Basic Electric Circuits
Module II
Mesh and Nodal analysis
Lecture-07
Topology
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Namashkar, so today we will discuss about the topology. What do you mean by topology? As we know that the network is not always series circuit or parallel connected, unlike what we have discussed so far. Actually in real scenario this electrical circuit will not be series or parallel, it will be combination of both. So, the objective of the electrical circuit analysis is that you need to find out the currents and the voltages across element which is connect to the network.

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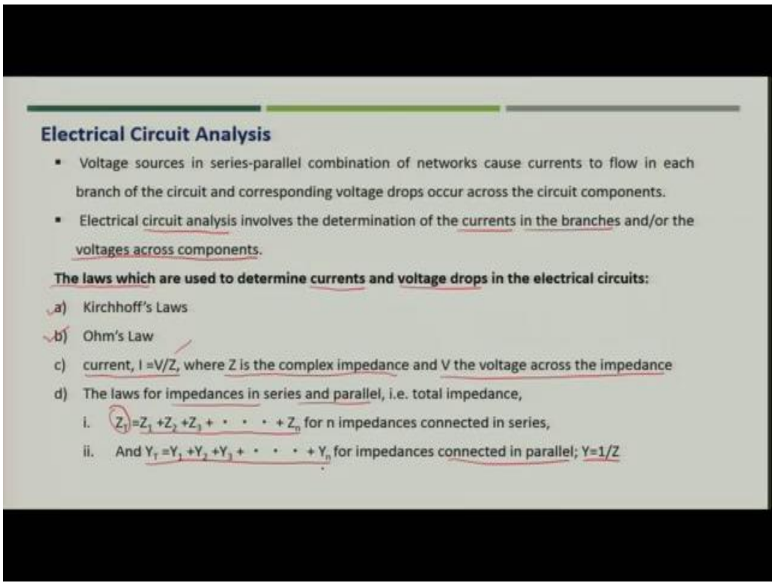

We will try to analysis the circuit and try to find out the currents which are there in the various branches of the network and the voltage across the components. Let us recap what we discussed till now related to circuit analysis. There are various laws which are used to determine the currents and voltage drops in the electrical circuit. We discussed about the Kirchhoff's law, we also discussed Ohm's law, Ohm's law is nothing but the finding out current $I = V/Z$.

Now here we are using Z instead of R, because Z is the complex impedance and is generally visible when you have the AC sinusoidal wave form in the circuit and then the voltage across the impedance. Now, if impedance is in series to find out the total impedance in a series connected it will be summation of all the impedances connected in series or if you have the parallel connected impedances then it is better to convert impedance into admittance.

Admittance is nothing but Y and it is reciprocal of the impedance jet and then you can some up to find out the admittance.

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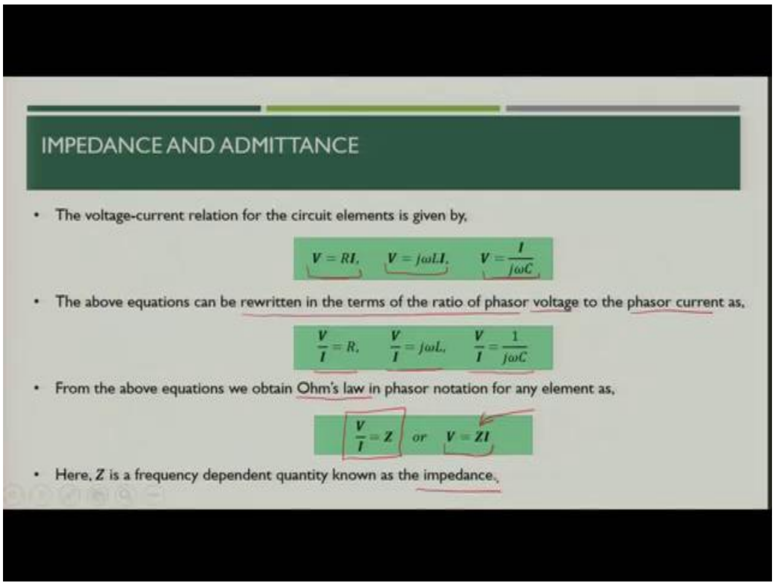

Now let us look at impedance and admittance in detail so that you can understand the concepts of these two entities with relations to the AC circuit. Now voltage current relation which we have discussed till now was something like this. We had $\boldsymbol{V}$ the phasor value is nothing but $\boldsymbol{I}R$, For this was for the case of resistor. For inductor we discussed , $\boldsymbol{V} = j\omega L\boldsymbol{I}$ and for capacitor the relationship was $\boldsymbol{V} = \frac{\boldsymbol{I}}{j\omega C}$.

