Basic Electric Circuits
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Module 10 - Sinusoidal Steady State Analysis 2
Lecture 50 - Unbalanced Three Phase System and Three Phase Power Measurement

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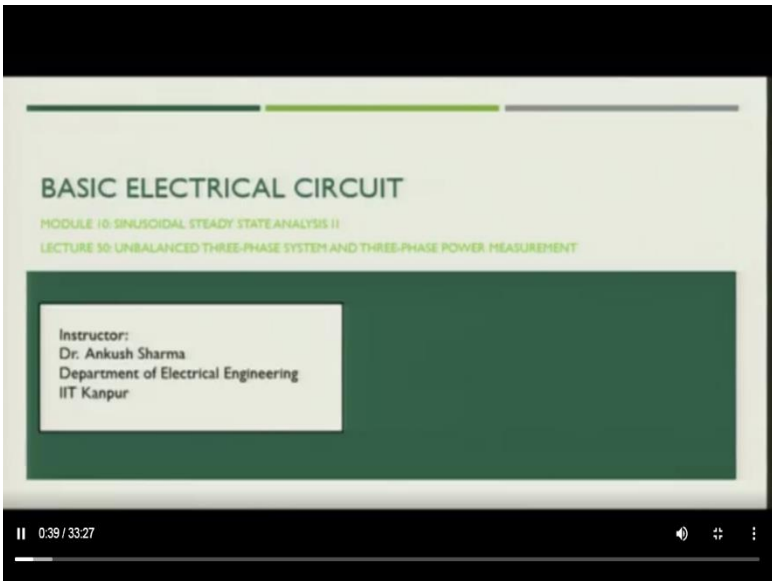

Namaskar. In last session we were discussing about the power measurement for a three-phase balanced system. Today we will start our discussion with the understanding about the unbalanced three phase system and then we will also discuss how to measure the power in case of unbalanced three phase system.

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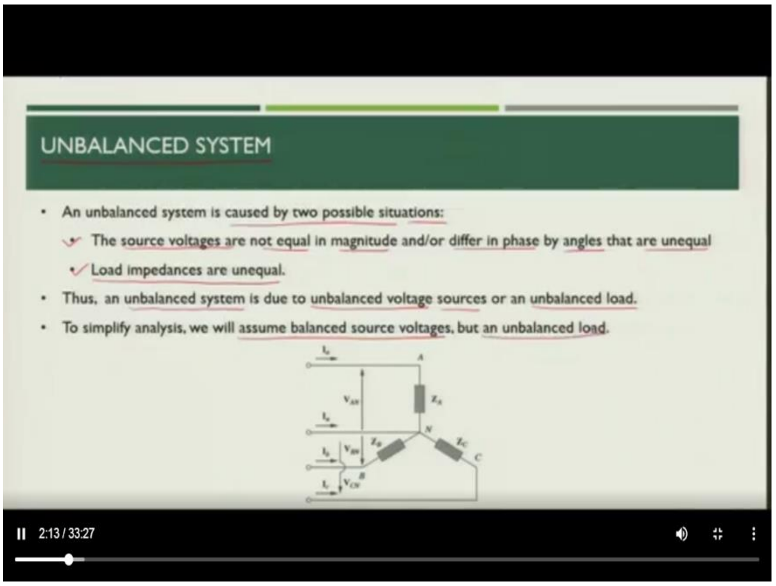

Let us start our discussion of today's lecture. First let us understand what unbalanced system is. So, unbalanced system is caused by two possible situations. What is that? The source voltage is not equal in magnitude and or differ in phase by angle that are unequal. That means for a balanced system we generally know that the phases are equally upon equally distant with respect to each other that is 120 degree apart from each other.

But in case of unbalanced system the phases will not be exactly 120 degree apart. There would be some phase difference. In that case, we will say that the system is unbalanced. Second case might be the case of unbalanced load where the load impedances are unequal. These are the two possible conditions under which we say that the three phase system is unbalanced.

So, therefore, we can say that unbalanced system is due to unbalanced voltage sources or unbalanced load. Now to simplify the analysis related to unbalanced system in this session we will assume that the source is balanced means the voltages, magnitude as well as angle are as per the balanced voltage source, but the load is unbalanced. It means that the load impedances that is $\mathbf{Z}_A$, $\mathbf{Z}_B$, and $\mathbf{Z}_C$ are not same.

