Basic Electric Circuits

Module II

Mesh and Node Analysis

Lecture-08

Star Delta Transformation and Mesh Analysis

By

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Namaskar. So, in today's session, we will discuss about 1 very important technique for solving the electrical circuit that is star delta transformation and then we will proceed for our math analysis.

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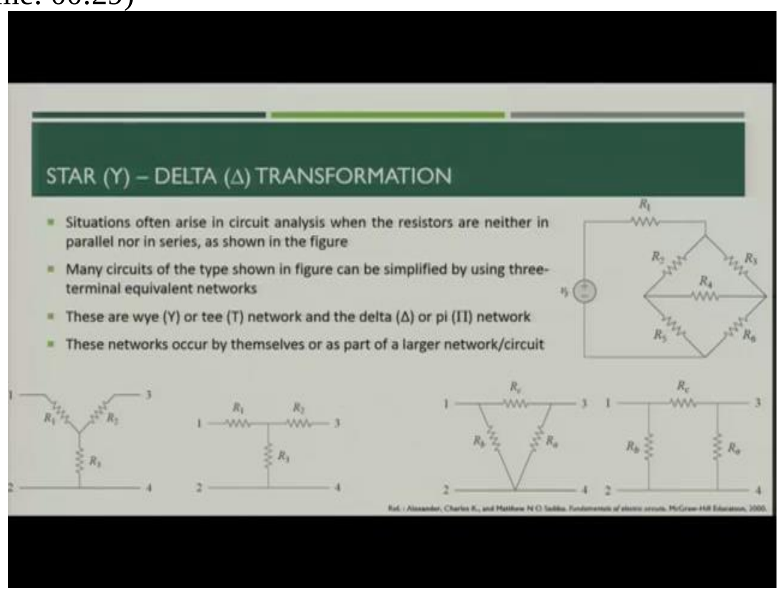

So now let us see, what do you mean by star delta transformation? Generally, you might have seen that the circuit is not so simple, where you have only had the resistors or inductors or capacitors in series or in parallel.

But it is a combination of series, parallel and sometimes it is having a different shape. Say in this particular circuit if you see the circuit is called a bridge circuit where the resistances are connected in bridge hence this is not either series or parallel. So, in these cases, the simplification of circuits can be done using 3 terminal equivalents of the networks. What do you mean by 3 terminal equivalents?

Basically, those circuits which are in the form of Y or t or delta or alternatively you could say pi networks are generally considered as 3 terminal networks. Now, if you see this particular figure, this is a Y or you could say, this is a star circuit where you see the resistances are connected in star form. At the same time, you could also, if you stretch it a little bit, it will become a T network. So basically, the star or T are the same type of circuits.

Similarly, if you see this particular figure, which is delta, their resistances are connected in the triangular format. If you expand or if you twist a little bit, you can convert a delta into a pi format. So essentially, delta and pi both are the same. The only thing is that the change in the orientation of the elements. So now, in all these circuits, what you can do, you can create the terminals like 1, 2, 3, 4 in this case.

And similarly, for T also, you can write the names of the terminals like 1, 2, 3, 4. And for delta and pi also. Now these circuits are, you can see them either part of very large network or they can be the circuit themselves.

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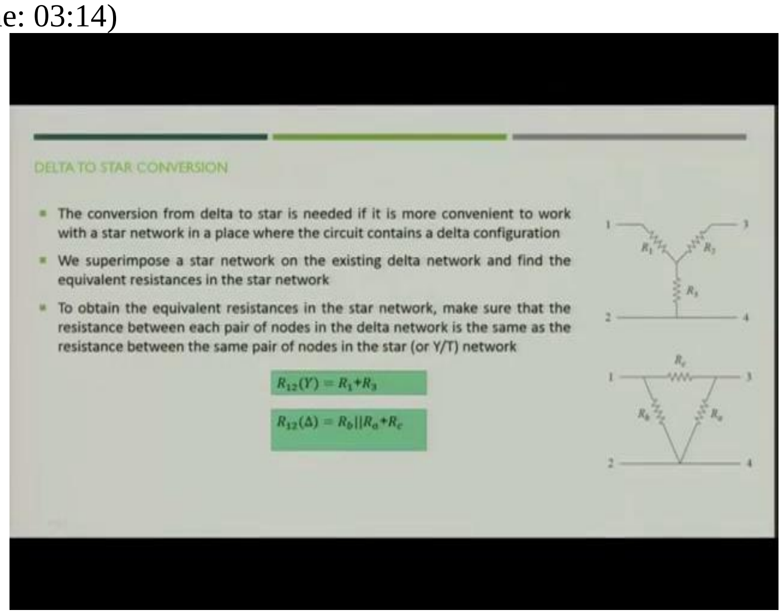

Now, the question is that how to solve it? Now if you convert delta to star then sometimes it is more convenient to work in star than in delta and you can solve the circuit which contain delta configuration.

So, what does it mean? It means that the circuit which has 1 particular segment in delta formation and it is difficult to solve, you can convert that delta formation into equivalent star and then you can solve the circuit which is more convenient to get the answers. Now what you have to do; you have to superimpose a star network on the existing delta network and find the equivalent resistances in the star network.

