Basic Electric Circuits
Module II
Mesh and Node Analysis
Lecture-09
Mesh Analysis
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Namaskar. So, today we will continue our yesterday's discussion on Mesh Analysis. So, let us first understand what we discussed yesterday.

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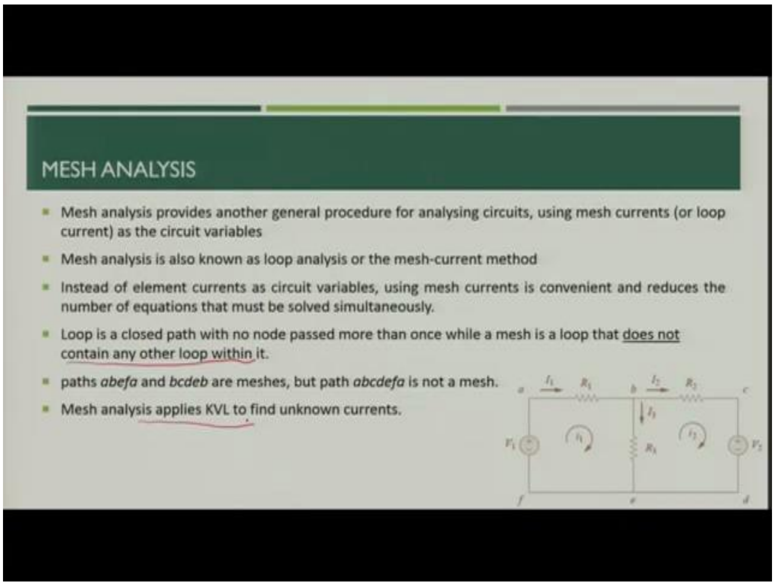

So, yesterday we saw the what is mesh analysis and we stabilized the difference between the mesh current and the branch current and we also saw that every loop cannot be considered as mesh because mesh is that loop which does not contain any other loop within it and we also saw that the mesh analysis can be done with the help of Kirchhoff Voltage Law.

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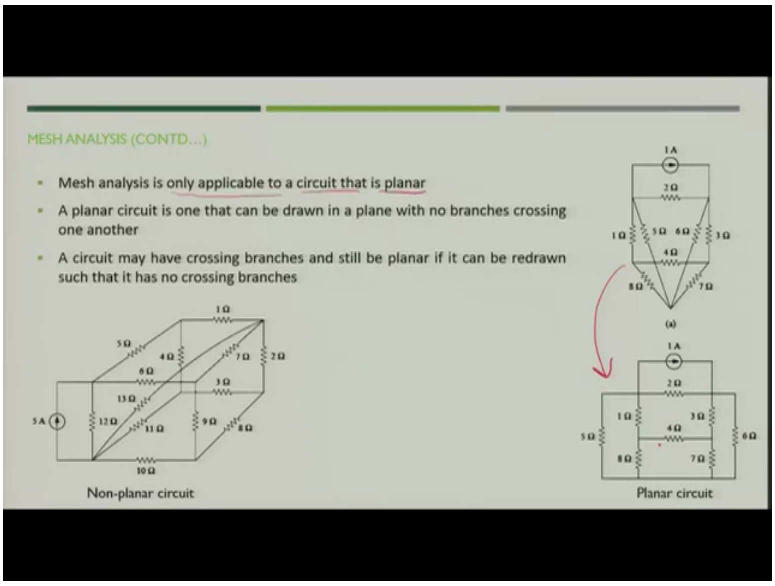

Yesterday we also discussed that the mesh analysis is only applicable to circuit that is planar, planar means the circuit which can be drawn on a plane and it does not have any branch which are crossing one another. So, like in this case if you see it is not a planar circuit because it is not drawn on a plane it is something like three-dimensional structure. Similarly, here also we saw that it is crossing the branches but we can reorganize and make it as a planar circuit.

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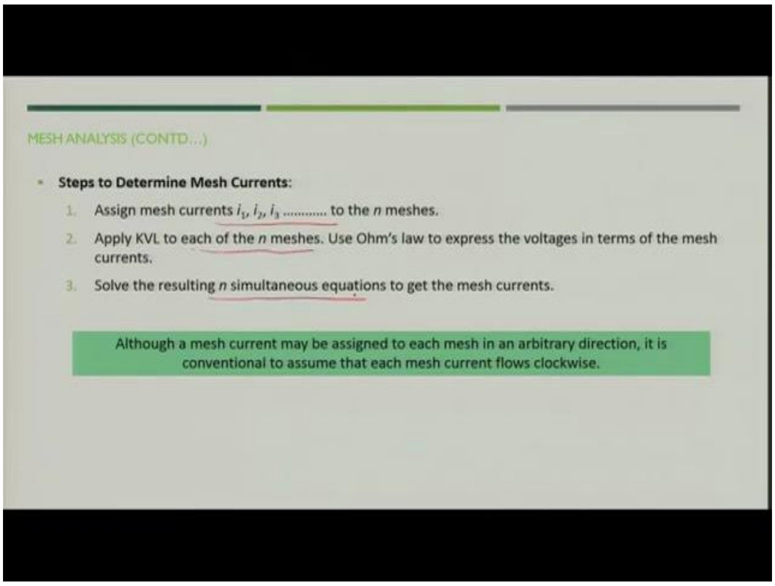

And we saw that the three steps are required for the mesh analysis we have you will first define the mesh currents then you apply KVL at each of the n mesh and then solve the n simultaneous equations.

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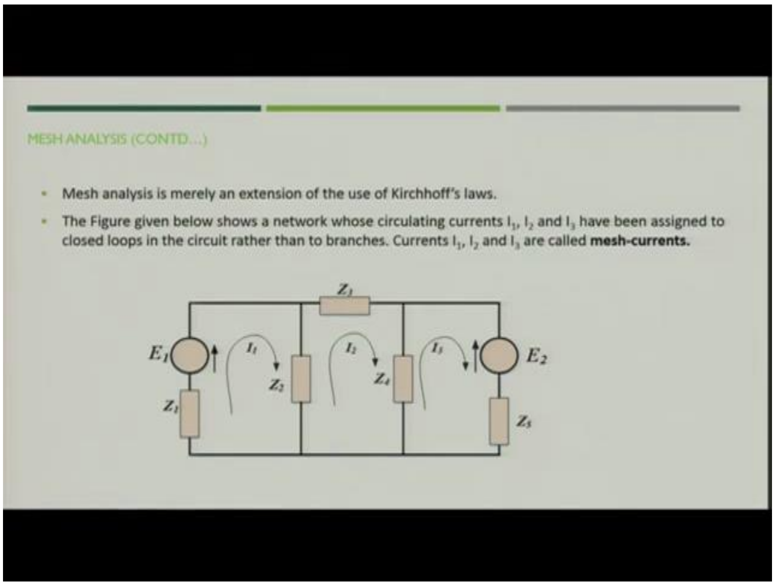

We took this circuit and we established that.

