Basic Electric Circuits
Module II
Mesh and Nodal Analysis
Lecture-06
RMS Voltage and Current
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Namaskar, so we are entering into the second week of this course. In this week we will try to understand the mesh and nodal analysis of various electrical circuits. Today we will specifically discuss about the RMS voltage and current values and how to derive these values.

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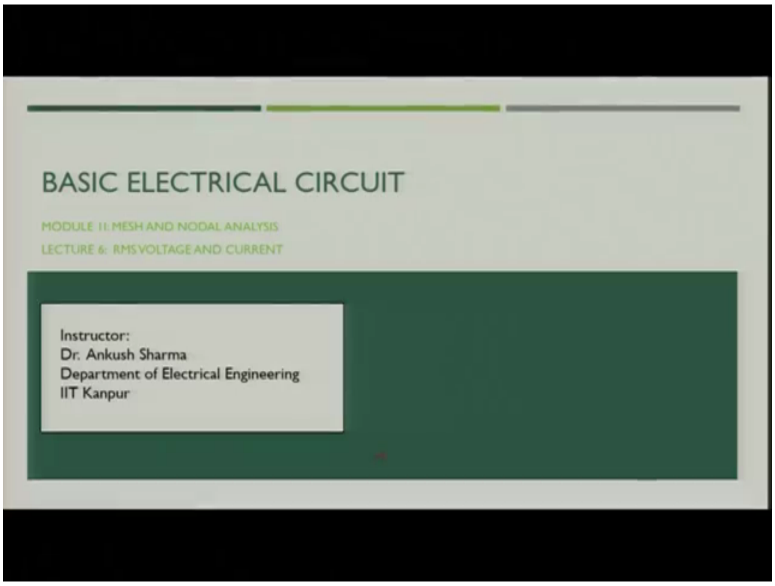

And how we can use RMS values in finding out the power output of a particular circuit?

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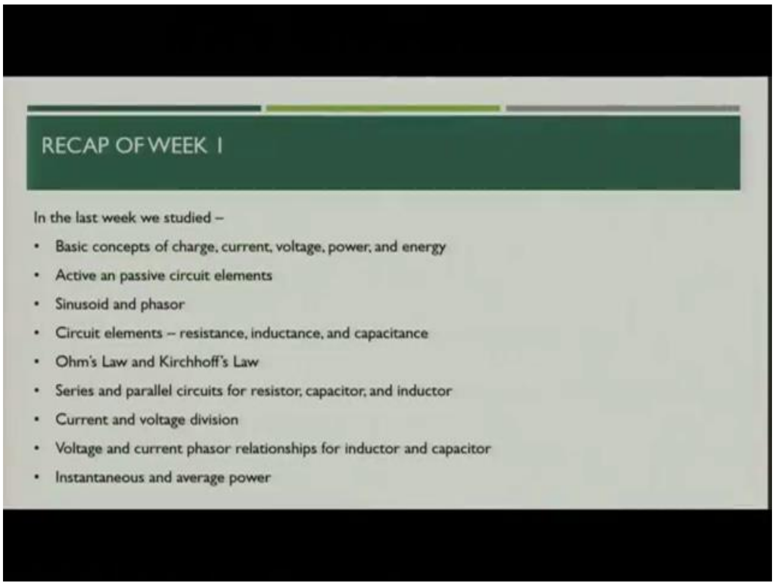

So before going into the details let us recap what we learnt in the previous week. We started with the basic concepts of charge then we spoke about the current, voltage, power and energy. Then we discussed about active and passive circuit elements where we discussed about the voltage and current sources. We also saw what are the various symbols of dependent as well as independent voltage and current sources.

Then we discussed about sinusoid and phasor, how can we find out the value of phasor with the help of given sinusoid. Then we discussed circuit elements that is resistance, inductance and capacitance. Thereafter we discussed some laws and Kirchhoff law which are the very basic and very important laws for the calculation of various current and voltage values in a given electrical circuit, so most of the time you would be using either ohms law or the Kirchhoff voltage or current law in your circuit analysis.

Then we discussed about Series and parallel circuits for resistor, capacitor and inductor. We also discussed current and voltage division then we discussed voltage and current phase relationships for inductor and capacitor and then we studied the instantaneous and average power calculation.

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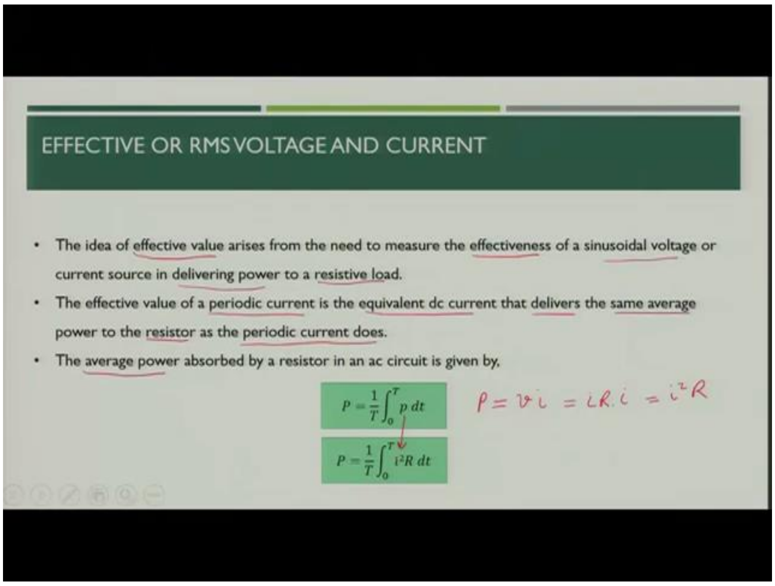

So today we will discuss the concept of root mean square voltage and current which is also called as effective voltage or current. Now the idea of effective value arises from the need to measure the effectiveness of a sinusoidal voltage or current source in delivering power to a resistive load.

The effective value means the value for a periodic current that is equivalent to that the DC source which delivers the same average power to the resistor as the periodic current does. It means that to find out the effective value of current we have to equate the power dissipated by an element when it is connected with AC source as well as the DC source. So, in that case when we connect this particular circuit to AC source and DC source the power should be equal in both of the cases, so we will equate the power consumed by the element when it is connected to AC and DC and try to find out what should be the value of effective current.

Similarly, you can say that if you connect the AC voltage source then how power would be dissipated by that particular element and when we equate the power for DC and AC we get the value of effective voltage. We will use that concept and try to find out what is the value of effective voltage and current. So now let's start with the average power definition.

$$P = \frac{1}{T}\int_0^T i^2R\,dt$$

Suppose you take a particular element, say let's take the resistor and it is connected to one AC source and then one DC source with a switch, so you can toggle that switch between AC and

DC, so first if you’re connecting to AC source, in that case what would be the average power? Average power would be given by the above equation.