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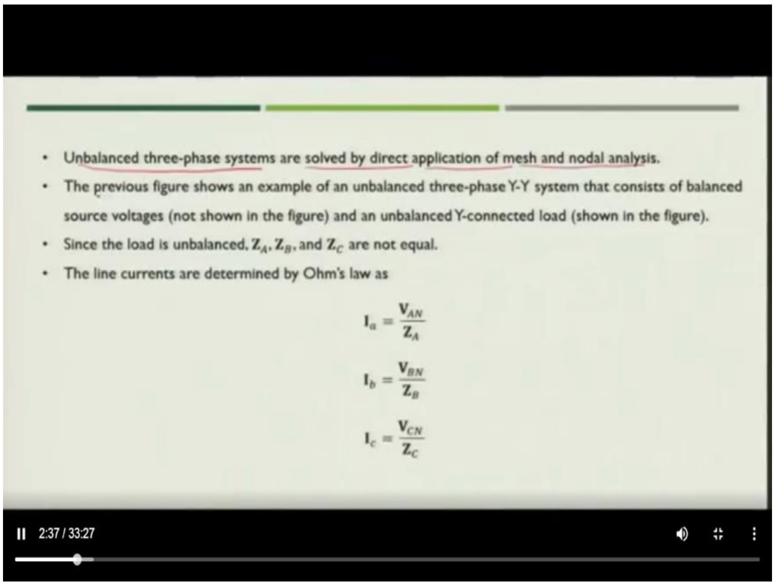

So in that case for the unbalanced three phase system, the various electrical parameters can be calculated by direct application of either mess of nodal analysis. So in the figure which we just saw in this slide this is an example of unbalanced three phase star star or you can also say Y Y system that consists of balance source.

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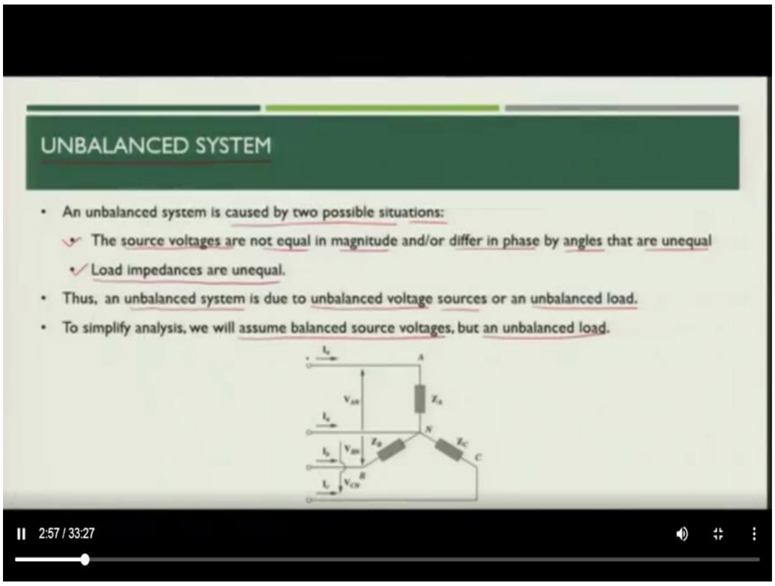

Balanced source is behind the particular nodes, which we are seeing so we are not showing that in the figure but since our objective is to analyze the circuit by assuming unbalanced load.

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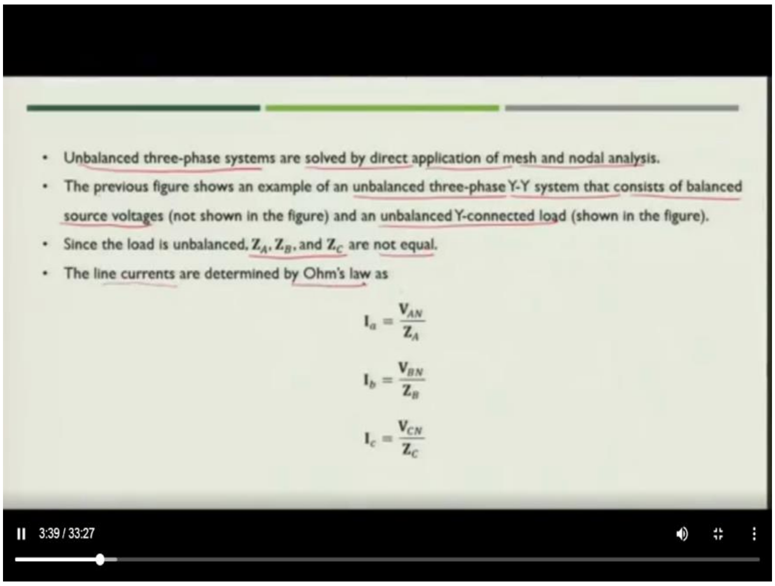

We are showing only the unbalanced load in the figure. So, we will see only the unbalanced load as we saw in the previous figure. Here in case of unbalanced load $\mathbf{Z}_A$, $\mathbf{Z}_B$, and $\mathbf{Z}_C$ are the phase impedances which are not equal. Now when we calculate the line current, we calculate the line current with the help of Ohm's law.

