Basic Electric Circuits
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Module-12
Analogous Systems
Lecture-59
Modeling of Electrical Systems

Namashkar, so, in today's session we will discuss the modeling of electrical system. Basically, this particular aspect you have already understood because we have discussed throughout our course. So, we will recap that what is the modeling of electrical system and then we will compare the electrical system with the mechanical system and we will try to identify what are the analogies in the mechanical system with respect to the electrical system. So, let us start the discussion of today's lecture.

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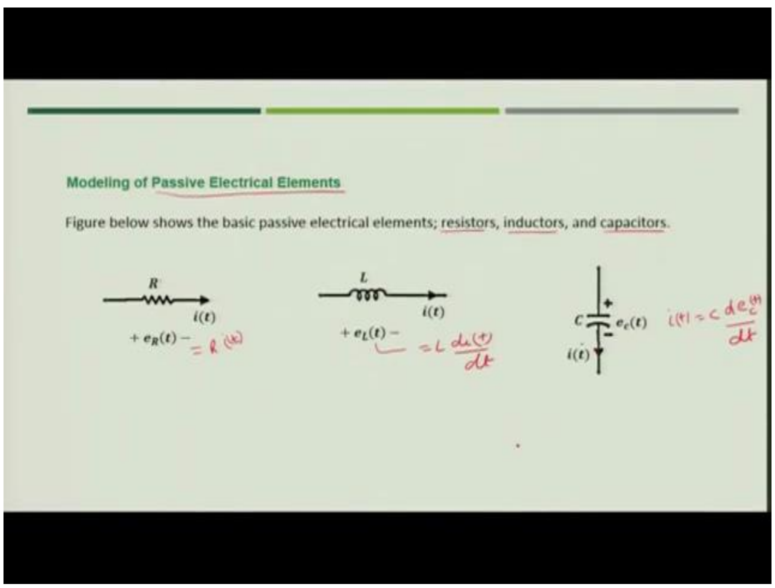

So, as we have already seen that modeling of passive electrical elements, we have discussed resistors, inductors and capacitors. So, resistors are you can say that the voltage drop across R is $e_R(t) = i(t)R$. Similarly, for the inductor voltage drop across L that is $e_L(t) = L\frac{di(t)}{dt}$ and for the capacitor we have seen the current i basically in terms of the voltage across capacitor, so, $e_c(t) = \int \frac{i(t)}{C} dt$. So, this is what we have already discussed.

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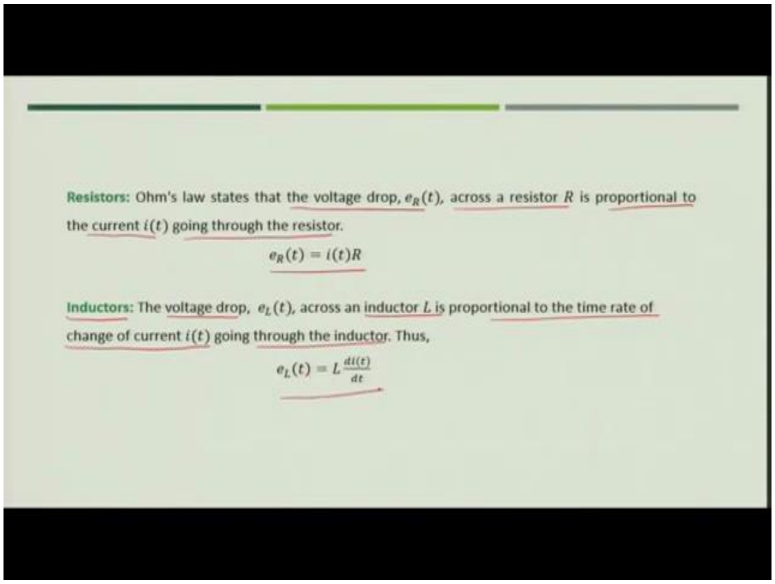

Now, let us see what the governing equations and the laws are we have discussed till now. So, far resistors, we discussed that the Ohms law basically defines the voltage drop across the resistors and it says that the voltage drop that is $e_R(t)$ across a resistor is proportional to the current i flowing through the resistor and as we just discussed the drop across resistance is given by current $i(t)R$.

In case of inductors, we just now discussed that the voltage drop that is $e_L(t)$ across the inductor L is proportional to time rate of change of current flowing through the inductor. So, we say $e_L(t) = L\frac{di(t)}{dt}$.

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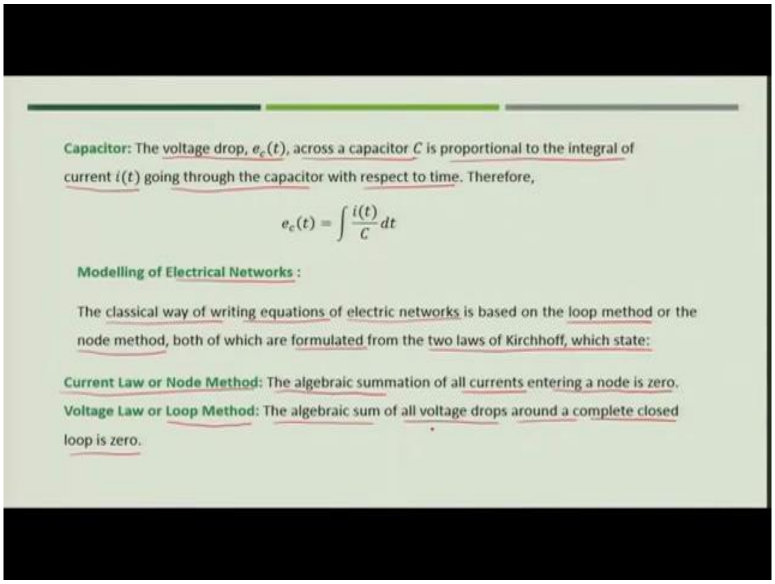

For capacitor we discussed in terms of current we saw that the i is nothing but $\frac{Cde_C}{dt}$ so if you take the voltage, if you want to represent the voltage in terms of current and capacitance what you can write the voltage drop $e_c(t)$ across the capacitor C is proportional to the integral of current going through the capacitor with respect to time. So, we $e_c(t) = \int \frac{i(t)}{C} dt$.