Now if you write the equation in terms of ratio of phasor voltage and phasor current

$$\frac{\boldsymbol{V}}{\boldsymbol{I}} = R, \quad \frac{\boldsymbol{V}}{\boldsymbol{I}} = j\omega L, \quad \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{1}{j\omega C}$$

So now if you are asked to find to represent all the three equation in a generic term what we write?

$$\frac{\boldsymbol{V}}{\boldsymbol{I}} = \boldsymbol{Z} \quad or \quad \boldsymbol{V} = \boldsymbol{ZI}$$

Now here Z is a quantity which is called impedance. What are the various properties of this impedance?

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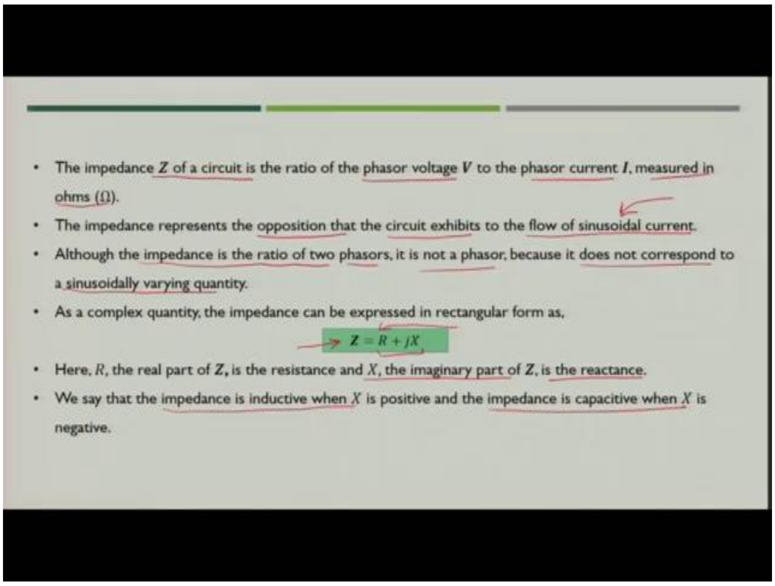

The impedance of a circuit is the ratio of voltage phasor and current phasor and it is also measured in ohms. Because in case of resistor or capacitor or inductor, whatever the reactance is which we calculated in previous lectures. The dimension of those was in ohms. Now impedance represents the opposition that circuit exhibits to the flow of sinusoidal current.

So, whenever the impedance terms into the picture it means that the circuit is AC circuit and the current is sinusoidal current. Now although impedance is the ratio of two phasor, that is phasor V divided by phasor I, but it is not a phasor, because it does not have the sinusoidal varying quantity. The complex quantity for impedance is generally represented as $\boldsymbol{Z} = R + jX$, where R is the real term and this real term is nothing but the resistance in the impedance term and then X which is imaginary part of Z and this imaginary part is called reactance.

Now impedance is inductive when X is positive. So here if you are writing $\boldsymbol{Z} = R + jX$ if this is the positive quantity, it means that the impedance is inductive while it would be capacitive when X is negative. So, in that case your inductance would be capacitive sorry the impedance would be capacitive impedance.

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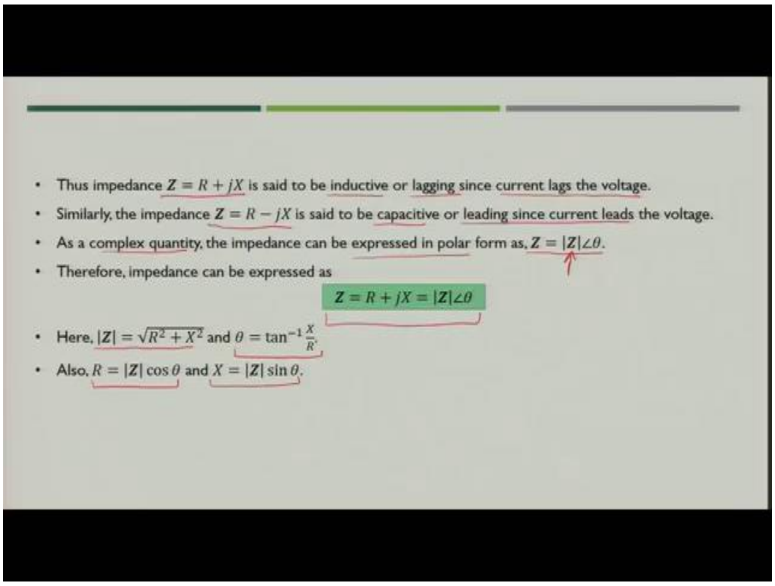

Now impedance is $\boldsymbol{Z} = R + jX$, so it would be inductive when it is lagging since the current lags the voltage.