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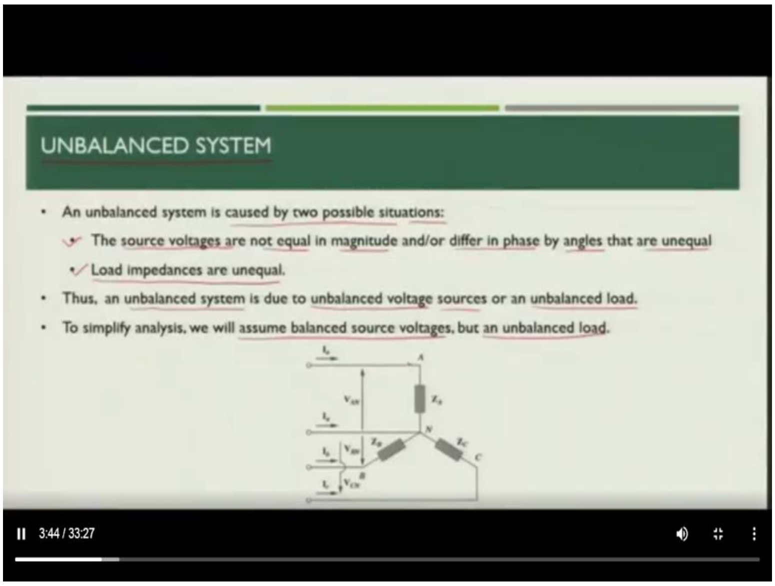

So,

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C}$$

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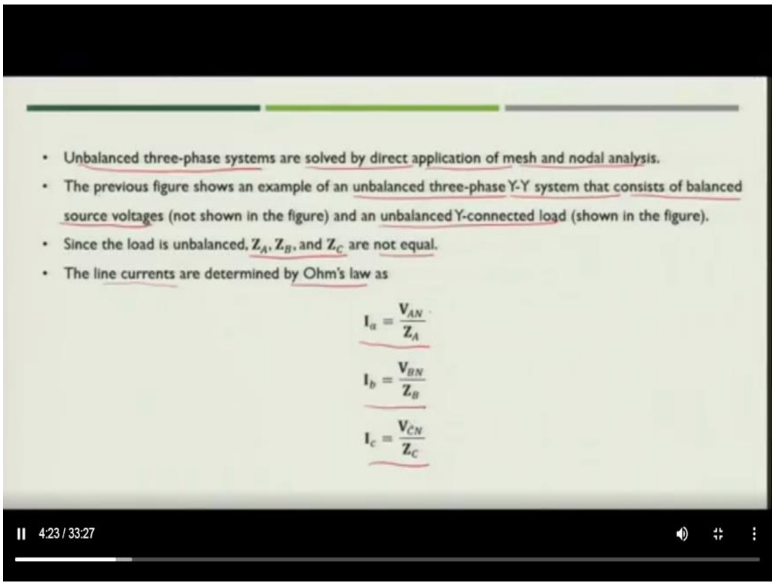

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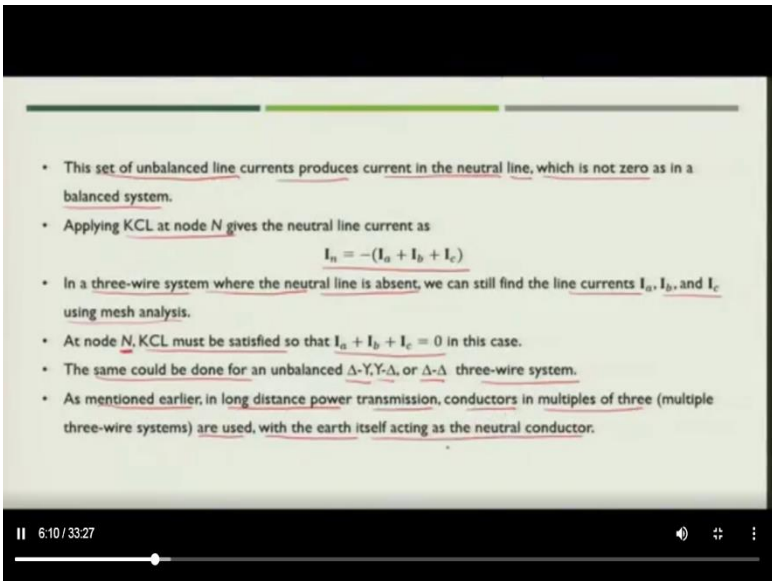

Now this set of unbalanced line current produce current in the neutral wire because the total sum of line currents, that is $\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$ will not be 0 which we saw in case of balance system. So in that case if we apply KCL at node N we get the value of neutral current that is

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$$

Now in case where we see the three-wire system, that means the neutral line is absent we can still find the line current $\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_c$ using mesh analysis. But in those cases when the neutral line is absent at node n KCL will still hold. That means $\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$.