Now, for modeling of electrical networks, we use the classical way of writing the equation of electrical network and it was based on the two methods one was loop method and second was nodal approach and these two approaches are formulated from the Kirchhoff's law. So, what we discussed? We discussed that the nodal method that is basically based on Kirchhoff's current law and says that the algebraic sum of all currents entering a particular node is zero. For Kirchhoff's voltage law, we also say that it is loop method in which the algebraic sum of all voltage drops around a complete close loop is zero. So, these were the two major laws which we used in finding out the governing equations for the electrical circuits.

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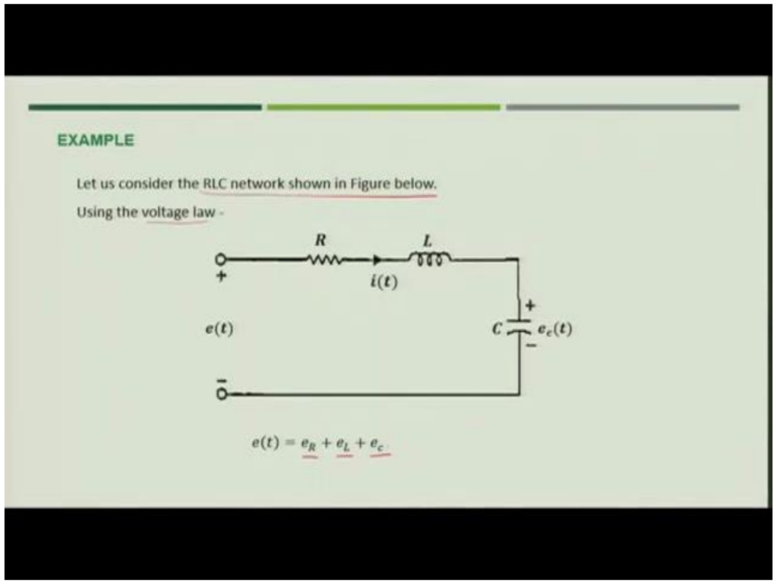

Now, let us take one example and we will see how the knowledge which we have just gathered related to electrical circuit how we can transform it into the model of the electrical circuit. So, let us consider the RLC circuit which is given in the figure. Now, when you use the voltage law so et is the applied voltage, we have resistance R in series with inductance L and the voltage across capacitance is e_c and current flowing through the circuit is $i(t)$. So, what we can write the applied voltage $e(t) = e_R + e_L + e_c$.

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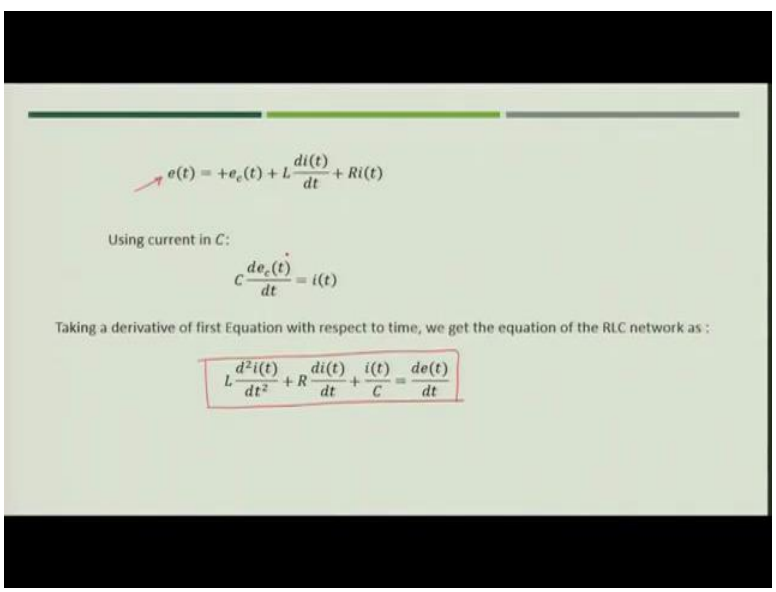

Now, the value of applied voltage $e(t) = +e_c(t) + L\frac{di(t)}{dt} + Ri(t)$. Now, we also know that the current which is flowing through the circuit is $C\frac{de_c(t)}{dt} = i(t)$. So, if you take the derivative of this equation with respect to time what we can write, we can write $L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = \frac{de(t)}{dt}$. So, this is equation which you get when you combine both of the equations.

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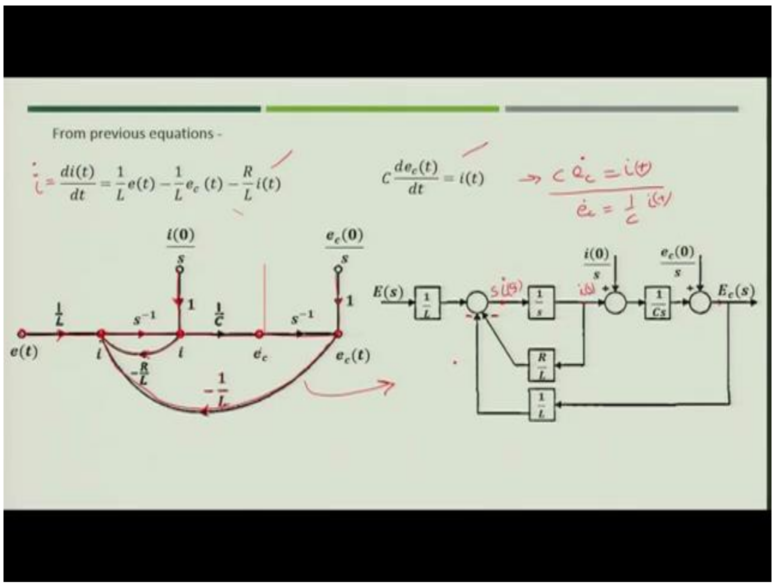

