Basic Electric Circuits
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Module 11 - State Variable Analysis
Lecture 51 - Introduction to Graphical Models

Namashkar, so today we will start our discussion on state variable analysis and particularly in this session we will discuss about the graphical model of the system.

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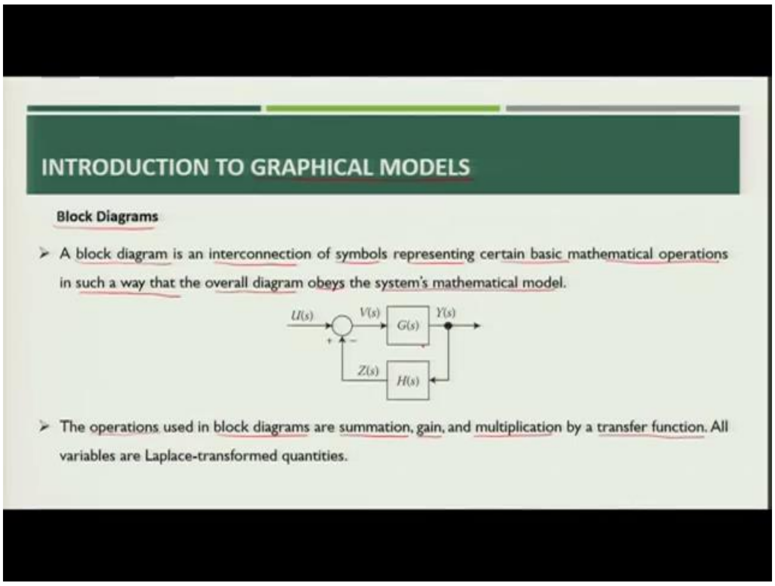

So, let us try to understand the various graphical models used in the control systems. So basically the graphical model is nothing but pictorial representation of the system generally we take only the linear system for the presentation. For example, if you see the figure shown in the slide it is a graphical representation of the system which has one input and the output and in between you have the mathematical representation of the system.

So, generally we use two different types of Graphical Models, one is Block diagram and another is signal flow graph. So now let us first understand what is block diagram. Block diagram is an interconnection of symbols representing certain basic mathematical operations in such a way that the overall diagram obeys the systems mathematical model. If you see this particular block diagram you have $U(s)$ as an input and $Y(s)$ as an output and inside you have the set of blocks which when you represent them mathematically will give the complete systems model.

So, in the block diagram we use the Laplace transform of various quantities whether these are signals or the processing blocks we used the Laplace transform to represent the systems model in block diagram. The operations used in block diagram are, can be the summations or maybe gain or multiplication by a transfer function. Say for example in this case if you see this symbol is for summation where you have the various signals summed up.

So here you have $U(s)$ as an added signal and $Z(s)$ is a negative, so $U(s) - Z(s)$ is the output which you will see as $V(s)$ and then the mathematical representation of the transfer function that is $G(s)$ you will have and then you have the $Y(s)$, so you will see different elements in this particular block diagram and we will discuss the different elements which we will see in the block diagram one by one.

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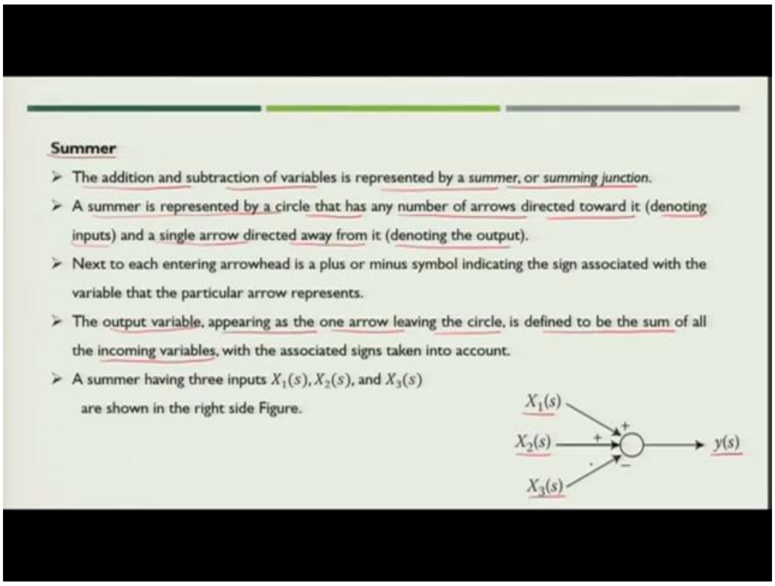

First let us try to understand what is summer? Summer means the addition and subtraction of variables. So, when you want to add or subtract the variables you represent them by a summer or you can also call it as summing junction. So, if you see this figure you will see one summing block you have where you have multiple inputs coming and then you have one unique output that is $Y(s)$.

The summer is represented by a circle. If you see this figure it is represented by a circle and number of arrows directed towards it which are nothing but the input for the summer and one single arrow which is nothing but the output of the summer. So here if you see the inputs are $X1(s)$, $X2(s)$, $X3(s)$ and in each and every arrow you will see this plus or minus sign is presented which indicates whether you want to be summing up or you want to subtract the signal.

If you see this figure you have two inputs as a summation and one as subtraction. So

$$Y(s) = X_1(s) + X_2(s) - X_3(s)$$

The output variable appearing as one arrow leaving the circle is defined has the total sum of the incoming variable. So summer is basically used for either adding or subtracting the variables.