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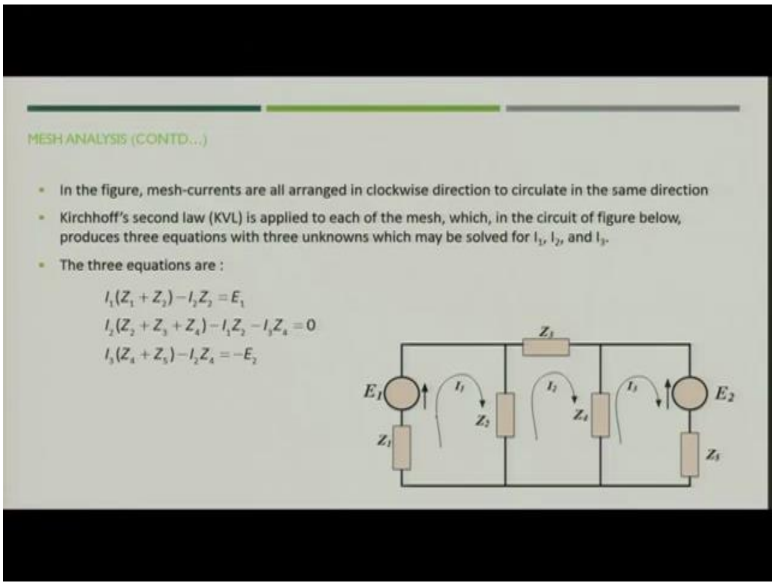

These are the equations which are the outcome of our mesh analysis and then we can finally solve it and get the answer.

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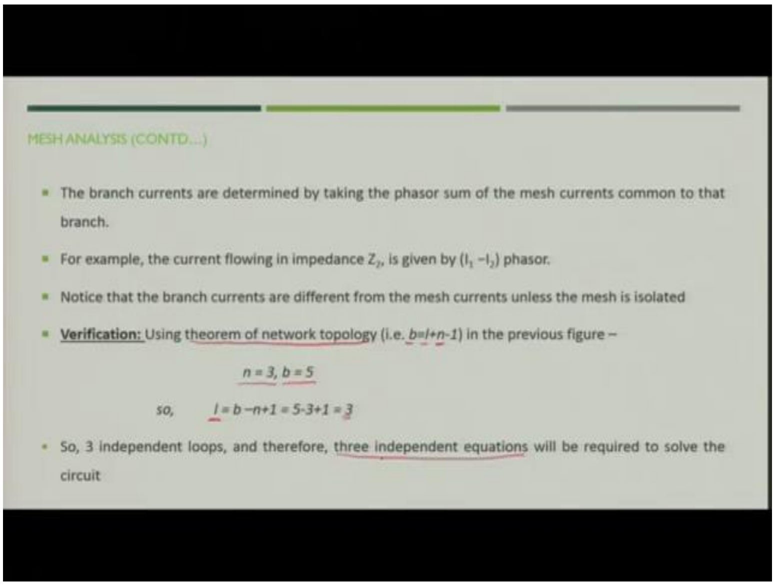

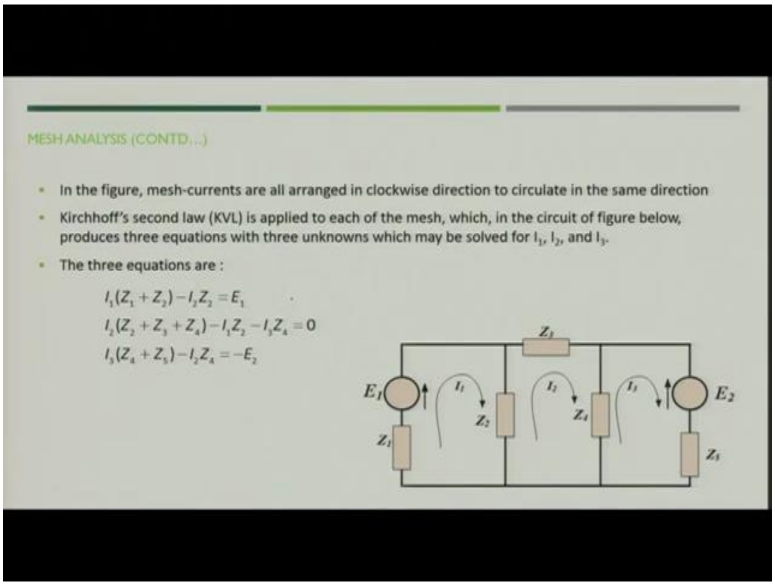

We also discussed that suppose if we do not know whether we have got equal means the required number of equations or not that can be stabilized with the help of theorem of network topology which says that number of branches equal to number of loops plus number of nodes minus 1. So, in the previous of figure the nodes for number of 3 and branches of 5, so we got number of loops equal to 3.

So, that means that we need minimum three independent equations to solve the circuit.

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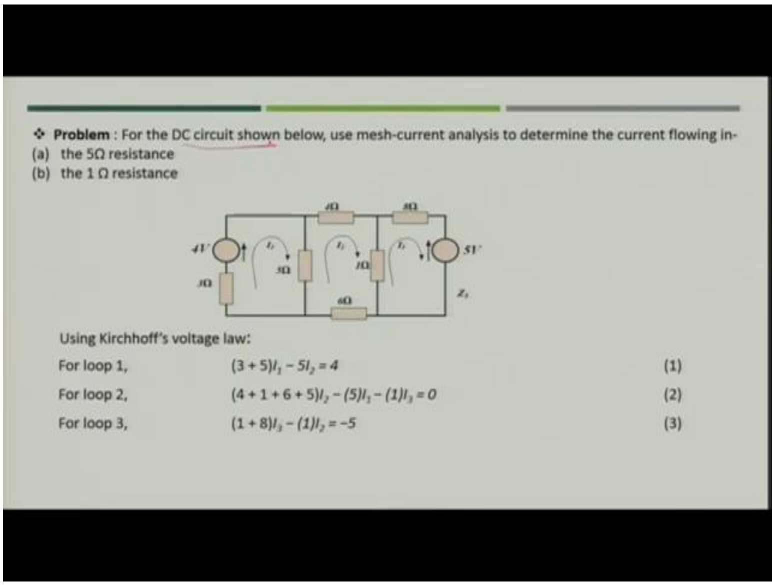

And then we discussed the example which was essentially a DC circuit.

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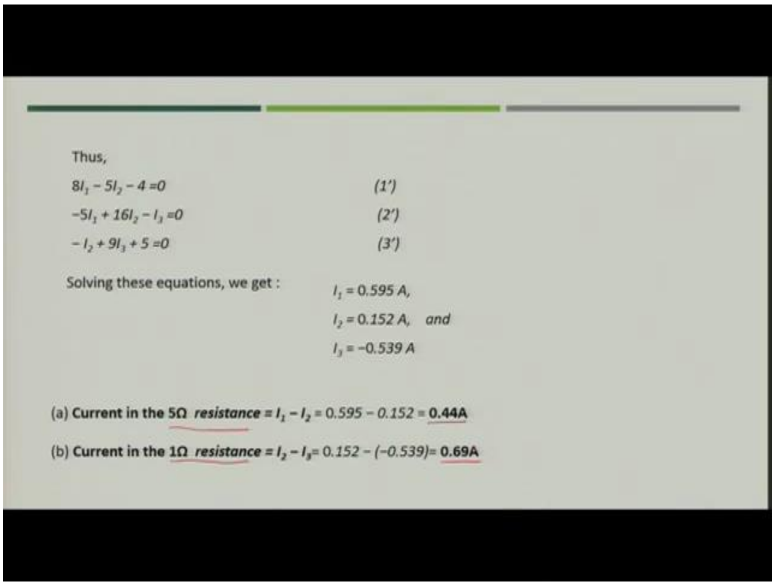

And we stabilized that the current in 5 ohm resistance is 0.44 ampere and 1 ohm resistance is 0.69 ampere.