Now this we have analyzed with the help of star star or Y Y connected system same we can do for the unbalanced Delta Y, Y Delta or Delta Delta three phase three wire systems. So, as we mentioned earlier the long-distance power transmission conductors in multiples of three, that is we have three phase three wire system so are used. In that case the earth itself will act as a neutral conductor. So, we do not lay the neutral conductor along the long transmission line. We use earth as a neutral conductor.

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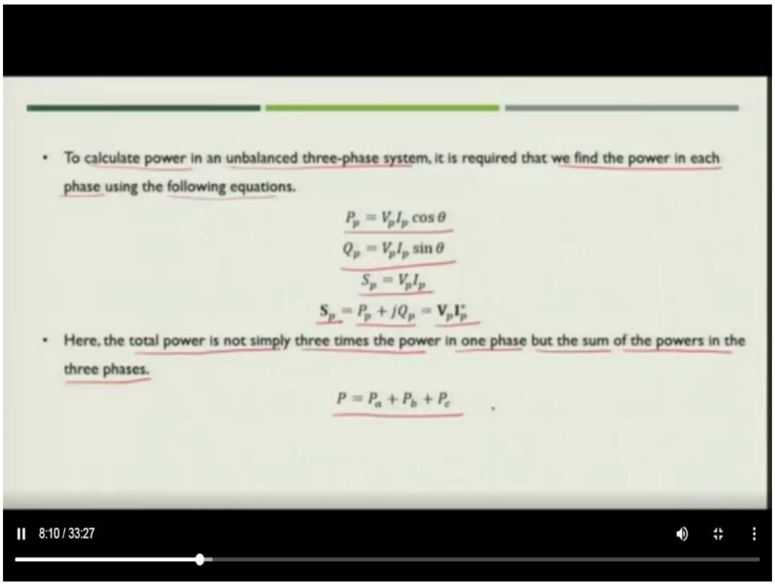

Now to calculate the power in an unbalanced three phase system, what we require? We require that we find the power in each phase using the equation which we have seen earlier also. So like real power in phase we can write as,

$$P_p = V_p I_p \cos\theta$$

$$Q_p = V_p I_p \sin\theta$$

$$S_p = V_p I_p$$

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$

The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

$$P = P_a + P_b + P_c$$

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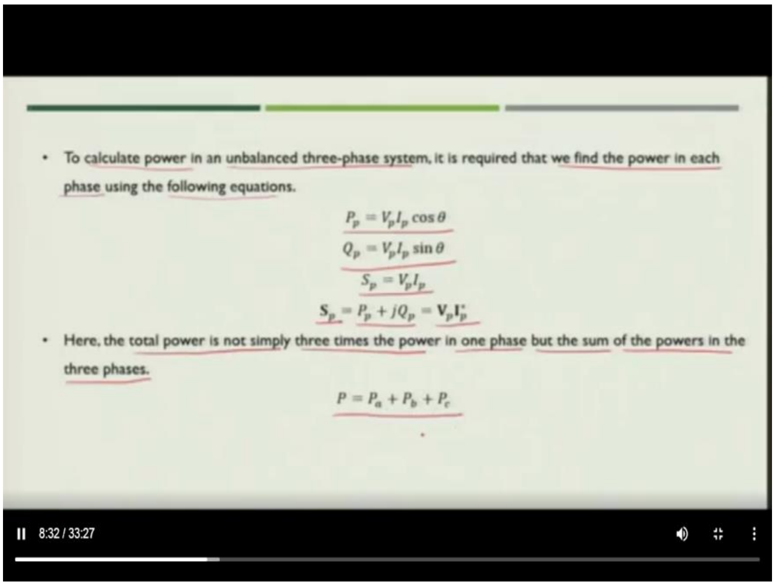

Now let us see how we will measure this power because we have found the condition that under unbalanced condition, we will get the value of power P as a sum of individual powers in each phase. How will calculate or how will measure this power?

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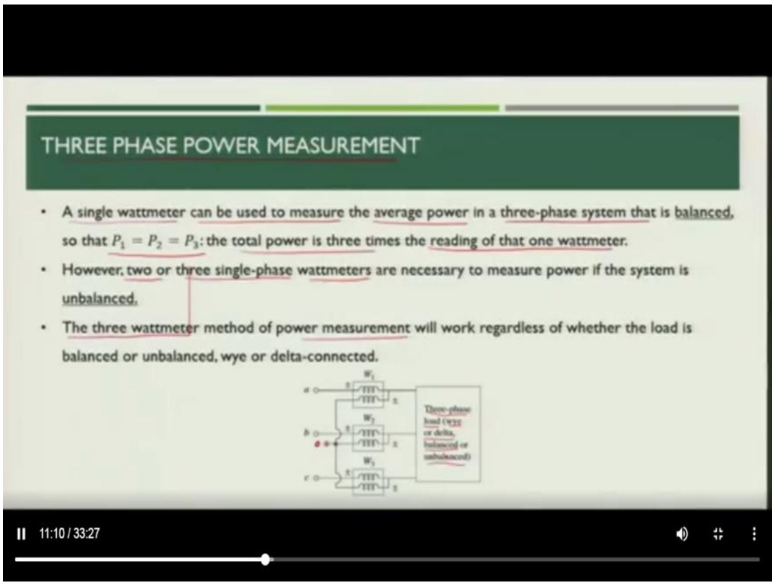

Will use different techniques for three phase power measurement. Let us discuss the techniques which we will use for the measurement of three phase power. So if we use the watt meter in case of balance system single watt meter can be used to measure the average power in three phase system because when we have the system completely balanced the power in each phase will be equal. So, if we measure power for one single phase the total power will be three times of the reading of 1 watt meter.