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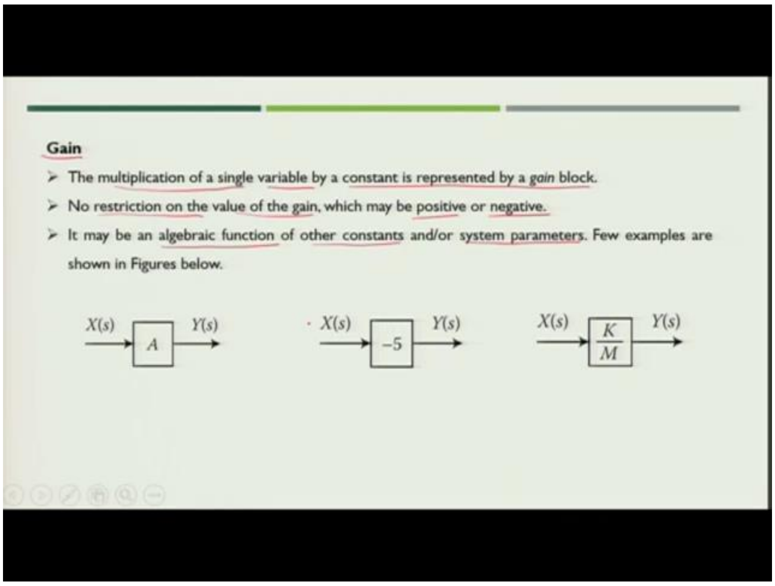

Now, let us talk about the Gain Block. The multiplication of a single variable by a constant is represented by the gain block. There is no restriction on the value of the gain, so it can be either positive or negative. Or it can be an algebraic function of other constants and, or system parameters. So, if you see this block, A is the gain block, so the $Y(s) = AX(s)$ in this case.

If you take the value of $A = -5$ then $Y(s) = -5X(s)$. At the same time you can represent the block as an algebraical function. In this case we have represented this gain as K/M. K and M are two different variables. So, if you put the variables of K and M you will get the total gain which you will get from this block. So, $Y(s) = \frac{K}{M}X(s)$. So, gain Block is basically used to amplify the signal input.

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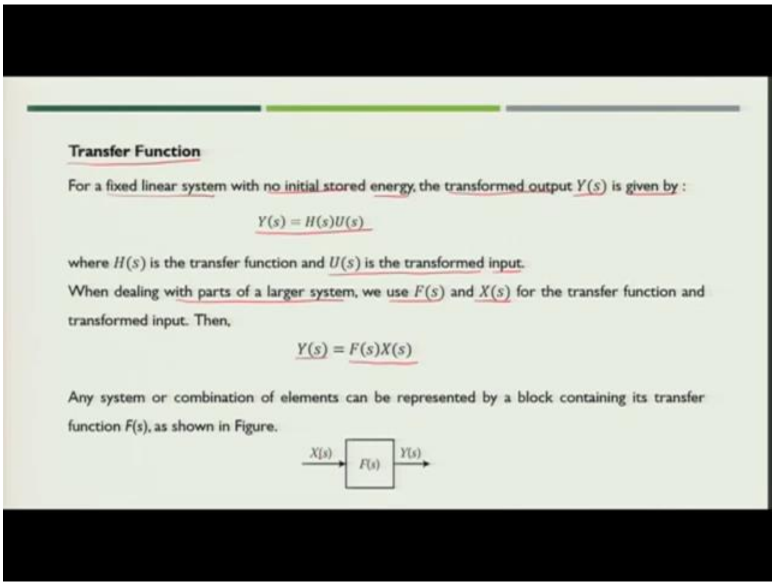

Now, let us talk about the Transfer Function. This transfer function concept we have already discussed when we were discussing about the Laplace transform. So, let us recap what we discussed at that time. So, for a fixed linear system with no initial stored energy the transfer function that is transformed output that is $Y(s) = H(s)U(s)$ where $H(s)$ is the transfer function of the system and $U(s)$ is the transform input represented in Laplace domain.

Now when dealing with the parts of the large system we may use subsystem Laplace transform. So $F(s)$ is the Laplace transform subsystem that is basically the part of a larger system and $X(s)$ is the input for that subsystem. In that case the intermediate output is $Y(s) = F(s)X(s)$. Now, how you will represent? You will represent this equation in the form of block diagram as $X(s)$ is the input to the block. $F(s)$ is the transfer function and the $Y(s)$ is the output, so $Y(s) = F(s)X(s)$.

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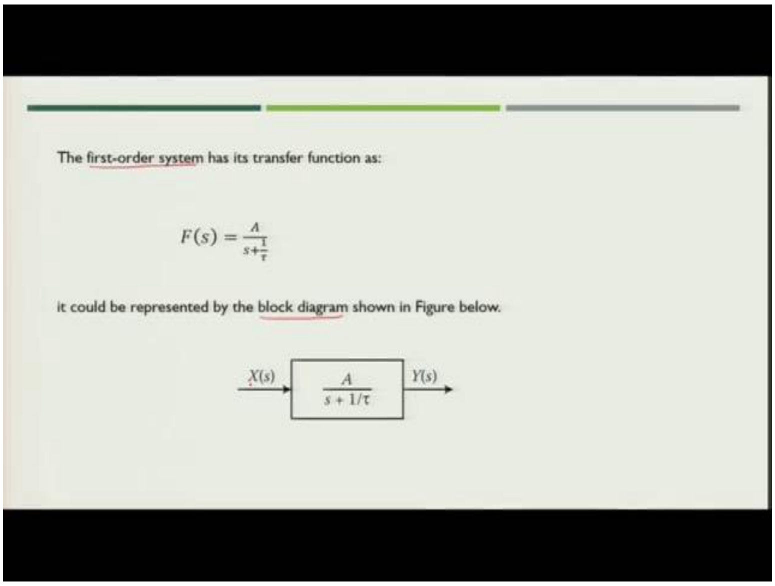

Now, if we have a first order system then in that case the first order system can be represented by the Laplace transform. So, transfer function for the first order system will become

$$F(s) = \frac{A}{s + \frac{1}{\tau}}$$

This will be the transfer function for a typical first order system and if you are asked to represent it in the form of block diagram, so you will give $X(s)$ as an input to the block. In the block you will have the transfer function of the system that is $\frac{A}{s+\frac{1}{\tau}}$ and $Y(s)$ will be the output.