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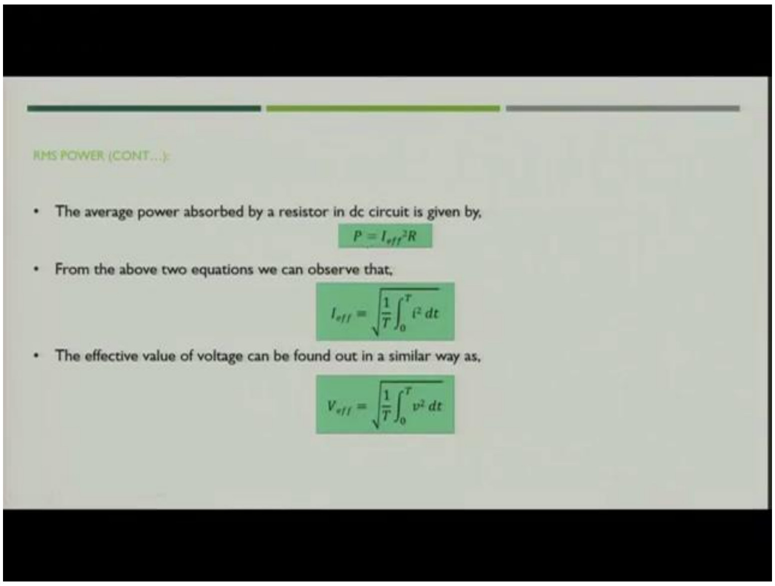

Now instead of AC if you’re connecting that element to DC, you will get another value of average power that would be absorbed by a particular element in this case we have the resistor but the source is DC, so we have some power dissipated by the resistor would be

$$P = I_{rms}^2 R$$

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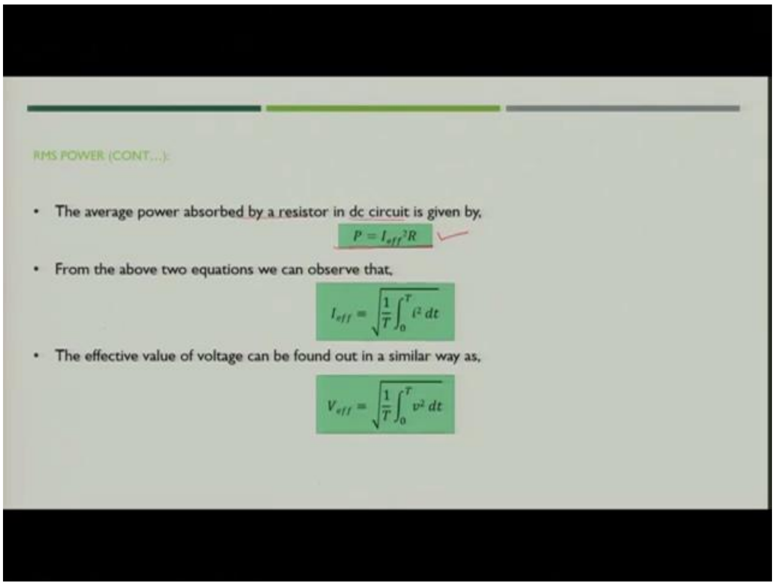

Now as per definition, our objective is to find out the effective value for the current it means that the power value should be equated when the DC current is flowing and when the AC current is flowing.

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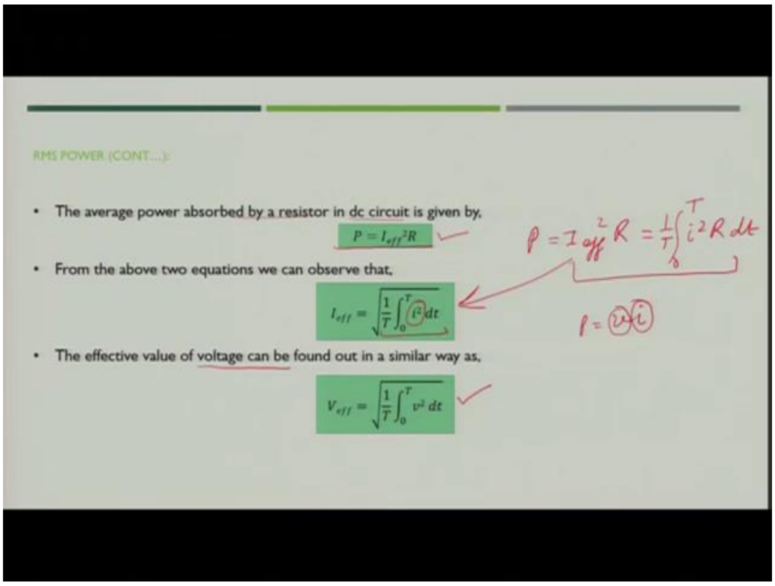

So, in that case what we can say, we can say P which is

$$P = I_{rms}^2 R = \frac{1}{T}\int_0^T i^2 R\,dt$$

So now if you see this particular equation using mathematical operations you can simply say

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}$$

Hence, I_{rms} is the current value which is nothing but root of the mean of the square values.

So, i^2 is the square value, so you take the integral means summation of all i^2 component and then you take the mean and then you take the under root of the complete output of this particular equation, so I_{rms} is nothing but root of mean square value of the current element.

Similarly, the effective value for voltage can also be written. You have to just simply replace the current i when we are saying $P = vi$, so instead of writing the value of v in terms of i we will write the value of i in terms of v and you will get this equation, so we can say

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\, dt}$$

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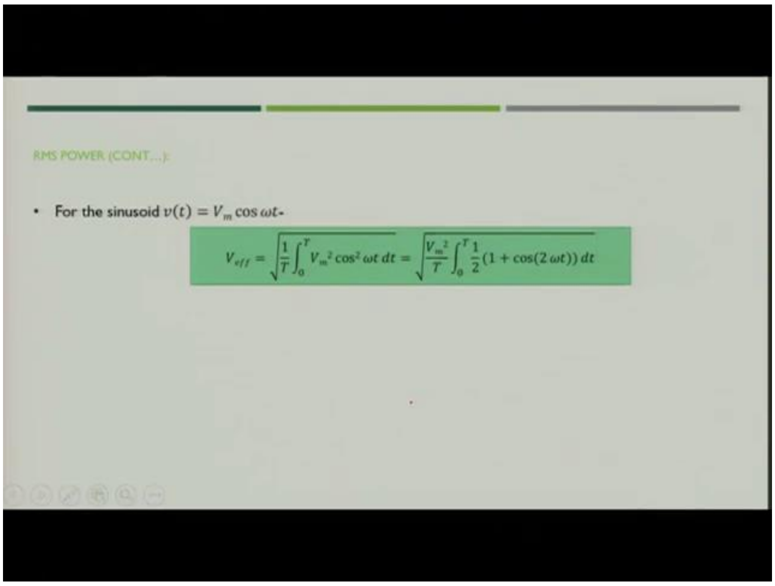