When $\boldsymbol{Z} = R\text{-}jX$, it is capacitive because the current leads the voltage. If we have to represent the complex quantity Z in the polar form we write $\boldsymbol{Z} = R + jX = |\boldsymbol{Z}|\angle\theta$. So, in summary you can say Z can be represented as $\boldsymbol{Z} = R + jX$ or in polar form $\boldsymbol{Z} = |\boldsymbol{Z}|\angle\theta$.

This two you can you interchangeably based on the requirement in the calculation and you can calculate $|\boldsymbol{Z}|$ using rectangular the equation of Z that is $|\boldsymbol{Z}| = \sqrt{R^2 + X^2}$, where $\theta = \tan^{-1}\frac{X}{R}$ or if you want to convert polar into rectangular, you can say $R = |\boldsymbol{Z}|\cos\theta$ and $X = |\boldsymbol{Z}|\sin\theta$.

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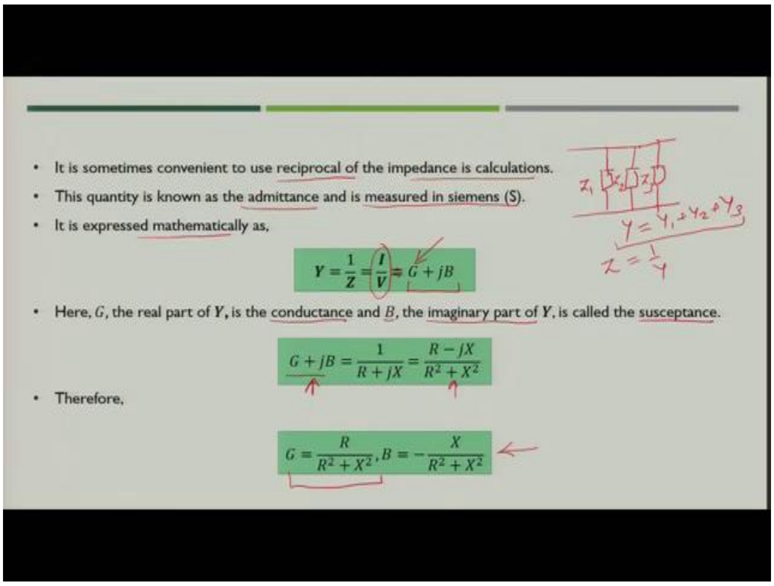

Now sometimes it is convenient to use reciprocal of impedances in calculation. This happens specifically when you have parallel connected circuits like if you have impedance connected in parallel like this.

So, if it is Z1, Z2, Z3 the equivalent impedance if you have to find out it is better to take the reciprocal of impedances. So, if reciprocal of Z1 is Y1, its Z2 reciprocal is Y2 and similarly for Z3 reciprocal is Y3. So, you can easily say that the equivalent admittance Y is equal to Y1 plus Y2 plus Y3 and if you have to find out the impedance then the equivalent impedance Z would be 1 by Y. So, in this case when you have parallel connected networks, it is better to convert impedance into admittance.

Now admittance is nothing but reciprocal of impedance and it is measured in siemens or you can say it is also measured in mho. So, mho and siemens both are same you can use them interchangeably and the admittance is expressed mathematically as $\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = G + jB$, where G is called conductance and B which is imaginary part of Y is called susceptance.

What is $G + jB$? $G + jB$ is nothing but $\frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}}$ and Z is nothing but $R + jX$. So, if you simplify, you will get

$$G + jB = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$$

So, if you compare real and imaginary terms from both of the components you get

$$G = \frac{R}{R^2 + X^2}, B = -\frac{X}{R^2 + X^2}$$

This admittance you can use whenever there is a requirement to analysis the circuit which like this, in this case.