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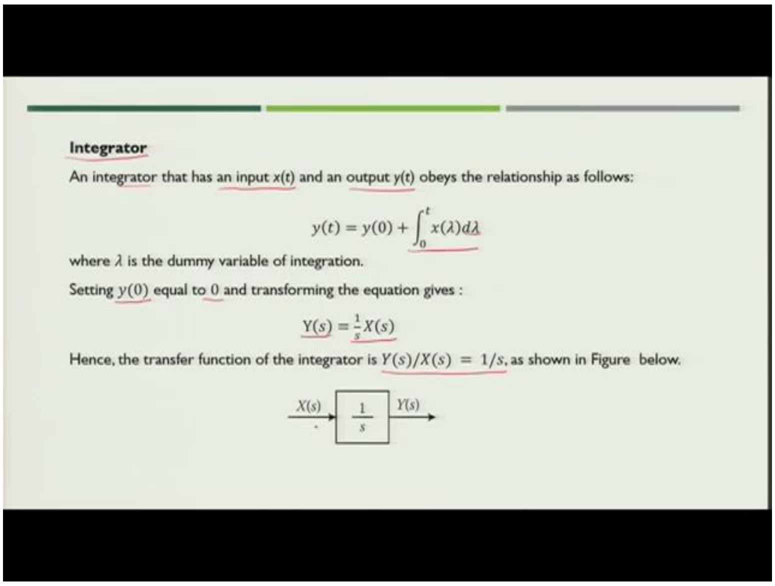

Now, let us talk about the integrator block. So, if you recall we discussed about the integration related when we were discussing about the Laplace transform so we used that discussion in this analysis where we want to find out the transfer function of the integrator block.

In that case if you consider $x(t)$ as an input and an output $y(t)$ as an output, so

$$y(t) = y(0) + \int_0^t x(\lambda)d\lambda$$

λ is the dummy variable to find out the integration of the input function xt.

Now, if you consider $y(0)$ equal to 0 that is initially this system does not have any value. So you can simply write

$$y(t) = \int_0^t x(\lambda)d\lambda$$

Now, if you convert into Laplace,

$$Y(s) = \frac{1}{s}X(s)$$

This is what we discussed when we discuss the Laplace transform for the integral term in the previous lectures.

We can write $Y(s) = \frac{1}{s}X(s)$ when you are asked to find the transfer function that means you need to find out $Y(s)/X(s)$. So, $Y(s)/X(s) = 1/s$ in case of integrator. Now, when you are asked to find, to draw the block diagram you can simply write $X(s)$ as an input to the block, within the block you will have integrator the Laplace transform of the integrator or you can say the transfer function related to integrator that will be $\frac{1}{s}$.

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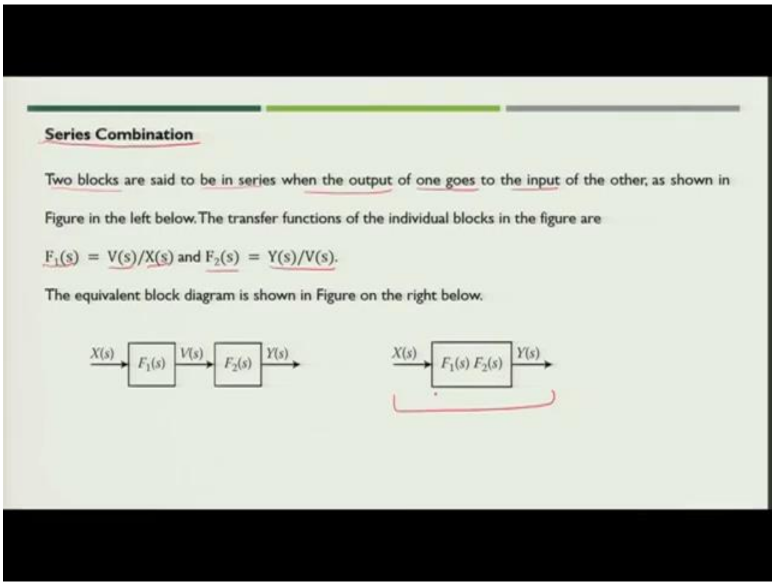

Now there are the blocks which may be in series combinations. Now, when you say that the blocks are in series combination it means that the blocks, the output of one block will go to other block as an input then we will see that these two blocks are in series. You can see in the figure the $\mathrm{X(s)}$ is the input to the block having transfer function $\mathrm{F1(s)}$. So, the intermediate output is $\mathrm{V(s)}$ for this block.

This output goes directly as an input to another block that is $\mathrm{F2(s)}$ and then finally you get the $\mathrm{Y(s)}$. So, $\mathrm{F_1(s) = V(s)/X(s)}$ and $\mathrm{F_2(s) = Y(s)/V(s)}$. So, if you use these 2 equations and simplify you can see that $\mathrm{\frac{Y(s)}{V(s)} = F_1(s)F_2(s)}$. When 2 blocks are in series you can combine both of them and represent by one single block and then the block will be $\mathrm{F_1(s)F_2(s)}$. So in case of series combination you can multiply the transfer functions and get the final transfer function of the system.

