independently, and we apply a constant DC voltage to maintain the field current  $I_f$  almost constant. The field resistance is also fixed, meaning  $I_f$  remains steady.

With the field current held constant, the armature circuit's resistance is left untouched, and we focus on varying the applied voltage  $V$ . This variation in voltage directly affects the armature current  $I_a$ . As we adjust  $V$ , the no-load speed, which is given by  $\frac{V}{k\phi}$ , changes accordingly, where  $k\phi$  represents a constant value since the flux  $\phi$  remains unchanged. Since the resistance is kept constant, only the applied voltage alters the motor's speed.

Thus, by controlling  $V$ , we effectively adjust the motor's speed without the inefficiencies caused by increasing resistance, making this a more energy-efficient method of speed control.

When we change the voltage  $V$ , the no-load speed  $\omega_{m0}$  adjusts accordingly. Initially, at voltage  $V_1$ , the no-load speed is  $\omega_{m01}$ . As we decrease the voltage to  $V_2$ , while keeping the slope constant, the no-load speed drops to  $\omega_{m02}$ . A further reduction in voltage to  $V_3$  results in a new speed,  $\omega_{m03}$ . When we continue lowering the voltage to  $V_4$ , the speed becomes  $\omega_{m04}$ .

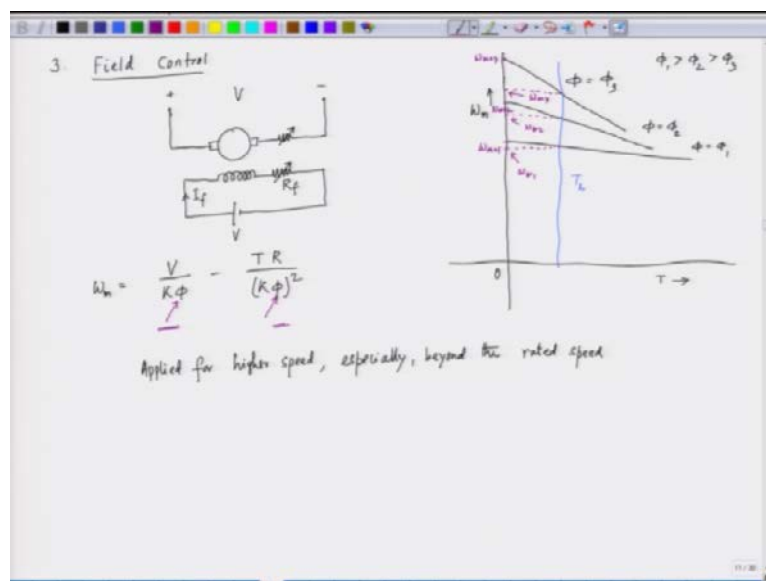
As the voltage is decreased from  $V_1$  to  $V_2$ , and subsequently to  $V_3$  and  $V_4$ , the motor speed reduces correspondingly. The key point here is that the slope of the speed-torque characteristic remains unchanged, but the no-load speed shifts. The intersection of the motor characteristic with the load torque characteristic  $T_l$  determines the operating point.

For  $V_1$ , the operating speed is  $\omega_1$ , and as we reduce the voltage to  $V_2$ , the speed drops to  $\omega_2$ . With further voltage reductions to  $V_3$  and  $V_4$ , the speed declines to  $\omega_3$  and then to  $\omega_4$ , respectively. This is how we manage the motor's speed by controlling the applied voltage.

Unlike the armature resistance control method, in this case, the armature resistance remains at its minimum value, making it a more efficient approach since we avoid the  $I^2 R$  losses associated with resistance control. However, to vary the voltage  $V$ , we need access to a variable DC voltage source, which we will discuss later.

Now, what happens if we vary the field flux  $\phi$ ? This is something we will explore next.

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The third method of speed control is through field control, also known as field current control. In this approach, we focus on controlling the field current, which directly influences the

magnetic flux  $\phi$  of the motor. Imagine we have a separately excited motor with the armature connected to a voltage source  $V$ , and the field winding with resistance  $R_f$  also connected to the same voltage source.

The key equation for the motor's torque-speed characteristic is given by:

$$\omega_m = \frac{V}{k\phi} - \frac{TR}{k\phi^2}$$

In this method, by adjusting the field current, we change the flux  $\phi$ . This results in both the no-load speed and the slope of the characteristic being affected simultaneously.