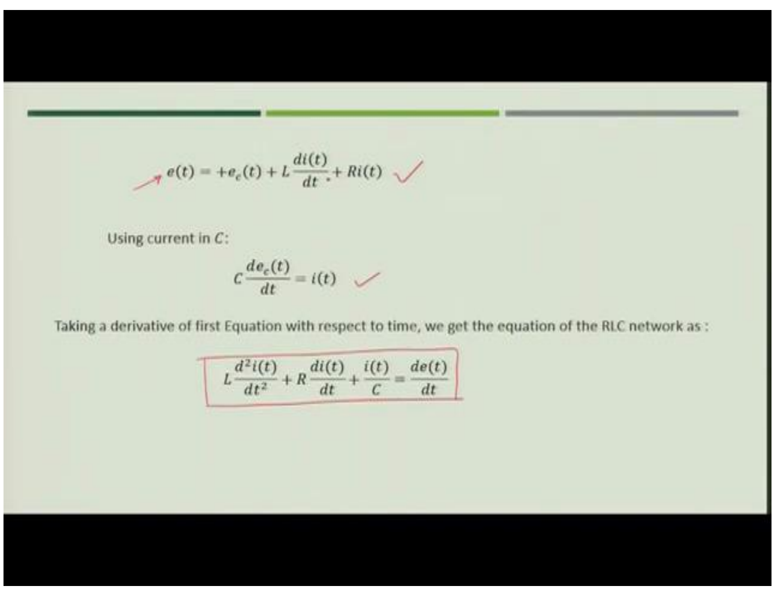

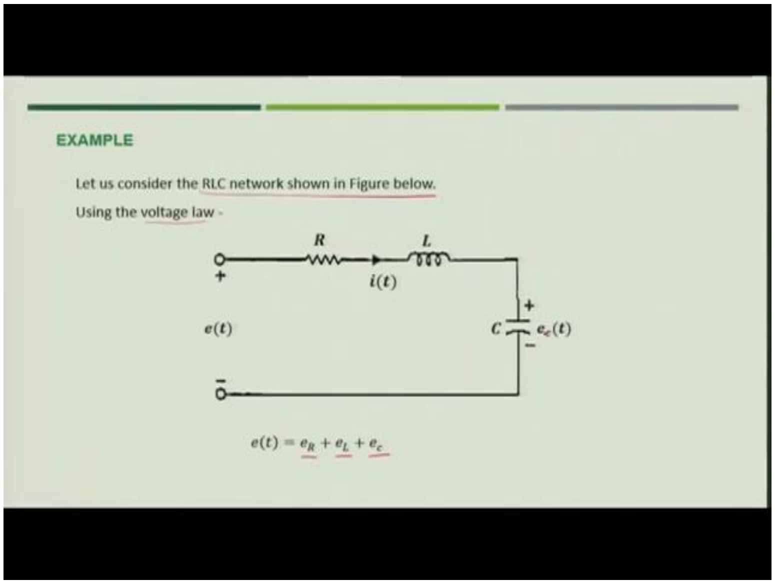

Now, the equations which we have got this main equation which we got from the Kirchhoff's voltage law and then this is the value of current which is flowing through the capacitor. So, we will use these two equations to model the electrical circuit using signal flow graph and the block diagram. So, let us see how we draw the signal flow graph of the series RLC circuit we just saw.

So, we have two equations, so, first is that is $e(t) = +e_c(t) + L\frac{di(t)}{dt} + Ri(t)$. So, let us take $\frac{di(t)}{dt}$ out, so, we take this on the left then we rearrange the equation, so, what we can write we can write $\frac{di(t)}{dt} = \frac{1}{L}e(t) \; - \frac{1}{L}e_c(t) - \frac{R}{L}i(t)$. So, this is the first equation we get and second we already have that is $C\frac{de_c(t)}{dt} = i(t)$.

Now, we have got these two equations. First we have to specify the nodes, so, as we know that generally we consider the voltage across capacitor and current flowing inductor as the two important state variable or the differential equations. If you write then you write in terms of the e_c and i, so, we take these two as a node. So, the output which is required from the circuit is e_c, so e_c we consider as an output also and then the other variable which we have is current i, so, we have got these two nodes. Then third one is the input, input is $e(t)$ so we apply at the node et also.

Now, to show this particular equation what we need, we need $\frac{di(t)}{dt}$. So, we add another node that $\frac{di(t)}{dt}$ and $\frac{de_c(t)}{dt}$. Now you have got all the nodes which are required to represent the signal flow graph. Now, from this equation you can see that $\frac{di(t)}{dt} = \frac{1}{L}e(t) \; - \frac{1}{L}e_c(t) - \frac{R}{L}i(t)$.

So, what you have got now, you have got the equation connected to this particular node. Now since you know that integral of $\frac{di(t)}{dt}$ you get i, so, you add the $1/s$ as a gain to represent theflow. Now, when you have i you need to specify the initial condition also because you have this particular aspect we discussed when we were talking about the signal flow graph then how to add the integrated block.

So, when we integrate the $\frac{di(t)}{dt}$ we specify the initial condition. So, initial condition we consider it as $i(0)/s$. So, that also we add. Now, next is how i is connected to e_c, if you take this equation i is connected to $\frac{de_c(t)}{dt}$ with the help of gain, so, if you write this as $\frac{de_c(t)}{dt} = \frac{1}{C} i(t)$. So, you connect this with $\frac{de_c(t)}{dt}$ with the gain $\frac{1}{C}$.

So now, you have connected this. Now, finally what you have to connect is $\frac{de_c(t)}{dt}$ with $e_c(t)$. So, again as we saw in case of $\frac{di(t)}{dt}$ and $i(t)$ the same we can apply here, so we use integrator $\frac{de_c(t)}{dt}$ when we pass through integrator that is $1/s$ we get the value of $e_c(t)$ and plus the initial condition so the value of $e_c(0)/s$ will be the initial condition which we add.

So, in this way with the help of these two equations, we can draw the signal flow graph of the electrical circuit which we just saw. Now, at the same time you can transform this signal flow graph into block diagram. So, if you see the block diagram, you use the Laplace transform of the input, so you get es then gain is $1/L$ this is the summation block. So, in this you get input from $1/L$ as a gain from e_c, so $E_c(s)$ will be now connected to this block with $1/L$ as a gain and here you get negative as a part of summation, so instead of sum it will be subtracted from the total value. Similarly, for the value of current i which you get here.