So, what you will do? You will put that star into the delta and you will compare the equivalent resistances which you will see through these terminals and you have to just equate them. And you will get the values of equivalent star or equivalent delta circuit for the system. Now what we have to do? We have to first try to convert delta into star. So, in this case, what we are going to do? We are going to find the value of the equivalent resistances seeing through these 2 terminals for delta as well as star.

Now if you see the star, the equivalent resistance, the equivalent resistance seen through this particular terminal would $R_1 + R_3$. And if you go to the delta circuit, the equivalent resistance would be Rb and Rb will have parallel of $R_a + R_c$. So that is why, $R_{12}$for delta would be Rb in parallel with Ra plus Rc. And R_{12}in star would be simply R1 plus R3.

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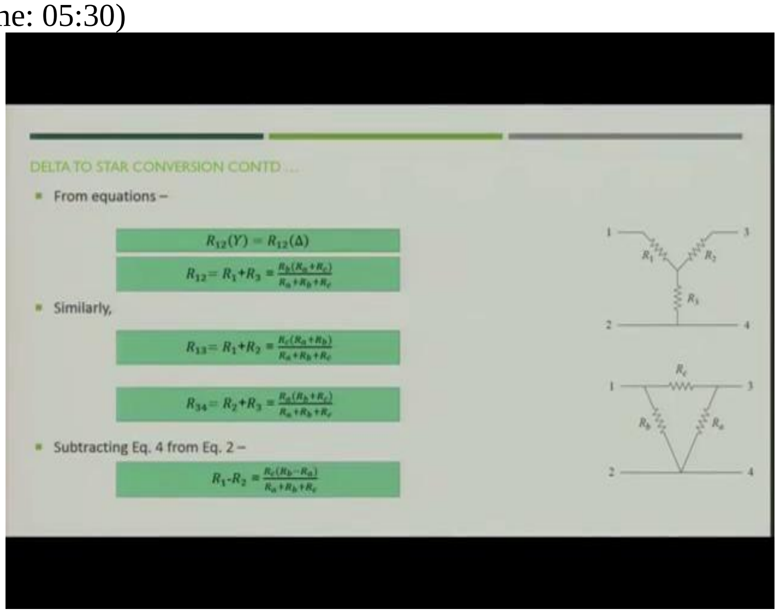

Now, the condition for conversion of delta into star is that for the equivalent resistance seen through both of the terminals, the value of equivalent resistances should be same. So that means that, you have to equate $R_{12}$for star equal to R_{12} for delta. Now, if you put the values of R_{12} star, we got $R_1 + R_3$. And for R_{12} delta, that was Rb in parallel with $R_a + R_c$. So, if you simplify, then it will become

$$\frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

Similarly, for R_{13}

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

Now, if you subtract equation R_{32} from R_{12}

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$$

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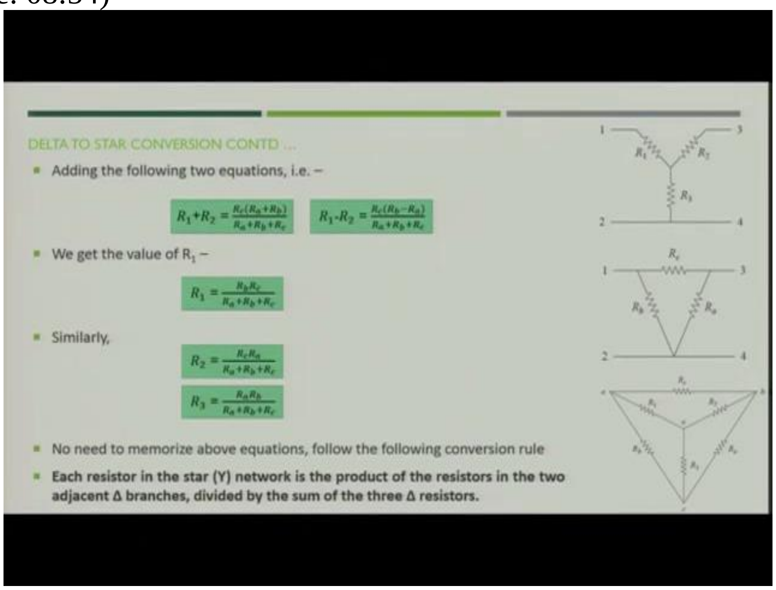

But,

$$R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

And the next which you have just calculated is $R_1 - R_2$. Now if you add both of them, you get,

$$2R_1 = \frac{2R_bR_c}{R_a + R_b + R_c}$$

You will get $2R_bR_c$, so 2 will cancel out from both of the sides. Finally, you will get

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

Similarly,

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Now, the important thing is that you need not to memorize the above equations because if you follow the simple conversion rule, you can easily get the value of the value of R1, R2 and R3. What is the rule? Now in each resistor in R, where that is the star connected network, each resistor is the product of the resistors in 2 adjacent delta branches divided by sum of the 3 star resistors. What does it mean? It means that if you put the star inside the delta you will get this kind of circuit where Rb, Ra, Rc are the branches of delta and R1, R2, R3 are the branches of star. So, if you want to find out the value of R1 in terms of delta branches, what you will do? You have to simply see what are the delta branches adjacent to R1. So in this case it is Rb and Rc. So, for R1, the value would be $R_b R_c$, that is the multiplication of the adjacent 2 branches divided by $R_a + R_b + R_c$.