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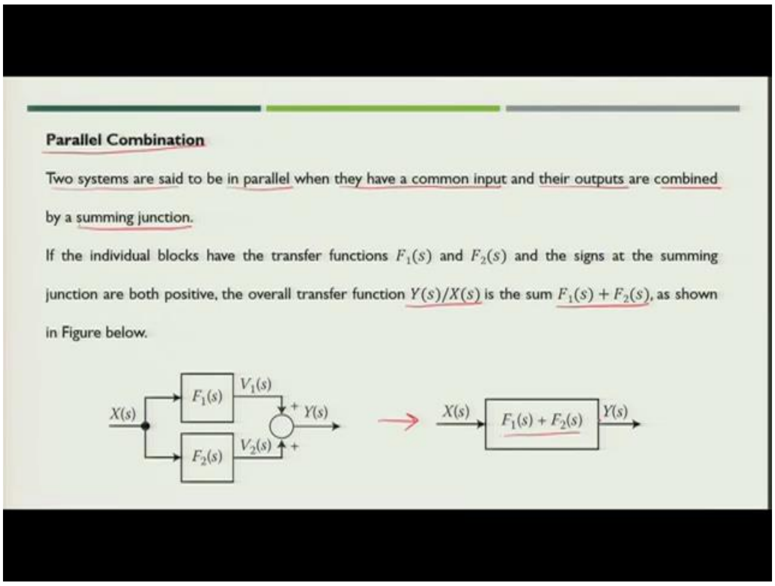

Now, if the blocks are in parallel combination means two systems are said to be in parallel when they have common input and their outputs are combined by a summing junction. So, if you see this figure here $\mathrm{F}_1(\mathrm{s})$ and $\mathrm{F}_2(\mathrm{s})$ are the two subsystems, the transfer functions of these two subsystems are $\mathrm{F}_1(\mathrm{s})$ and $\mathrm{F}_2(\mathrm{s})$ and these two systems are given the same input that is $X(s)$. So you will get to the output $Y(s) = V_1(s) + V_2(s)$. In that case if you sum up you will get $Y(s)/X(s)$ is the sum $F1(s) + F2(s)$.

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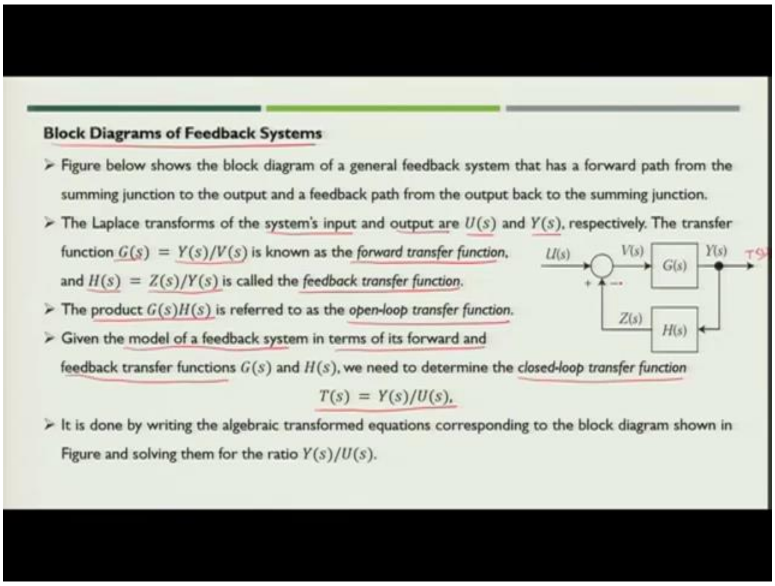

Now, let us talk about the block diagram of the feedback system. So this is basically a typical the block diagram of the system having one feedback loop. So $U(s)$ is the input and the $Z(s)$

is the output of the feedback loop. So, $V(s)$ you can simply see as $U(s) - Z(s)$ and this goes into the transfer function $U(s)$ and you get the output as $Y(s)$. in this case the system has one feedback loop also. We will see what the various components are we have in this particular block diagram.

So, Laplace transform for the system's input and output are $U(s)$ and $Y(s)$,. The transfer function $G(s) = Y(s)/V(s)$ which is called as forward transfer function. So $G(s)$ is called as forward transfer function. While in case of $H(s)$ that is the transfer function for the feedback loop. You will have $H(s) = Z(s)/Y(s)$. So, this transfer function is called feedback transfer function.

When you combine $G(s)H(s)$ you get another term called open loop transfer function. So, these three are the most important transfer functions which we use frequently to represent the feedback system. Now given the model of the feedback system in terms of its forward and feedback transfer function we can determine the transfer function of the complete closed loop system.

So, we can say, we can determine the closed loop transfer function. The closed loop transfer function will be given as say the total output is $T(s)$ so you can write the $T(s) = Y(s)/U(s)$.

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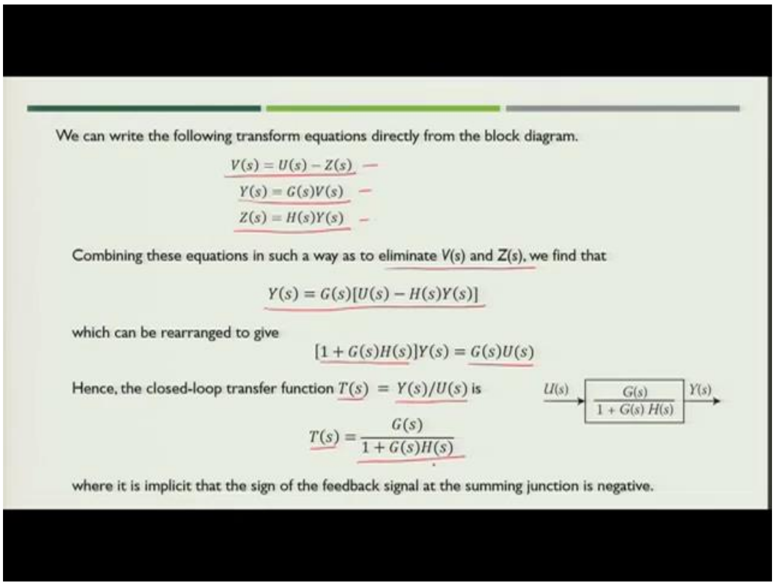