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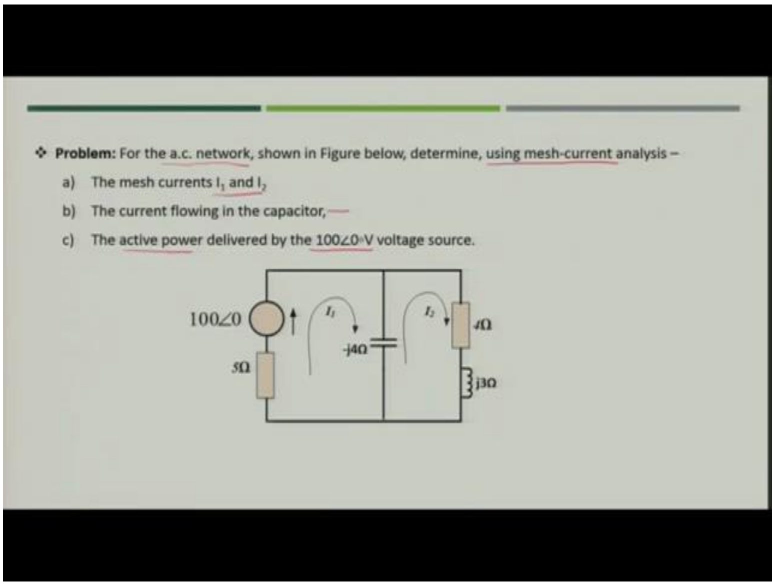

Now, let us talk about another example. Now, this example is not the dc network it is related to ac network as shown in the figure below and now the objective is to determine the mesh current I 1 and I 2 and the current flowing in the capacitor and active power delivered by 100 voltage with reference phasor as 0. So, we will use this circuit and try to apply the mesh analysis technique and find out the result.

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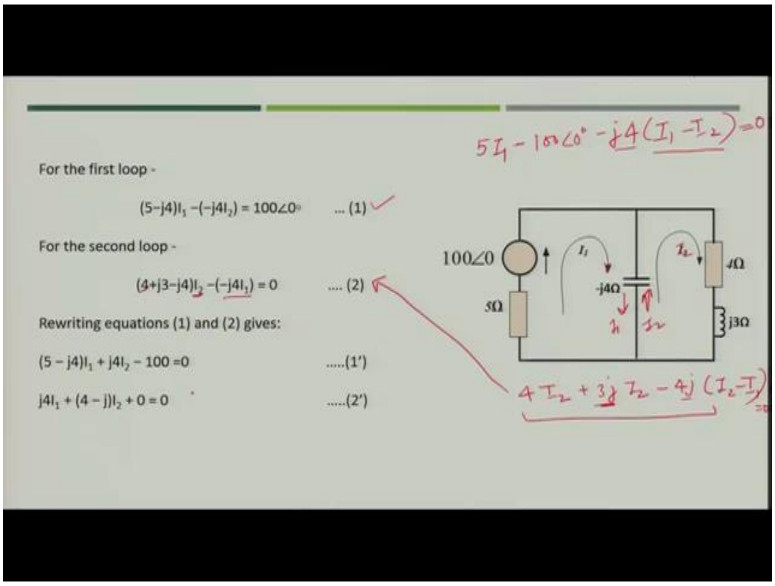

Now, in this particular case you can create two loops because these are the two independent mesh which you can create. So, let us say that in this particular mesh the current is I 1, in this mesh is current is I 2. So, we can write two equations for both of the meshes one for each mesh.

So, for first mesh if you see the current which is flowing in this particular mesh you can apply Kirchhoff Voltage Law and find out the governing equation of this particular mesh.

So, start with the 5 ohm resistance. So, if you say current is flowing in 5 ohm so you can simply write $5 * I_1$ then you are going from minus to lower voltage level to upper voltage level in the case of voltage source you will simply write $100\angle 0^\circ$ for this and then this current will flow through capacitor so you will write $-j4(I_1-I_2)$, why $-I_2$? Because I_1 is in this direction but I_2 is flowing in opposite direction.

So, for this particular capacitor you will have net current as I_1-I_2. So, now you got this equation, if you rearrange you will get $(5-j4)I_1-(-j4I_2) = 100\angle 0^\circ$, so you will get first equation for the mesh 1. Now, if you see this the second mesh the equation you can write if you start with 4 ohm you can say $(4+j3)I_2$ and then you come here $-j4(I_2-I_1)$, and that would be equal to 0.

If you rearrange what you will get? You will $(4+j3-j4)I_2-(-j4I_1) = 0$. So, if you rearrange this equation you will simply get this equation. Now, what you have to do? You have to just simply rearrange in such a way that you can solve it.

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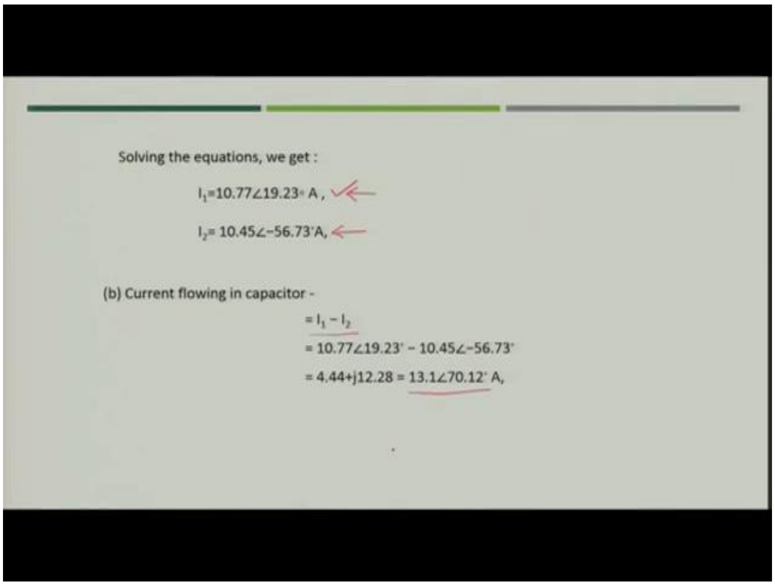

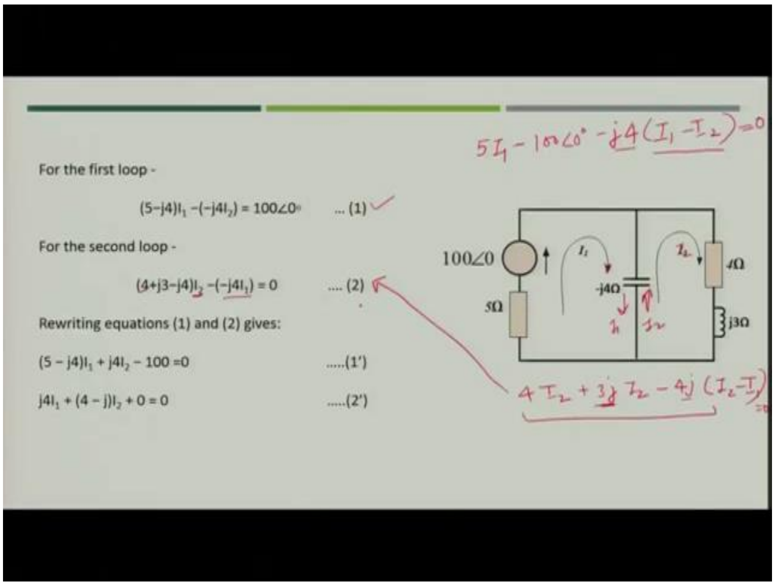

If you rearrange and solve this equation the current $I_1 = 10.77\angle 19.23^\circ$ A = $10.8\angle -19.2^\circ$A and $I_2 = 10.5\angle -56.7^\circ$. Now, here you can see that the result would come in the form of rectangular coordinate where you will have real component as well as imaginary component. So, using your previous knowledge of phasor calculation you can find out the value of these currents in terms of phasor also.