Similarly, for R2, if you see the adjacent branches across R2, those are Rc and Ra. So what you will get? The value of R2 will be $R_c R_a$ divided by $R_a + R_b + R_c$. Similarly for R3, the adjacent delta branches are $R_a R_b$. So, $R_a R_b$ is the term that which will come in numerator and divided by $R_a + R_b + R_c$. So, if you follow this convention, you need not to worry about how to memorize the formulas for R1, R2 and R3.

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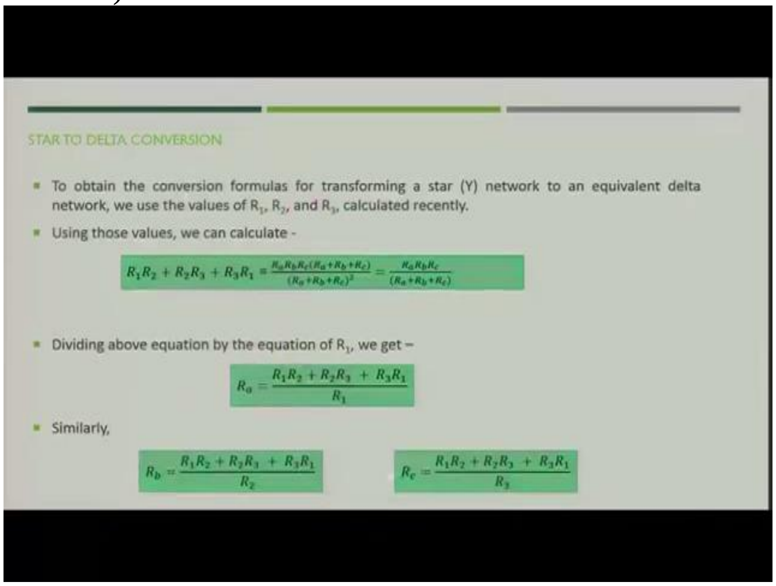

Now, let us talk about star to delta conversion. Now, to obtain the conversion formulas for transforming a star network into an equivalent delta network, we use the value of R1, R2, R3. We just calculate it. Now, if you use those values and put those values in $R_1R_2 + R_2R_3 + R_1R_3$ you get

$$R_1R_2 + R_2R_3 + R_1R_3 = \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} = \frac{R_aR_bR_c}{R_a + R_b + R_c}$$

It is simple. You just put the value and simplify it. You will get this expression. Now, you need to find the equivalent value of Ra. So what you will do? You will simply divide the equation by the value of R1. Now, what is the value of R1?

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

Now in this case, this will cancel and also it will cancel out Rb Rc from numerator and denominator and finally what you get is Ra. So that means,

$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$$

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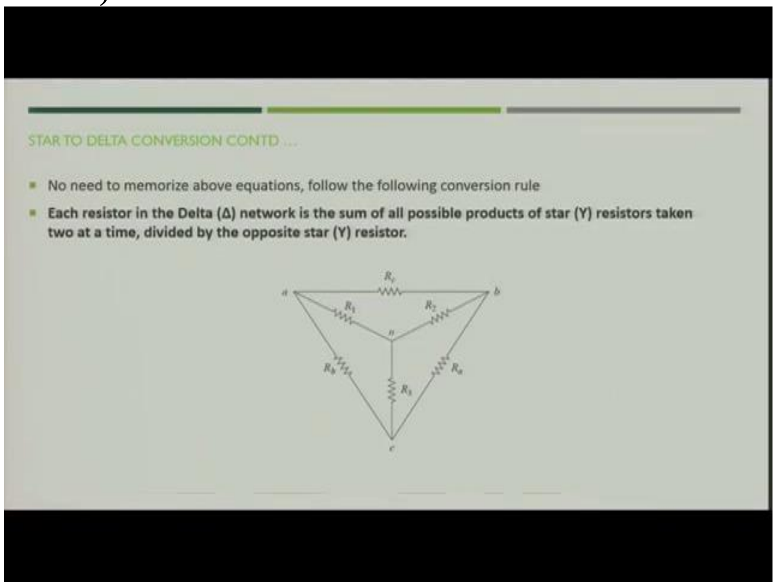

Now here again, the important thing is that you need not to memorize the above equations. You have to just follow the conversion rule. What is that conversion rule? Each resistor in delta network is the sum of all possible products of delta resistors taken 2 at a time and divided by opposite star resistor. That means suppose you are going to find out the value of Ra. In the numerator would be the product of, sum of all the product of star resistors taken 2 at a time. That means the numerator will be $R_1R_2 + R_2R_3 + R_1R_3$. What would come in denominator? Denominator would be opposite star resistor. So, in this case, what is the opposite star resistor? The opposite star resistor is R1.