Let's plot the speed-torque characteristic with speed  $\omega_m$  on the y-axis and torque  $T$  on the x-axis. Suppose we have the curve for a given flux  $\phi_1$ . Now, as we reduce the flux, the no-load speed increases, and the slope becomes steeper. This gives us a new profile for the characteristic at a lower flux  $\phi_2$ . If we further reduce the flux to  $\phi_3$ , we get yet another profile, with a higher no-load speed.

So, for each reduction in flux, we observe corresponding increases in the no-load speeds,  $\omega_{m01}$ ,  $\omega_{m02}$ ,  $\omega_{m03}$ , and so on. This method of speed control is particularly useful for achieving speeds higher than the motor's rated speed, making it ideal for applications requiring operation beyond rated conditions.

Here, the fluxes are ordered as  $\phi_1 > \phi_2 > \phi_3$ , meaning that as we decrease the field current and thereby reduce the flux, the motor speed increases. This control method is specifically applied for higher-speed operations.

If we consider a constant load torque profile, the intersection points between the motor's speed-torque characteristic and the load torque curve determine the operating speeds. For instance, at  $\phi_1$ , the operating speed is  $\omega_{m1}$ , while at  $\phi_2$ , the speed rises to  $\omega_{m2}$ . When the flux is reduced further to  $\phi_3$ , the speed increases to  $\omega_{m3}$ .

Thus, by controlling the field, we achieve higher speeds in shunt or separately excited motors.

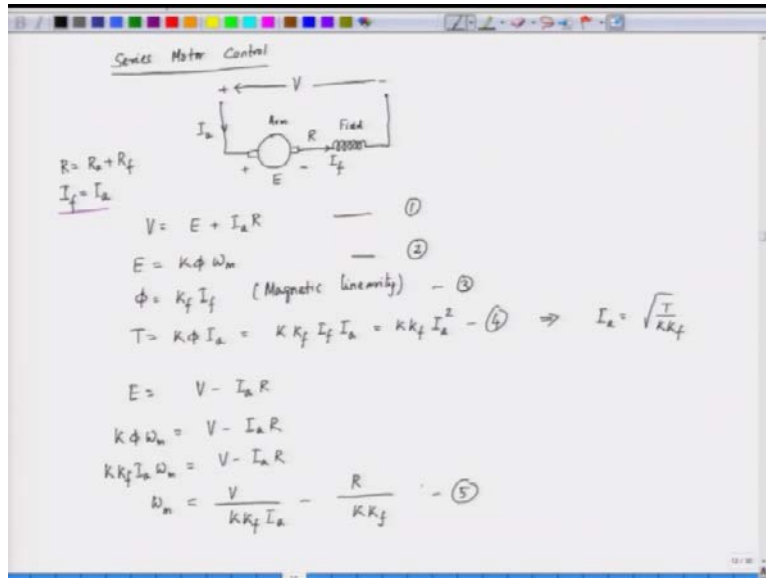
Now, let's move on to examine how we can control the speed of a series motor.

Now, let's consider the operation of a series motor. In this type of motor, the field winding is connected in series with the armature winding. So, when we apply a voltage across the motor

terminals, the same current  $I_a$  flows through both the armature and the field winding. The total resistance of the circuit, denoted by  $R$ , is the sum of the armature resistance and the field resistance:

$$R = R_{\text{armature}} + R_{\text{field}}$$

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The back EMF  $E$  appears across the armature, and the voltage equation for the series motor can be written as:

$$V = E + I_a R$$

Now, the back EMF  $E$  is proportional to the product of the flux  $\phi$  and the motor speed  $\omega_m$ . Specifically, we can express the back EMF as:

$$E = k\phi\omega_m$$

Here,  $\phi$  is the magnetic flux, and in a series motor, the flux is generated by the series field winding. Since the same current  $I_a$  flows through both the armature and the field, the flux is directly related to the armature current. Therefore, the flux  $\phi$  in this motor is a function of the field current, which is the same as the armature current  $I_a$ :

$$\phi = k_f I_a$$

Where  $k_f$  is a constant representing the relationship between the current and the flux in the

magnetic circuit. It's important to note that this relationship assumes magnetic linearity, meaning the magnetic circuit behaves linearly with respect to the current. Under this assumption, as we vary the armature (or field) current, the flux changes proportionally.

