Basic Electric Circuits
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Module-12
Analogous Systems
Lecture-60
Solving Analogous Systems

Namashkar, so today we are coming towards the end of our course and today we will discuss about the force current analogy and then we will try to solve the analogous system and we will see how the analogous system can be equally represented with the help of electrical circuit. So, let us start the today's discussion.

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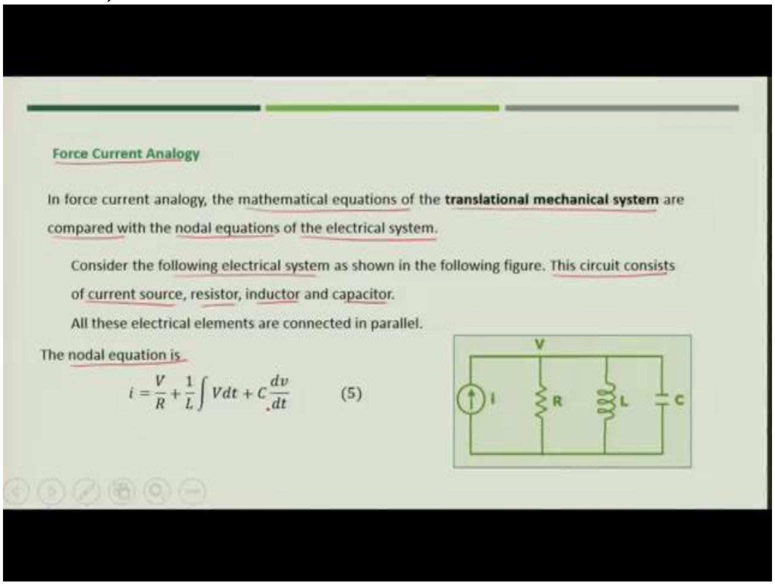

So, first we will see what is the force current analogy? So, the force current analogy the mathematical equation of the translation mechanical system are compared with the nodal equations of the electrical system. Now, consider the following electrical system which is shown in the figure. Now, this circuit consist of current source, resistor inductor and capacitor. So, all are connected in parallel.

So, in this case the nodal equation you can write as the current

$$i = \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{dv}{dt} .$$

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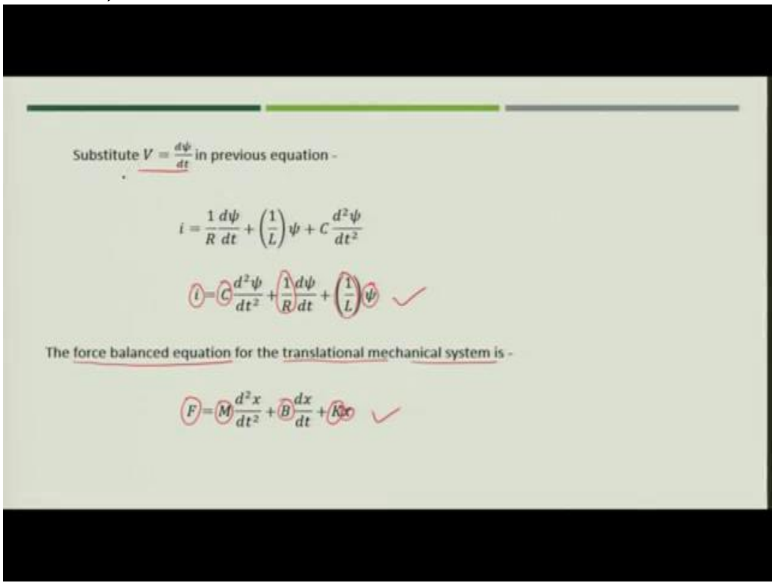

Now, if we substitute $V = \frac{d\psi}{dt}$ or let the other say $\frac{d\psi}{dt}$ which is nothing but the magnetic flux. So, what we can write, $i = \frac{1}{R}\frac{d\psi}{dt} + \left(\frac{1}{L}\right)\psi + C\frac{d^2\psi}{dt^2}$. Now, if you rearrange $i = C\frac{d^2\psi}{dt^2} + \frac{1}{R}\frac{d\psi}{dt} + \left(\frac{1}{L}\right)\psi$. Now, the force balance equation for translational mechanical system we derived yesterday in the class that we have shown that the force $F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$. Now, if you compare these two equations what we get?

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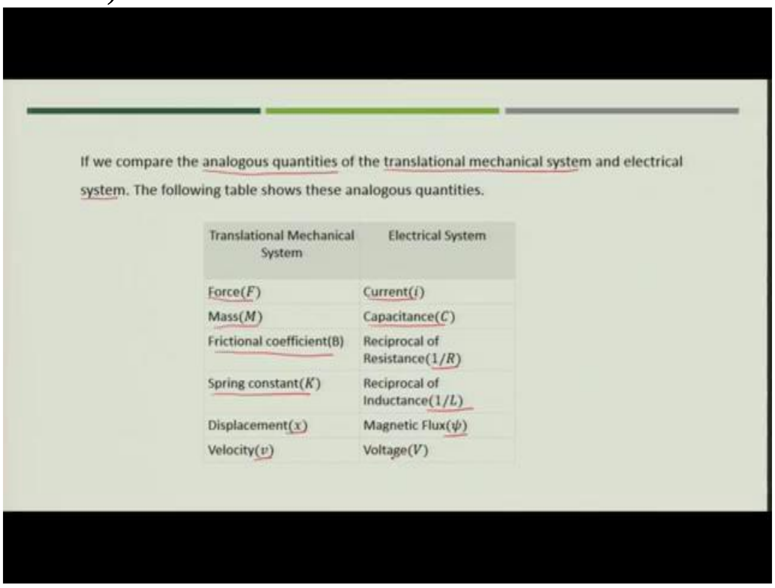