So we have these three equations,

$$V(s) = U(s) - Z(s)$$

$$Y(s) = G(s)V(s)$$

$$Z(s) = H(s)Y(s)$$

Now with the help of these three equations we will eliminate $V(s)$ and $Z(s)$. So when we eliminate we get the value

$$Y(s) = G(s)[U(s) - H(s)Y(s)]$$

which can be rearranged to give,

$$[1 + G(s)H(s)]Y(s) = G(s)U(s)$$

The closed loop transfer function, $T(s) \ = \ Y(s)/U(s)$ is

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

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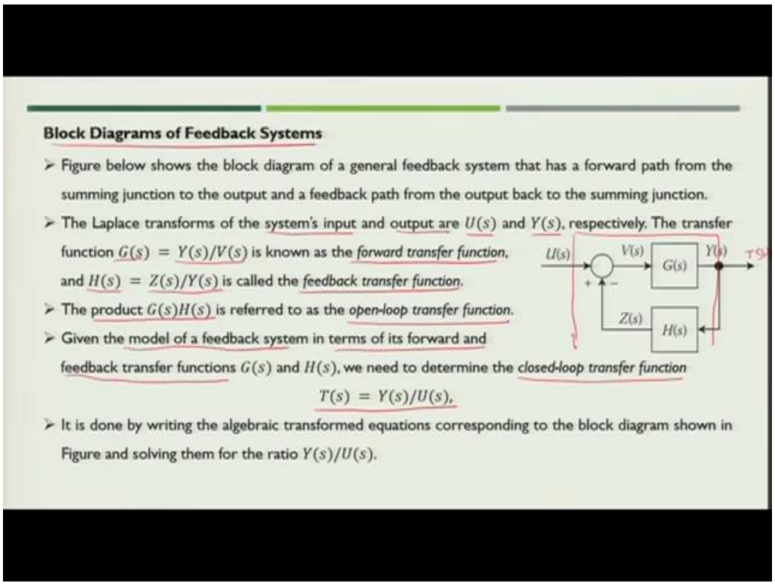

So the figure which you saw in this particular case, so if you take the complete system and see the transfer function of the system, this transfer function will become $T(s)$.

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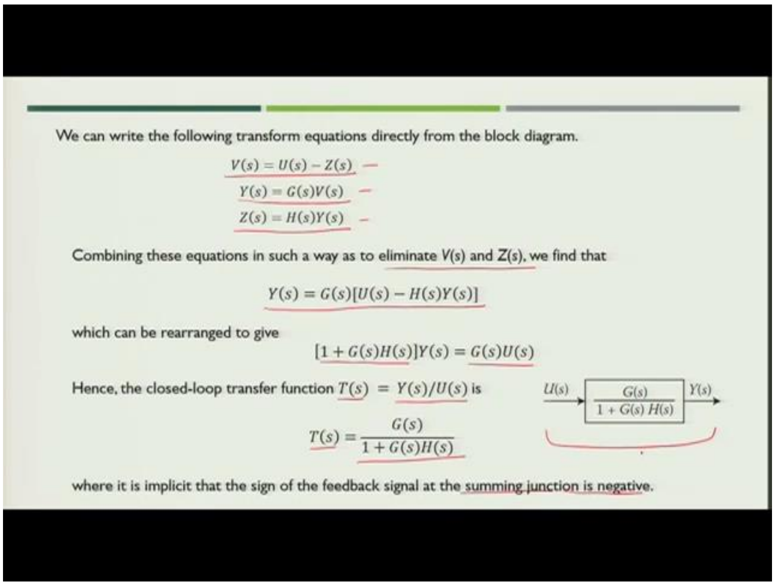

You can equivalently represent this system as the simple block having the transfer function of the feedback system that is $\frac{G(s)}{1+G(s)H(s)}$ and the $U(s)$ as input and $Y(s)$ as the output. Now, here in this case we say that in the previous figure we can observe that the summing junction will have negative feedback signal.

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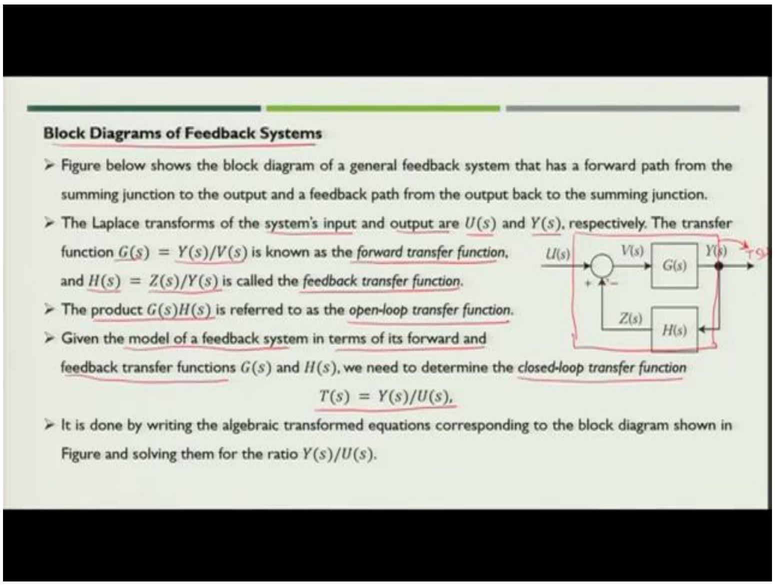

So here we have added negative as a feedback signal.