So, for Ra you will get $R_1R_2 + R_2R_3 + R_1R_3$ divided by R1. And this is what we got for Ra. Simply for Rb also, you have numerator same, that is $R_1R_2 + R_2R_3 + R_1R_3$ divided by, now for Rb, what is the resistor opposite to Rb in star? That would be R2. So, you will get simply the value of Rb. Similarly, for Rc, it will be $R_1R_2 + R_2R_3 + R_1R_3$ divided by; see the value opposite to Rc that is R3. So, in this way you can easily calculate the value of the corresponding delta elements using star connected elements.

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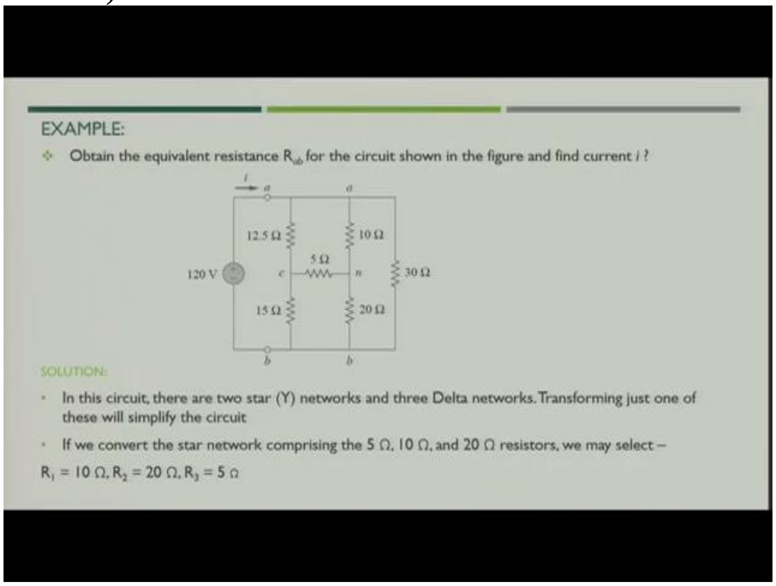

Now, to get the idea clearer, let us try to find out the value of current I in this particular circuit. Now before that we have to first find out the value of Rab, that is equivalent resistance across terminal AB. Now if you see this particular circuit, it looks complex. But if you follow the star delta transformation technique, which we just discussed, this will be very easy for you to analyse. How? Now if you see the circuit, there are 2-star networks and 3 delta networks. How? If you see this element, this element and this element it will create one sub network. Similarly, this element, this element and again this element will create another star subsection of the network.

Similarly, if you see these 2 and this particular element, it is like a pi, means this is again a delta connected section. And similarly, this also is a delta connected section. Third one is, this this and this is again a delta connected subsection of the network. So it means that we have 3 delta sub-networks and 2 star sub-networks. Now, to simplify it, you have to just transform onE1 of these so that you can simplify. Now let us convert the star network comprising of 5, 10- and 20-ohm resistors. So, 5, 10 and 20, so these are the 3 resistances. Let us put the value (R)

This is R1 that is 10 ohm. R2 is 20 ohm and R3 is 5 ohms. So if you have to create equivalent delta there will be one segment here, one segment here and third segment would come here. So what would be the value of this? We just calculated the value of value for delta element. This would be the sum of the product of 2 elements. All possible products of 2 elements in a star. So here what would be the first element, 5 into 10 plus 10 into 20 plus 20 into 5 divided by what we write in the denominator, we write the opposite star element. So what would be in this case? Opposite star element is 20. For this particular segment, the opposite star element is 20 ohm. Right.

So, what would be the value of this element? If you simplify, this will become 350 divided by 20 is nothing but 17.5. So, the value of this would be 17.5. Right? Similarly, for this segment also you can calculate and (your) this will always remain same. You have to simply divide it by 5 ohm. You have to simply divide 350 by 5 ohm. So, this segment value would be 70 and for this again you have to simply divide 350 by opposite element that is 10 ohm, so you will get 35.

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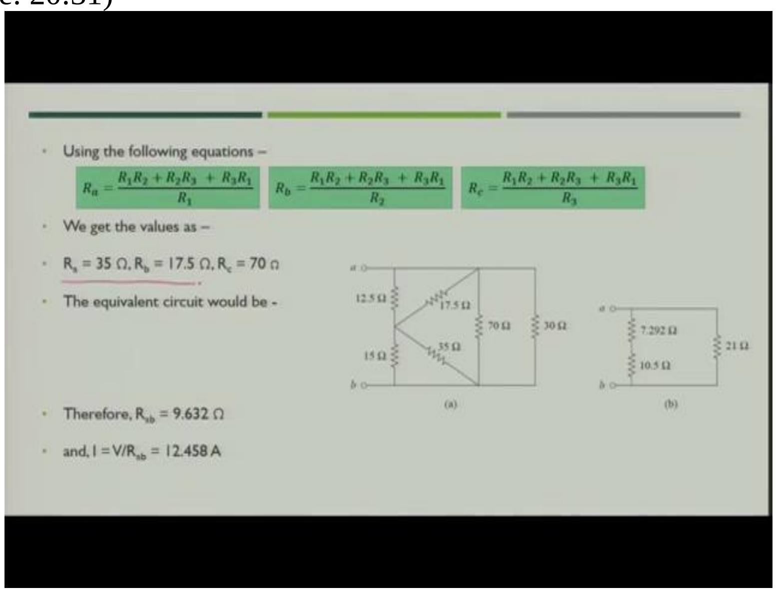