Now we know that the sinusoid

$$v(t) = V_m cos\omega t$$

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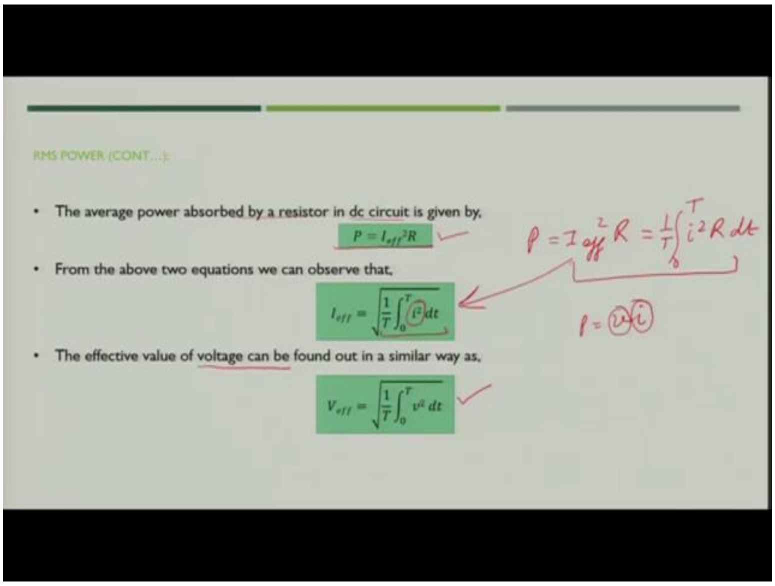

Now if you put this value of v because this is the instantaneous voltage and it is represented in the form of sinusoid.

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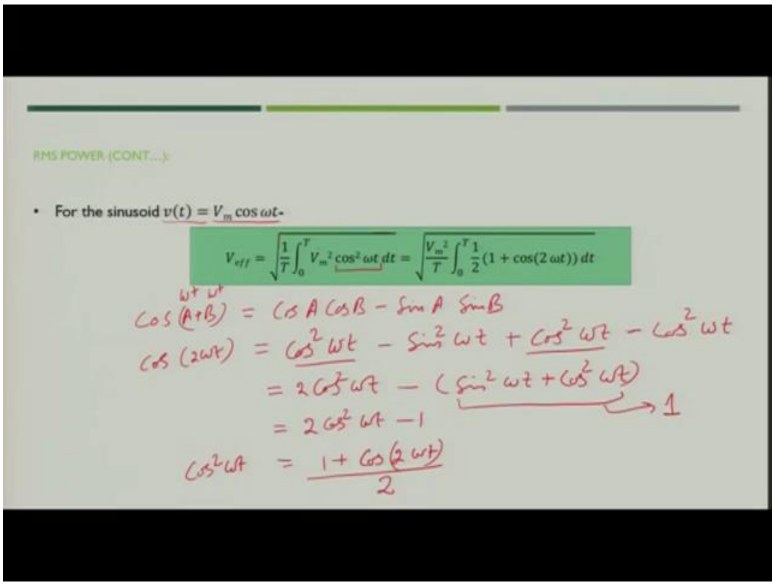

So, you will write

$$Vrms = \sqrt{\frac{1}{T}\int_{0}^{T} V_m^2 cos^2\omega t dt}$$

Now if you see this particular term this is nothing but, if you recollect we discussed

$$cos(A + B) = cosAcosB - sinAsinB$$

If $A = B = \omega t$, so this will become $cos2\omega t$ and that will become

$$cos^2\omega t - sin^2\omega t = 2cos^2\omega t - 1.$$

Now if you rearrange, what you will get

$$cos^2\omega t = (1 + cos2\omega t)/2$$

so, this value which we have just derived, you can put in place of

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 cos2\omega t dt} = \sqrt{\frac{V_m^2}{T}\int_0^T (1 + cos(2\omega t)\} dt}$$

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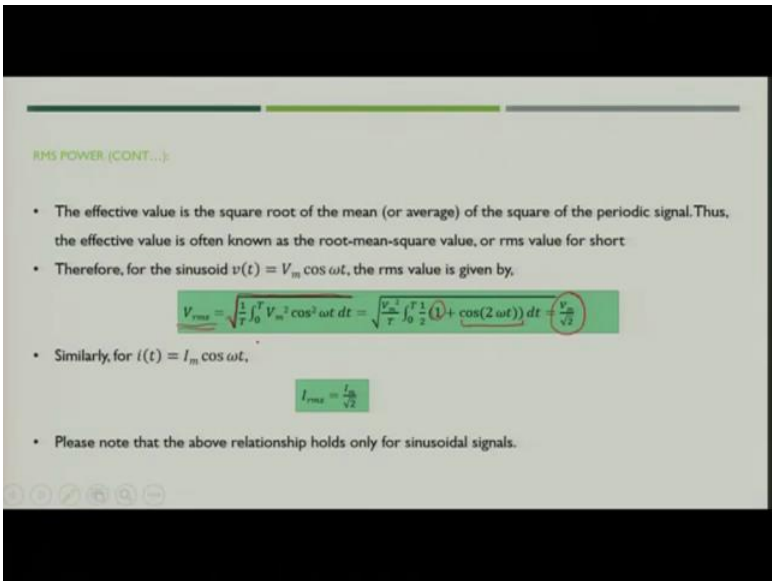

Now if you see this particular expression what you will get. Since this is a sinusoidally varying term, so the value of this term over one particular time period that is between 0 to T will become 0, so eventually what we have left with this only term 1. So, if you simplify just by integrating dt this particular constant term with dt you will get the output as $\frac{Vm}{\sqrt{2}}$ which is the rms value of voltage.

$$Vrms = \sqrt{\frac{1}{T}\int_0^T Vm2\cos 2\,\omega t\,dt} = \sqrt{\frac{Vm2}{T}\int_0^T (1 + \cos(2\,\omega t))\,dt} = \frac{Vm}{\sqrt{2}}$$

Why it is rms? Because from the initial convention we have seen that this particular value is nothing but root of square of the mean value.

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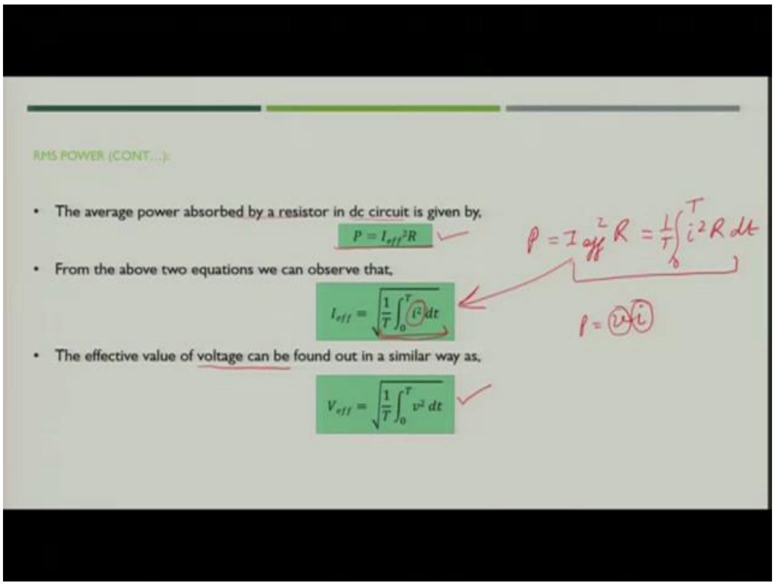

So here square value is there to sum up and take the mean and take the under root of the term.