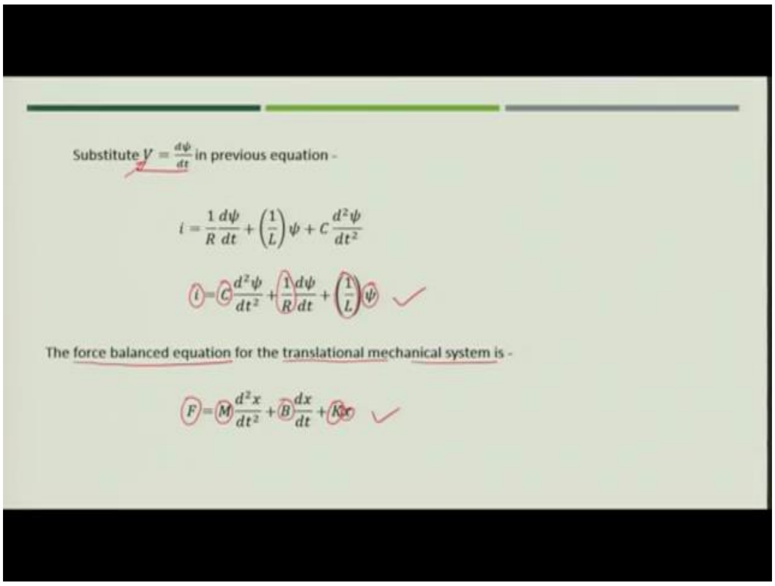

We will get the analogous quantities of the translational mechanical system and the electrical system. So, if you see these two equations you can say that force and current has the analogy M that is mass is analogous to C, B is analogous to $1/R$, K is analogous to $1/L$, x is analogous to ψ and you can also say dx/dt that is velocity v is analogous to $\frac{d\psi}{dt}$ that is nothing but voltage V.

So, this is what we have compiled in the form of table force F is analogous to current, mass analogous to C, B that is frictional coefficient is reciprocal of resistance 1 by R, spring constant K is reciprocal of inductance 1 by L, displacement x is analogous to magnetic flux ψ and velocity v is analogous to voltage V.

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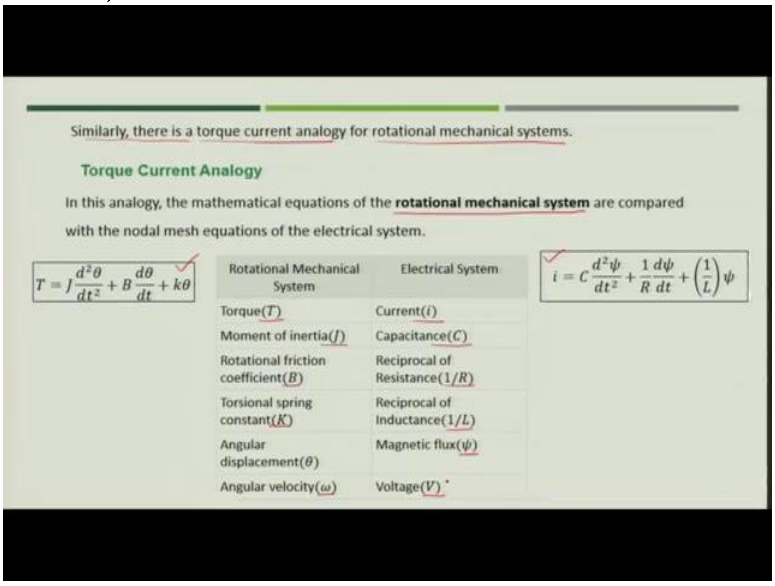

Now, the same concept we can utilize for torque current analogy also that is basically applicable for rotational mechanical system. When we apply torque current analogy then the mathematical equation for rotational mechanical system we get

$$T = T_J + T_B + T_K = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + k\theta$$

and the nodal equation for electrical circuit we just now compiled that is

$$i = C\frac{d^2\psi}{dt^2} + \frac{1}{R}\frac{d\psi}{dt} + \left(\frac{1}{L}\right)\psi$$

Now, if you compare these two equations, you can say that torque is analogous to current i, moment of inertia J is analogous to capacitance, rotational frictional coefficient B is analogous to reciprocal of resistance 1/R, torsional spring constant K is analogous to reciprocal of inductance that is 1/L, angular displacement θ is analogous to magnetic flux ψ, angular velocity ω is now analogous to voltage V. So, this is how we stabilize the analogy between mechanical and the electrical systems.

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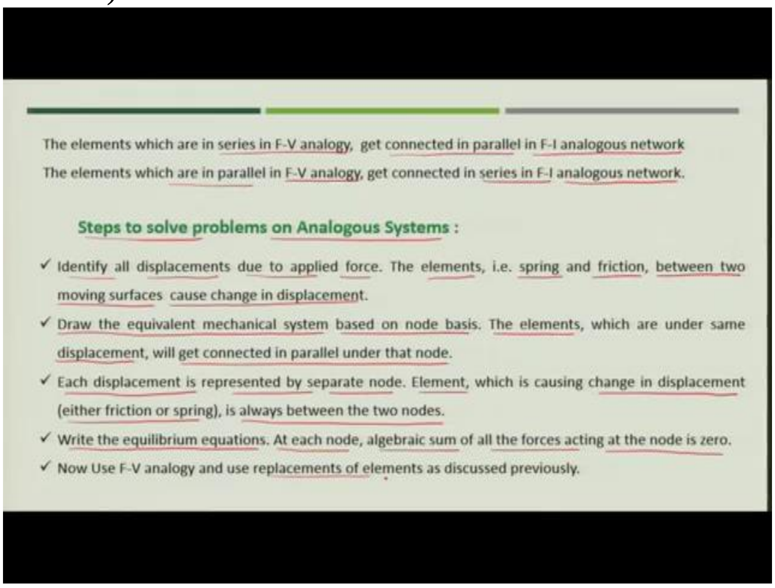

Now, the elements which are in series with force voltage analogy get connected in parallel in force current analogous network. Similarly, the elements which are parallel in force voltage analogy get connected in series in force current analogous system. Now, let us try to understand how to solve the problem on analogous systems. So, what are the various steps that we can follow to solve the problem?