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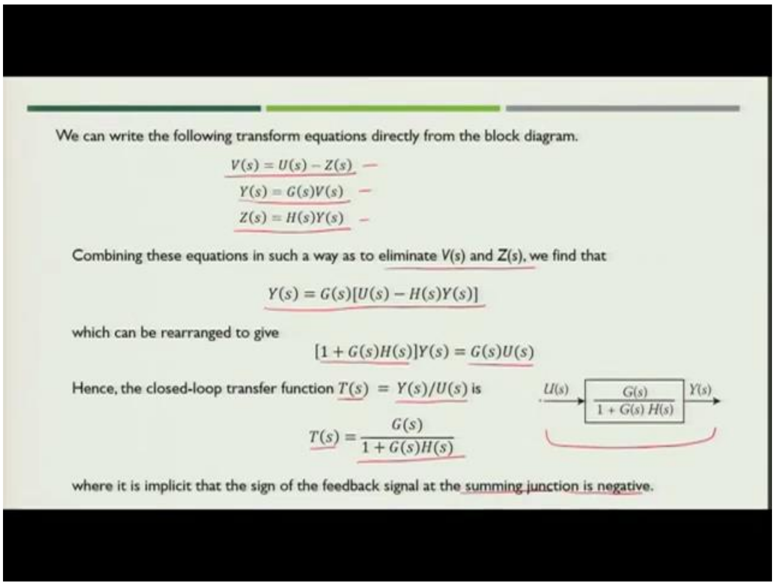

To find the Laplace transform of the complete feedback system.

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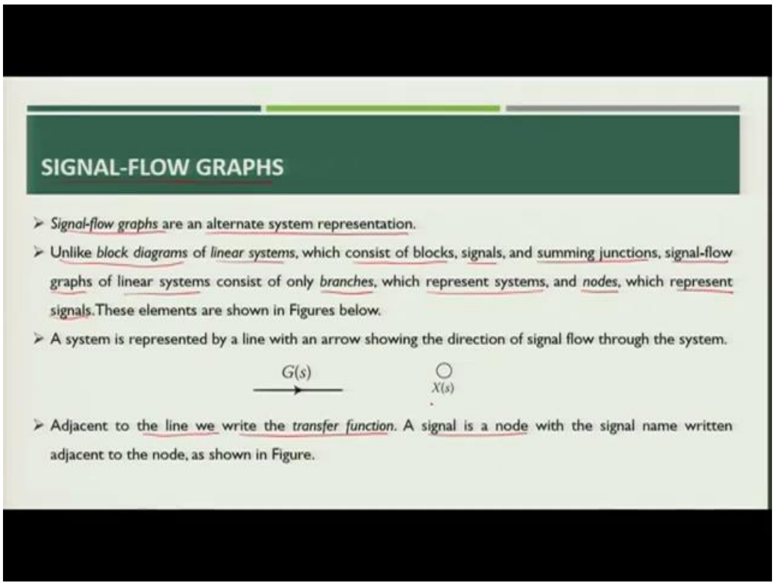

Now, let us talk about another the graphical representation of the system that is Signal Flow Graph. So, Signal Flow Graphs are also an alternative system representation. Unlike the block diagram which we discussed for the linear system which consist of blocks, signals and summing junctions. In case of signal flow graph we will see, for the linear system we will see only branches which represent the system and the nodes which represent the signals.

So, if you see the branch if you see in the signal flow graph the $G(s)$ is a transfer function of the system and node is represented by a circle where $X(s)$ is the signal that can be either input output or the intermediate signal of the system. Now adjacent to line we write the transfer function. So, this is a transfer function of the system and in case of signal we write the transfer function near to that particular node.

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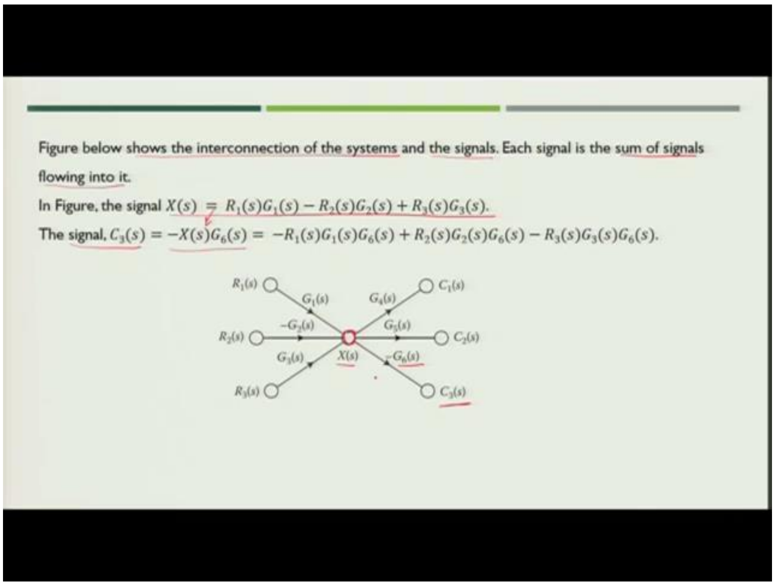

Now, the figure which you see below will represent the interconnection of the systems and signals. Now each signal here is the sum of signals flowing into it. So, if you see this figure and try to find out the value of $X(s)$. So $X(s) = R1(s)G1(s) - R2(s)G2(s) + R3(s)G3(s)$.

Now the value of signals $C3(s) = -X(s)G6(s) = -R1(s)G1(s)G6(s) + R2(s)G2(s)G6(s) - R3(s)G3(s)G6(s)$. So, this signal flow graph property will help in finding out the transfer function of the system more easily.

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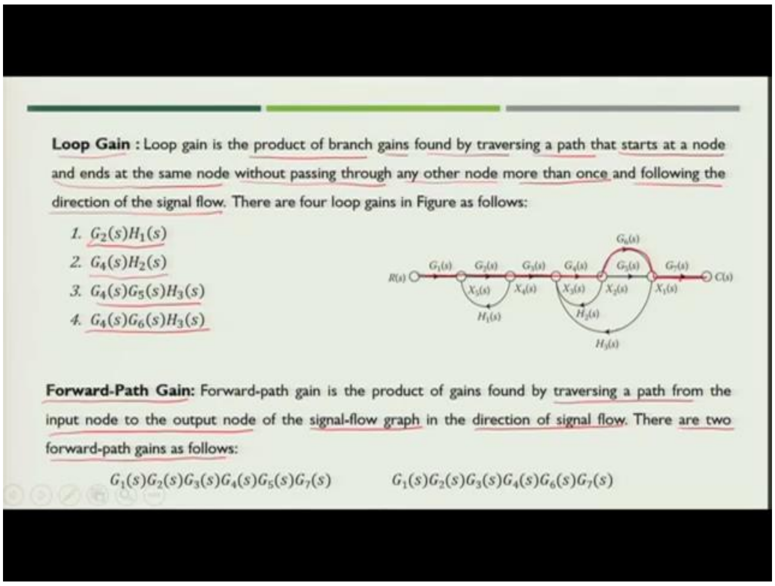

Now, before going into finding out the transfer function of a system let us understand few terms which we will use for finding out the transfer function of the system using signal flow graph.