Now if the system is unbalanced, in that case the single watt meter method cannot be utilized. In that case, we need either two or three single phase watt meters to find out the total power in case of unbalanced three phase system. Let us first try to understand how we will use three-watt meter method for power measurement. We will use the watt meters connected in each phase of the line.

We will connect the current coil in series and potential coil in parallel. So parallel means the zero is neutral or you can consider any line through which we are able to find out the difference potential difference between a and o so V_{ao} will be the voltage which will be given by the potential coil and will get the total power from the meter W_1 .

Similarly, for W_2 and W_3 meters will connect them to individual phases that is b and c and the potential coil that is voltage coil will be connected between phases and the reference node. So, we are considering reference node as o in this arrangement. Now, we need to find out how we will calculate the total power in case of three phase load which may be connected as Y or Delta or it can be either balanced or unbalanced.

So now let us see how we will find the value of the total power being consumed by this load with the help of the reading which we get from these three-watt meters.

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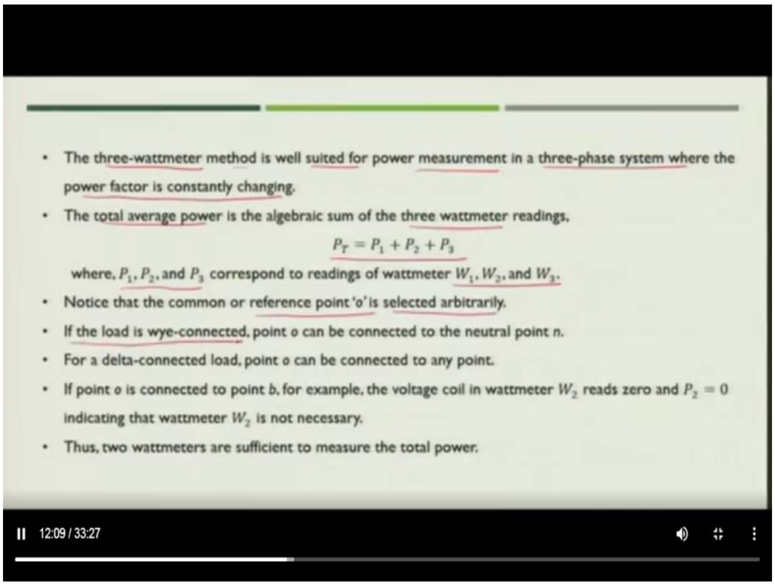

Now the three-watt meter method is well suited for power measurement in a three phase system where power factor is constantly changing. So the total average power in the case of three watt meter method will be the algebraic sum of three watt meter readings,

$$P_T = P_1 + P_2 + P_3$$

where, P_1, P_2, and P_3 correspond to readings of wattmeter W_1, W_2, and W_3.

Now we have taken the reading with reference to common point o, so we have selected it arbitrarily. But generally, what happens in case of Y connected the o will be nothing but the neutral point so that the voltage which you see across the potential coil will be the phase voltage.

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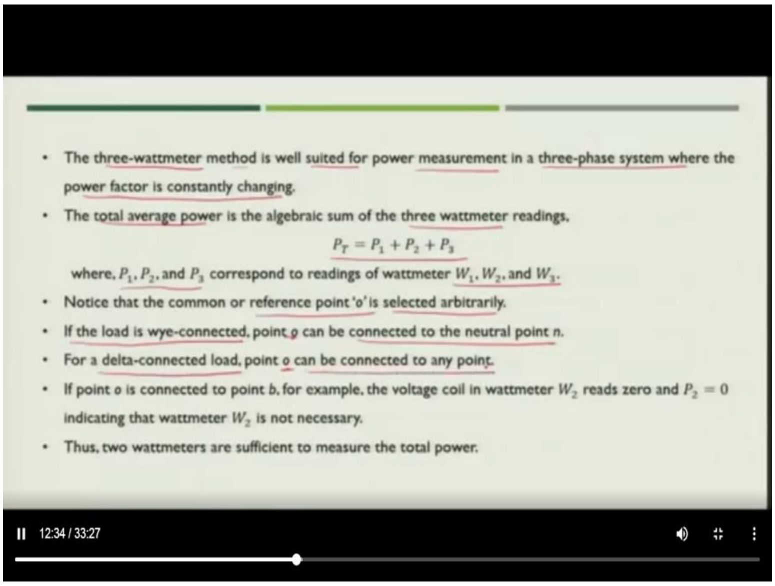

Now if the load is Delta connected, in that case the neutral point will be absent. In that case the reference point o can be connected to any phase. We can connect to either a, b or c if the load is Delta connected. For example, if point o is connected to the phase b that is point b, the voltage coil in the watt meter W 2 will read 0. So therefore, the reading which we get from watt meter W 2 will be 0. That means that in this case, the watt meter W 2 is not necessary to be connected. We can use only 2 watt meter for getting the power consumption details. We will also see that how two watt meter is sufficient to measure the total power.

