Fundamentals of Electric Drives

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Lecture - 04

Steady State Stability, Load Equalization

Hello and welcome to this lecture on fundamental of electric drives. Today, we will be discussing about the steady state stability of the drive system.

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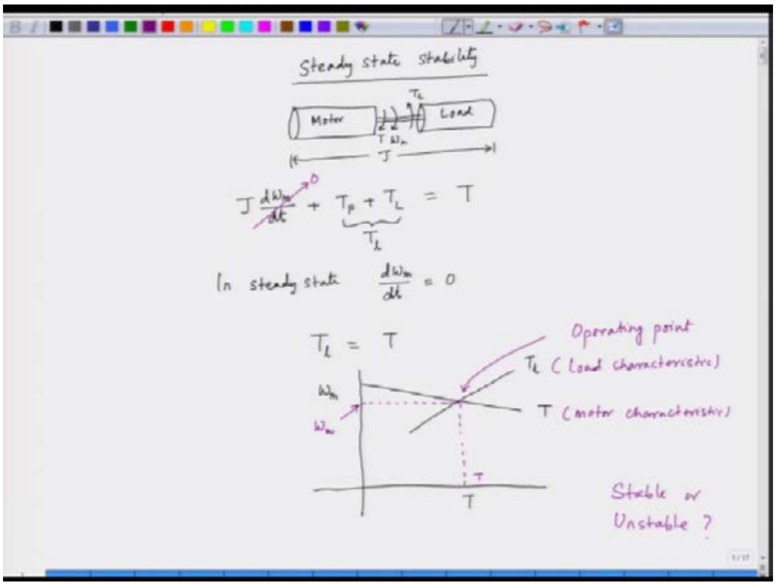

We know that when a motor drives a load, the speed remains constant in steady-state operation, meaning there is no acceleration, or mathematically, $\frac{d\omega_m}{dt} = 0$. Picture a motor driving a load via a mechanical shaft, which couples the motor to the load. The motor speed ω_m matches the load speed, while the motor torque T is counteracted by the opposing load torque T_l. The total moment of inertia of the entire system is denoted by J.

In the previous lecture, we discussed that for such a motor-load system, we can express the dynamics with a fundamental equation of motion. The equation is written as:

$$J\frac{d\omega_m}{dt} + T_F + T_L = T$$

Here, $J\frac{d\omega_m}{dt}$ represents the inertial torque (proportional to angular acceleration), T_F is the viscous friction or frictional torque, and T_l is the useful load torque. Together, these terms must balance the motor torque T.

To simplify, we assume there is no friction, or we can combine the frictional torque into the load torque T_l, which gives us the simplified version of the equation. Now, in steady state, since $\frac{d\omega_m}{dt} = 0$, the inertial torque vanishes. This leads us to the steady-state condition:

$$T_L = T$$

In other words, the load torque equals the motor torque when the system is in steady-state equilibrium.

To determine the operating point of the motor-load system, we look at the speed-torque characteristic curves. On a speed-torque plane, with ω_m on the x-axis and torque T on the y-axis, the operating point is found at the intersection of the motor's characteristic curve and the load's characteristic curve. The motor will operate at the speed corresponding to this intersection, with the corresponding torque as well.

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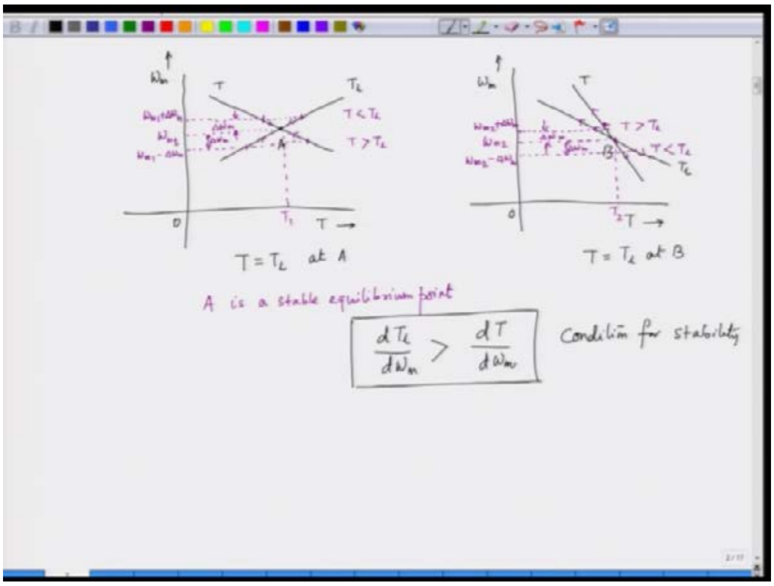