So, you get R/L as a gain and then you again subtract from this. So, when you see this particular segment the value which you get from here is actually the value of $\frac{di(t)}{dt}$, so you can simply write $sI(s)$ here. Now, when you transform this through $1/s$ as a block you get the value of $I(s)$ here. Now, $I(s)$, here you will add the initial condition that is $i(0)/s$ and then you pass on through $1/Cs$, so you get the value of the e_c and plus you add the initial condition that is e_c/s.

When you sum up both of them you will get the value of $E_c(s)$. So, in this way using this you can transform the series RLC circuit which we discussed into mathematical model of the circuit. So, this block diagram will represent the series RLC circuit. Now, next is to basically we have represented in the form of transfer function. So, the alternate way can be that, we represent the same into state variable methods.

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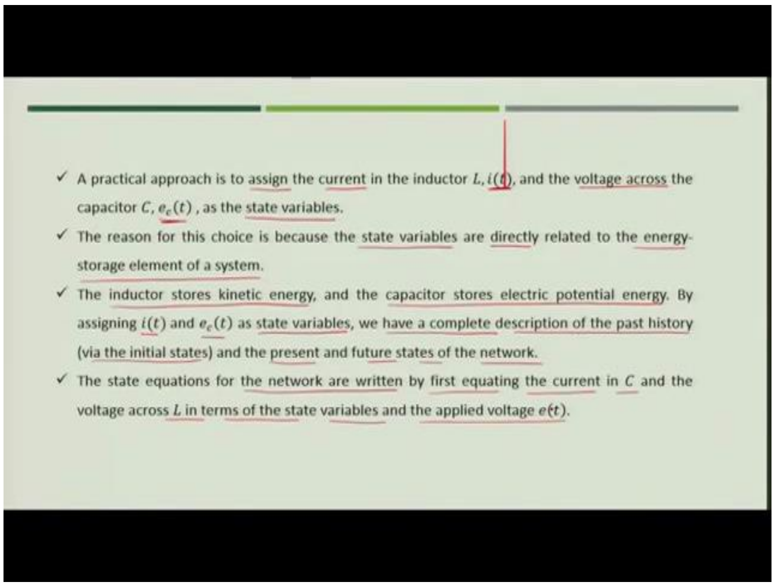

So, let us understand how we can represent that series RLC circuit into state variables. So, the practical approach for us would be to assign the current in the inductor that is $i(t)$ and voltage across capacitor that is $e_c(t)$ as the state variables. Now, why we are choosing this because the state variables are directly related to energy storage element of the system and in that L and C are considered to be the energy storage elements.

Now, the inductor stores the kinetic energy and capacitor stores the potential energy that is electrical potential energy. So, by assigning i and $i(t)$ and $e_c(t)$ at any time t as the state variable, we can have the complete description of the past history that is the initial states of the system and the present and future states of the RLC network. Now, the state equations for the network are written by first equating the current in C and voltage across L in terms of state variable and the applied voltage.

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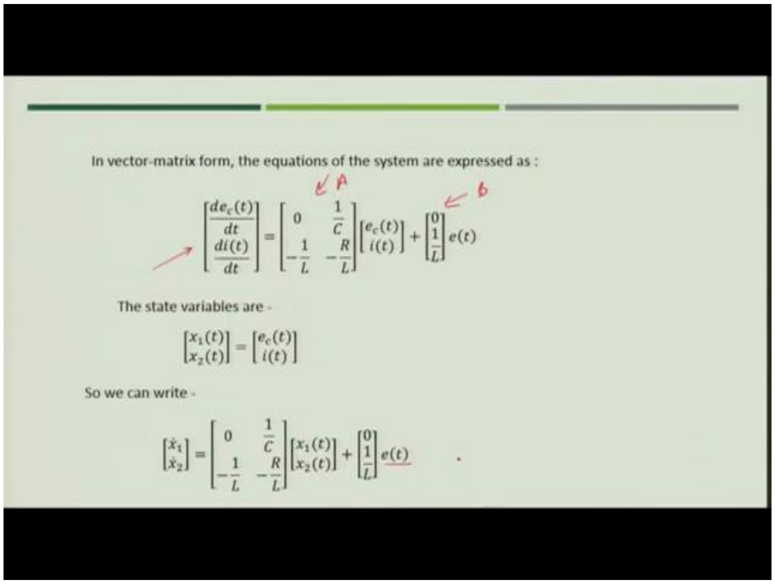

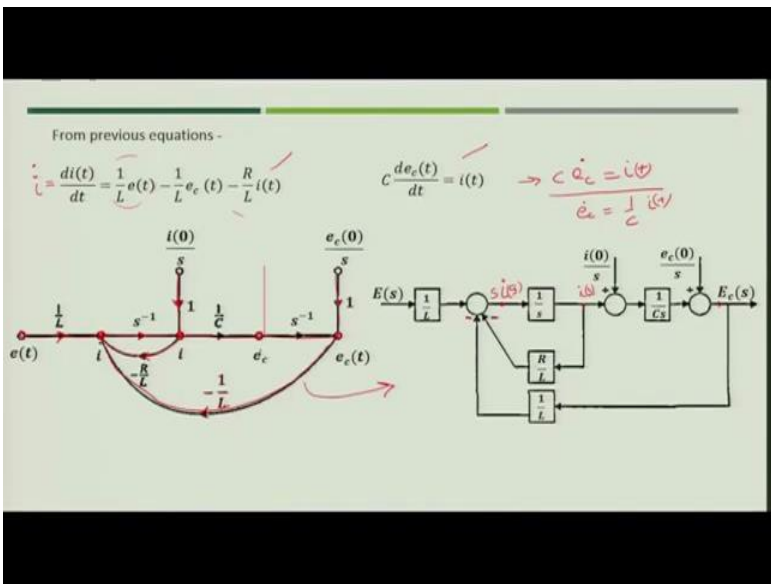

So, using the previous equations, which we just saw in this case, we can recollect and we can write in terms of the state variables, so we can write that $\frac{de_c(t)}{dt} = i/C$. So, this you can easily get from this equation and we do not have any forcing function in that equation. So, the components of A matrix will be 0 and $1/C$, in this case.