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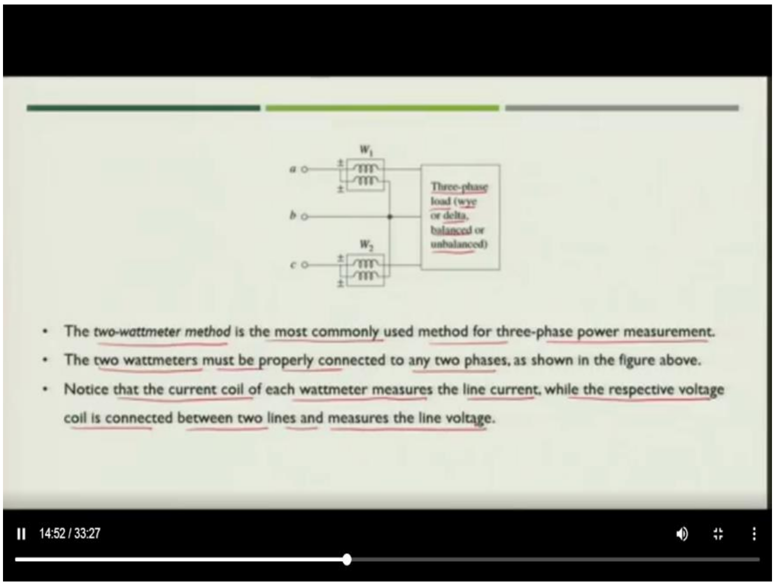

So now this is the arrangement which you will see in case of two-watt meter method when you are asked to find out the total power consumed by the load. Here again, the load is three phase it can be star or Delta connected or it can be balanced or unbalanced. In this case if you see the arrangement of watt meters the current coil is connected in series with the phase a and phase c. And the potential coil is connected between two phases.

So for W_1 the potential coil is connected between a and b and for W_2 the potential coil is connected between c and b. So the two watt meter method is most commonly used method for three phase power measurement. Two watt meters must be properly connected as we see in this figure for two any two phases which we have seen.

Now notice that the current coil of each watt meter measures the line current. So because it is connected in the line while the respective potential coil or we also say voltage coil is connected between two lines. That means you will get the line voltage.

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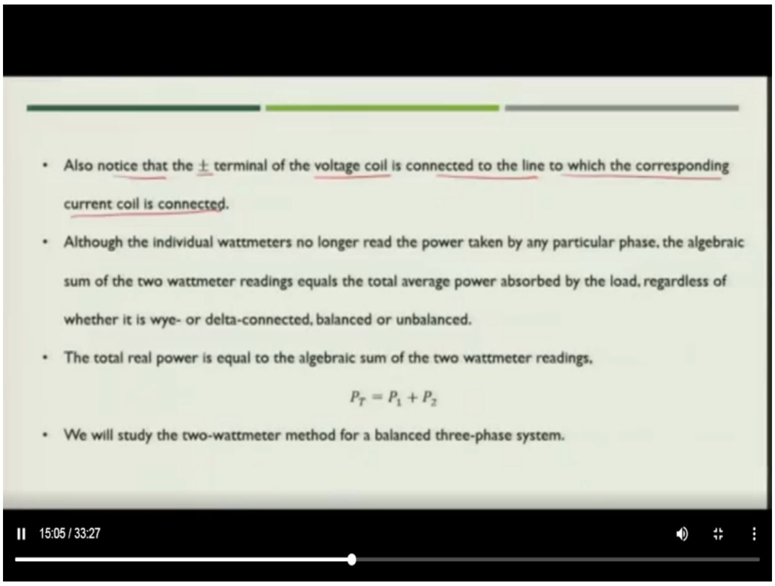