First is the identification of all displacements due to applied force in the mechanical system the elements that is spring of friction and so on which are applied between two moving surfaces will cause the change in the displacement. Now, draw the equivalent mechanical system based on node basis. So, we will rearrange the mechanical system and redraw based on the number of nodes we identify and nodes are nothing but we see the various displacements and based on that we will define the nodes.

Then the elements which are under the same displacement will get connected in parallel under that node. Each displacement is represented by separate node. Element which is causing change in the displacement that is may be either friction or spring is always between the two nodes. Then we write the equilibrium equations. At each node the algebraic sum of all the forces acting at the node will be 0 and then we will use force voltage analogy and use the replacements of the elements what we have just now discussed how to create the analogy.

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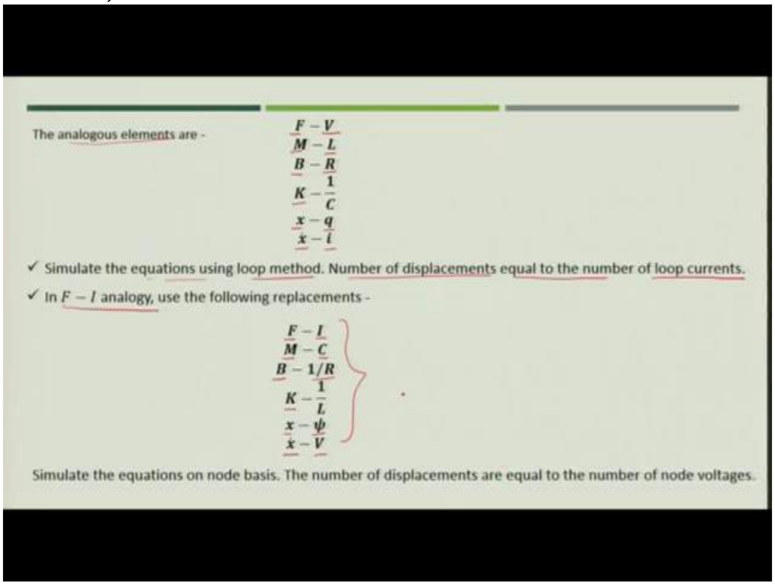

So, what are the analogous elements? So, analogous elements we discussed in the last class and this session also that force is analogous to voltage, M is analogous to inductance, the viscous friction coefficient is analogous to R, spring constant is analogous to 1/C, x is analogous to q, dx/dt is analogous to i. Now, simulate the equations using loop method, the number of displacements will now be equal to number of loop currents in the electrical circuit. If you are using force current analogy then we will use these analogous proper quantities.

This is we just discussed force is analogous to current i, M is analogous to C, B is analogous to 1/R, K is analogous to 1/L, x is analogous to ψ and dx/dt is analogous to voltage V. Now, we convert the mechanical system in two equivalent electrical circuit and then we will solve it.

In case of force current analogy the number of displacements will be equal to number of node voltages.

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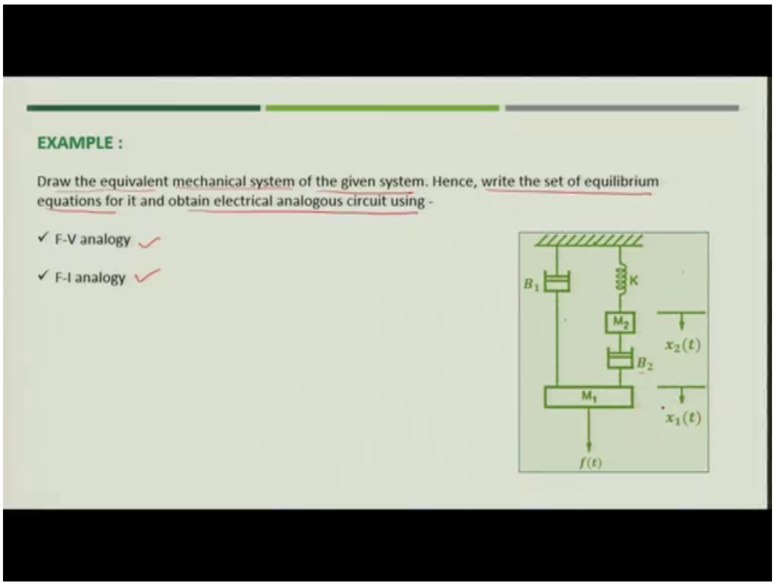

Now, let us try to understand what we have just now discussed with the help of one example. So, in this example we need to draw the equivalent mechanical system of the given circuit, given mechanical system. Now, write the equilibrium equations for it and obtain the electrical analogous circuit using force voltage and force current analogy.

So, if you see this figure we have one dash pot having viscous friction coefficient B_1 which is connected to M_1 and force $f(t)$ is applied and the M_1 is also connected to spring k through M_2 and B_2 as a another viscous friction coefficient and we say that the M_2 is having the displacement of x_2 any time t and M_1 is having displacement at displacement of x_1 at any time t. So, now we have this mechanical system we need to convert it into equivalent analogous electrical system.