So, what you have got now? You have got these 3 values that what we calculated just now. Now, if you see this particular subsection of the network, you can see that these 2 are in parallel. These 2 are again in parallel and these 2 are again in parallel. So, if you combine both all the parallel elements, what you will get? You will get, simply if you take the equivalent of these 2 elements, you will get 7.292 ohm. If you take the equivalent of these 2 elements, you will get 10.5 ohm and again if you take the equivalent of these 2 elements, you will get 21 ohm. So now this is even very simple circuit which you can consider for the calculation of equivalent resistance across A and B terminal.

So what you get? You get, Rab is nothing but 9.632 ohm. So these 2 are in series and their summation would be in parallel with 21 ohm. So you will simply get the value of Rab and now what would be the current? Current you have to just simply divide the resistor, the voltage by equivalent resistance which we have just calculated. So Voltage is 120 volt. If you divide by 9.632, you will get current as 12.458 Ampere.

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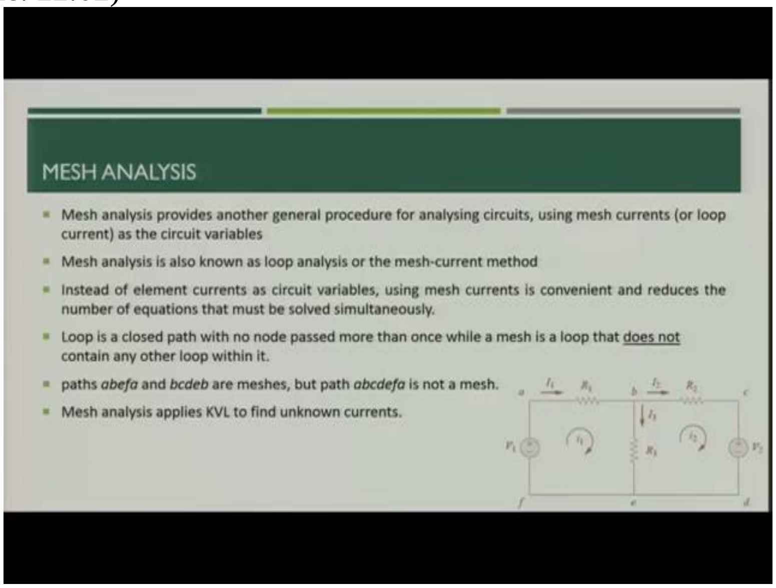

Now, let us talk about mesh analysis. This is also one of the important components in our circuit analysis. Why? Because it provides another general procedure for analysing the circuits, using mesh currents. Mesh currents or you can say loop current as a circuit variable.

So, if you see in this figure, these currents I_1 and I_2 are called as mesh currents. And it is different from the branch current. Because branch current flows in a particular branch of the circuit and loop will be same current flowing through a particular loop which we have just saw in the particular case. Now this is; mesh analysis is also called loop analysis or you can say mesh current method. Now, here instead of element current as circuit variable, we use mesh current as a circuit variable because it is very convenient and it reduces the number of equations that must be solved simultaneously.

Like in this case if you see, there are only 2 variables related to mesh current. But if you take the branch variable, you will have 1 for branch current for R1. Similarly, 1 branch current for R3 and 1 branch current for R2. So here, you have 3 variables if you use branch current method. While in case of mesh current method, you will have only 2 variables. So it means that you can reduce the number of equations required to solve the circuit. Now, there is a little difference between loop and mesh. Loop can be \considered as a close path with no node passed more than once. So here, this is a loop and this is also a loop because it is not tracing 1 node (at) 1 node 2 times. But mesh is a loop that does not contain any other loop within it.

So mesh can be considered as a special kind of loop which does not contain any other loop within it. Now what does it mean? Let us see the example for better understanding. In the particular circuit, you will see abefa, abefa is 1 mesh and bcdef, bcdef is another mesh. But abcdefa is not a mesh. Abcdefa so outer loop is not a mesh. Why? Because it contains 2 loops inside. So that is why mesh is considered to be a special kind of loop which does not contain any other loop within it. Now you can simply apply the Kirchhoff's voltage law to find the unknown currents.