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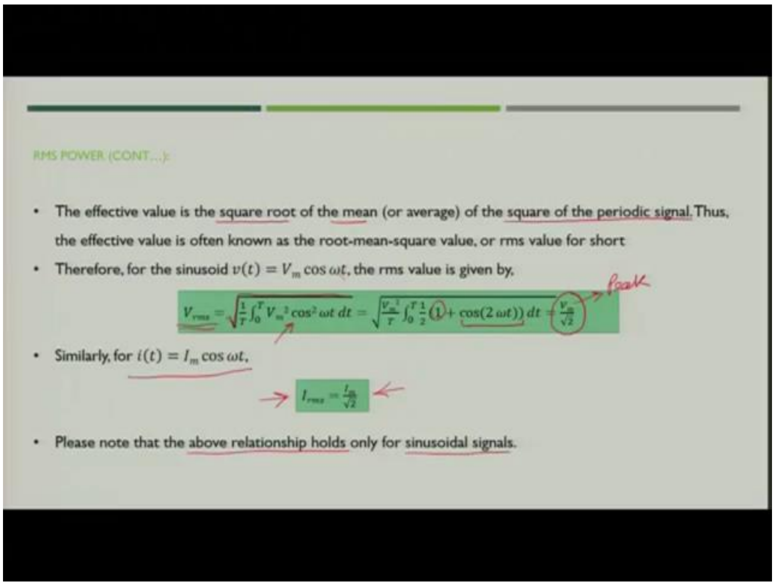

So, this will become the root mean square value that means the square root of the mean of the square of the periodic signal. In this case this periodic signal is V, so the output which we get from here is Vrms and this value is equal to $\frac{V_m}{\sqrt{2}}$ and Vm is the peak value of the signal. Similarly, if you take current that is it and we represent it in terms of sinusoid as $i(t) = I_m \cos \omega t$ we will get the rms value of current that is Irms equal to $\frac{I_m}{\sqrt{2}}$.

Now you have to note the important point here that this particular value is applicable only for sinusoidal signals. If you do not have sinusoid signal it means that you have to compute from the integral and then calculate the complete rms value with the help of the original function which we just used for derivation of rms value.

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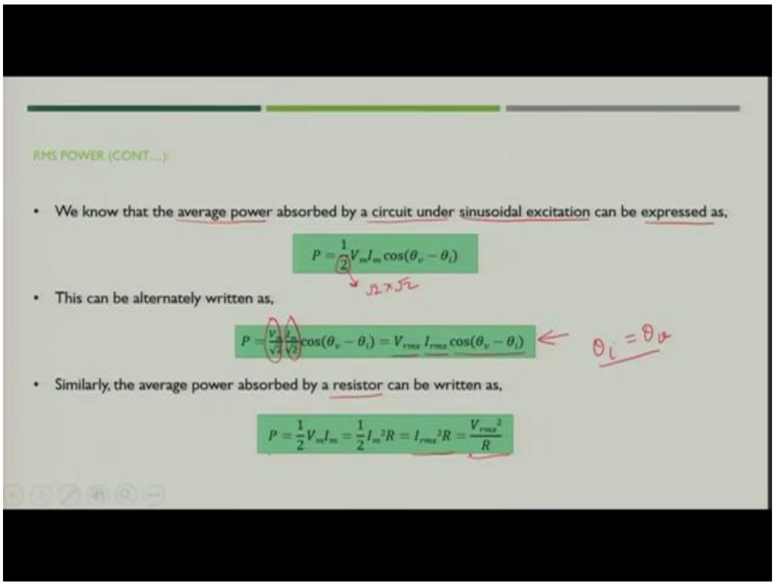

So now from the previous lectures we can recollect that the average power absorbed by a circuit under sinusoidal excitation can be represented as

$$P = \frac{1}{2} VmIm \cos(\theta v - \theta i)$$

Now how can you represent it in the form of rms values.

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) = V_{rms}\, I_{rms} cos(\theta_v - \theta_i)$$

Now this is the rms value which we just derived and similarly im by root 2 is also the rms current we just saw. So average power you can write as $V_{rms}\, I_{rms} cos(\theta_v - \theta_i)$. Now, this is the general expression, if the circuit element is purely resistive it means that the because your current and voltage both phasor will be in phase, then $\theta_v - \theta_i = 0$ and finally the average power absorbed by the resistor would be

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

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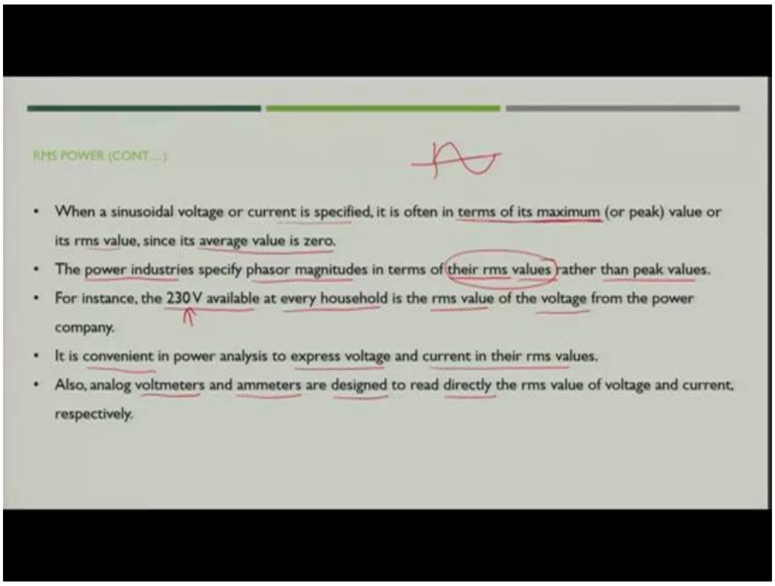

So, when the sinusoidal voltage or current is specified it is often specified in terms of its maximum value or the rms value because its average value will always be 0. Being a sinusoidal you will always get the average value of either voltage or current as 0, so that’s why it is not advisable to keep the value of voltage and current in terms of average value, that’s why you will always see either the value of voltage and current is specified as maximum or rms value.

But in our day-to-day life in the power industries specify phasor magnitude in terms of their rms value rather than the peak values, so that’s why the rms term is very common. It is mostly used in almost all common equations when we talk about the various voltage and current signals in the electrical engineering or in the power system area. Say for example if you see to 230 volts which is coming to every household this is rms value now to the peak value.

So, whenever we say that the voltage in our house is 230 volts it implies that you are talking about the rms value not the peak value. It is convenient in power analysis to express voltage and current in their rms value because it is more convenient to do the calculations and the power calculations which is discussed is also easy when we use the rms value. And our other measuring instruments like voltmeter and ammeter are designed to read the rms value directly, so that’s why the rms value is very important for our circuit analysis.