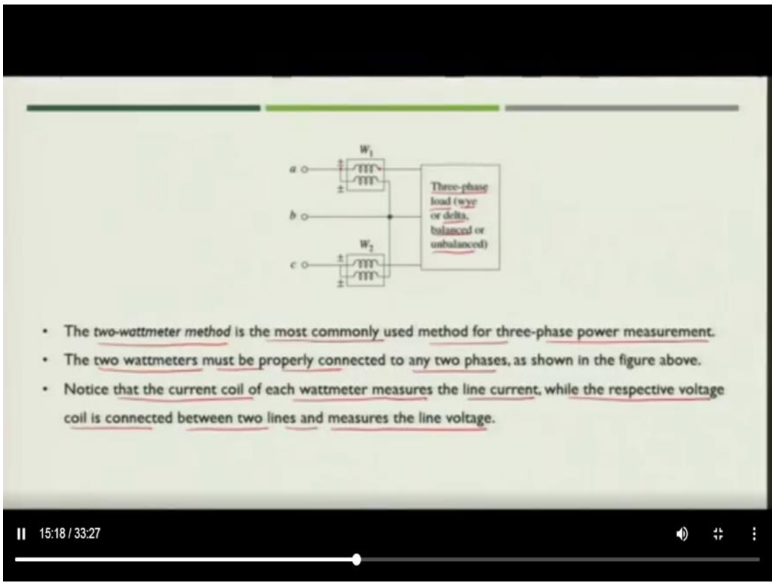

Now you also notice that the plus minus terminal of the voltage coil is connected to the line to which the corresponding current coil is connected. So if you see the direction of the watt meter connection, you will connect potential coil from where the current coil is connected. Now, although the individual watt meters no longer read power taken by any phase because these are the voltage which is coming across the potential coil is not the per phase voltage. In that case the individual watt meters will not give you the reading of power consumed per phase or in a particular phase where it is connected, but the algebraic sum of two watt meters will give us the reading which will be equal to total average power absorbed by the load and this is regardless of whether the load is wye or Delta connected or whether it is balanced or unbalanced. So, the total power will be equal to the algebraic sum of two watt meter readings.

Now, let us understand how we can justify the total power consumed is equal to the sum of the two watt meter readings. For simplicity we will take the two watt meter method for a balanced three-phase system so that we can stabilize the relationship of the total power consumption versus the individual watt meter reading.

So, will justify that the total power consumption is nothing but algebraic sum of two watt meter readings.

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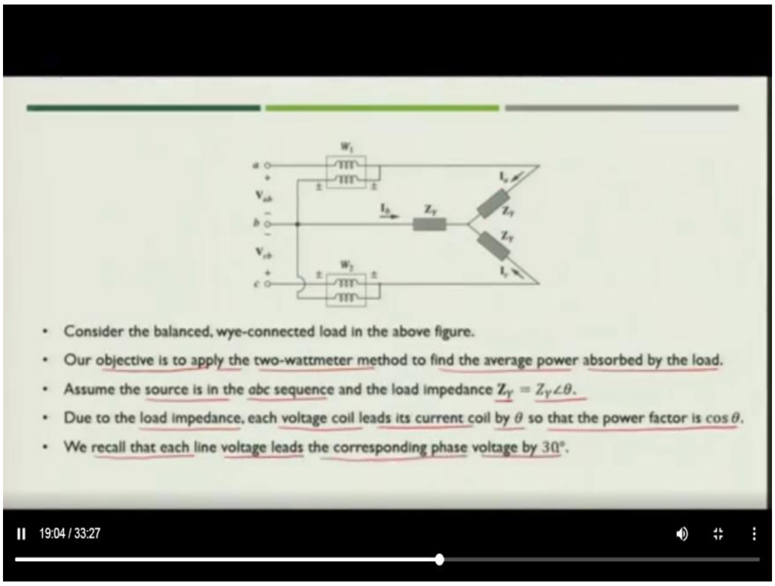

So for that lets consider this particular circuit arrangement. In this you will see the voltage difference between phase a and b is $\mathbf{V}_{ab}$, between phase b and c is $\mathbf{V}_{cb}$. V_{cb} is written because the potential coil is connected from c to b. So, we will consider the c as plus point and b as minus so V_{cb} will be the potential across the potential coil because we consider b as a reference point for the connection of the watt meter coils. Now $\mathbf{I}_b$ is the current which is flowing in b phase $\mathbf{I}_a$ in a phase and $\mathbf{I}_c$ in c phase.

So these $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$ are also the line currents. Now for simplicity we have assumed that the load is balanced. So we will say the impedance of the load is $\mathbf{Z}_Y$ in all three phases. Now the objective for this particular study is that will apply two watt meter method to find the average power absorbed by the load.

Now since we have considered the Y connected load and we assume the source is in a b c sequence. So in that case the $\mathbf{Z}_Y$ if you convert it into phasor it will be $\mathbf{Z}_Y = Z_Y\angle\theta$. So, we assume that the load impedance will cause the voltage coil to lead its current coil by θ because this is the load which is applied. So, in that case what we can say that the power factor is cos Theta.

Also, if we recall what we discussed in case of balance star star or Y Y connected network. We saw that the line voltage leads the corresponding phase voltage by 30 degree.