For second equation that is the value of $\frac{di(t)}{dt} = \frac{1}{L}e(t) - \frac{1}{L}e_c(t) - \frac{R}{L}i(t)$. So, we can compile it. So, against $e_c(t)$ we write $-\frac{1}{L}$ and against $i(t)$ we write $-\frac{R}{L}$. So, in this way we get matrix A plus B matrix you get 0 in the as a first element in B and then next one is $1/L$, because in the second equation we have the forcing function present, so that is anyway the input, so you get B also as $\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$.

So, now the state variables in this case are basically,

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} e_c(t) \\ i(t) \end{bmatrix}$$

Then you can simplify this equation as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} e_c(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} e(t)$$

So, in this way you can see that, with the help of the equations which you write for the series RLC circuit using Kirchhoff Voltage Law can be easily transformed into state equation.

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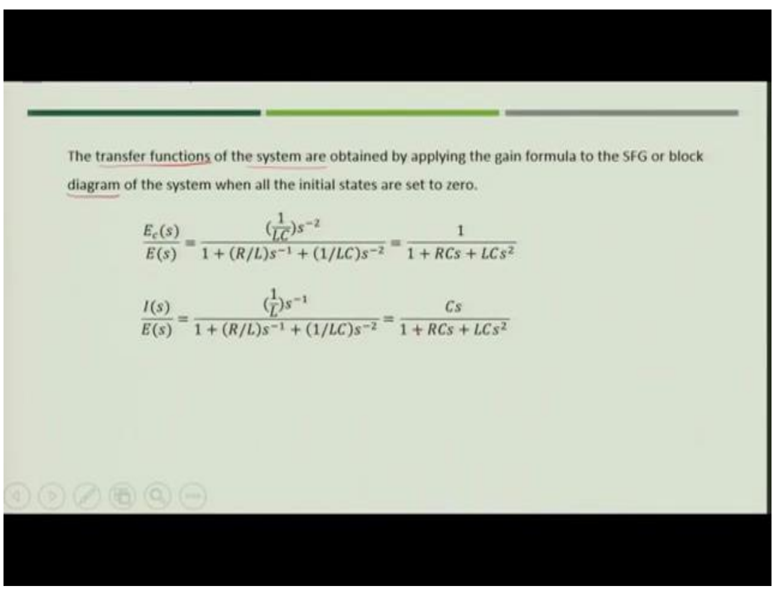

Now, when you are having all that information in hand, you can now write the transfer function of this system. Now, since we have here 2 variables, so, we will, we can write the transfer function in terms of e_c that is the voltage across capacitor and i that is current flowing through the circuit.

So, with the help of signal flow graph and or may be the block diagram which we just saw, we can compile this information, we can write $\frac{E_C(s)}{E(s)}$ that is the transfer function.

So, when you write and simplify you get transfer function as,

$$\frac{E_C(s)}{E(s)} = \frac{(\frac{1}{LC})s^{-2}}{1 + (R/L)s^{-1} + (1/LC)s^{-2}} = \frac{1}{1 + RCs + LCs^2}$$

$$\frac{I(s)}{E(s)} = \frac{(\frac{1}{L})s^{-1}}{1 + (R/L)s^{-1} + (1/LC)s^{-2}} = \frac{Cs}{1 + RCs + LCs^2}$$

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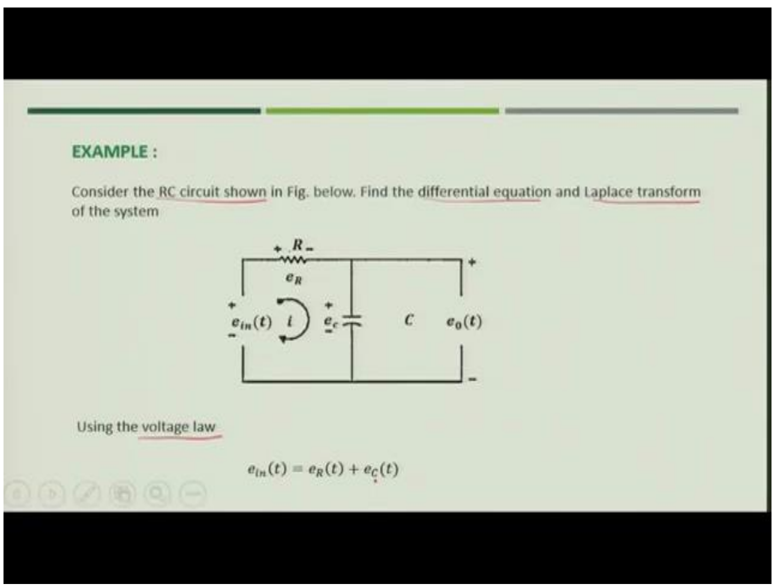

Now, let us take quickly the example, that if you have the RC circuit in the figure, we need to find out the differential equation and then the Laplace Transform of the system. So, what we can do? Basically, we need to find out the transfer function of the system finally. So, using Kirchhoff Voltage Law, you can write the input $e_{in}(t) = e_R(t) + e_C(t)$.