So, in a series motor, the field flux  $\phi$  increases or decreases in direct proportion to the armature current, which affects both the torque and speed of the motor accordingly.

Now, let's apply this to the torque equation. We know that torque  $T$  is given by the product of flux  $\phi$  and armature current  $I_a$ . Mathematically, this is expressed as:

$$T = k\phi I_a$$

Since in a series motor the flux  $\phi$  is directly proportional to the armature current  $I_a$ , we can substitute  $\phi = k_f I_a$ , giving us:

$$T = k k_f I_a^2$$

This means that in a series motor, the torque is proportional to the square of the armature current. So, as the current increases, the torque increases quadratically. In other words, doubling the current will quadruple the torque, which makes the series motor ideal for high-torque applications.

Now, let's derive the motor's speed as a function of torque. We already have the following key equations:

1.  $E = V - I_a R$  (voltage equation for the series motor, where  $E$  is the back EMF)

2.  $E = k\phi\omega_m = k k_f I_a \omega_m$

Substituting the expression for  $E$  into the voltage equation, we get:

$$k k_f I_a \omega_m = V - I_a R$$

From this, we can solve for the motor speed  $\omega_m$ :

$$\omega_m = \frac{V}{k k_f I_a} - \frac{R}{k k_f}$$

This equation relates the speed  $\omega_m$  with the armature current  $I_a$ . However, we are ultimately

interested in expressing speed as a function of torque. Using the torque equation:

$$T = k k_f I_a^2$$

We can solve for  $I_a$  in terms of torque:

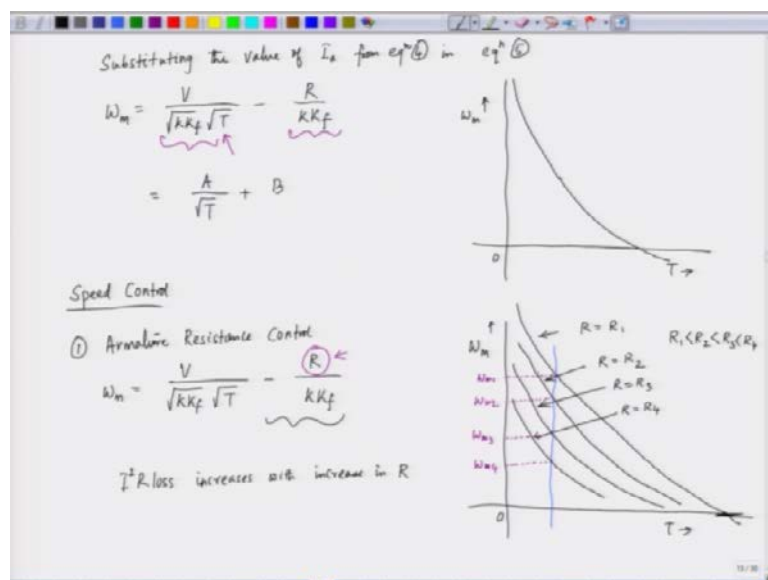
$$I_a = \sqrt{\frac{T}{k k_f}}$$

Now, substituting this expression for  $I_a$  into the speed equation, we get:

$$\omega_m = \frac{V}{k k_f \sqrt{\frac{T}{k k_f}}} - \frac{R}{k k_f}$$

This equation now expresses the speed of the series motor as a function of the torque. It shows that as the torque increases, the speed decreases, reflecting the typical behavior of a series motor, which delivers high torque at low speeds.

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By substituting the value of  $I_a$  from equation (4) into equation (5), we get the expression for motor speed  $\omega_m$  as:

$$\omega_m = \frac{V}{kk_f\sqrt{T}} - \frac{R}{kk_f}$$

Let's break this down. In the previous step, we replaced  $I_a$  with  $\sqrt{\frac{T}{kk_f}}$ , so when we substituted this into the equation, some terms canceled out, and the final form of the equation becomes:

$$\omega_m = \frac{V}{\sqrt{kk_f} \cdot \sqrt{T}} - \frac{R}{kk_f}$$

This equation shows the relationship between the speed  $\omega_m$  and the torque  $T$  for a series motor. Now, let's visualize this by plotting the speed-torque characteristic, where torque  $T$  is on the x-axis, and speed  $\omega_m$  is on the y-axis.