Now, next is what is the current flowing in the capacitor? The current flowing in the capacitor would be

$$I_1 - I_2 = 10.77\angle 19.23^\circ - 10.45\angle -56.73^\circ = 4.44 + j12.28 = 13.1\angle 70.12^\circ \text{ A}$$

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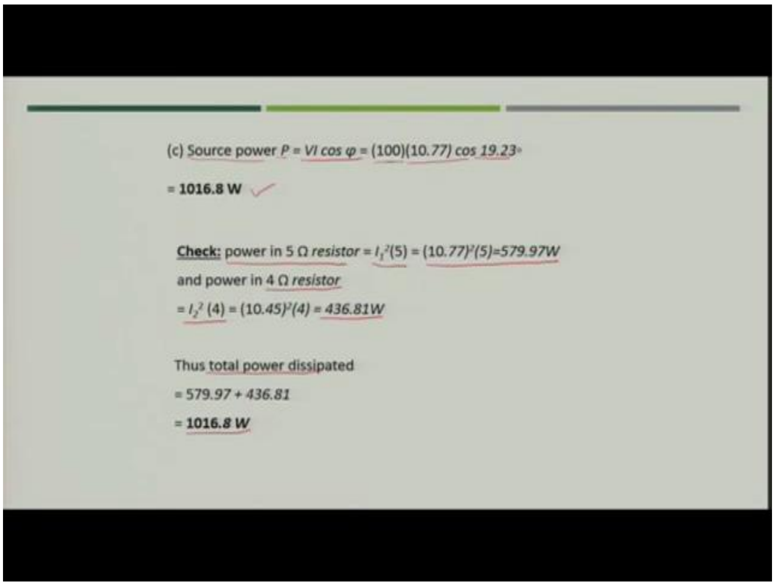

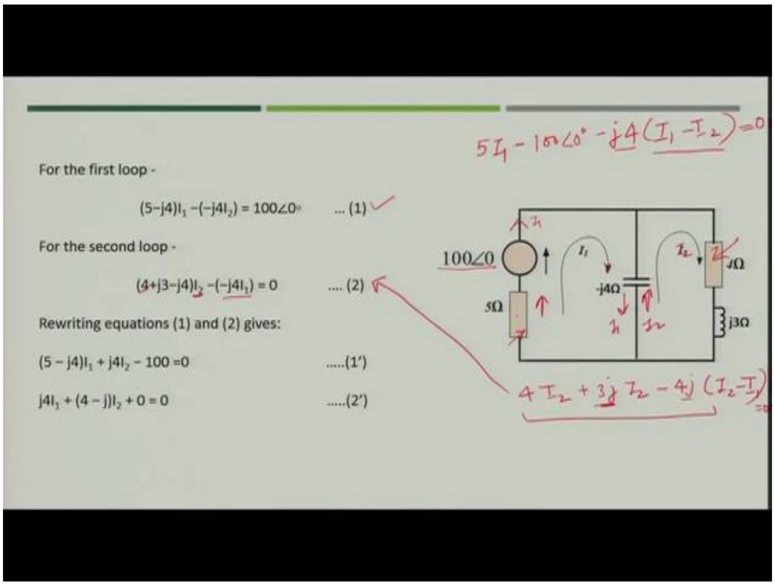

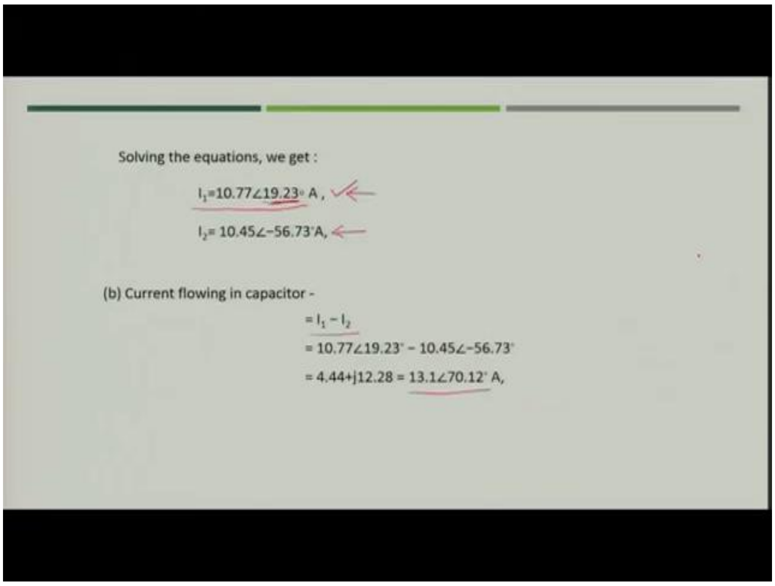

Now, next the source power that is basically the active power you need to calculate. So, active power is $P = VI \cos \varphi$, here voltage V is a difference phasor because it is generally 0 so we are considering it as a reference phasor and then you have to simply put the value of current, current would be what? Current is flowing from the source, so what is the current? Current would be essentially the value of I_1 because this is the current which is flowing from this particular site.

So, you put the value of by V and I $\cos \varphi$ is nothing but the angle for current I_1 which you will get source power delivered as

$$P = VI \cos \varphi = (100)(10.77) \cos 19.23^{\circ} = \mathbf{1016.9W = 1020W}$$

Now, you can do verification also like if you are interested to find out whether our power calculation is correct or not you have to simply check the circuit. Here the active power can only be absorbed by the resistor 5 ohm and the resistor 4 ohm.

So, if you want to know that what the power active power being delivered by the source you have to simply find out what the power being absorbed by these two resistances. So, now power in 5-ohm resistor is nothing but $I_1^2(5)$ because if you see the current flowing through 5-ohm resistance is simply I 1 and here for 4 ohm the power the current flowing is I_2.

So, we have to just simply put the value of I_1 in case of 5 ohm, so $I_1^2(5)$ you will get the value of 579.97 watt. Similarly, for 4-ohm resistor, the power absorbed the resistor would I_2^2 (4) and that would be 436.81 watt. So, therefore the total power you have to just add both of them and you will get the same value as we calculated previously. So, in this way you can cross verify that whatever you have calculated in previous case is correct.