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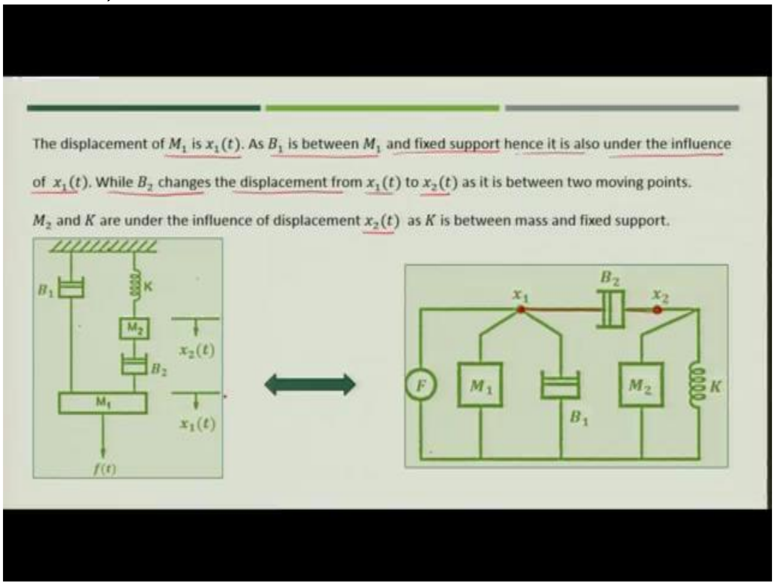

So, what we will do? The displacement of M_1 is $x_1(t)$ and as B_1 is between M_1 and fixed support hence it is also under the influence of $x_1(t)$. While B_2 changes the displacement from $x_1(t)$ to $x_2(t)$ as it is between two moving points., and M_2 And K are under the displacement $x_2(t)$ as K is between mass and fixed support. So, force is now connected to x_1 . Then M_1 and B_1 these are the two elements which are under the influence of the x_1 displacement. So, what we will do, we will connect both of them in parallel. So, M_1 is connected now, B_1 is connected and these all 3 elements are now connected to node x_1. Now, in case of x_2 the B_2 changes the displacement from x1 to x2. So, since B2 is responsible for changing the displacement from x1 to x2, so, we connect B2 between x1 and x2.

Now, M2 and K these are directly under the influence of displacement x2 because K is directly connected to fixed side of the mechanical system and then through K you are connected to M2. So, when you get the displacement in M2, M2 and K would be directly under the influence of displacement x2. So, for x2 we will connect M2 and K in parallel. So, now this mechanical system which we have now we saw in case of the example have been converted into equivalent node representation of the mechanical system.

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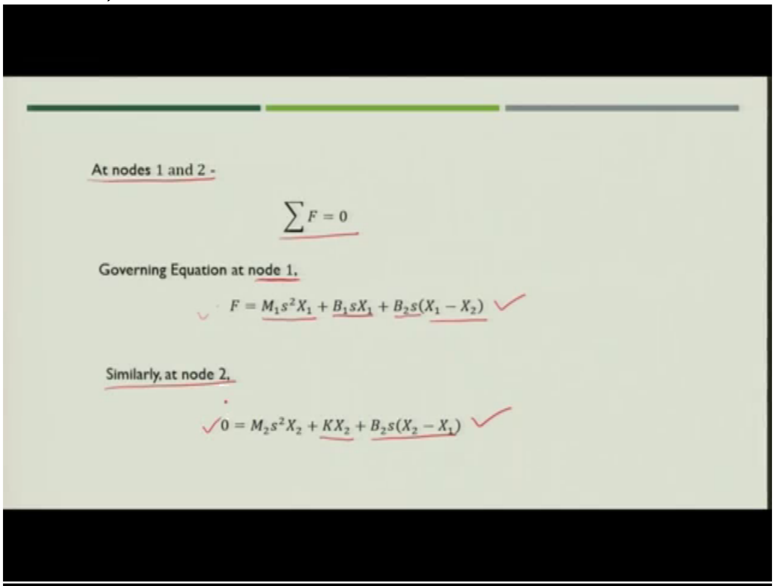

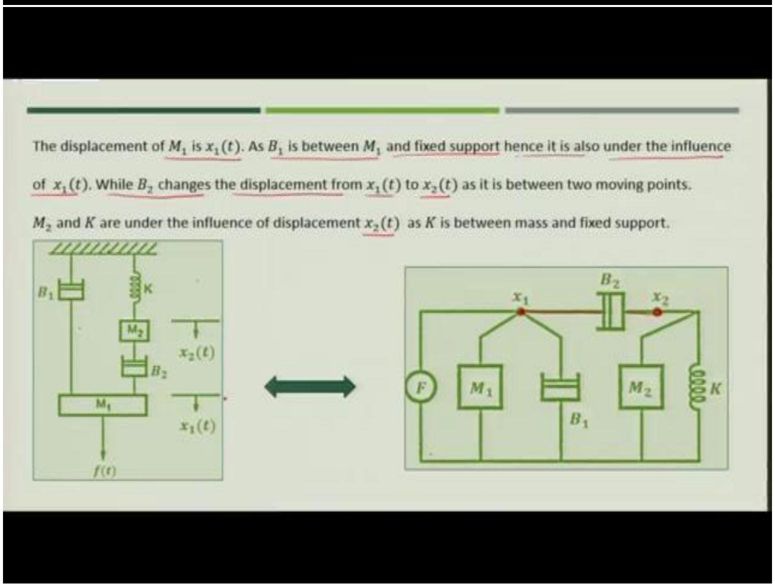

Now, what we will do? We will say that at node 1 and 2 your sum of the forces should be equal to 0. So, in that case for node 1, let us write them into the Laplace domain. So at node 1,

$$F = M_1 s^2 X_1 + B_1 s X_1 + B_2 s(X_1 - X_2)$$

At node 2,

$$0 = M_2 s^2 X_2 + KX_2 + B_2 s(X_2 - X_1)$$

So, now you have got 2 mechanical equations for node 1 and node 2.