First is Loop gain, so loop gain is the product of branch gains formed by traversing the path that starts at a node and ends at the same node without passing through any other node more than once and following the direction of the signal flow.

So that means if you are asked to find out the loops, loops will be, first loop you will see as you travel in this fashion. This is one loop where you are starting from the same node and coming back to the same node without tracing the nodes more than once. So the loop gain of this loop will be $G_2(s)H_1(s)$. Similarly, we can find other loops also, one such loop is this one.

So this will become $G_4(s)H_2(s)$, another loop can be this one. So this will become $G_4(s)G_5(s)H_3(s)$. Similarly, there is another loop which you can trace using this section. So this will become $G_4(s)G_6(s)H_3(s)$. So, you see in this signal flow graph we have four independent loops.

Now, let us talk about Forward Path Gain. So, what is Forward Path Gain? Forward Path gain is the product of gain found by traversing the path from input node to the output node of the signal flow graph and that is in the direction of signal flow. So, if you see this signal flow graph there are 2 forward Path gains for the particular signal flow graph. What are those forward Path?

One such path is the direct path which you can see which is connecting input to output. So the gain of that particular forward Path is $G_1(s)G_2(s)G_3\ (s)G_4(s)G_5(s)G_7(s)$ so this is one straight path. What can be the other path? Other path can be the path which is going via $G_6(s)$. So, you get another forward path gain that is $G_1(s)G_2(s)G_3\ (s)G_4(s)G_6(s)G_7(s)$. So, you will get two forward Path gains with respect to the signal flow graph which we see in this figure.

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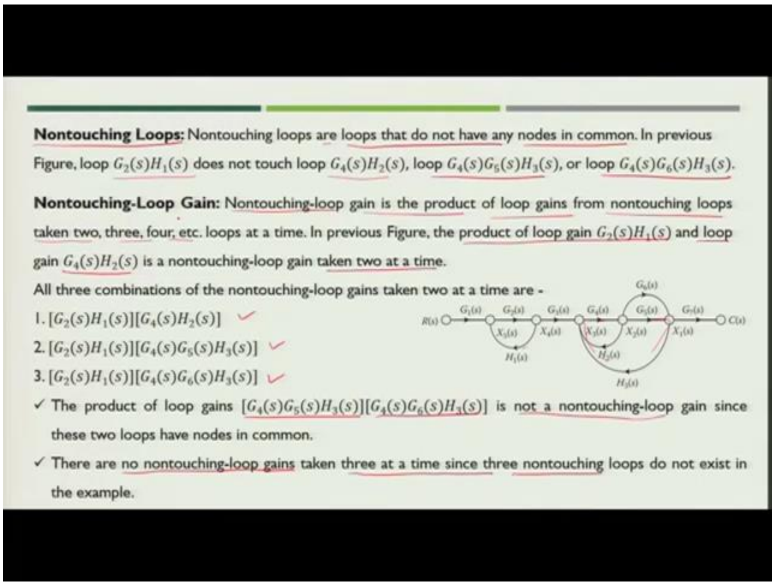

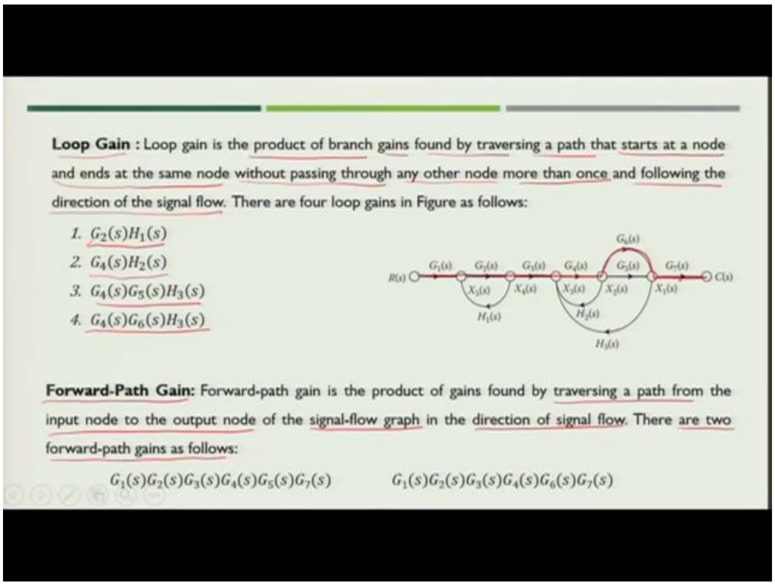

Next is to find out the non-touching loops. So non-touching loops are the loops that do not have any node in common. So, if you see the previous case we found 4 loops out of that the slope is the loop which is not connecting through any of the other 3 loops. Then we can write that $G_2(s)H_1(s)$ that is the first loop is not touching other 3 loops.

So those other 3 loops $G_4(s)H_2(s)$, loop $G_4(s)G_5(s)H_3(s)$, or loop $G_4(s)G_6(s)H_3(s)$. So, the combination of these loops will be the non-touching loops gains. When you identify non-touching loops you will be, you can easily find out the non-touching loop gains from the non-touching loops taken two at a time or three or four at a time.