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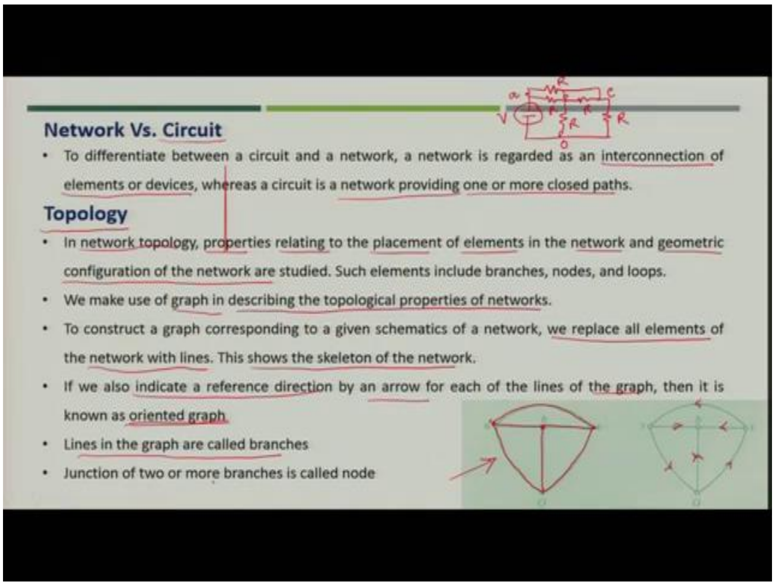

Now let us talk about the network. Till now we have discussed like every time we use electrical circuit not the network. So, what are the difference between network and circuit? The difference between a network and circuit is that the network is an interconnection of elements or devices. That means that it is not necessary that it will create a close loop, so it can have the close loops but there may be some open loops also.

So, network is the broader term for circuits, while in case of circuit its network which providing one or more closed path. So, the difference between network and circuit is that network is generally regarded as a more generic term for the electrical circuit, where the circuit can be open or closed. While in case of electrical circuit we make sure that there is a return path for current to flow. So, in Electrical Engineering we generally used both of them interchangeably, because network is more generic terms.

We, in this particular course also we will use some time network some time circuit, but that means the same thing. Now let us talk about the topology. So, for network topology what is the topology? The property is which (which) is relating to the placement of elements in the network and the geometric configuration of the network. So, what does it mean, some generally when you see a particular network like if you represent here there is a resistor, if you take the circuit like this.

So, suppose all are of equal resistance R and this is voltage V, so this is the circuit you may be interested to find out the connectivity information. So, connectivity information means you need to find out how the various elements are connected. Now if you give this node as a, this node as b, this node as c and this is o that is the ground or may be origin you can say. Now if you are asked to find out the topology of this circuit, what you will do? You have to simply construct the graph which will replace all the elements of the network with lines.

Why? Because the graph what we are creating is describing the topological properties of the network. So, from the circuit if you create a graph which describe the topological property that means the connectivity information of all the elements in a particular network, then that particular network will be the topology of that particular circuit. So here if you see a is connected to c, so instead of adding R we are simply adding the line, a is connected to c then between ab resistor R, so we are again creating a line, b to c we are connecting the line, c to o that is again we have one line, so for this R it is the line, for b to o we are again creating the line and between a to o because this is the voltage source we are again creating the line.

So, this particular circuit will be equally represented as a graph in this session which will remove all the elements there may the sources that may be the voltage or current sources or the resistive, capacitive or inductive elements in the circuit. What we get is called the graph of the circuit or graph of the network. This is very important because we have various topological properties associated with it which will help us to analyze the circuit.

Now what we got is the skeleton of the network. Here the elements which you will see would be like nodes, we will see the loops or we will see the branches. Now if you add the reference directions by an arrow for each of the lines of the graph then it is called as oriented graph. That means that you when you analysis the circuit you imagine a the initial direction of may be the current, so then if you show the direction of the current then this will become a oriented graph. Now lines in the graph are called branches and junction will be called as node.