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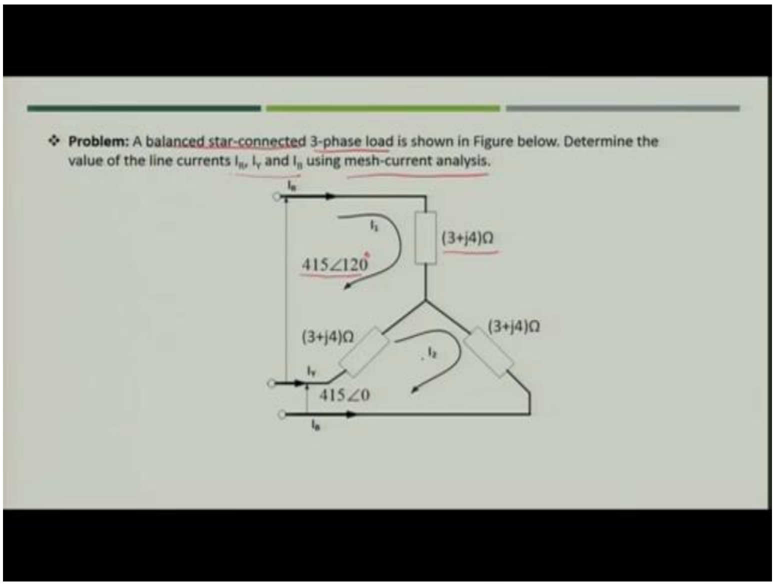

Now, another example which you can take and this is very common example you will see in book, this is balanced star connected 3-phase load and we need to determine the value of currents flowing through R, Y and B phase. So, here also we can apply the mesh current analysis, the voltage a line to line voltage that is voltage across R and Y is given as 415∠120 and between Y and B is 415∠0 and this is balanced 3-phase load, so the impedance is across the loads is 3 + j4.

Now, we have to apply mesh current analysis, so we will have essentially two meshes which we can create say let us say that in this particular mesh current is flowing as I 1, in this mesh current is flowing as I 2.

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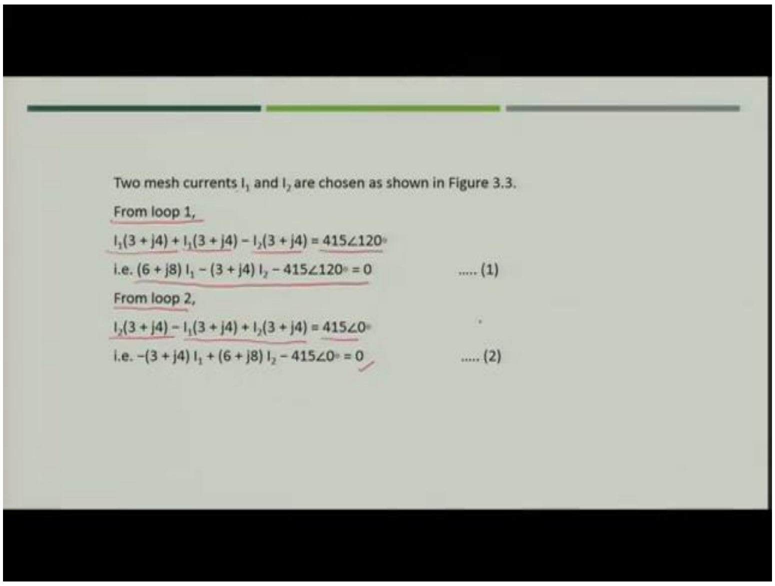

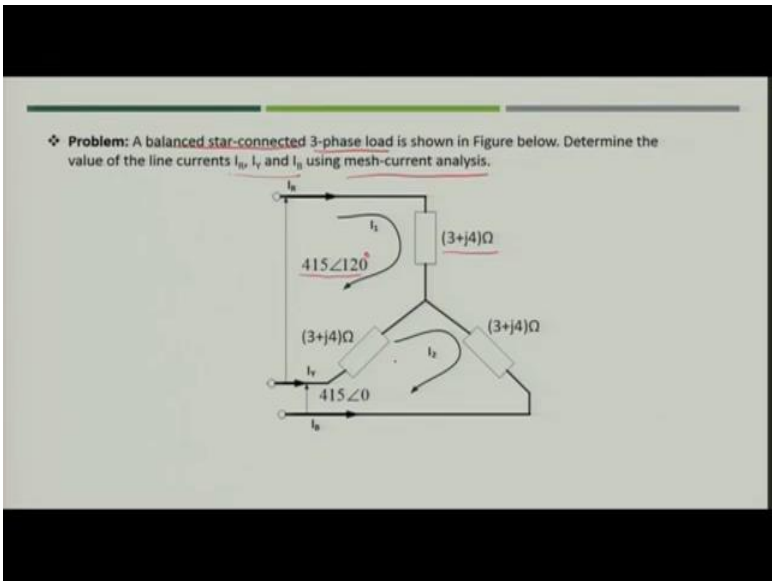

So, now the two mesh currents you can use and apply KVLs for both of the loop. For loop 1 if you applied what will be the value? From loop 1,

$I_1(3 + j4) + I_1(3 + j4) - I_2(3 + j4) = 415\angle 120^\circ$

i.e. $(6 + j8) I_1 - (3 + j4) I_2 - 415\angle 120^\circ = 0$

Similarly for second loop also,

From loop 2,

$I_2(3 + j4) - I_1(3 + j4) + I_2(3 + j4) = 415\angle 0^\circ$

i.e. $-(3 + j4)\, I_1 + (6 + j8)\, I_2 - 415\angle 0^\circ = 0$

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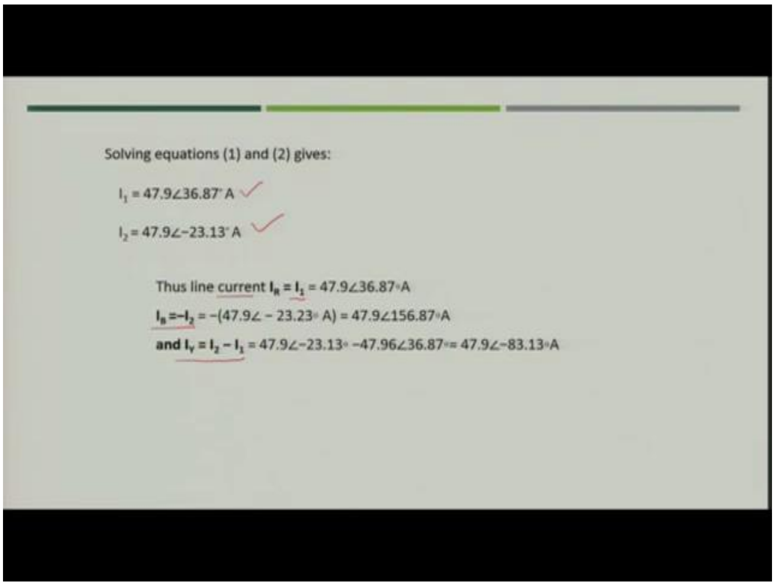