Now, the important question arises: Is this operating point stable or unstable? This is critical to determine because a stable operating point means that if there are slight disturbances, the system will return to equilibrium, while an unstable point would cause the system to deviate further from equilibrium.

Next, let's explore how we can analyze and decide whether this intersection point is stable or unstable.

Let's consider two examples to illustrate this concept. We have a graph with speed, ω_m, on the y-axis and torque, T, on the x-axis. In both cases, the motor's characteristic curve is represented by T, and the load characteristic is represented by T_l. In the first example, the intersection of the motor and load curves occurs at point A, and in the second example, the intersection is at point B. These points, A and B, represent potential operating points of the system.

Now, our task is to determine which of these equilibrium points, A or B, is stable, and which one is unstable. At both points, we have the condition $T = T_l$, meaning the motor torque matches the load torque, indicating an equilibrium. However, we need to analyze how the system behaves if we introduce a small disturbance around these points, specifically a change in speed.

Let's say we slightly increase the speed from its equilibrium value at one of these points, for example, from point A. This disturbance in speed results in a change in torque. If we increase the speed by a small amount, $\Delta\omega_m$, we move from the initial speed ω_{m1} to a new speed $\omega_{m1} + \Delta\omega_m$.

Now, examine the situation after this speed change. The motor torque T decreases, but the load torque T_l remains higher than the motor torque, creating a situation where $T < T_l$. This imbalance means that the load is exerting more torque than the motor can provide.

So, what happens in this situation is that the system exhibits a tendency for the speed to decrease when disturbed because the load torque exceeds the motor torque. Since the motor torque is insufficient to match the load, the load naturally slows the motor down, causing the speed to drop. As the speed decreases, the system will eventually return to the original equilibrium point.

Now, consider disturbing the speed in the opposite direction by the same amount, $\Delta\omega_m$, but this time decreasing it. This brings us to a new equilibrium point at $\omega_{m1} - \Delta\omega_m$. At this lower speed, the motor torque is higher than the load torque, meaning the motor now exerts more force than the load requires. As a result, the motor will accelerate the load, causing the speed to increase. This behavior will continue until the speed stabilizes back at the original equilibrium point.

Therefore, in this case, point A is a stable equilibrium point. Any disturbance, whether increasing or decreasing the speed, will naturally bring the system back to this point, making it stable.

Now, let's analyze point B. Here, we again observe the intersection of the motor and load characteristics at an equilibrium point. Initially, the system operates at ω_{m2} and the corresponding torque T_2, with the motor torque matching the load torque. When we introduce a disturbance by increasing the speed slightly by $\Delta\omega_m$, the new speed becomes $\omega_{m2} + \Delta\omega_m$. At this increased speed, we find that the motor torque is greater than the load torque.

However, at this point, the motor will try to further accelerate the load because the motor torque exceeds the load's demand. This means the speed will continue to rise instead of stabilizing, causing the system to move away from the equilibrium point. This behavior indicates that point B is an unstable equilibrium point because any disturbance causes the system to deviate from its original state.

Thus, while point A is a stable equilibrium point, point B is unstable due to the system's tendency to move away from it when disturbed.

So, in this scenario, when the motor torque T is greater than the load torque T_l, the motor exerts more power than the load requires. This imbalance causes the speed to keep increasing, leading to what we call divergence. The system, therefore, moves away from the equilibrium, and the speed will continuously rise, driving the system further from point B. This behavior shows that point B is an unstable equilibrium point.

Now, consider what happens if we decrease the speed slightly by an amount $\Delta\omega_m$. The new speed becomes $\omega_{m2} - \Delta\omega_m$, and in this case, the motor torque is less than the load torque. Since the motor provides less torque than what the load demands, the speed will continue to drop. As a result, the system will move away from point B, confirming that the operating point will diverge further, reinforcing the conclusion that point B is unstable.