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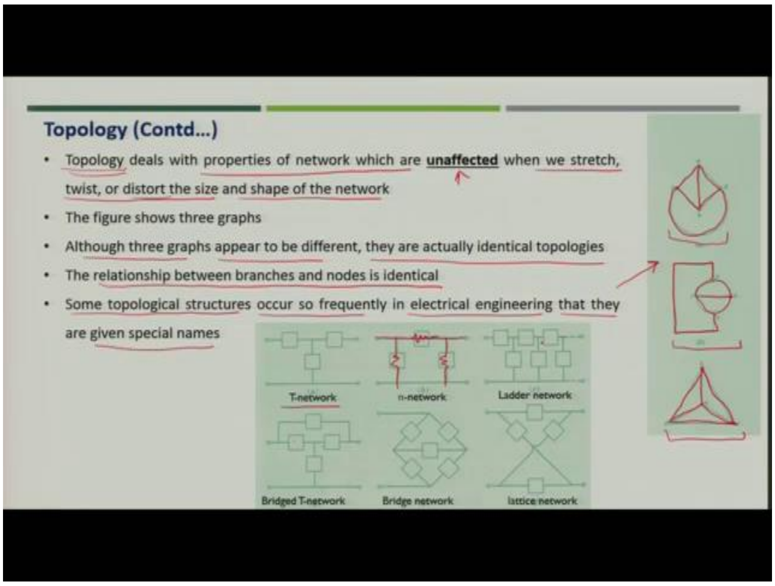

Now, why the topological information of the network is important? Because the topology shows us the property of the network which is unaffected when we stretch, twist or distort the size and shape of the network. That means that if you have the network and you change the orientation of the graph by stretching twisting or distorting its size if you keep the connectivity information intake all the different types of stretches or distort you have created with that particular circuit will always remain same because the topology of the network is same.

So, in this example if you see this the topology of one particular circuit is like this where a, c, b, d and a b are connected in this fashion and there is another circuit where you create the topology in this fashion where these elements are connected like this and third one where the circuits are connected in a triangular fashion. Now if you see all this three from first look it looks likes that all three are different, but eventually all the three circuits are same.

Why because the connectivity information in all the three topologies are same. So in this example if a is connected to c here, a is connected to c here and a is connected to c here, a is connected to d, a is connected to d and a is connected to d, c is connected to b, here c is connected to b and here also c is connected to b, b is connected to d here, b is connected to d here and b is connected to d here. Now c is connected to d in this fashion here c is connected to d in this fashion and here c is connected to d in this fashion. Finally we are connecting a with to b through straight line. Here we are connecting a to b like this and similarly a is connected a is connected to b like this.

Now since connectivity information is same it means that topology is same. Why it is important because sometimes in the circuit if it is given a the figure is given in a different way you can better convert it into a similar type of a circuit where it is very easy to analysis the current flow and the voltage drop across the element. So that is why the topology information is very important where you can understand the topology configuration of the circuit and you can stretch, twist or distort that particular circuit in such a way that it is easier you to analyze.

So, although the three graphs which we discussed are different but they are actually the identical topologies. Why because relationship between branches and node is identical in this case. Sometimes the topological structure in the electrical circuit occur so frequently that in Electrical Engineering we have given them some special names, like if you see circuit like this we called them as T-network, if it is like this were you may have resistor connected like this then it is called pi network.

If the circuit topology is like this it is called a ladder network. When the topology is like this where two resistances are connected parallelly with one resistor and in between you have another resistor or maybe you can say all of them are impedances then it is called Bridged T-network. Similarly, if it is connected in bridge fashion like this, it is called bridge network and this is called the latest network. So, these are the most common topologies you will encounter in the electrical circuits. So, if you can understand the topological information, it is very easy for you to analyze the circuit.

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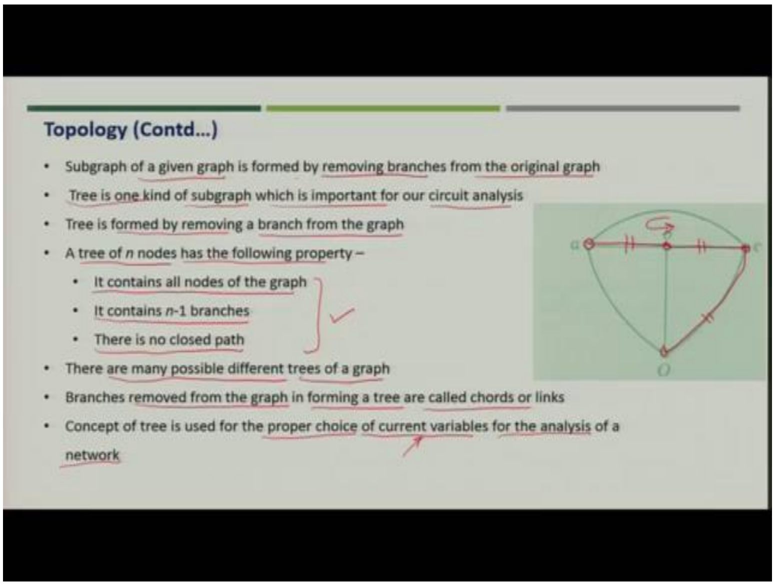