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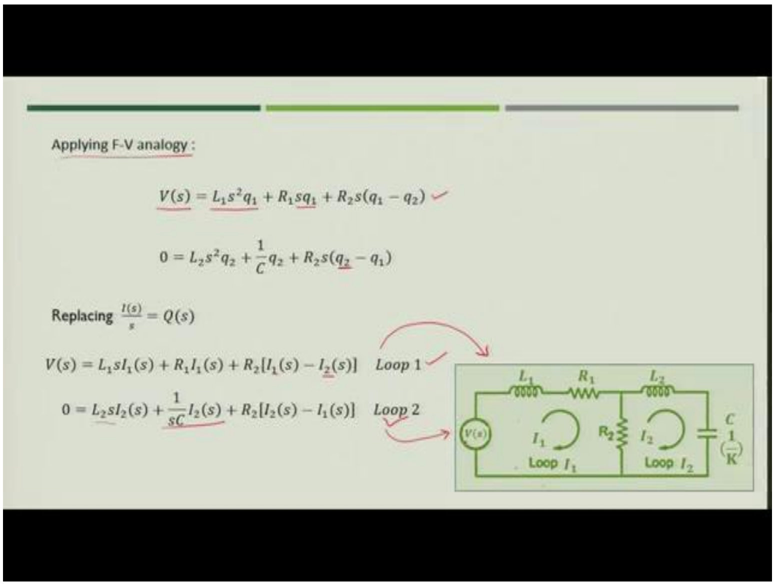

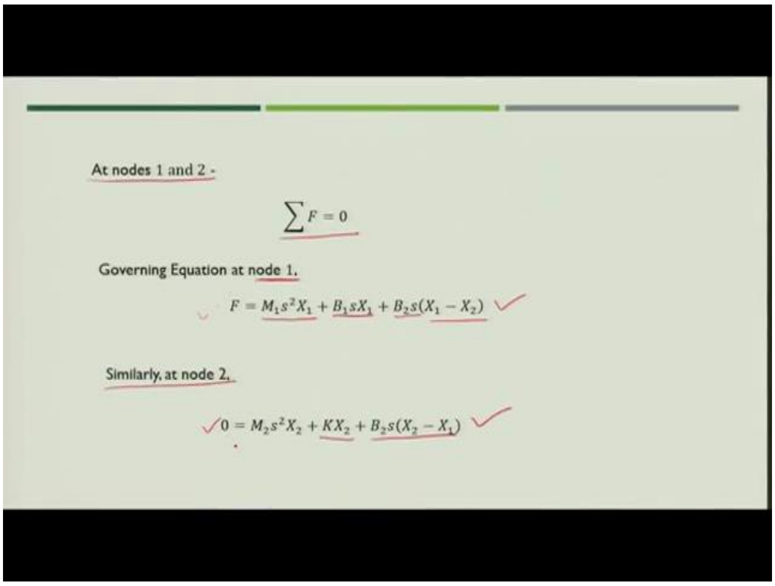

Now, if you use force voltage analogy, you can simply write these two equations in the corresponding analogous voltage equations. So, what we can write, we can write,

$$V(s) = L_1 s^2 q_1 + R_1 s q_1 + R_2 s(q_1 - q_2)$$

$$0 = L_2 s^2 q_2 + \frac{1}{C} q_2 + R_2 s(q_2 - q_1)$$

Now, let us replace $\frac{I(s)}{s} = Q(s)$.

So,

$$V(s) = L_1 s I_1(s) + R_1 I_1(s) + R_2[I_1(s) - I_2(s)] \quad Loop\ 1$$

$$0 = L_2 s I_2(s) + \frac{1}{sC} I_2(s) + R_2[I_2(s) - I_1(s)] \quad Loop\ 2$$

So, using these two equations you can create the equivalent electrical circuit which will represent the mechanical system which was given in the example.

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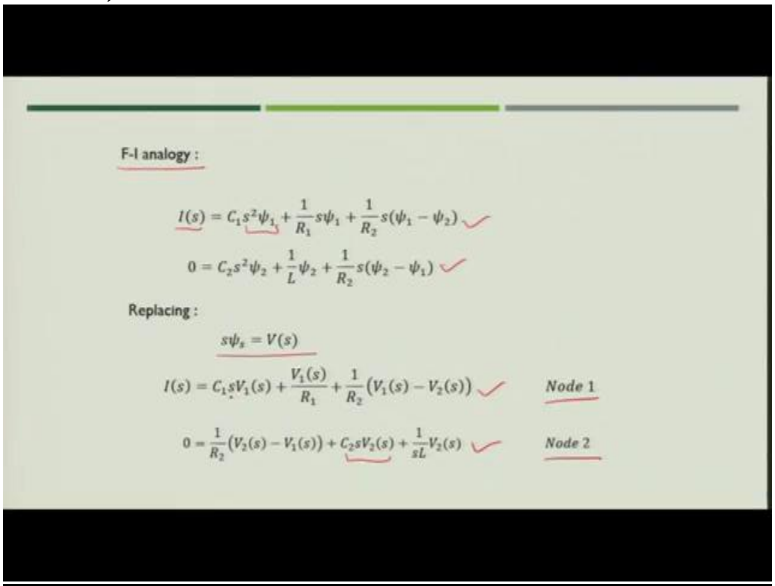