To summarize, we have two distinct scenarios. In the first case, the system reaches a stable equilibrium point where any disturbance, whether an increase or decrease in speed, will bring the system back to its original state. In the second case, the system reaches an unstable equilibrium point, where any deviation causes the system to drift away from the equilibrium.

This observation means that not all motor and load combinations are stable. Some combinations will lead to a stable system, while others will result in instability. We've just explained and understood this behavior graphically, but can we also determine stability mathematically?

Indeed, there is a mathematical rule for determining stability. The condition for stability is that if $\frac{dT_l}{d\omega_m}$ (the slope of the load torque curve) is greater than $\frac{dT}{d\omega_m}$ (the slope of the motor torque curve) at the equilibrium point, then the equilibrium point is stable. This condition serves as a criterion for stability.

If the condition is satisfied, we classify the equilibrium point as stable. If the condition is not met, we identify the equilibrium point as unstable. Now, let us proceed to prove this mathematically.

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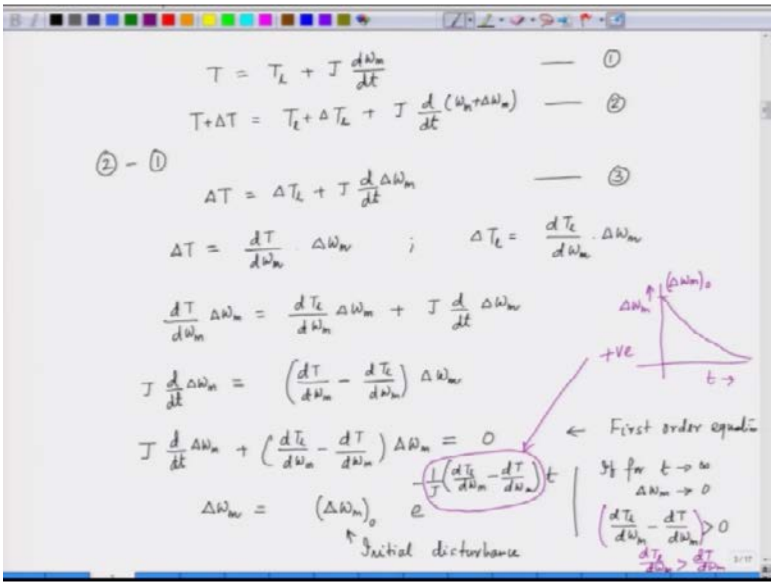

Let's examine the motor torque equation, which is represented by the load torque plus the inertial torque $J\frac{d\omega_m}{dt}$, i.e.,

$$T = T_l + J\frac{d\omega_m}{dt}$$

Now, let's introduce a small perturbation, a slight disturbance to the system. This results in a modified, perturbed equation, which reflects the change in system behavior. If we call the original equation as equation (1) and the perturbed equation as equation (2), we can subtract equation (1) from equation (2) to derive the small signal model.

When we perform this subtraction, we obtain the small signal equation:

$$\Delta T = \Delta T_l + J\frac{d}{dt}(\Delta\omega_m)$$

This equation captures the small changes, or perturbations, in motor torque ΔT, load torque ΔT_l, and angular velocity $\Delta\omega_m$. This small signal equation plays a critical role in determining the stability of the system.

Next, let’s analyze the changes in torque due to perturbations. We know that ΔT, the change in motor torque, can be expressed as:

$$\Delta T = \frac{dT}{d\omega_m}\Delta\omega_m$$

This represents the slope of the motor characteristic at the equilibrium point multiplied by the small perturbation $\Delta\omega_m$. Similarly, for the load torque, we have:

$$\Delta T_l = \frac{dT_l}{d\omega_m}\Delta\omega_m$$

This represents the slope of the load characteristic at the equilibrium point. These equations describe how small disturbances in speed affect the motor and load torques.