At zero torque (i.e., when  $T = 0$ ), if we substitute  $T = 0$  into the equation, the first term  $\frac{V}{\sqrt{kk_f} \cdot \sqrt{T}}$  approaches infinity, which indicates that the no-load speed is theoretically infinite. This means that when no torque is required, the motor can run at very high speeds, which is a characteristic of series motors.

The form of the equation can be rewritten as:

$$\omega_m = \frac{A}{\sqrt{T}} + B$$

Where  $A = \frac{V}{\sqrt{kk_f}}$  and  $B = -\frac{R}{kk_f}$ , with  $B$  being a negative constant. This implies that as torque increases from zero, the motor speed decreases. When torque is initially zero, the speed is very high (infinity). As the torque increases, the speed drops steadily. Eventually, when the two terms in the equation become equal (i.e., when the torque is large enough), the speed approaches zero.

So, in summary, we arrive at the torque-speed characteristic of a series motor. It is quite fascinating to note that at  $T = 0$ , the speed theoretically approaches infinity. This presents a critical point: a series motor should never be started under no-load conditions. When the torque is zero, the speed could reach dangerously high levels, potentially causing severe damage to the mechanical components due to the immense centrifugal forces generated.

Hence, it is imperative to avoid starting a series motor without any load attached. This brings



us to the discussion of the speed-torque characteristic. Now, how do we vary the speed? We can achieve this by controlling both the armature voltage and the armature resistance. Let's first focus on armature resistance control.

Recall the equation:

$$\omega_m = \frac{V}{\sqrt{k_f k}} \sqrt{T} - \frac{R}{k_f}$$

In this context, we are attempting to change the resistance  $R$ . As we adjust  $R$ , we need to consider how the characteristic curve is affected. Torque is plotted on the x-axis, while speed is on the y-axis. Let's visualize the original characteristic curve, which intersects the x-axis at a certain point corresponding to  $R = R_1$ .

Now, if we increase the resistance, say to  $R = R_2$ , the second term becomes a negative quantity that subtracts from the first term. This means that for the same voltage and torque values, the resulting speed decreases. In other words, a higher resistance leads to a reduced speed. As a result, the new characteristic curve will appear somewhat like this, shifting downwards from the original profile for  $R = R_1$  to a new profile for  $R = R_2$ .

If we further reduce the resistance, making it  $R = R_3$ , the characteristic will shift again, this time moving towards the left. This illustrates how decreasing the resistance continues to affect the speed-torque relationship of the series motor, providing a clear demonstration of the impact of armature resistance control on motor performance.

As we continue to increase the resistance, the profile continues to shift in this manner. For example, this curve corresponds to  $R = R_3$ , and the next one corresponds to  $R = R_4$ , where the resistances follow the order  $R_1 < R_2 < R_3 < R_4$ . So, as we progressively increase the resistance  $R$ , we naturally achieve speed control.

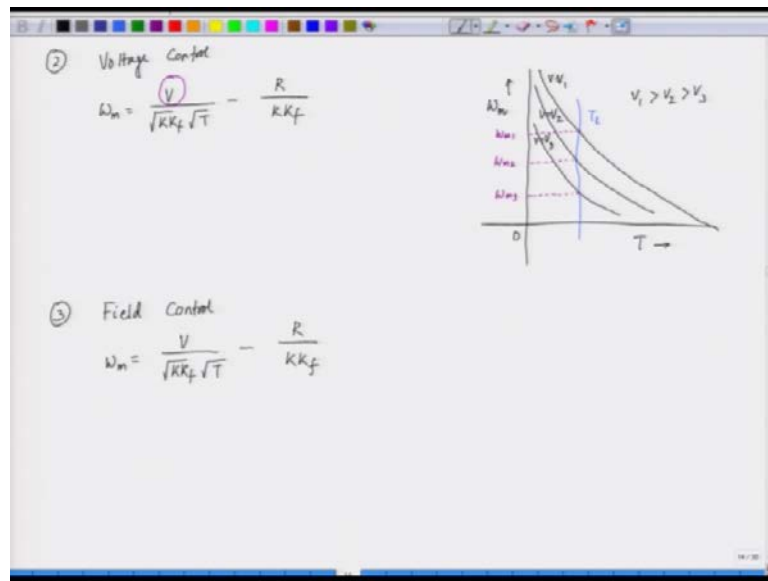
Now, if we consider a load torque profile like this, the speed is determined by the intersection of the load profile and the motor characteristic. Initially, we have a speed value  $\omega_{m1}$ . As the resistance increases to  $R_2$ , the speed reduces to  $\omega_{m2}$ . With further increase to  $R_3$ , the speed decreases to  $\omega_{m3}$ , and finally, for  $R_4$ , we obtain  $\omega_{m4}$ . This demonstrates how speed control is achieved as resistance is varied.