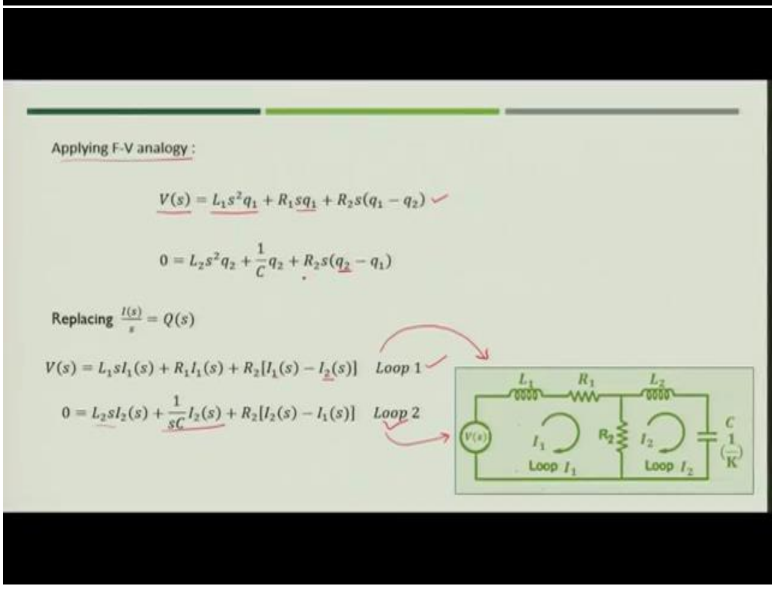

Now, the second case is that you can create the force current analogy also. So, in that case what you can do? You can instead of writing the equation for voltage analogous we will write the equation for current analogous. Then we can write,

$$I(s) = C_1 s^2 \psi_1 + \frac{1}{R_1} s\psi_1 + \frac{1}{R_2} s(\psi_1 - \psi_2)$$

$$0 = C_2 s^2 \psi_2 + \frac{1}{L}\psi_2 + \frac{1}{R_2} s(\psi_2 - \psi_1)$$

So, you have now got the two analogous equations related to the mechanical system. Now, we can replace $s\psi_s = V(s)$. So, for if you put these two in the above equations you can simplify,

$$I(s) = C_1 s V_1(s) + \frac{V_1(s)}{R_1} + \frac{1}{R_2}\big(V_1(s) - V_2(s)\big) \qquad Node\ 1$$

$$0 = \frac{1}{R_2}\big(V_2(s) - V_1(s)\big) + C_2 s V_2(s) + \frac{1}{sL} V_2(s) \qquad Node\ 2$$

So, now you have got two node equations.

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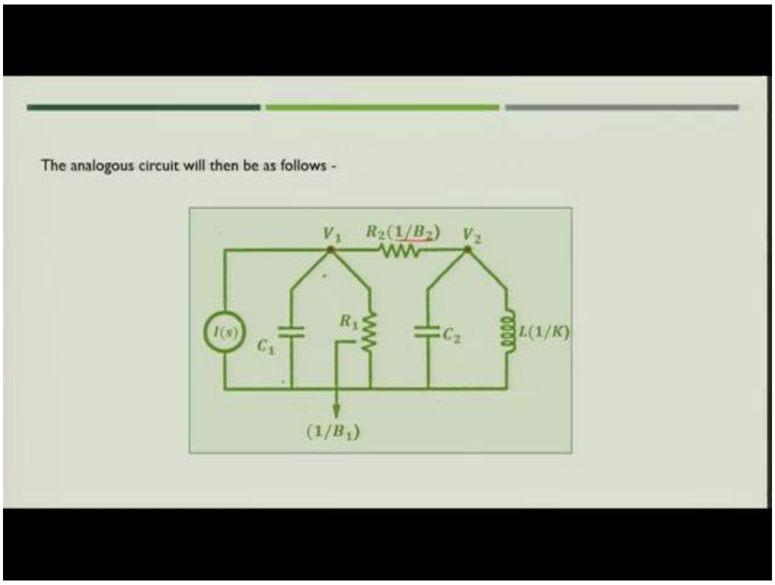

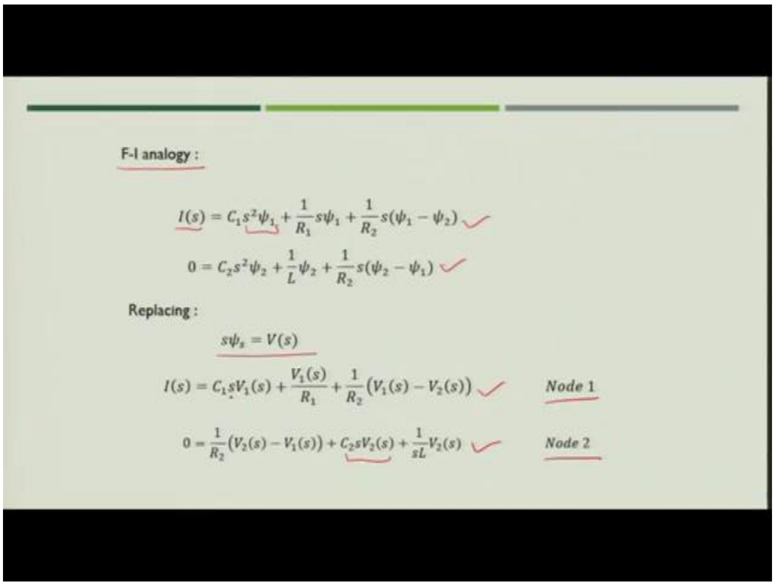

Which is basically can be represented with the help of this particular circuit. So, if you see here you have the C1 and R1 these are in parallel. So, C1 R1 in parallel then you have current source I1 which is now connected to V1, between V1 and V2 you have the resistance that is 1 /R2.