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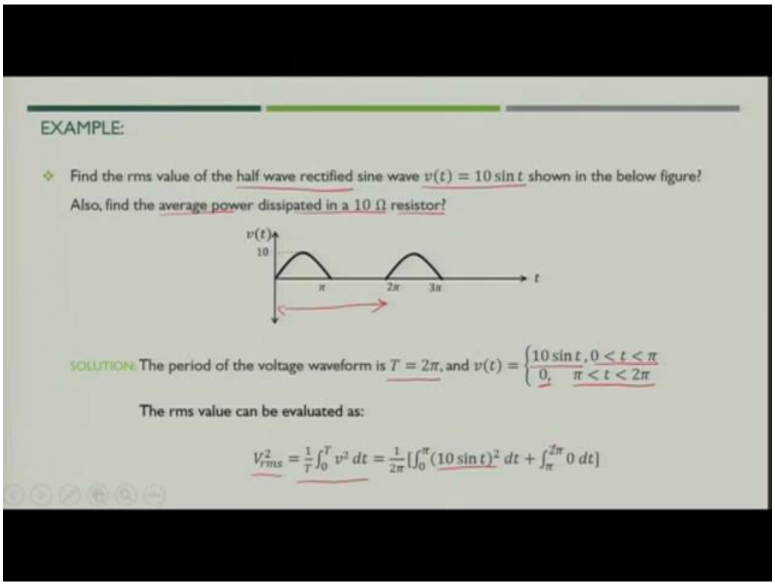

Now let's take one example, so that you can understand the phenomena. Now let's see that there is some voltage $v(t) = 10\sin t$ which is shown in this figure. It is half wave rectified, means the negative value is not there and we want to find the average power dissipated in 10 ohms resistor. So, if you see this figure you will come to know that the time period of this particular waveform is also 2π and the voltage is defined as

$$v(t) = \begin{cases} 10\sin t\,, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

So how you will calculate the rms value? Since it is not a pure sinusoidal you cannot directly use Vrms is equal to Vm by root 2 you have to calculate with the help of the standard definition of root mean square, so

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\,dt$$

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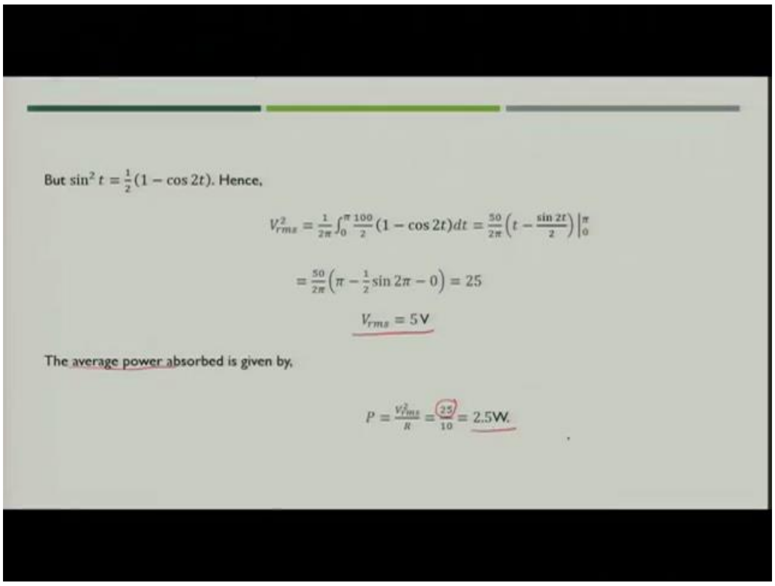

So, when you calculate simplify tis expression, you will get rms value of voltage as 5 volts. So the average power absorbed by the resistor you can say that P is nothing but rms square by R, so here rms value which we derived is 5 volts, so Vrms square is 25, resistor value is 10, so output would be 2.5 watt.

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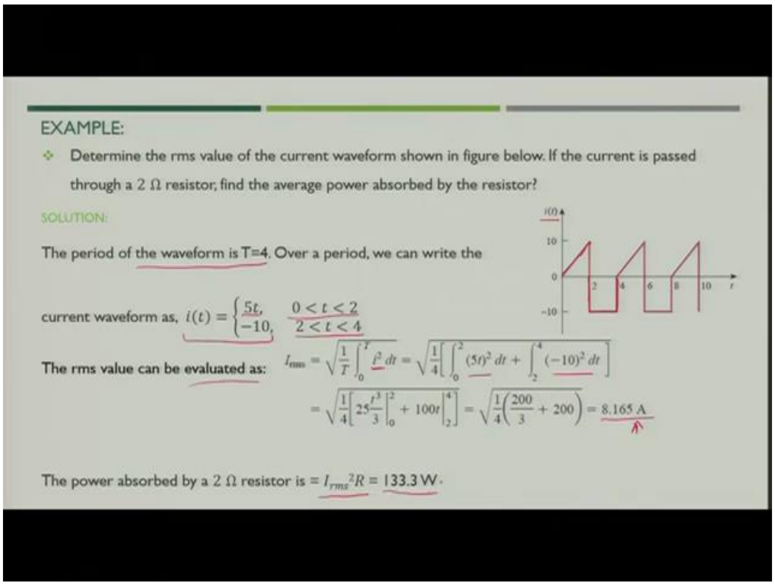

Let’s take another example suppose the signal is like this which is shown in the figure between 0 to 2 it is regular wave and between 2 to 4 days square wave, so the period of the wave form is 4 and waveform can be defined as because we are talking about the current

$$i(t) = \begin{cases} 5\text{t}, & 0 < t < 2 \\ \text{-10}, & 2 < t < 4 \end{cases}$$

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2\,dt = \frac{1}{4}\left[\int_0^2 (5\text{tc}\,dt + \int_2^4 (-10)^2\,dt\right]$$

In that case when you simplify this integral, you will get the output of Irms as 8.165 ampere. So, when you get this current can simply put the value in the average power P that is nothing but Irms square of and you will get the power absorbed by the resistor that is 133.3 watt.

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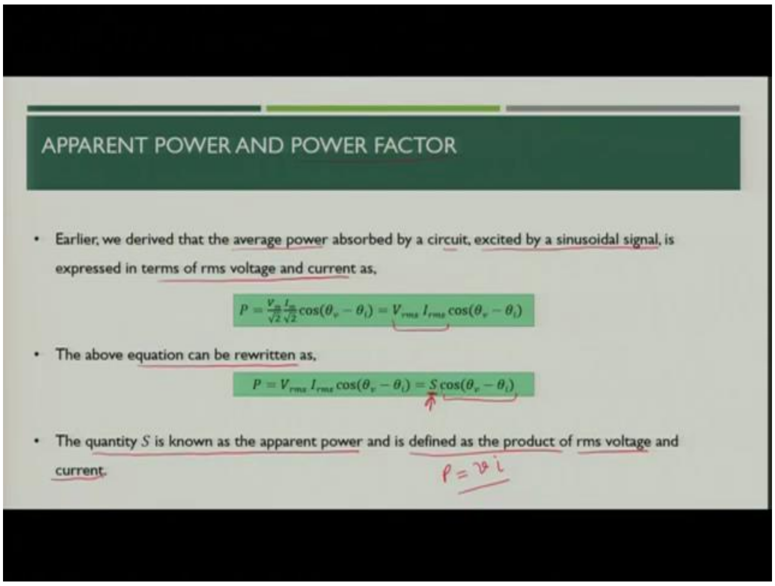

Now let's talk about another term called apparent power and the power factor. So earlier what we derived was the average power absorbed by the circuit which is excited by a sinusoidal signal and we express them in the form of voltage rms voltage and current. So, power

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i) = V_{rms}\, I_{rms} cos(\theta_v - \theta_i)$$

So, what you can do, you can rewrite the above expression as

$$P = S\cos(\theta_v - \theta_i)$$

What is S? S is called as apparent power because it is defined as a product of rms voltage and current. So, if you corelate with the DC voltage source power you see that in DC we define power as v into i, so if you correlate the apparent power would be the Irms into Vrms, so that you get a feel of like this is apparent as compared with the DC source.