So, in this particular figure the product of loop gain that is $G2(s)H1(s)$ and loop gain $G4(s)H2(s)$ is the non-touching loop gain taken two at a time. Similarly, for others also you can write, so first we will take non-touching loop gain by considering this loop and this loop then we take the loop gain by taking this loop and the other loop and then third one you take this loop and this loop.

So you will have 3 sets of non-touching loop gains where you will take two at a time. Now the product of loop gains like if you take this loop and this loop, this cannot be considered as a non-touching loop gains because the nodes are common. So we will only consider the nodes which are not, the loops which do not have common notes for non-touching loop gain.

Now, we can also identify that there is no non-touching loop gains where we can take three loops at a time because we cannot find out three non-touching loops which do not have the common nodes. So we will have only two sets, two non-touching loop gains means when we take two loops at a time then only we will have non-touching loop gains

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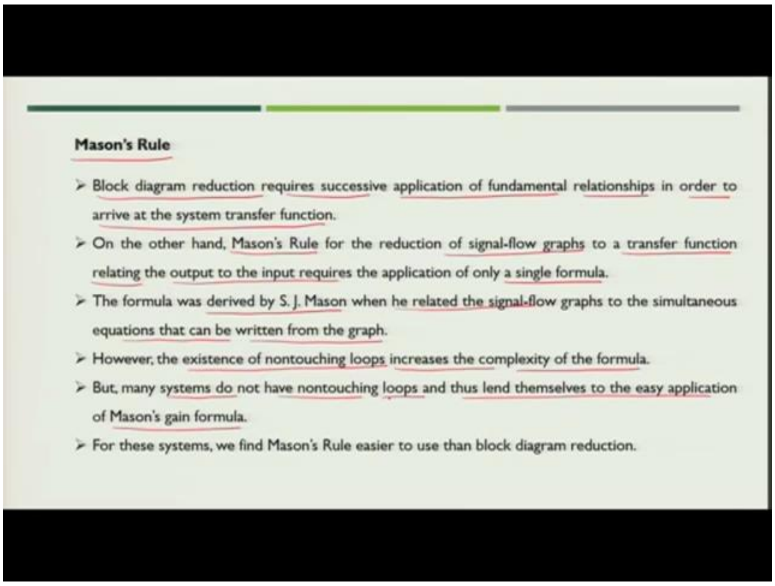

Now, let us talk about the Mason's rule which is important to find out the transfer function of the system. Generally, the block diagram reduction requires the successive application of fundamental relationships in order to arrive at the system transfer function. This is what we have seen in case of block diagram where we have to apply the fundamental relationships to come at the final transfer function of the system.

Now with the help of Mason's rule we can utilize the rule reduce the signal flow graph to a particular transfer function which can relate output to input and this require only one single formula to convert signal flow graph into the transfer function and this formula was derived by

SJ Mason when he related the signal flow graph to the simultaneous equations that can be written from the graph.

Now, it has complexity involved when you have multiple non-touching loops. So, if you have larger number of non-touching loops the complex of the formula will increase. This we can understand when we see the formula which was given by Mason. So many systems generally we do not have the non-touching loops, so in that way this will be very easy to find the transfer function. Generally, when you have no non-touching loop or only few non-touching loop you can utilize the Mason's formula easily find out the transfer function.

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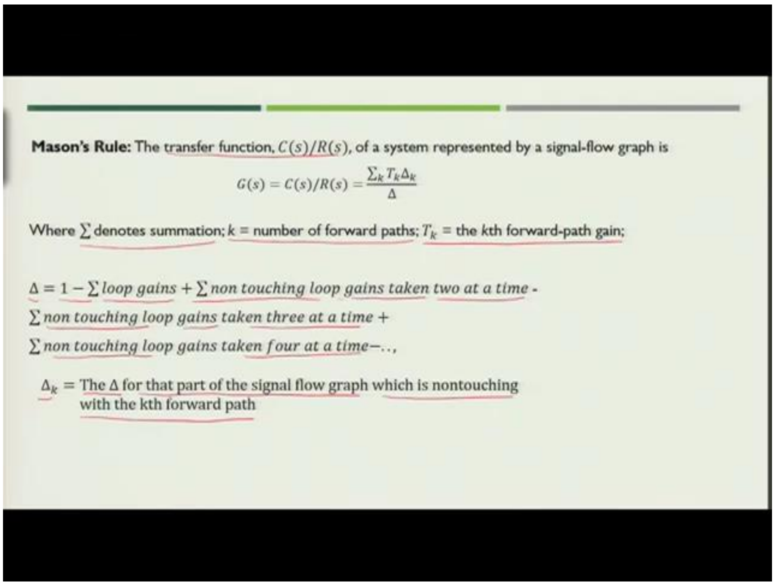

So, let us see what was the Mason rule? So suppose, if $C(s)$ is the output and $R(s)$ is the input Laplace transforms so the transfer function of the system can be represented as $C(s)/R(s)$ which can be provided with the help of Mason's rule

$$G(s) = C(s)/R(s) = \frac{\sum_k T_k \Delta_k}{\Delta}$$

$\sum$ denotes summation; k = number of forward paths; T_k = the kth forward-path gain.

$\Delta = 1 - \sum loop\ gains + \sum non\ touching\ loop\ gainstaken\ two\ at\ a\ time$ -
$\sum non\ touching\ loop\ gains\ taken\ three\ at\ a\ time +$
$\sum non\ touching\ loop\ gains\ taken\ four\ at\ a\ time - ..,$. So Δ will give you the loop gains related to individual loop gains as well as the non-touching loop gains when you take two at a time or three or four and so on at time.

Now,

$$\Delta_k = \Delta - \sum \text{loop gain terms in } \Delta \text{ not touching the } k^{\text{th}} \text{ forward path}$$

So we close our discussion at this point of time and we will continue tomorrow, thank you.