So, you will just represent with 1 by basically this is 1/B2 which is, which you can compare with the mechanical system.

Similarly, for the second code C2 and the L these two are in parallel which you can see from here. So, in this way you can create the node analogous system where you apply instead of voltage source as a current source and then you get the circuit which is analogous to the mechanical system which we saw in the example.

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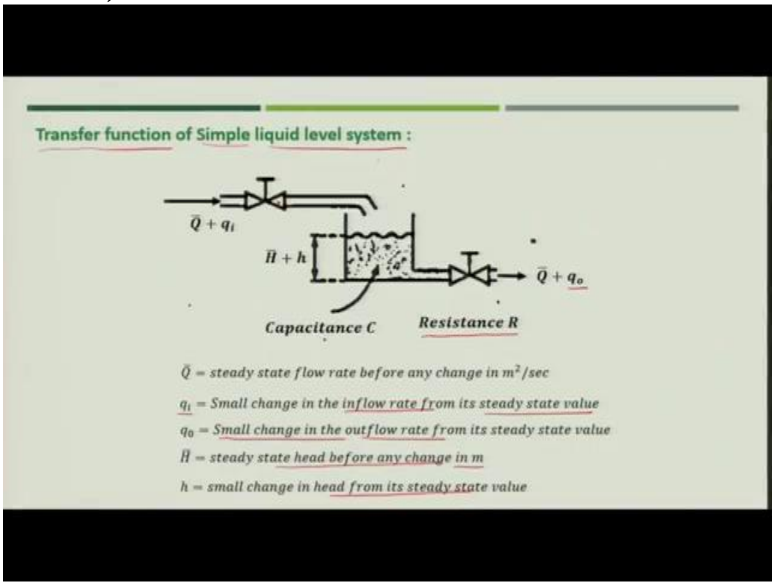

Now, let us see the transfer function development for say simple liquid level system where you have some steady state flow Q which is going inside the tank, the water is flowing and suppose there is a small change in the inflow rate from its steady state value which will cause the small change in the head from a steady state value. So, when you change the inflow rate the head will increase.

So, now head is say example for example you say head is now $\bar{H} + h$, $\bar{H}$ is the steady state head before any change in the flow. Now, that will show that the capacity of the tank will increase because, now flow of the water has changed. The outlet which you can control will have some resistance R. So, when you increase the inflow rate you equivalently change the small outflow rate also. So, suppose if this is q_0 so you will change in inflow as well as change in outflow which is causing change in the capacitance of the tank.

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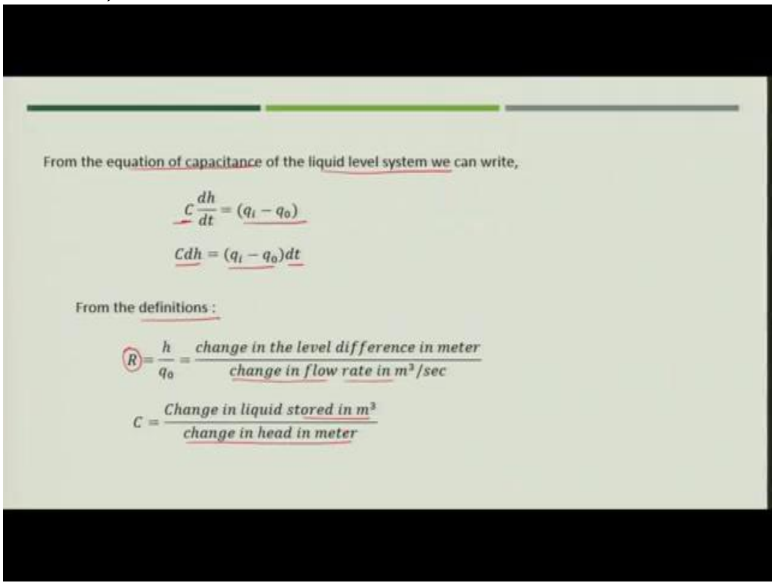

Now, from the equation of capacitance of the liquid level system, we can write that the capacitance of the tank into $\frac{dh}{dt} = q_i - q_0$ or we can write $C\frac{dh}{dt} = (q_i - q_0)$. Now, from the definition what is the resistance $R = \frac{h}{q_0}$ that is change in the level difference in the tank and then change in the flow rate that is in meter cube per second. What is the value of C? C is nothing but change in liquid stored in the tank divided by change in head in the meter.

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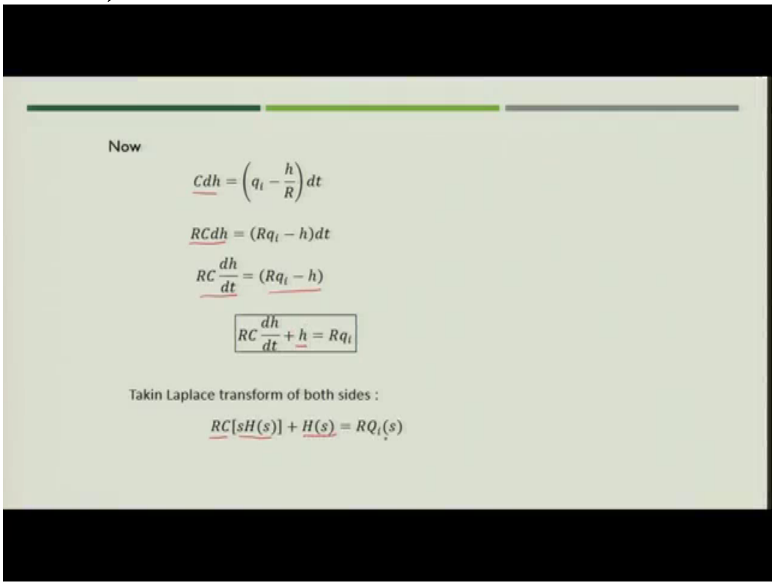

Now, you have all those information in your hand then you can write $Cdh = \left(q_i - \frac{h}{R}\right)dt$. So, what you can write now $RCdh = (Rq_i - h)dt$ or $RC\frac{dh}{dt} = (Rq_i - h)$ or you can say $RC\frac{dh}{dt} + h = Rq_i$. Now, if you take the Laplace transform then it will become,

$$RC[sH(s)] + H(s) = RQ_i(s)$$

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Neglect the initial conditions.