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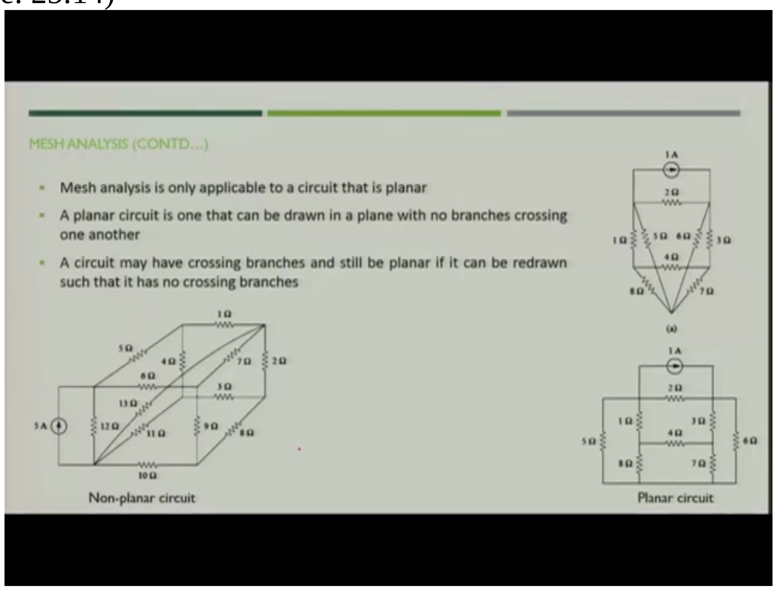

One of the important properties of mesh analysis is that, mesh analysis is only applicable to the circuit that is planer. Planar means the circuit is the planer which can be drawn in a plane with no branches crossing one another. So, it means that if you see this particular figure, the branches are separate and no any branch is cutting any other branch. While in this particular case if you see this is non-planar circuit because the branches are cutting across and it is not in a plane. It is now become a 3-dimensional circuit. So that is why the mesh analysis cannot be done for this type of circuits but it can be done for planar circuits.

Now, another important thing is that sometimes when you see the circuit it looks like this circuit right and you will see that lot of branches are cutting across. So, at the first instance you will consider it as a non -planar. But if you can redraw the circuit in such a way that it can become a planar circuit then you can use the mesh current method to analyse it. What does it mean? If you see this particular circuit then you will see that this 5 ohm is cutting across. Instead of putting 5 ohm like this if you put 5 ohm as shown in this figure, it will become non cutting branch. Similarly (3 ohm) this particular 6 ohm is also can be arranged in this fashion and if you remove this 2 and rearrange you will eventually get a planar circuit.

So you need to analyse the circuit once before doing the mesh analysis whether it can be converted into a planar circuit or not. So for this you can utilize the network topology information which we discussed previously. So when you create the topological network of this circuit and if you rearrange the nodes and branches in such a way that their connectivity information is not changed then you can easily find out whether that particular non planar circuit can be converted into planar circuit or not. And if it can be converted into planar circuit you can simply run your mesh analysis.

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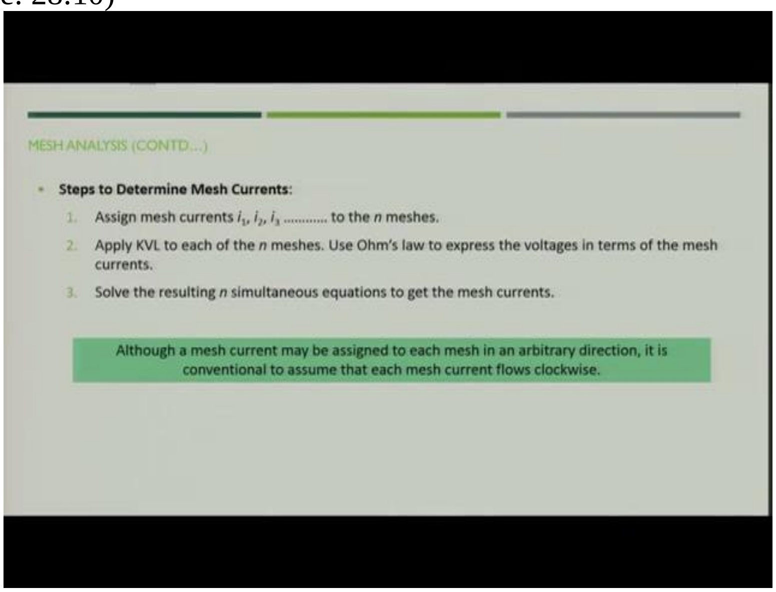

Now, what are the various steps to determine the mesh current in the mesh analysis. You have to assign first mesh currents I1, I2, I3 and so on for number of meshes. Suppose in this case there are n mesh and then you apply Kirchhoff's voltage law to each of the n mesh and use Ohm's law to express the voltages in terms of the mesh currents and then solve the resulting and simultaneous equations to get the mesh currents.

Now one important thing to remember is that although a mesh current maybe assigned to each mesh in an arbitrary fashion or in an arbitrary direction. But it is conventional to assume that each mesh current flows clockwise because this will be easier for you to analyse the circuit and there would be less chances of errors. Like in this case we have done the, we have taken the direction of the mesh currents as clockwise.