Now let us talk about the sub-graph. Sub-graph of a given graph is formed by removing branches from the original graph. So if you remove one branch, like this if you remove it will become sub-graph of the original graph. Now what is tree? Tree is one kind of sub-graph which is important for our circuit analysis and it is form by removing a branch from the graph. So tree can also be consider as a sub-graph, but it has some special properties.

What are those properties? Tree of n nodes, suppose if there are n nodes in a particular graph and we are creating tree containing all the n nodes then it will have the following property. It will contain all nodes of the graphs, it will contain n minus one branch and there will be no closed path. So these are the three major properties of tree and let us understand this property from this figure. If you ask to find out the tree then first is that it will contain all nodes.

It means that node a, b, c and o should be part of the graph. It will contain n minus one branch. So we have 1,2,3,4 nodes, it means that it will have n minus one that is three branches. What would be the three branches? The three branches can be this because it makes sure that there is no closed path or alternatively what you can do, you can have the nodes but your branches can be one that is second and it can be third.

In this case also you know that all the nodes have been traced but there is no closed loop or similarly you can do one more thing that you can connect through some other fashion like this, this and that, that again a tree, so there may be many possible different trees of a graph. But if you connected through some session like if you connect from o to c, c to b and b to a then you will again find the tree.

In all the cases if you see there would be precisely three branches because it will contain n minus one branch for four nodes it will always have three branches, if you connect more, then definitely you will have one closed loop which is not allowed in this case so tree will have precisely no closed path. Now the branches are forming a tree are called chords or the links. So, if you removed this particular branch then you get one tree so this is called chord.

The concept of tree is used were we have to carry out the proper choice of current variable for the analysis of the network. So, this is very important when you have to choose the current variables for circuit analysis properly.

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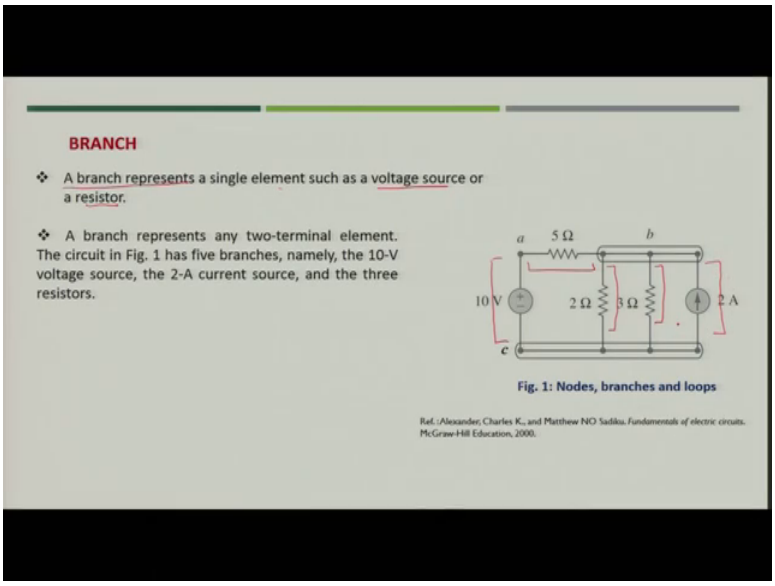

Now let us talk about the elements which we just saw in the network topology, one is branch. Branch represents a single element such as voltage source or a resistor. So, in this case this can be consider as branch, this can be considered as a branch, this can be considered as a branch this can also be consider as a branch and this can also be considered as a branch. So basically, were ever we see the electrical elements present that particular segment is called a branch.