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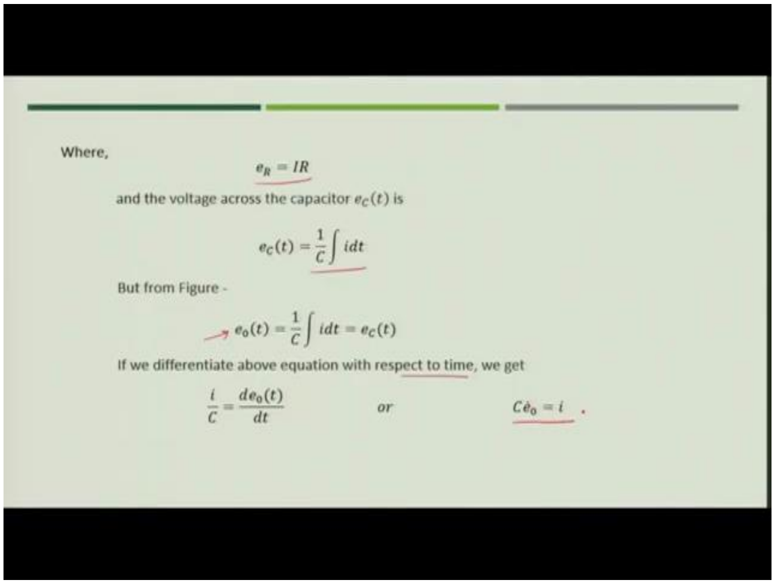

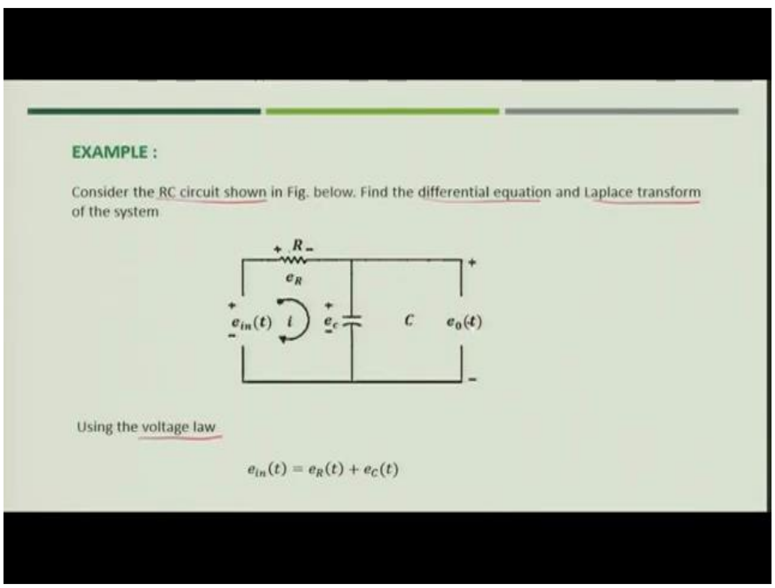

So, $e_R = IR$ and $e_C(t) = \frac{1}{C}\int idt$ or if you see this figure, $e_0(t) = \frac{1}{C}\int idt = e_C(t)$ that is output. Now, if you differentiate the above equation with respect to time, you can simply write $\frac{i}{C} = \frac{de_0(t)}{dt}$ or you can write in short form as $C\dot{e}_0 = i$.

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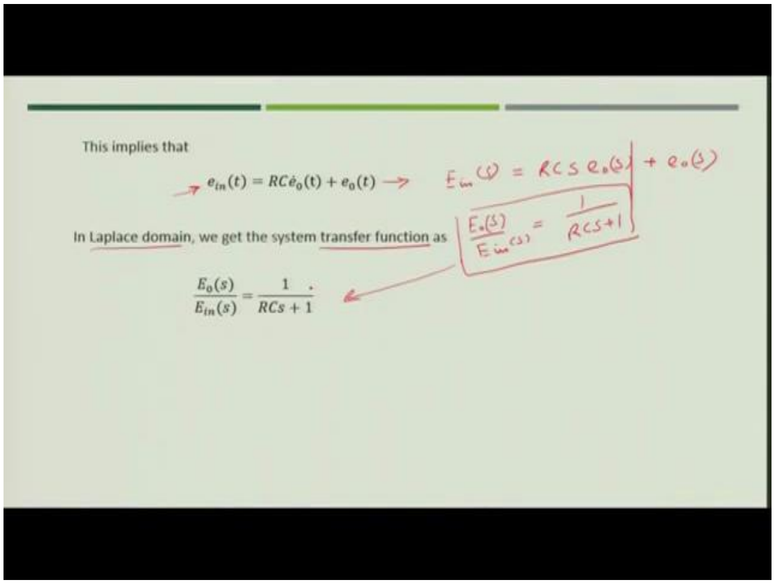

So, now we have got 1 equation, then we put this into the Kirchhoff Voltage Law equation that is $e_{in}(t) = RC\dot{e}_0(\mathrm{t}) + e_0(t)$. So, here what you can write in terms of Laplace, you can write, Ein S equal to RCs e naught s plus e naught S. So, with the help of this in Laplace domain we can get easily the transfer function. Transfer function is

$$\frac{E_0(s)}{E_{in}(s)} = \frac{1}{RCs + 1}$$

So, with the help of the conventional equations related to Kirchhoff Voltage Law or Kirchhoff Current Law, you can convert your circuit into the Laplace domain and then find out the transfer function of the system.

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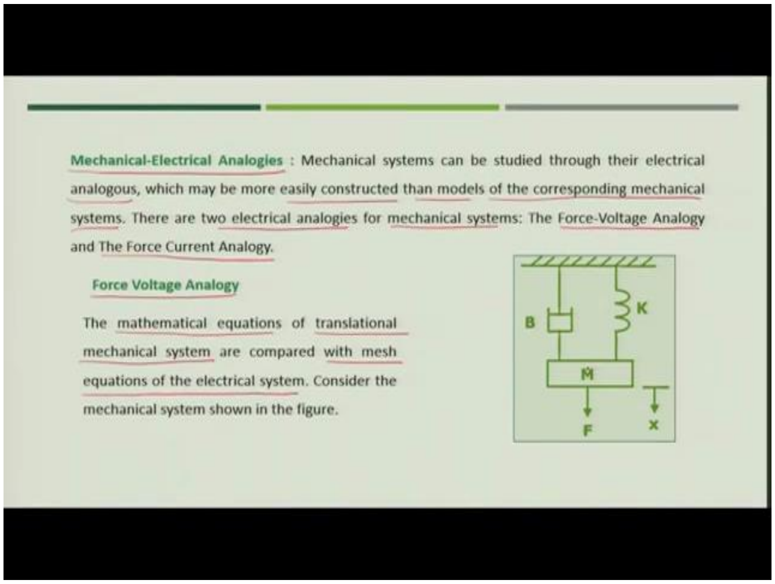

Now, let us talk about the analogies. So, first we will discuss about the mechanical, electrical analogies. So, the mechanical system can be studied through their electrical analogous, which may be more easily structured constructed than the models of corresponding mechanical systems. So, basically, we will study 2 electrical analogies for mechanical systems. First one is force voltage analogy and second is force current analogy. So, let us first understand the force voltage analogy. If you see the mechanical system shown in the figure, we have one dashpot and one spring at which you see the mass M connected and force F is applied which is giving the displacement x.