This method is relatively straightforward, as you can simply place a rheostat in the armature

circuit, adjust it, and the motor speed will change accordingly. However, the major drawback of this approach remains the same as before, there is a significant increase in  $I^2 R$  losses as resistance increases. This inefficiency makes it less desirable.

Now, let's move on to another method: the applied voltage control, or more specifically, the terminal voltage control.

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Let's delve into the details regarding the speed-torque characteristics of the motor. The relationship can be expressed as

$$\omega_m = \frac{V}{\sqrt{k k_f}} \sqrt{T} - \frac{R}{\sqrt{k k_f}}$$

In this scenario, we focus on changing the voltage  $V$  while keeping the resistance  $R$  constant. When we adjust the voltage, we observe the impact on the motor speed. The speed  $\omega_m$  is plotted on the y-axis, and the torque is represented on the x-axis.

Now, let's consider the profile for an initial voltage  $V_1$ . If we increase the voltage to  $V_2$ , the no-load speed will change accordingly. Conversely, if we decrease the voltage, the speed will also decrease. Thus, as the voltage changes from  $V_1$  to  $V_2$  and then to  $V_3$ , where  $V_1 > V_2 > V_3$ , we can visualize the corresponding characteristics for these voltages.

For a constant load torque profile  $T_L$ , the speed is determined by the intersection of this load

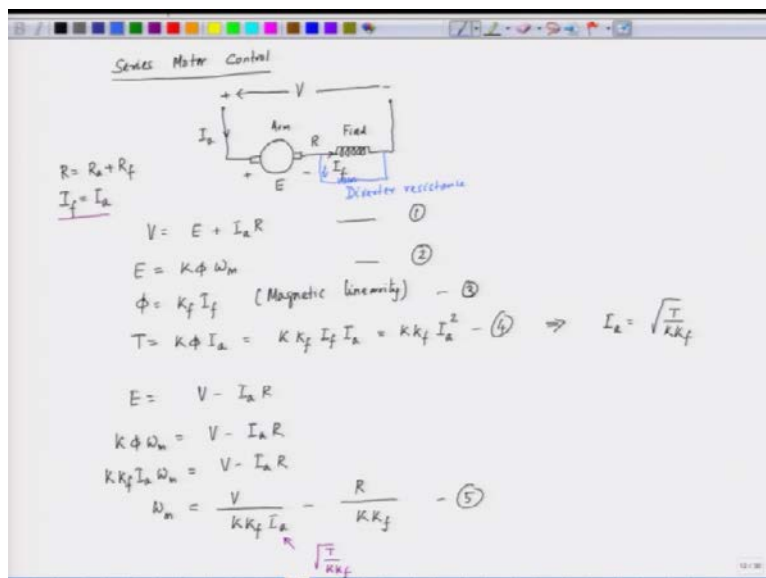
with the motor characteristic. This intersection corresponds to the initial speed  $\omega_{m1}$ . If we decrease the voltage to  $V_2$ , the speed adjusts to  $\omega_{m2}$ , and further reduction to  $V_3$  yields  $\omega_{m3}$ . This illustrates how we can control the motor speed by managing the terminal voltage  $V$ .

Next, we move to the third method of control, known as field control. The equation remains the same:

$$\omega_m = \frac{V}{\sqrt{k k_f}} \sqrt{T} - \frac{R}{\sqrt{k k_f}}$$

So, what does field control entail? In this configuration, the field winding is connected in series with the armature. By implementing field control, we utilize a diverter resistance to effectively reduce the field current. This means we connect a resistance in parallel with the field winding, thereby decreasing the field current flowing through it. This approach is essential for controlling the motor's performance while optimizing its efficiency.

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When we introduce a diverter resistance into the circuit, the effective field current is diminished. This happens because the diverter resistance draws some current away from the field winding, resulting in a decrease in the current flowing through it. Consequently, this reduction in current leads to a decrease in the magnetic flux generated by the field winding. Thus, field control emerges as an effective method for regulating the motor's speed. We will discuss this topic and explore its implications in our next lecture.