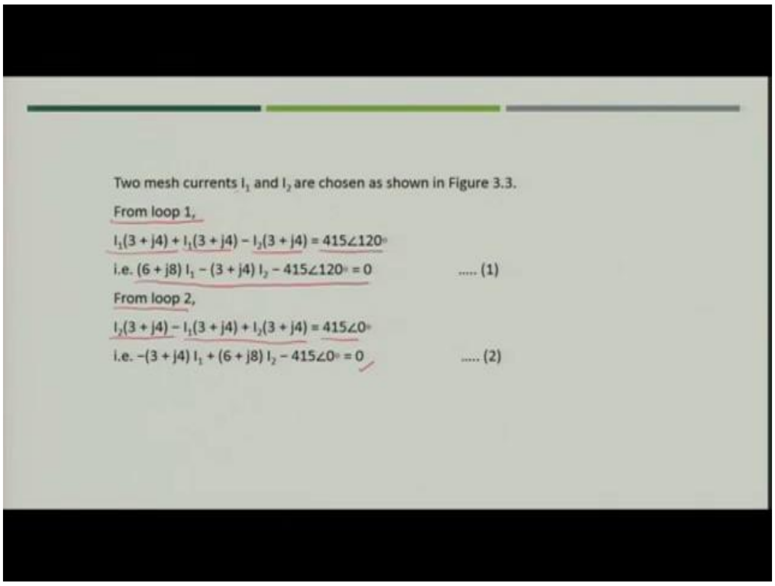

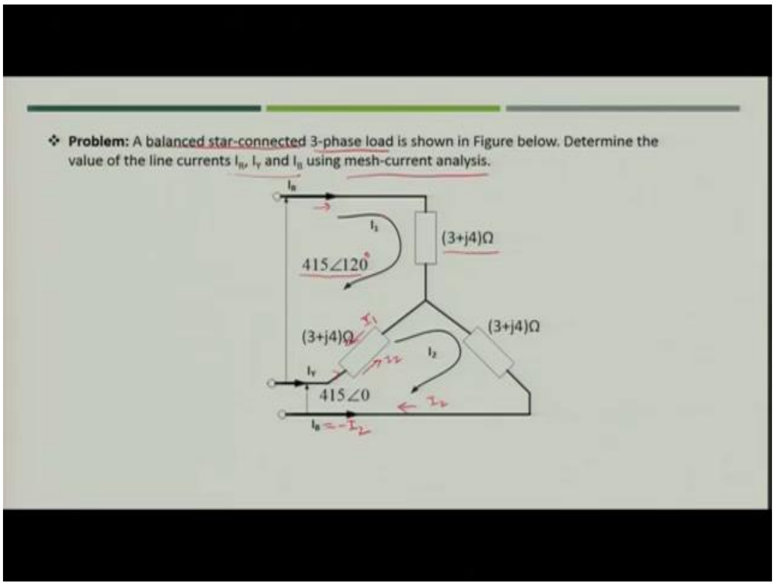

Now, you have both of the equations available you have to just solve these two equations and you will get the values for current I 1 and I 2. Now, line current which you will see in this figure, $\mathbf{I_R = I_1}$, $\mathbf{I_Y = I_2 - I_1}$ and $\mathbf{I_B = -I_2}$.

So, these would be very important example for you because most of the time you will see in the electrical circuit analysis you would be given these kinds of load to identify the line currents which are flowing across the line terminals.

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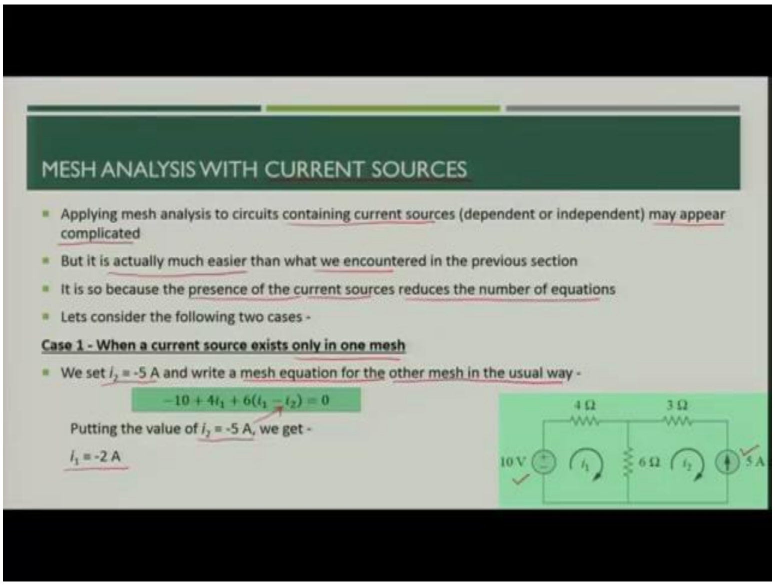

Now, let us come to another important concept of mesh analysis where you not only voltage source but also the current source. If you apply mesh analysis to the circuit only the current source it looks that it is very complicated. If you see this figure where current source is there, voltage source is also there so at first instant you see that this is a little bit complicated circuit and difficult to solve but actually it is much easier than what we discussed till now, how? Because the presence of the current source reduces the number of equations.

So, because of this the number of equations would be reduced and it is much easier to solve this kind of circuit. Now, to analyze these kinds of circuits let us take two examples rather let us take two cases first so that you can understand two different types of circuits containing the current sources. First one is that when current source exists only in one mesh, so in this particular case you will see that the current source exists only in this particular mesh.

So, this is 5 amperes for this particular mesh, this mesh contains voltage source. So, if you see these kinds of circuits what you will do? You will set current $i_2 = -5$ ampere because the direction of i_2 we have taken as clockwise as we have taken for i_1 but the current source is opposite to what we have assume the current for i_2, so that is why what we will say? We will say i_2 is nothing but minus 5 amperes.

So, this you can easily verify from the figure because i_2 will flow in this direction and the current will from the current source will flow in this direction, so i_2 would be nothing but minus of 5 ampere. Now, we got the value of i_2 straightaway through inspection using this figure, what we have to do next? We have to next find out the current in other mesh, so you have to just write the mesh equation for this particular mesh.

So, what you will write? You will write simply minus $-10+4i_1+6(i_1-i_2) = 0$ for this particular mesh. Now, since we already know that $i_2 = -5$ ampere you can simply put the value of i_2 in this equation and you will get current $i_1 = -2A$.

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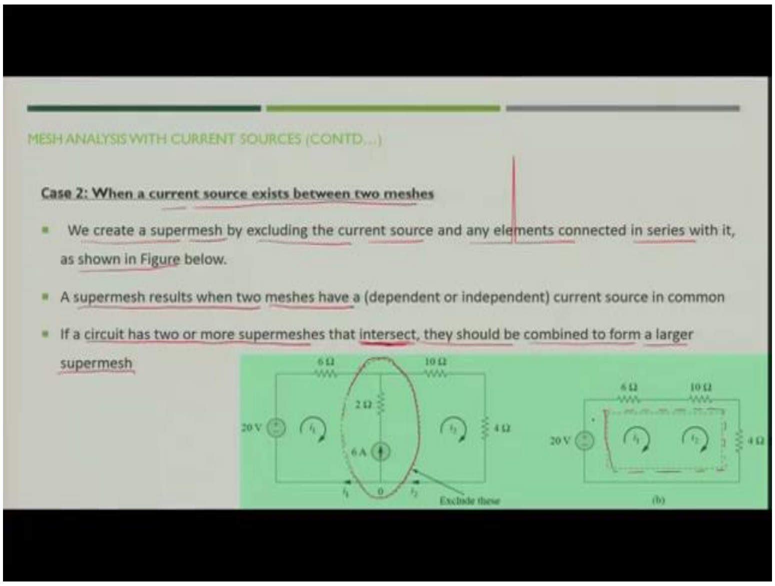