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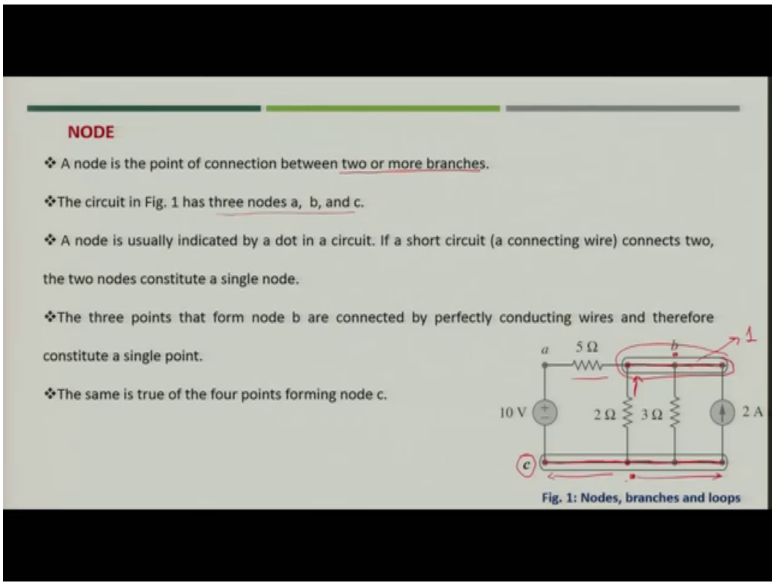

Now let us talk about node, node is the point of connection between two or more branches. Now in this case this is the node because it is connecting this and this branch. So, node will be the connection between the two or more branches, here in this figure we have three nodes, node a, node b and node c.

Now in this figure c is not a single load it contains all four nodes which are there in the circuit, but we represented by a single node c while connect two nodes whit a short circuit or may be the direct wire both will always be at a same potential. So, when it is like this you can join all of them and create one single node. So eventually if you have four different nodes it is nothing but you are talking about one single node, because you can join then without any change in the circuit configuration.

Similarly, in this also, although you have three nodes but since all are connected through direct wire means all three nodes are short circuited then you can equal entry represent all the three nodes with one single node. So here you will have, in b you will have three nodes but actually it is considered as one node. Similarly, for this also you have four nodes but it is considered as a single node because of the wires which are connecting these nodes are not having any elements and it can be consider as a short circuit node.

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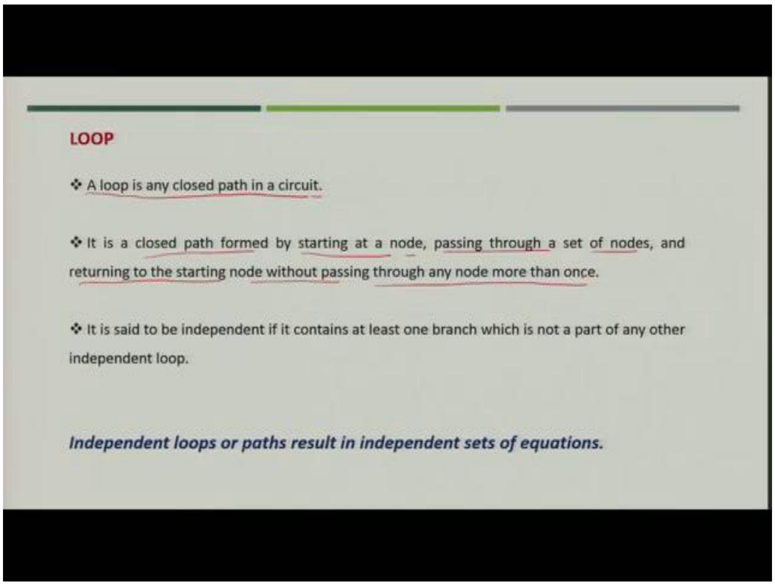

Now what is loop? In any closed path loop is basically a closed path in any circuit. The closed path formed by starting at a node, passing through a set of nodes and finally returning to the starting node without passing through any node more than once.

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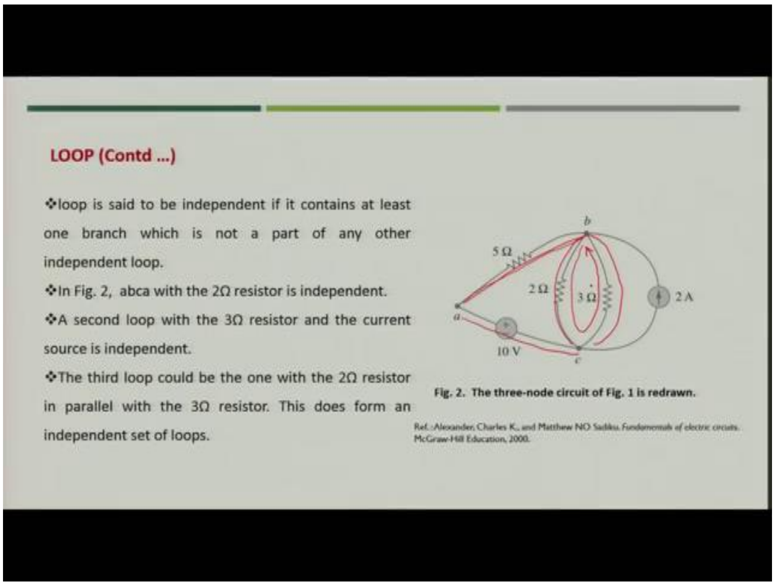