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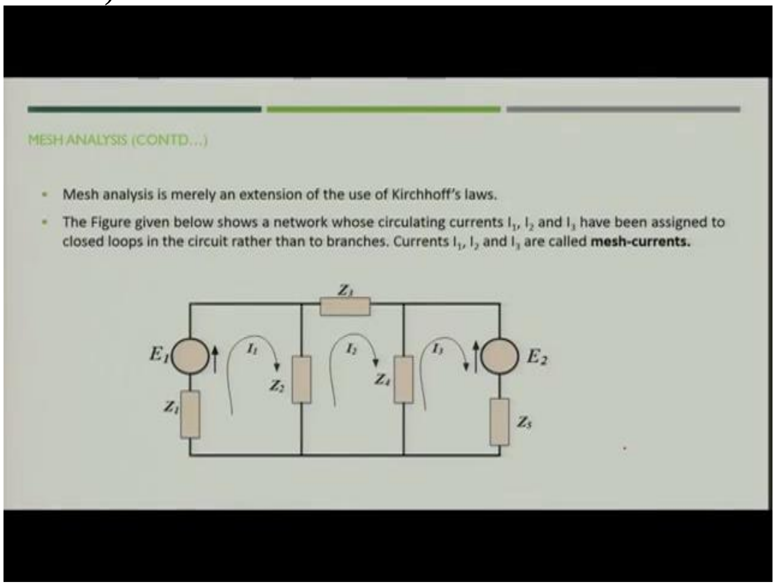

Now, how you will do the mesh analysis? It is if you see the mesh analysis technique it is merely an extension of Kirchhoff's law. Because here also you have to apply the Kirchhoff's law to find out the value of circulating currents.

Now if you consider this particular figure, there you will see that network has 3 circulating currents that is I1, I2, and I3. Now, all have been assigned a clockwise direction for not to take risk of error and then we have to analyse these currents which are called mesh currents.

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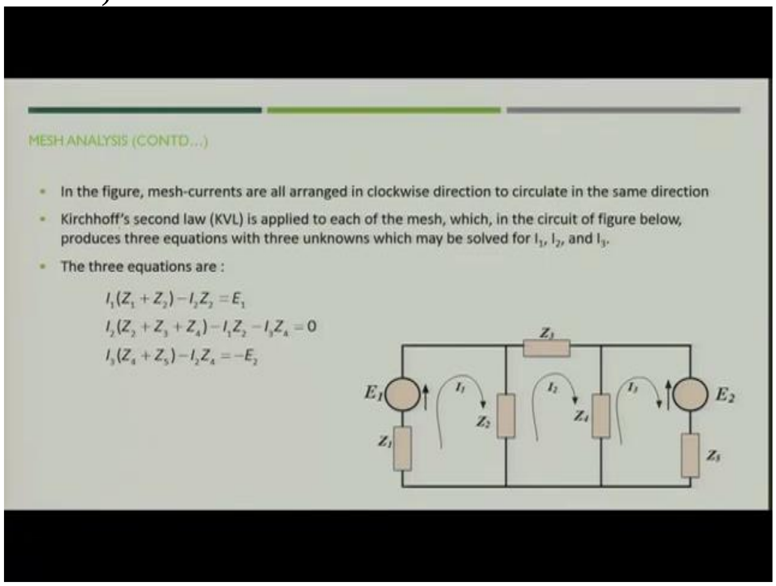

So, if you see the circuit and analyse, what you will do? You will use Kirchhoff's voltage law in each mesh and then you will try to find out what would be the equations for those mesh currents. So if you see in this figure the mesh the equation for the first mesh would be

$$I_1(Z_1 + Z_2) - I_2Z_2 = E_1$$

Similarly, for loop two and three you can also write

$$I_2(Z_2 + Z_3 + Z_4) - I_1Z_2 - I_3Z_4 = 0$$

$$I_3(Z_4 + Z_5) - I_2Z_4 = -E_2$$

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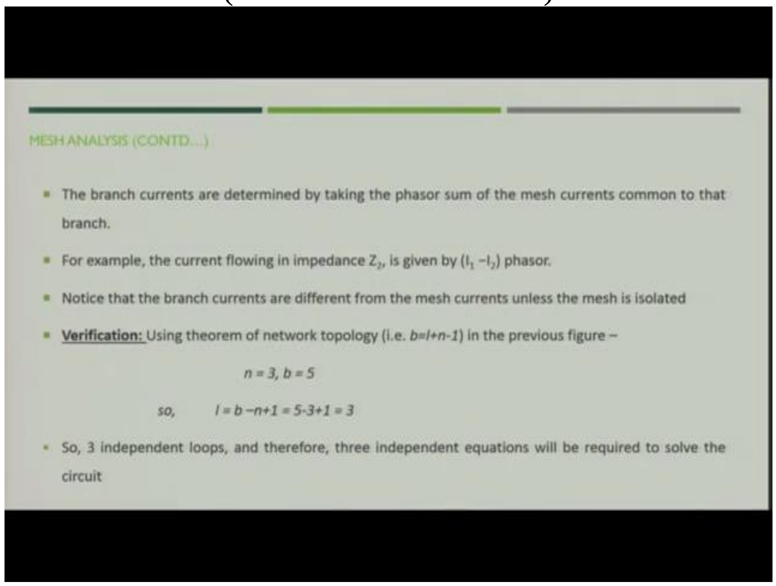

Next step is that you have to analyse these particular equations and find out the value of current I1, I2 and I3. And the important thing is that branch currents are determined by taking the phasor sum of mesh currents common to that branch. That means if you see this figure for this particular branch, the branch current would be phasor sum of these two.