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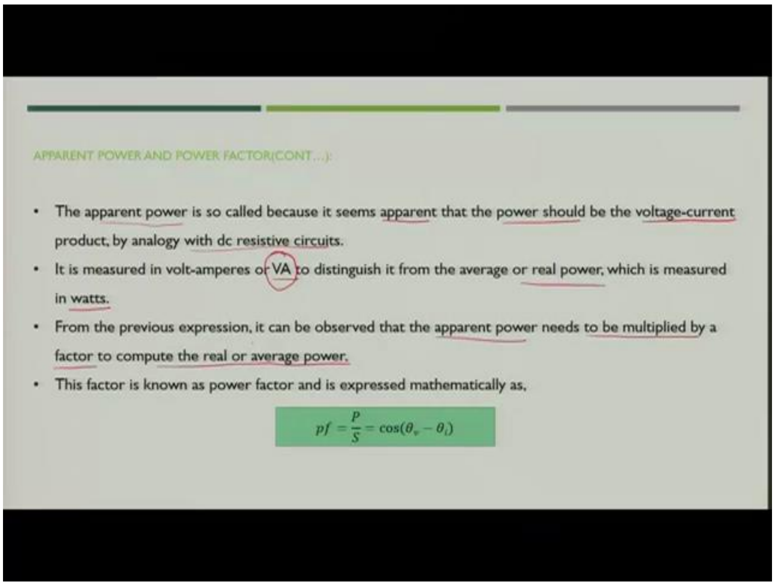

So, it is apparent power because it seems apparent that the power should be the voltage current product which is by analogy with the DC resistive circuit. Now the apparent power is measured in volts ampere just to distinguish it from the real power because we say real power in terms of watts. So just to differentiate between apparent power and the real power we use volt-ampere as a quantity for apparent power and watt as quantity for real power.

Now from the previous expression you can also observe that apparent power needs to be multiplied by a factor to compute the real or average power.

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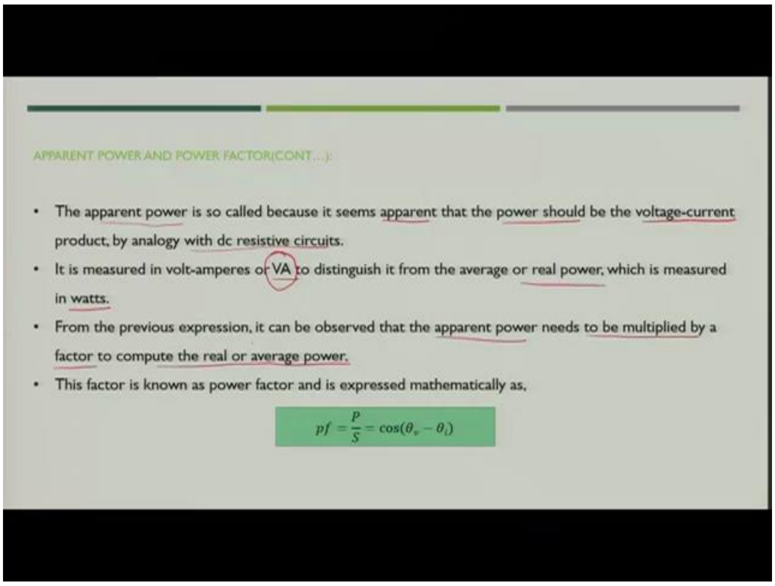

So what was the term? That term was $\cos(\theta_v - \theta_i)$.

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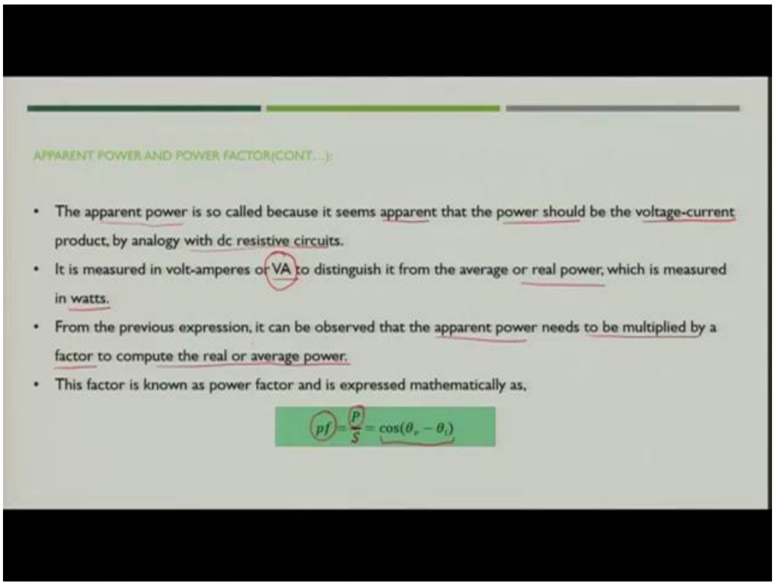

So,

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

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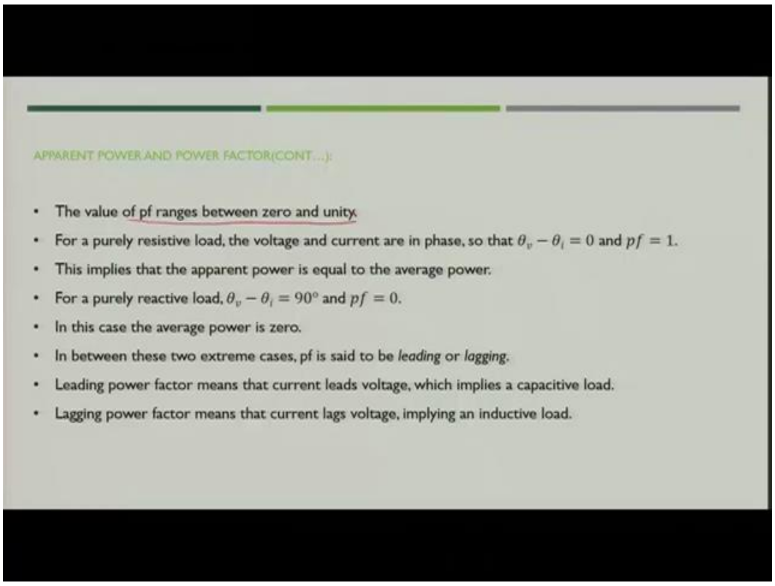

Now this power factor ranges between 0 to unity. So, if you have the purely resistive load, the voltage and current would be in phase in that case $\theta_v - \theta_i = 0$. So, what will happen?