Now, let's substitute these expressions for ΔT and ΔT_l into our small signal equation. This gives us:

$$\frac{dT}{d\omega_m}\Delta\omega_m = \frac{dT_l}{d\omega_m}\Delta\omega_m + J\frac{d}{dt}(\Delta\omega_m)$$

We can further simplify this equation by isolating the terms involving $\Delta\omega_m$:

$$J\frac{d}{dt}(\Delta\omega_m) = \left(\frac{dT}{d\omega_m} - \frac{dT_l}{d\omega_m}\right)\Delta\omega_m$$

Finally, we move everything to one side:

$$J\frac{d}{dt}(\Delta\omega_m) + \left(\frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m}\right)\Delta\omega_m = 0$$

This is the simplified equation that we use to assess the stability of the system.

Now, this is a crucial equation, as it is a first-order differential equation with respect to $\frac{d}{dt}(\Delta\omega_m)$. To solve this equation, we must consider its structure. The solution will take the following form:

$$\Delta\omega_m = \Delta\omega_{m0}\left(1 - \exp\left(-\frac{1}{J}\left(\frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m}\right)t\right)\right)$$

Here, $\Delta\omega_{m0}$ represents the initial disturbance in speed, and this expression gives the evolution of the speed disturbance over time.

Now, let's analyze the meaning of this result. If we disturb the system by a small amount $\Delta\omega_{m0}$, eventually, as time progresses, the disturbance should vanish, meaning the system will return to its original equilibrium point. For that to happen, as $t \to \infty$, $\Delta\omega_m$ must tend to zero.

In this case, for $\Delta\omega_m \to 0$ as $t \to \infty$, the coefficient in the exponential must be positive. This means the term $\frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m}$ should be positive. In other words:

$$\frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m} > 0$$

This condition ensures that the exponential term decays to zero over time. Thus, if the motor speed is disturbed and left to stabilize, it will naturally return to the equilibrium point as $t \to \infty$, provided this condition holds.

Additionally, since J, the moment of inertia, is always positive, the only requirement for stability is that:

$$\frac{dT_l}{d\omega_m} > \frac{dT}{d\omega_m}$$

Therefore, we have mathematically proven that for the system to be stable, the slope of the load torque curve $\frac{dT_l}{d\omega_m}$ must be greater than the slope of the motor torque curve $\frac{dT}{d\omega_m}$.

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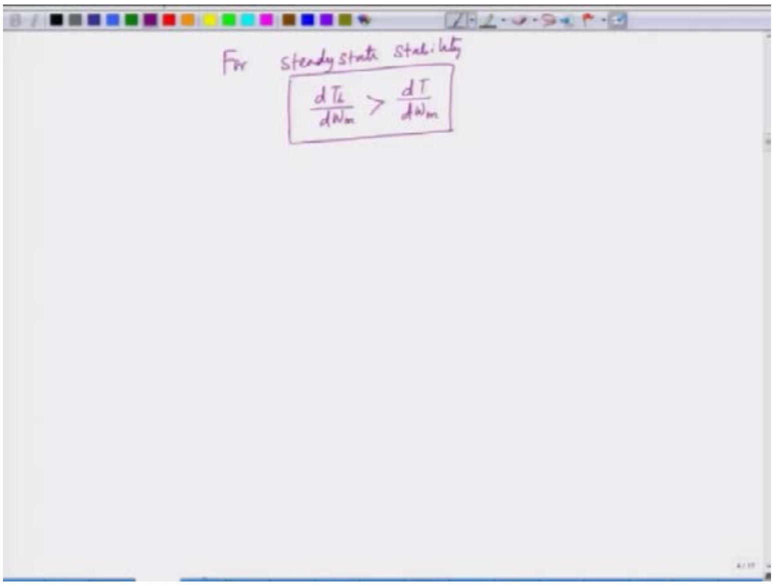

For steady-state stability, we can express the condition as $\frac{dT_l}{d\omega_m} > \frac{dT}{d\omega_m}$. This means, at the equilibrium point, we must check whether this inequality holds. If it does, we can confidently say that the operating point is stable. Conversely, if the condition is not satisfied, we know the operating point is unstable.

In an unstable scenario, any small disturbance will cause the speed to diverge, meaning it won't return to that equilibrium point. Therefore, it's essential to avoid unstable equilibrium points. Our goal should always be to ensure that the motor and load torque combination remains stable. Stability ensures that the system responds predictably and returns to equilibrium after any disturbances.