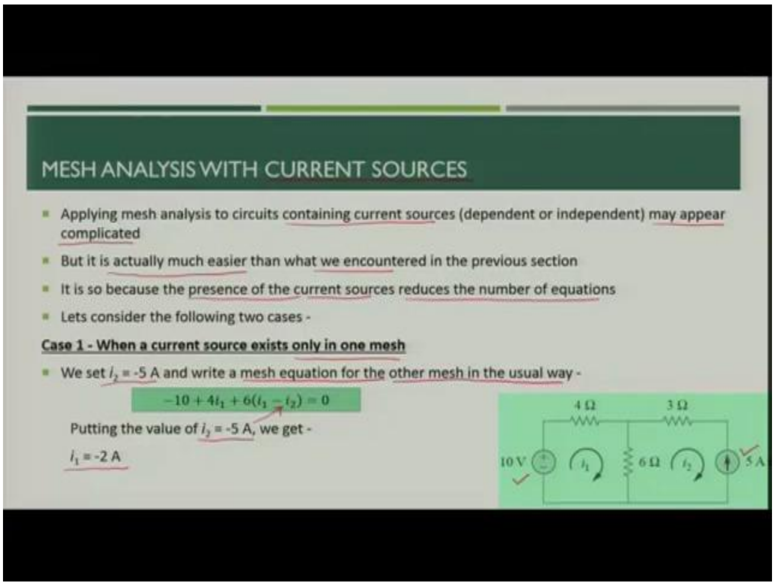

Now, that particular case was relatively simple because mesh the current source was in only one mesh. But suppose if the current source exists between two meshes where the both of the meshes are sharing the current source then what you have to do? You have to analyze the circuit by creating a super mesh, super mesh means? You have to exclude the current source and any element connected in series with that particular source.

So, in this case if you see this is the segment which has current source and a 2-ohm resistor connected in series with the current source. So, for our analysis what we will do? We will remove this particular branch from the circuit while keeping the mesh currents same. So, like in this case we have two meshes we have assumed that both are in the clockwise direction with i_1 and i_2 values, so current will remains same but the link between these two meshes have been removed because it was containing the current source.

Now, what do we get? We get a super mesh which contains two meshes and we have removed the current source and created the super mesh. So, what will happen in this case? Super mesh would be the mesh having all the elements except the branch which we have removed. So, in this case this would be the super mesh but there might be few cases where we may get two or more current sources and then in that case we may get two or more super meshes.

So, in that case what we have to do? We have to combine the super meshes who are intersecting, so this is important thing that if two super meshes are intersecting then only we can create a larger super mesh. So, this particular aspect we will discuss with one example. Now, let us understand how the super mesh will help in solving the circuit.

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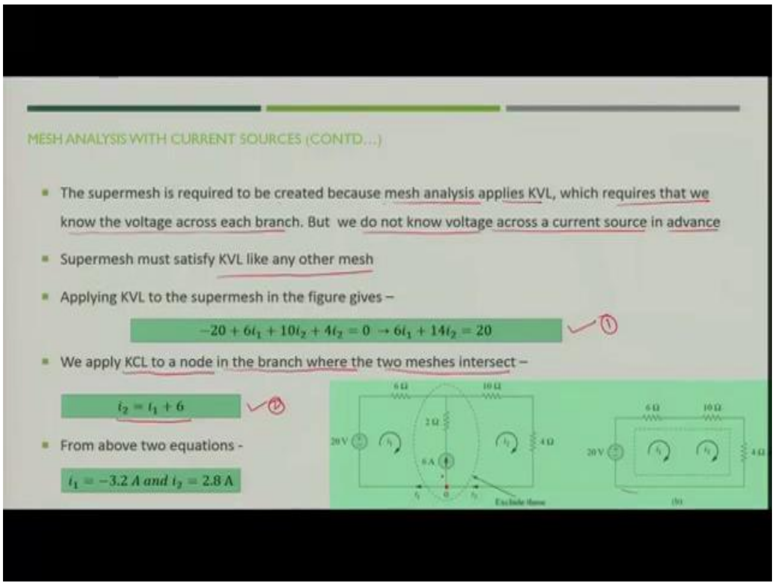

Now, super mesh is required to be created because mesh analysis applies to KVL and the KVL requires that we should know the voltage across each branch but when we have the current source we it is difficult to find the voltage across the current source in advance. So, what we will do? We will use the super mesh and apply KVL to that super mesh. So, in this case if you apply to super mesh what will happen? It will start with -20 $+ 6i_1 + 10i_2 + 4i_2 = 0$.

Now, if you solve it, if you simplify it you get $6i_1 + 14i_2 = 20$. Now, you have got one equation using super mesh but since we have two variables we need more than one equation to solve it. So, how we will get the second equation? We will get the second equation with the help of Kirchhoff Current Law which we will apply to a node where two meshes intersect that means we will apply at this particular point.

So, now if you see this particular node, what you can see? You can see i_2 is coming in and i_1 and 6 ampere current source are going out, so you can simply apply Kirchhoff Current Law and say that $i_2 = i_1 + 6$. So, now have got another equation first is this equation, second you have got from the Kirchhoff Current Law. Now, if you see these two equations it is very easy to solve you get the value of $i_1 =$ -3.2A and $i_2 =$ 2.8A.

So, with this you can easily solve these kinds of circuits, where you have current sources connected in the network.

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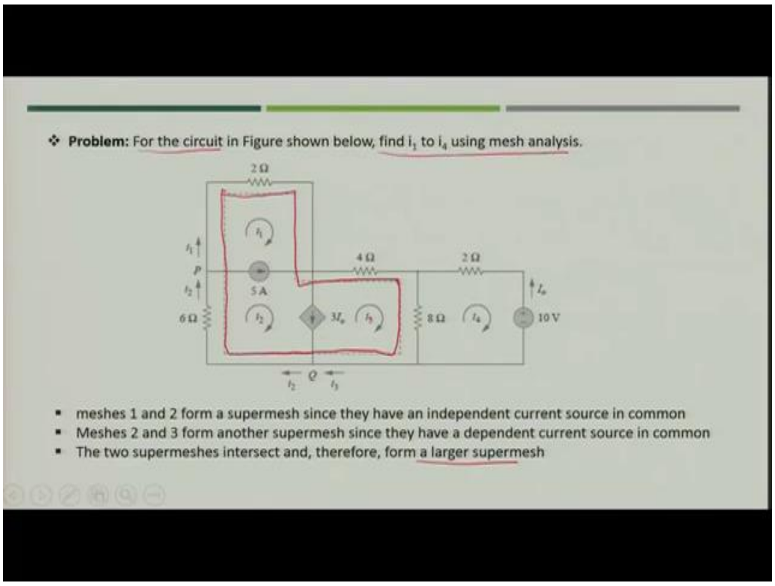

So, let us see one example so that you can understand what we discussed about the super mesh. So, let us take this circuit, this circuit contains one dependent current source and one independent current source, right. Now, we have to find the values from i_1 to i_4. If you see this circuit you will have four meshes created, so that is why we have four mesh currents like i_1, i_2, i_3 and i_4.

So, now what you will observe? You will observe that there are two current sources, so what we discussed previously if you apply that particular technique to find out the super mesh? You will first remove this particular link. So, what will happen? You will get one super mesh like this. Now, we know that there is another current source, so if you remove this current source you will get one super mesh as this one.