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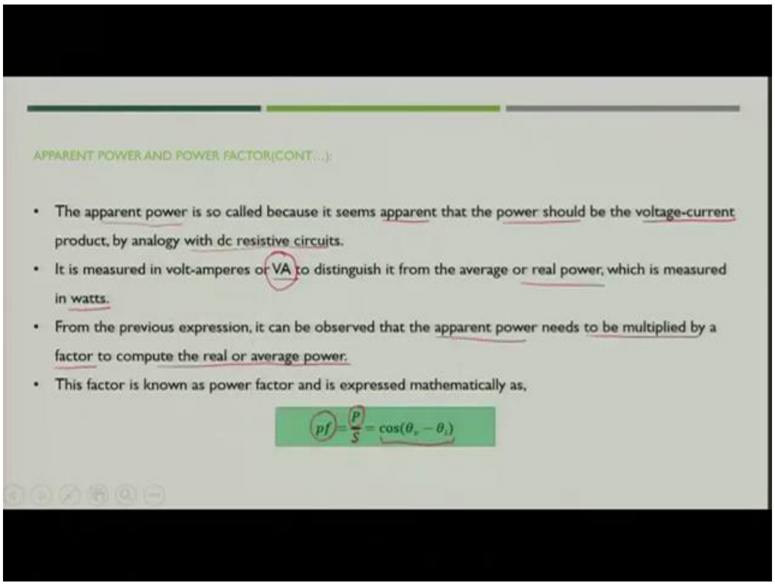

In that case the value of $\cos(\theta_v - \theta_i) = 1$.

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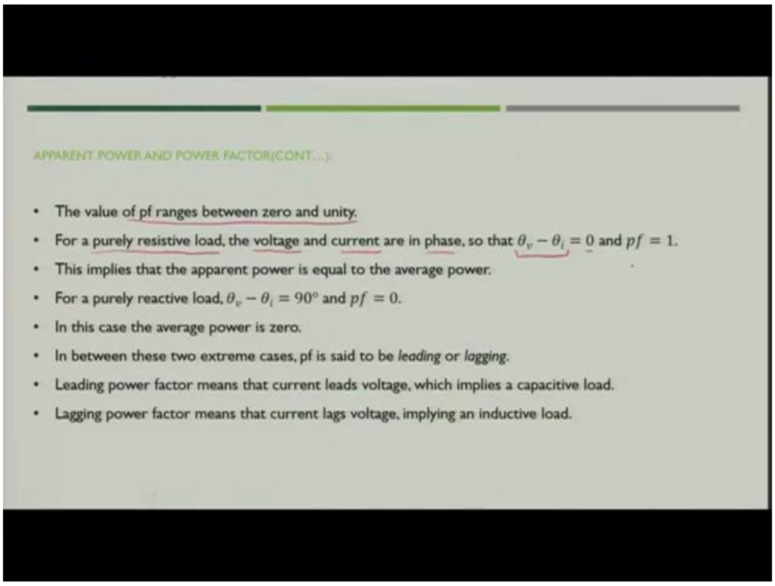

So the power factor would be 1. So that implies that apparent power is equal to the average power in this case. In case of purely reactive load when $\theta_v - \theta_i = 90$ degree, so that can be either inductive load or the capacitive load, the power factor would be 0. So in this case your average power would be 0. So, these are the 2 extreme cases in which power factor is said to be either leading or lagging.

Leading power factor means that currently leads voltage which implies that it is having a capacitive load profile in case of lagging power factor it means that current lags voltage and that implies an inductive load. So, this particular aspect you will keep on using in your circuit analysis because the circuit will compose of resistor, capacitor, inductor all components would be there, so when you will say that current is leading with respect to voltage it means that the circuit is capacitive or even the current is lagging with respect to voltage you will say the circuit is inductive. So, in that case when you have capacitive circuit you will have leading power factor when you have inductive circuit when you will have lagging power factor.

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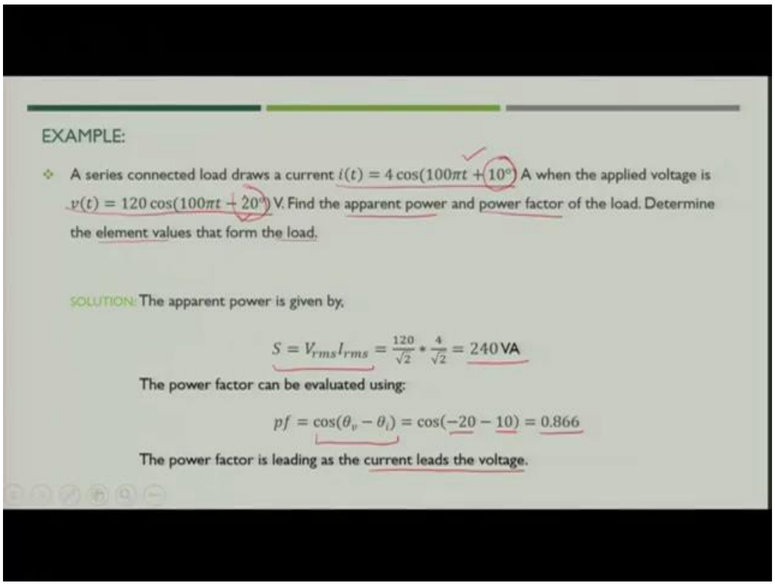

Now let's take one example. Suppose a series connected load draws current given by this expression $i(t) = 4\cos(100\pi t + 10°)$ when they apply a voltage is $v(t) = 120\cos(100\pi t - 20°)$. Now we have to find out the apparent power and power factor of a load and determine the value of element from the load. Now the apparent power would be

$$S = V_{rms}I_{rms} = \frac{120}{\sqrt{2}} * \frac{4}{\sqrt{2}} = 240 \text{ VA}.$$

Now power factor you can simply calculate with the help of these 2 expressions because power factor is nothing but $\cos(\theta_v - \theta_i)$. Here $\theta_v = -20$ and $\theta_i = 10$, so you will get power factor as cos(-30) = 0.866. Here power factor is leading because current leads the voltage. Here the phase angle is positive and for voltage the phase angle is negative that you can easily visualize that current is leading with respect to voltage, so it means that the circuit is capacitive

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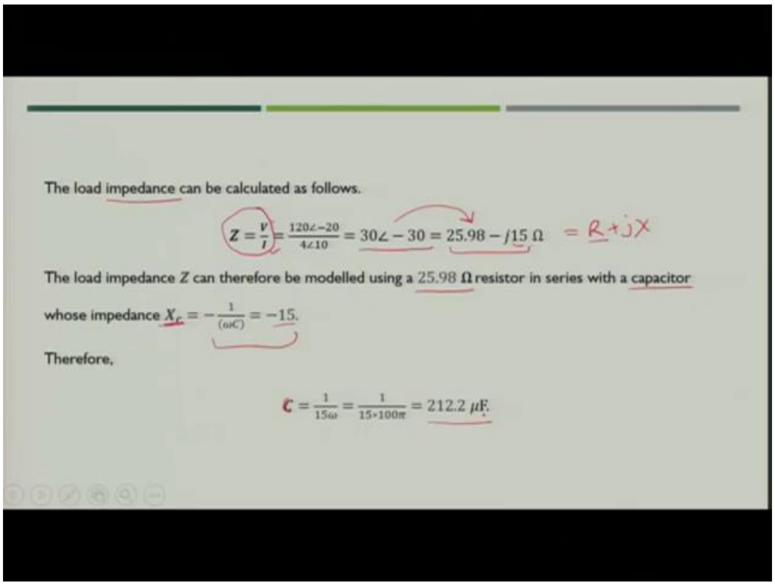

So the load impedance can be calculated by this formula that is

$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{120\angle-20}{4\angle10} = 30\angle-30 = 25.98 - j15\ \Omega$$

From this if you correlate with respect to the term called R plus jx you will get that the Z is composed of 25.98 ohm as a resistive load and in series with capacitor whose impedance is Xc that is equal to -15. So, using this expression you can simply find out the value of capacitance that comes out to be 212.2 microfarad.