For q_i as the input and h as the output, the transfer function for the system is $\frac{H(s)}{Q_i(s)}$. Hence,

$$\frac{H(s)}{Q_i(s)} = \frac{R}{1 + sRC}$$

If q_o is the output of the system, then we can write :

$$Q_o(s) = \frac{H(s)}{R}$$

$$\boxed{\frac{Q_o(s)}{Q_i(s)} = \frac{1}{1 + sRC}}$$

Now

$$Cdh = \left(q_i - \frac{h}{R}\right)dt$$

$$RCdh = (Rq_i - h)dt$$

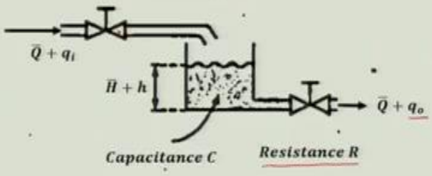
$$RC\frac{dh}{dt} = (Rq_i - h)$$

$$\boxed{RC\frac{dh}{dt} + h = Rq_i}$$

Takin Laplace transform of both sides :

$$RC[sH(s)] + H(s) = RQ_i(s)$$

Transfer function of Simple liquid level system :



$\bar{Q}$ = steady state flow rate before any change in m^2/sec

q_i = Small change in the inflow rate from its steady state value

q_o = Small change in the outflow rate from its steady state value

$\bar{H}$ = steady state head before any change in m

h = small change in head from its steady state value

Now, if you neglect the initial conditions and we want to find out the transfer function. So, for q_i as the input and h as the output, the transfer function for the system is $\frac{H(s)}{Q_i(s)}$. Now, if you use the previous equation you can simply find out the value of

$$\frac{H(s)}{Q_i(s)} = \frac{R}{1 + sRC}$$

So, in this way you can easily find the transfer function of the system. Now, in case instead of h you are taking q naught as an output and you know $Q_0(s) = \frac{H(s)}{R}$.

So, in that case $\frac{Q_0(s)}{Q_i(s)} = \frac{1}{1+sRC}$. So, in this way the given system that is liquid level system can be equivalently represented with the help of the transfer functions. So, with this we can close our today's session and of course the closer of our course. So, let us recap what we discuss in this course.

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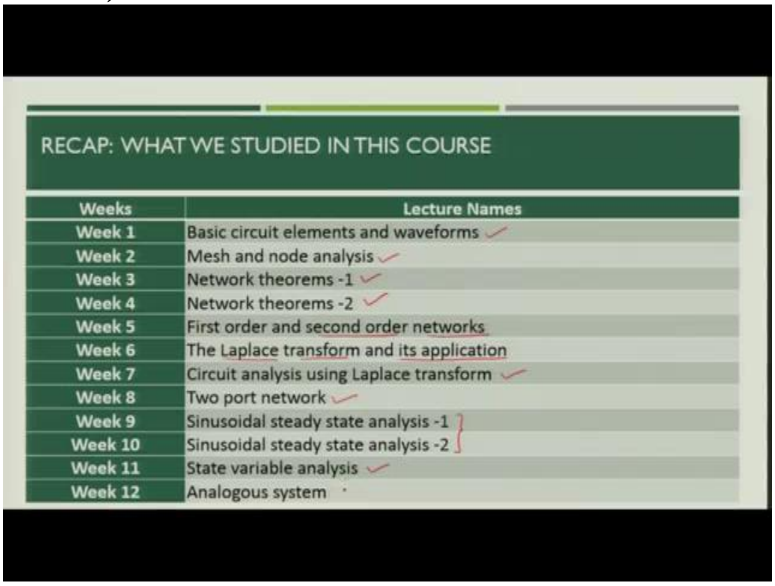

So, basically, we started our discussion from week 1 where we discussed about the basic circuit elements and waveforms. There we discussed the relationship and found the way to calculate the value of power P, then we studied mesh and nodal analysis then we discussed about the various network theorems in two weeks there we discussed about the Thevenin's, Norton's, Maximum power transfer theorem and the super position theorem and so on.

So, these two weeks we devoted related to the network theorems. Then in 5th week we discussed about the first order and second order and networks then sixth week we discussed the Laplace transform and its application then we used the knowledge we acquired in the sixth week and applied the knowledge of the Laplace transform in the circuit analysis using Laplace transform in week 7.

Then we discussed two port network in week 8. Week 9 and 10 we focused on the AC circuit analysis where the network theorems which we discussed in case of DC circuit we discussed that in view of the AC circuit and we try to solve few examples, so that we can understand the network theorems with respect to the AC circuits.

Then in week 11 we discuss about the state variable analysis and we try to solve couple of examples related to electrical circuit with the help of state variable analysis and this week we discussed about the analogous system where we stabilize the relationship between various mechanical quantities with respect to the electrical quantities.

So, I hope you have enjoyed this course and you have got sufficient knowledge related to the basic electrical circuit and I hope you will do well in your exams also, so all the best for your exam and thank you.