Now, let’s move forward and discuss the concept of load equalization. What exactly do we mean by load equalization? As we’ve seen, loads can vary in nature, whether it's a fan load, a traction load, or a constant power load. But there are also special types of loads that are pulsating in nature. These loads don't maintain a constant torque; instead, the load torque fluctuates over time, increasing and decreasing periodically.

Take, for instance, a sugarcane juice-making machine. When the sugarcane is being pressed through the machine, there’s a significant load torque applied. But when the sugarcane exits, the load torque drops to zero, and this cycle repeats continuously. A similar situation occurs in a steel rolling mill. As the ingot moves in and out of the rollers, the load torque pulses accordingly, resulting in a fluctuating or pulsating load.

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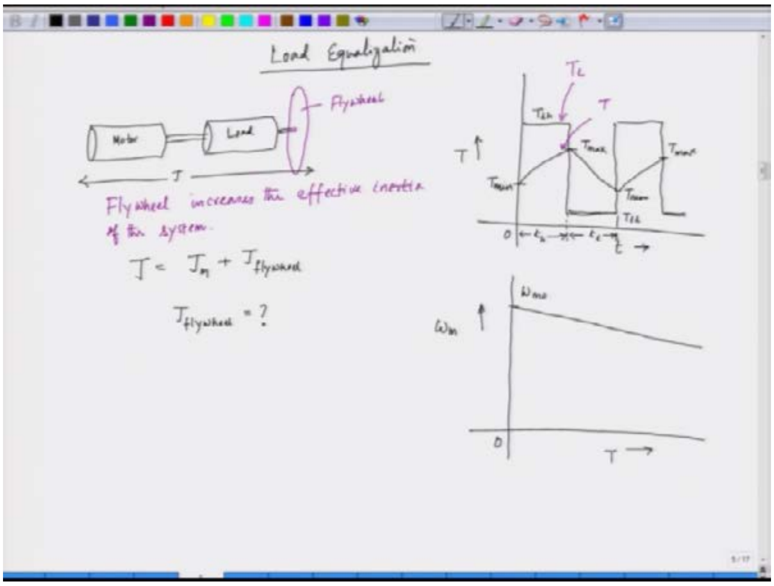

To visualize this, imagine a load torque that oscillates over time. We can plot this on a graph, where the load torque varies between a high value and a low value over time. On the y-axis, we have the load torque, and on the x-axis, we have time. The torque reaches a high value, T_h, for a period t_h, and then drops to a low value, T_l, for a period t_l. This is a typical example of a pulsating load torque.

Now, the important question arises: if we have such a pulsating load, how should we size the motor? Should the motor be selected based on the high value of the load torque or the low value? Ideally, the motor torque should be balanced somewhere between the high and low values, allowing it to supply the load torque in a smooth and manageable way. Therefore, the motor's torque response should fluctuate in alignment with the varying load, ensuring it can handle both the high and low points effectively.

The motor torque should neither be excessively high nor too low. We denote the motor torque as T, and the load torque as T_l, with the motor torque fluctuating between a minimum value, $T_{\min}$, and a maximum value, $T_{\max}$. In steady-state operation, the motor torque oscillates between these two limits. But how do we achieve this controlled variation in torque?

We achieve this by connecting a flywheel to the motor-load system. Imagine a motor connected to a load, and to stabilize the system, we attach a flywheel. Now, what exactly does the flywheel do? Essentially, it acts as a load equalizer. Even though the load torque fluctuates significantly, the motor torque variation is much less extreme because the flywheel absorbs and supplies the differential torque.

In technical terms, the flywheel increases the system's effective inertia. With higher inertia, the motor-load combination can handle larger variations in load torque. The total system inertia, denoted as J, is now the combined inertia of both the motor and the flywheel. So, when we add a flywheel, we need to carefully calculate its required inertia. The key question becomes: what should the flywheel's inertia be to achieve the desired values of $T_{\min}$ and $T_{\max}$?

To answer this, we can refer to the motor's characteristic curve, which shows the relationship between speed and torque. On a graph where speed ω_m is plotted on the y-axis and torque on the x-axis, the motor's behavior typically follows a drooping profile. Understanding this behavior is crucial to determining the appropriate flywheel inertia.

With that, we wrap up this discussion, and we will continue from this in the next lecture.