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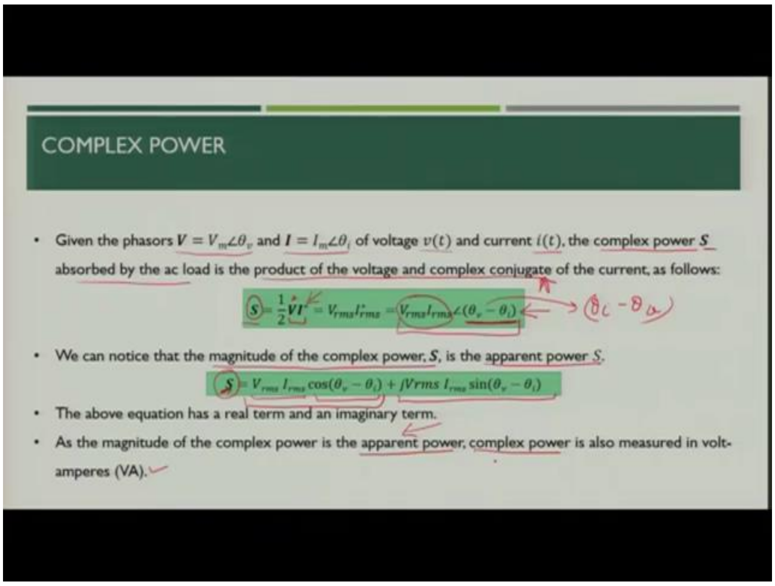

Now let’s come to another term called Complex power. Given the phasor that is $\boldsymbol{V} = V_m\angle\theta_v$ and $\boldsymbol{I} = I_m\angle\theta_i$, we represent V and I in the form of phasors because this phasors are for voltage and current which is sinusoidally varying, so complex power is nothing but the power S which is absorbed by the ac load and it is the product of voltage and complex conjugate of the current.

So how we will write complex power S?

$$\boldsymbol{S} = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^* = V_{rms}I_{rms}^* = V_{rms}I_{rms}\angle(\theta_v - \theta_i)$$

So, you may ask that why this particular complex power is in the form of conjugate? Because you're putting value of current as a current conjugate not be voltage as a conjugate. The reason is that if you compare it with the average power we calculated we got the term of theta v minus theta i. So that particular average power we calculated with the help of the fundamentals of the trigonometric equations that is the voltage we took in the sinusoidal form, current we took in the form of sinusoidal form and when we analyzed that particular value of average power we got value of average power as 1 by 2 VmIm cos of theta v minus theta i.

So since we already have that particular quantity derived if we put V as a conjugate and not I is a conjugate we will get this term as theta I minus theta v which would be contradictory to what we derived in previous case for the average power that why we use VI conjugate not V conjugate I. So the conjugate is not having any physical interpretation or physical significance in this particular depression this is just a mathematical representation, so that we can represent the equations effectively.

Now you can notice that the magnitude of complex power S is apparent power because if you see this term Vrms into Irms this is our apparent power. So if you write into rectangular form

$$\boldsymbol{S} = V_{rms}\, I_{rms} cos(\theta_v - \theta_i) + jV_{rms}\, I_{rms} sin(\theta_v - \theta_i)$$

this is something which we have already derived and that is called average power or real power and this is imaginary term which we have got in derivation of the complex power.

So, complex power is also measured in the voltampere as we saw in case of apparent power. Just to distinguish between real power that is what we measure in watts, we measure complex power in terms of volt-ampere.

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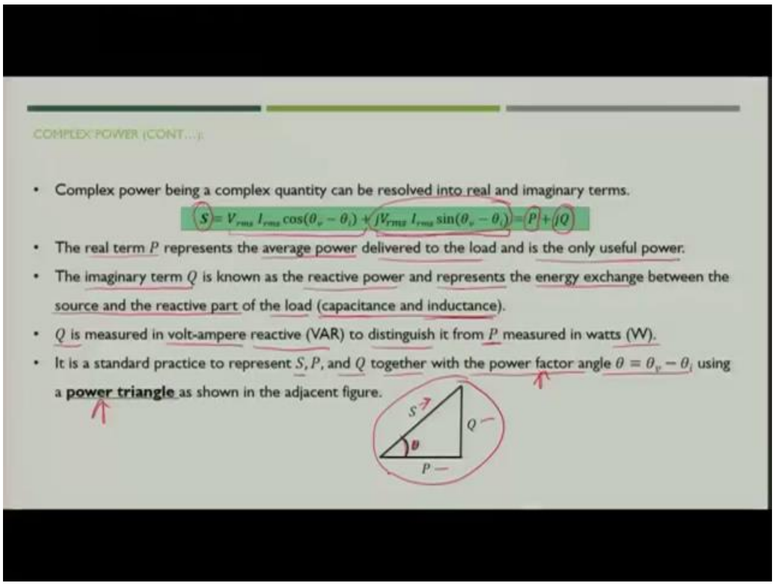

So let's look at the expressions once again $V_{rms}\, I_{rms} cos(\theta_v - \theta_i)$ would be the real power and $V_{rms}\, I_{rms} sin(\theta_v - \theta_i)$ will be written as jQ.

Here the real term is P which represents the average power delivered to the load and is only the useful power in the system while the imaginary term Q is known as reactive power and represents the energy exchange between source and the reactive part of the load. If you see the equation your reactive power would not be contributing in the overall power calculation when you have only the resistive load. Because in this case $\theta = \theta_v - \theta_i = 0$ and sin 0 will be 0, so in that case you will have Vrms Irms only the term for purely resistive load.

Similarly, for purely capacitive and inductive you will only have this term that is the reactive power. So, it means that the reactive power is only visible when we have either capacitive or inductive load available in the circuit. Now Q is measured in voltampere reactive just to distinguish it from real power P which is measured in watts. Now in standard practice S, P and Q are represented together with the power factor angle that is $\theta_v - \theta_i$ using power triangle.

And how we represent power tangle? Power triangle is nothing but the relationship between P, Q and S and the angle between P and S is the θ degree which is nothing but the power factor angle, so this power tangle is very important and most of the time you will be asked to show what is the power triangle, so you should remember that the power tangle is nothing but this particular triangle where we co-relate real power, reactive power and the complex power. So with this we close our today's session and in the next session we will discuss more about the other certain aspects like what is mesh? What is node? And then we will further build up the relationship for calculation of various electrical circuit parameters, thank you.