Now, this mathematical, this mechanical system, the basically this is translational mechanical system, so we must first write the mathematical equation. So, when you write the mathematical equation you will compare this with the mesh equation of the electrical system and then you can stabilize the analogous quantities of mechanical and the electrical system. So, let us first write the mathematical equation for the translational mechanical system, which you are seeing in the figure.

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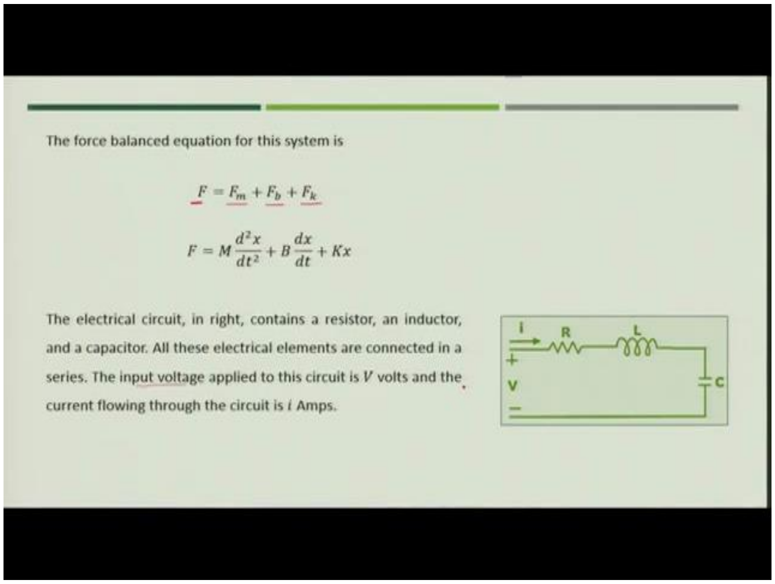

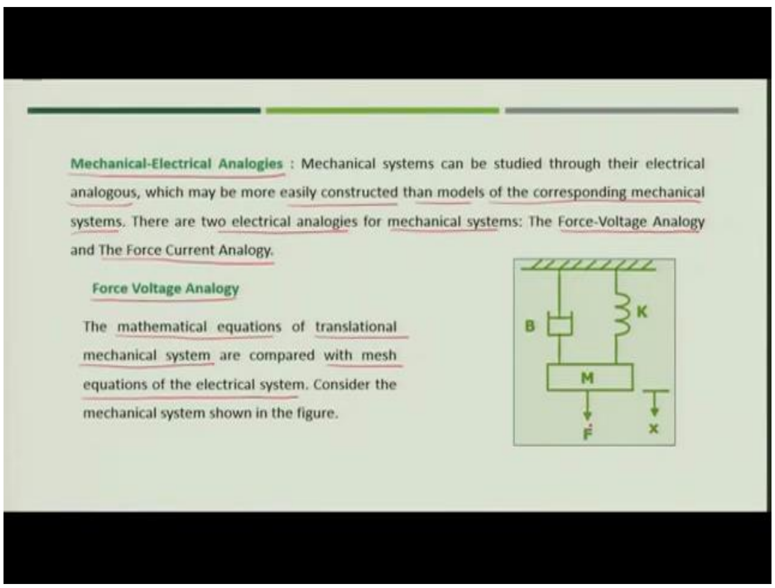

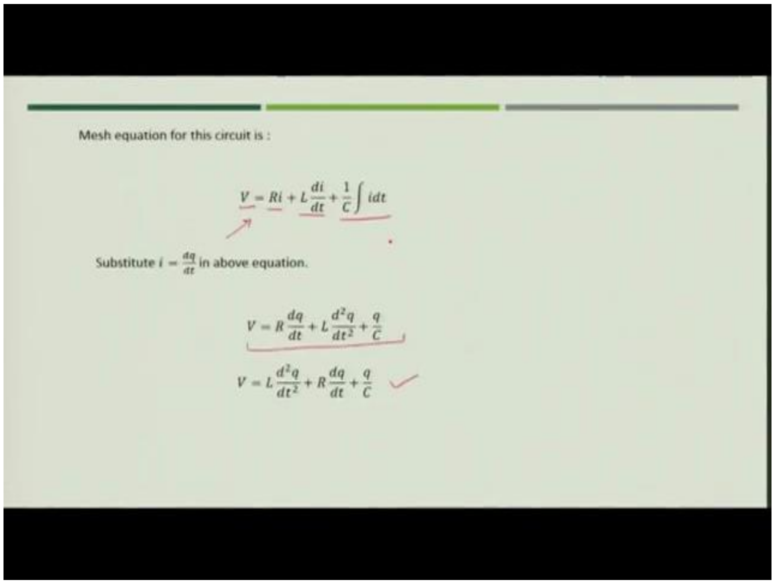

So, what you can write, so, you have $F = F_m + F_b + F_k$. F_m is say equal to $M\frac{d^2x}{dt^2}$, this is because of the mass, then F_b is $B\frac{dx}{dt}$ because of the viscous friction and then $F_k = Kx$ because of the presence of spring. So, you can write force,

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

Now, let us consider a series RLC circuit where you apply voltage V and current is flowing as i. So, the electrical system which you see we will write the equation for that,

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int idt$$

Let us substitute $i = \frac{dq}{dt}$ in the above equation. So, what you can write now,

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

So, this equation we have now got, so if you rearrange you take higher order term first, so you can write,

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}$$

So, now you have the mathematical equation related to electrical system and mathematical equation related to mechanical system.