So that means if you see the circuit like this when you are ask to find out the loop, it means that if you trace out various nodes and come back to the original node then this will consider to be one loop, right.

Now this particular loop is said to be independent if it contains at least one branch which is not part of any other independent loop. Now, why the loop is important? Because independent loops or path result in independent set of equations. So, when you have given a complex circuit and you are asked to execute the Kirchhoff's voltage law, then you have to first find out what are the various independent loop. So that you can precisely identify what are the various equations or unique equation you have to find out so that you can find out you can get the answer of that particular circuit.

Now if you see this figure if you connect a, b, c and a in this fashion a, b, c and a this will be considered one independent loop or alternatively you can go with a, b and c which is through two ohm and then a that will be another independent loop or you can have two and three ohms connected that is also another independent loop. So, in all the three cases you will see that at least one element in the loop is not part of other loop, So in that way you can create the number of loops.

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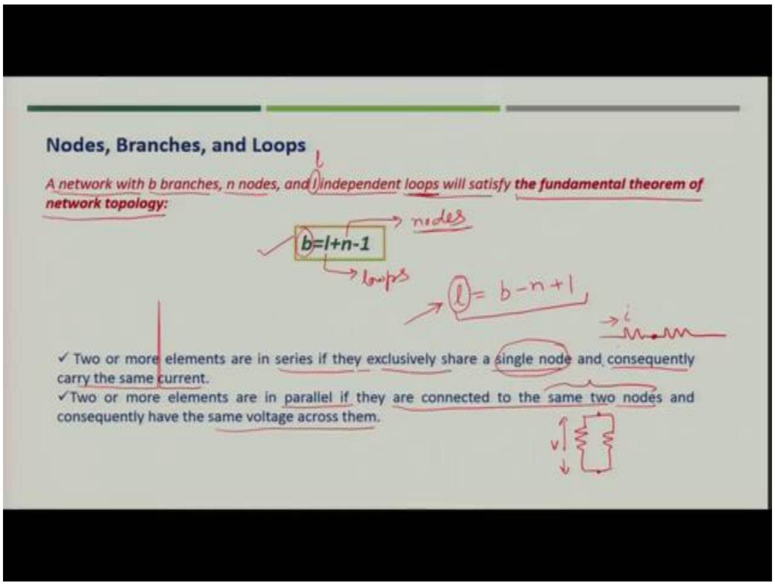

Now the question would be, how you will find out whether you have got the correct number of loops or not? For that we use the fundamental theorem of network. So, b=l+n-1, l is the number of loops, n is number of nodes and b is number of branches. So, you know you can find out the numbers of nodes, you can find out the number of branches, so you can easily find out what would be the numbers of loops, so l would be nothing but b minus n plus one.

So, you can count number of loops which you have created and try to find out whether number of loops are satisfying this particular theorem or not. If it is satisfying it means that you have precisely identified the number of loops in the circuit. Now there are two or more elements which are in series they exclusively share a single node and then you can say that (those) those will carry the same current.

So that we have (also) already seen in the series and parallel circuit analysis that if the elements are connected in series means they will share a single node and they will carry the same amount of current. Now if there are two or more elements which are connected in parallel then they are connected to same two nodes and consequently have the same voltage across them. So it means that if the nodes are connected in series then they both elements will connected through one single node, So in that case you have the same current flowing in the both of the elements.

Now if those are connected in parallel then these elements would be connected through the same two nodes and in that case the voltage across both of the resistors will be same. So, this will give you an idea that if you have node configuration like this it means that the current will be same if they are connected in this fashion and voltage will be same if they are connected in this fashion. So, with this we close today's session. In next session we will to try to analyze some other circuits which are like a circuit combination of series and parallel and how to make them easier to understand and how to analyze so that it is very easy for us to find the solution of those circuits. Thank you!