Now, the problem is that these two meshes are crossing each other. If you have this kind of scenario where two meshes are crossing each other we have to merge both of them and create a larger super mesh. So, what will happen in that case? You will get another super mesh which is the union of both of the super meshes and you will call it as larger super mesh.

So, now you get this larger super mesh, what you have to do? Now, larger super mesh can be used to apply Kirchhoff Voltage Law.

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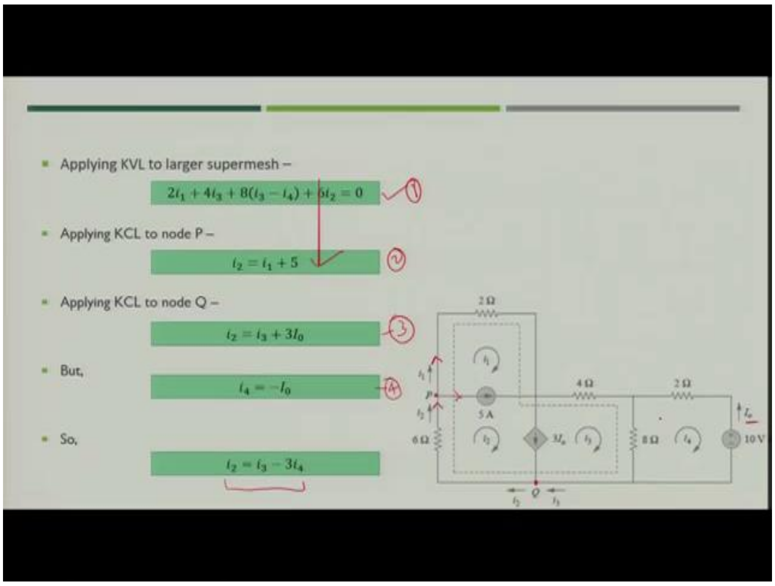

So, if you apply Kirchhoff Voltage Law for the larger super mesh what you will get? Let us start with 2 ohm resistance you will get $2i_1 + 4i_3 + 8(i_3\text{-}i_4) + 6i_2 = 0$. So, this is what you got from the Kirchhoff Voltage Law. Now, you have now two current sources, so what you have to do? You have to apply KCL at two nodes where these two current sources are connected.

So, for node P if you apply Kirchhoff Current Law and if you see the mesh current is i_2 which is going in to node P and the mesh current i_1 is coming out of node P and another current source of 5 ampere is coming out of node P, so what you can write? You can write $i_2 = i_1 + 5$. Now, you have got the second equation this was your first equation, the third which you will get is using KCL at node Q.

So, here what you can say for node Q? $i_2 = i_3 + 3\text{I}_0$ because these are coming inside the node and i_2 is going outside the node. So, $i_2 = i_3 + 3\text{I}_0$, so here you got three equations. Now, unknowns are how many? Unknowns are four i_1, i_2 i_3 and i_4, so where we will get the fourth equation? Fourth equation can be created by writing the equation for this mesh.

Now, if you see this mesh $i_4 =$ -I_0 because i_4 and I_0 are in opposite direction. So, if you see this you will simply get $i_4 =$ -I_0. So, now if you put these values you will get $i_2 = i_3 - 3i_4$, so it is simplified now but still we need the equation for this mesh, so what we will write?

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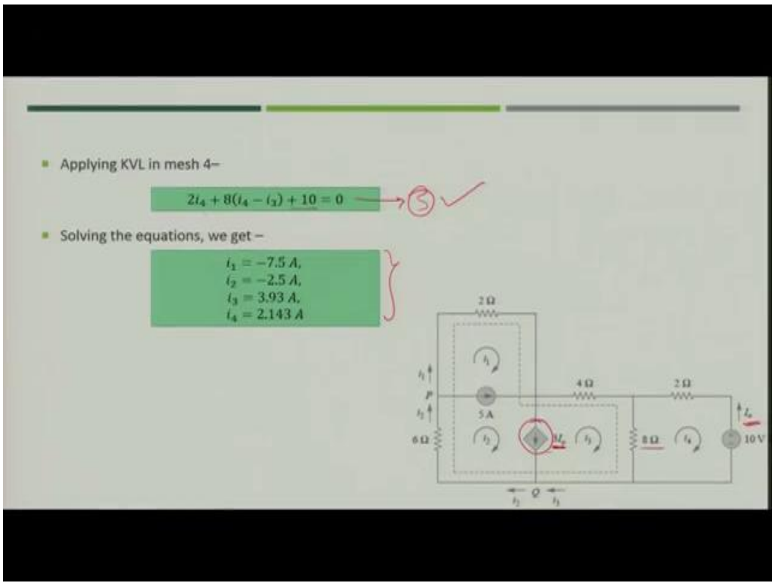

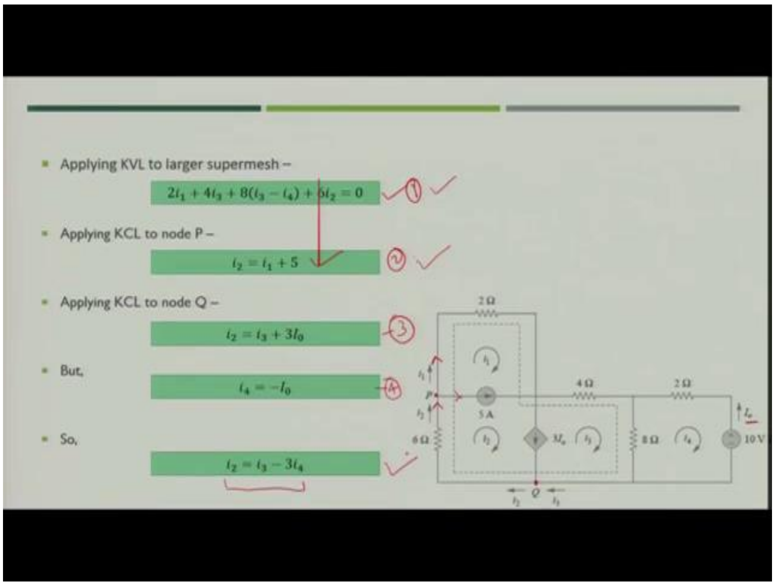

We will write KVL for this mesh, so what we can write? $2i_4 + 8(i_4 - i_3) + 10 = 0$, so, you will get another equation here. Now, what you have to do? You have to utilize this, this, this and anywhere we have utilized this and put the value inside the equation 3 and we get $i_2 = i_3$-$3i_4$.

So, you have four final equations and you have four unknowns. So, you solve these four equations you will get the values for current i_1, i_2, i_3 and i_4. So, if you follow the concept of larger super mesh and the super mesh it is very easy to solve these kinds of circuits which looks little bit complicated when you see them for first time but it is very easy to handle it. The important thing which you have to remember is that since it is dependent current source you have to find out how the value of this dependent current source would be calculated.

So, here the dependent current source value is depending upon the current which is flowing from the voltage source. So, with this value that is what we calculated $i_4 = -I_0$. it was easier to put the value in the dependent current source and do the analysis. So, with this we close our today's session. So, today we discussed about the mesh analysis and particularly the case where current sources available in the mesh analysis.

So, in the next class we will discuss about the node analysis because that is where we will apply our Kirchhoff Current Law and analyze the circuit where voltage sources are there, thank you.