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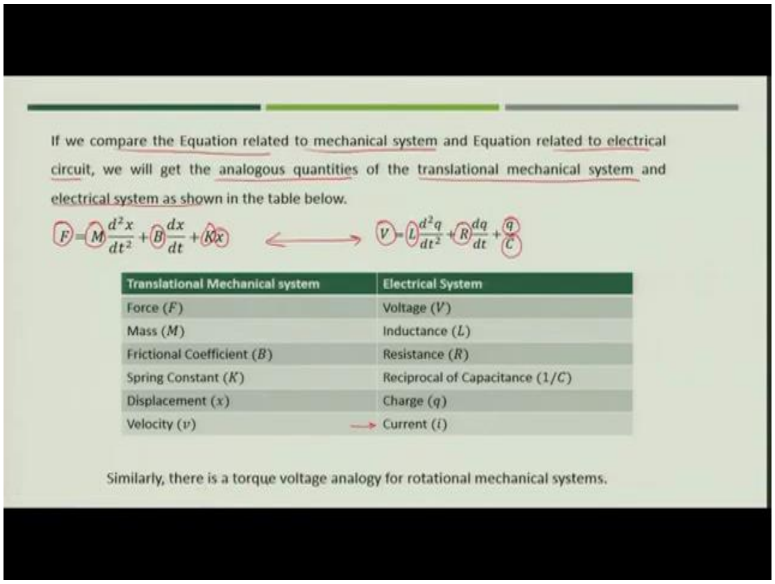

Now, let us compare both of them, so, if you compare the equations related to mechanical and the electrical circuit, we will get the analogous quantities of the translational mechanical system and electrical system. So, if you compare both of the equations, you can easily find out that F that is force in case of translational mechanical system is analogous to voltage V in the electrical system, mass M is analogous to inductance, frictional coefficient that is B is analogous to resistance R.

Spring constant K is analogous to reciprocal of capacitance that is 1/C, displacement that is x, you see will be analogous to charge q and then if you differentiate x that is dx/dt, you will get velocity v and when you differentiate q that is dq/dt you will get current i, so velocity v is analogous to current i. So, this example shows that how the different quantities of mechanical system are analogous to the electrical system. Let us take the rotational system also. So, where you instead of force you compare torque with the voltage.

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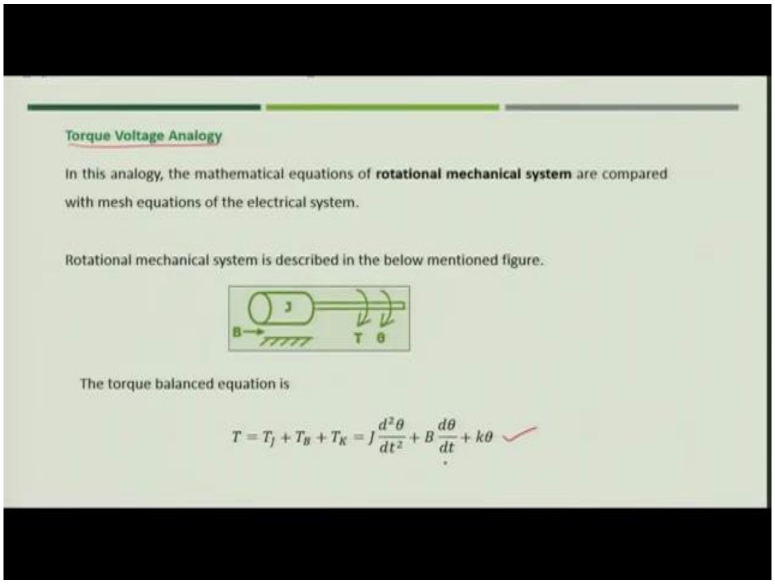

So, the analogy which you get, in case of torque voltage, you first consider one rotational system, where g is the inertia of rotating body. Torque T is the applied torque, B is viscous friction and it is giving the displacement of angle θ. Now, what you can write torque

$$T = T_J + T_B + T_K = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + k\theta$$

So, now you get the equation related to rotational mechanical system. Now, you will compare this again with the series RLC circuit equation.

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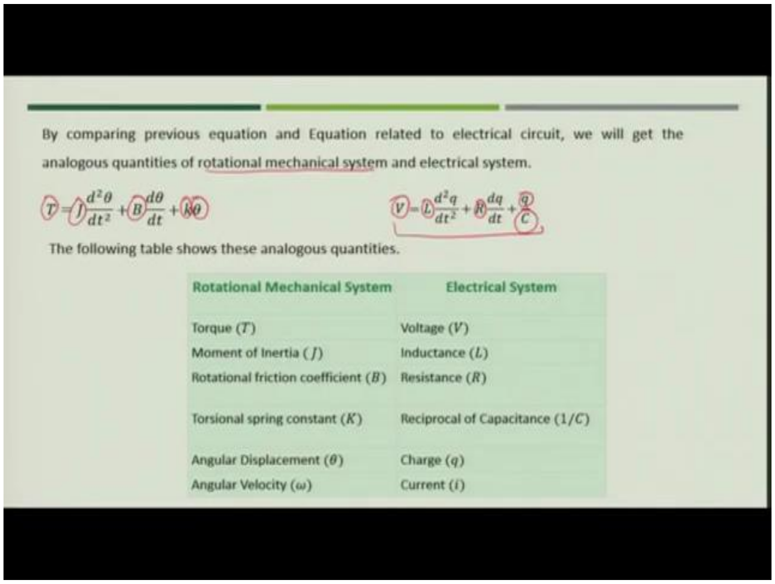

So, what we got from series RLC circuit, we got

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}$$

Now, if you compare both of them, what you can say, that torque T is analogous to voltage V, moment of inertia that is J is analogous to inductance L, rotational friction coefficient that is B is nothing but analogous to resistance R.

Torsional spring constant that is K, you can say that it is analogous to reciprocal of capacitance C, angular displacement θ is analogous to charge q and then again if you take the derivative of θ, you get angular velocity ω that is equal to $d\theta/dt$ which is analogous to dq/dt that is nothing but the current i. So, we have seen the translational as well as rotational mechanical system and we described the analogies with respect to the electrical system in today's discussion.

We will continue our discussion tomorrow where, we will discuss about the further mechanical relationship. So, in this we discussed force voltage analogy, tomorrow we will discuss force current analogy and then we will wind up our course. Thank you.