Phasor sum means you have to consider the direction. So, for this particular segment the branch current would be I1 minus I2. Now you can notice that the branch currents are different from the mesh currents and this will be the case unless mesh is an isolated mesh. Now the question would come how you will verify whether you have written the all the equations which are required for the analysis. You have to take the help of theorem network topology.

So if you see the theorem of network topology it says that the number of branches are equal to number of loops plus number of nodes minus 1. So in this particular figure if you see you will have 1, 2, 3 nodes and 1,2,3,4 and 5 branches. So number of nodes are 3, number of branches are 5. So how many loops are there? Loops in that particular circuit would be equal to 3. That means that 3 independent loops and therefore 3 independent equations are required to solve the circuit. So here you will see that there are 3 independent equations we have created to solve this particular circuit.

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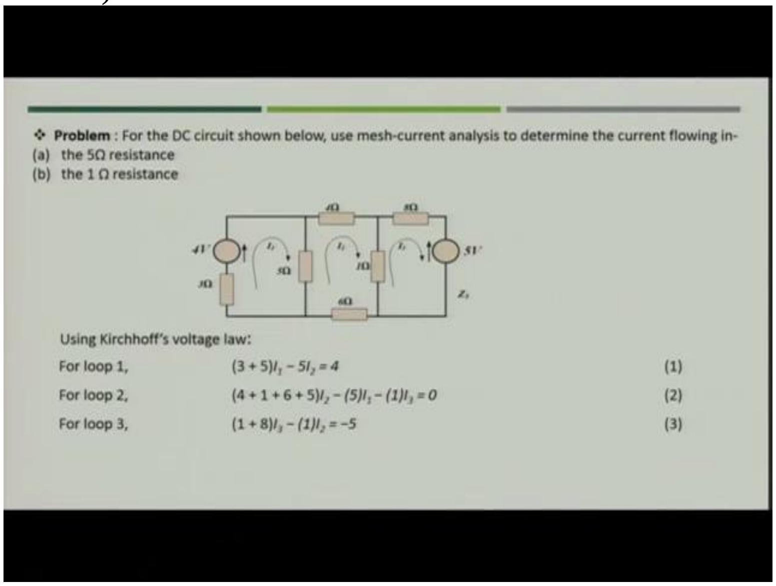

So now let us take one example quickly so that you can understand what we discussed till now. So if you see this particular circuit and you write the equations for all 3 mesh currents what you can write.

For loop 1, $(3 + 5)I_1 - I_2 = 4$ (1)

For loop 2, $(4 + 1 + 6 + 5)I_2 - (5)I_1 - (1)I_3 = 0$ (2)

For loop 3, $(1 + 8)I_3 - (1)I_2 = -5$ (3)

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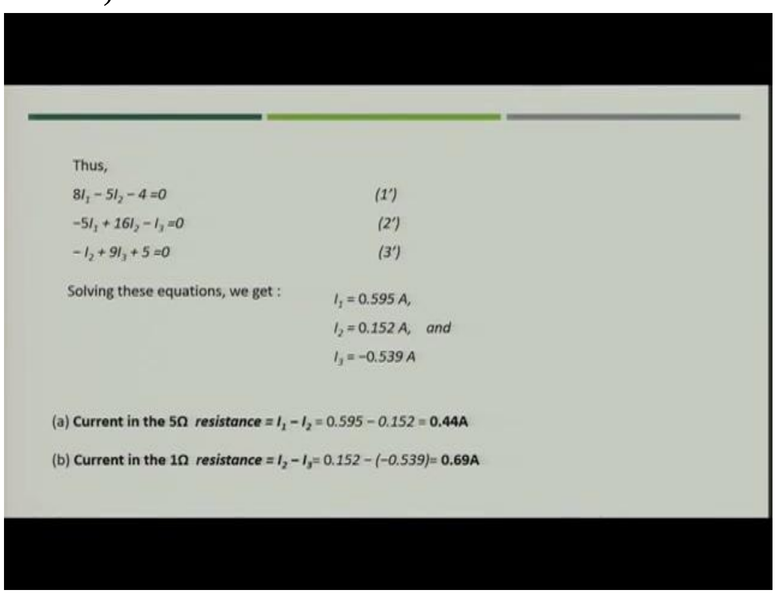

Now, if you rearrange you will get these 3 equations. And after solving you can get

$I_1 = 0.595\ A$,

$I_2 = 0.152\ A,\quad and$

$I_3 = -0.539\ A$

So similarly, you can calculate the value of current in 5 ohm and 1 ohm resistor because these are the branch currents and 5 for 5 ohm resistance the branch current would be $\boldsymbol{I_1 - I_2} = 0.595 - 0.152$ = **0.44A** and current in 1 ohm resistance would be $\boldsymbol{I_2 - I_3} = 0.152 - (-0.539)$= **0.69A**. Now in this case we have taken all the current direction as clockwise, here we have got current in negative with negative sign. That means that our initial direction of current was clockwise but finally the direction of current is anti-clockwise. So, from this you can get but for better analysis we first keep all of them in one direction and solve it. So, with this we close our today's session and we will continue the analysis using mesh analysis in the next session also. Thank you.