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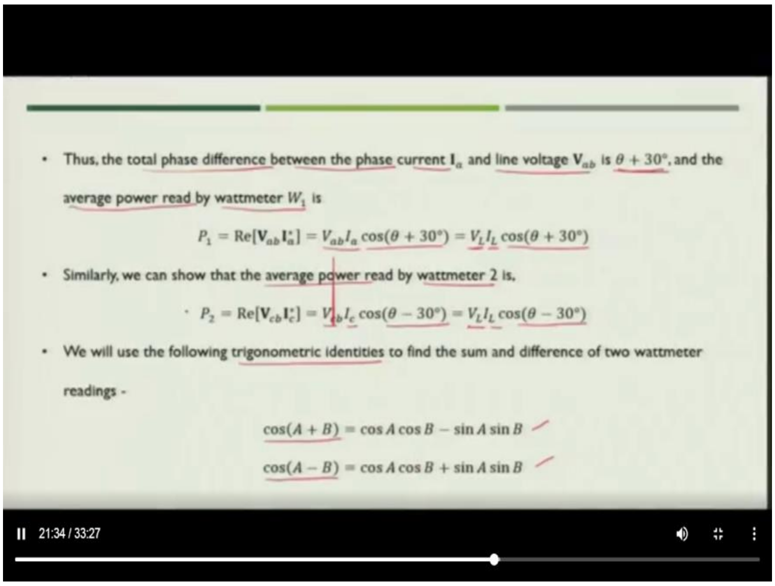

In that case what we can write that the total phase difference between phase current $\mathbf{I}_a$ and the line voltage $\mathbf{V}_{ab}$ will be $\theta + 30°$. This is we have got when we take the reference as a phase voltage, so your line voltage will be leading by 30 degree.

In that case the average power, which watt meter W_1 is

$$P_1 = \text{Re}[\mathbf{V}_{ab}\mathbf{I}_a^*] = V_{ab}I_a\cos(\theta + 30°) = V_L I_L \cos(\theta + 30°)$$

Similarly, we can show that the average power read by wattmeter 2 is,

$$P_2 = \text{Re}[\mathbf{V}_{cb}\mathbf{I}_c^*] = V_{cb}I_c\cos(\theta - 30°) = V_L I_L \cos(\theta - 30°)$$

We use the trigonometric identities,

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

to find the sum and difference of two wattmeter readings.

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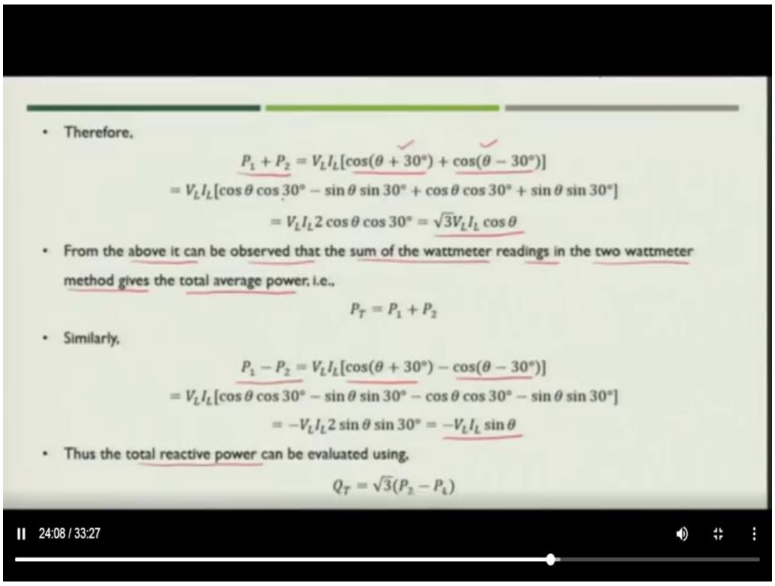

So we will use these two formulas to find out the total value of power,

$$P_1 + P_2 = V_L I_L[\cos(\theta + 30°) + \cos(\theta - 30°)]$$

$$= V_L I_L[\cos\theta \cos 30° - \sin\theta \sin 30° + \cos\theta \cos 30° + \sin\theta \sin 30°]$$

$$= V_L I_L 2 \cos\theta \cos 30° = \sqrt{3} V_L I_L \cos\theta$$

From the above it can be observed that the sum of the wattmeter readings in the two wattmeter method gives the total average power, i.e.,

$$P_T = P_1 + P_2$$

Similarly,

$$P_1 - P_2 = V_L I_L[\cos(\theta + 30°) - \cos(\theta - 30°)]$$

$$= V_L I_L[\cos\theta \cos 30° - \sin\theta \sin 30° - \cos\theta \cos 30° - \sin\theta \sin 30°]$$

$$= -V_L I_L 2 \sin\theta \sin 30° = -V_L I_L \sin\theta$$

Thus, the total reactive power can be evaluated using,

$$Q_T = \sqrt{3}(P_2 - P_1)$$

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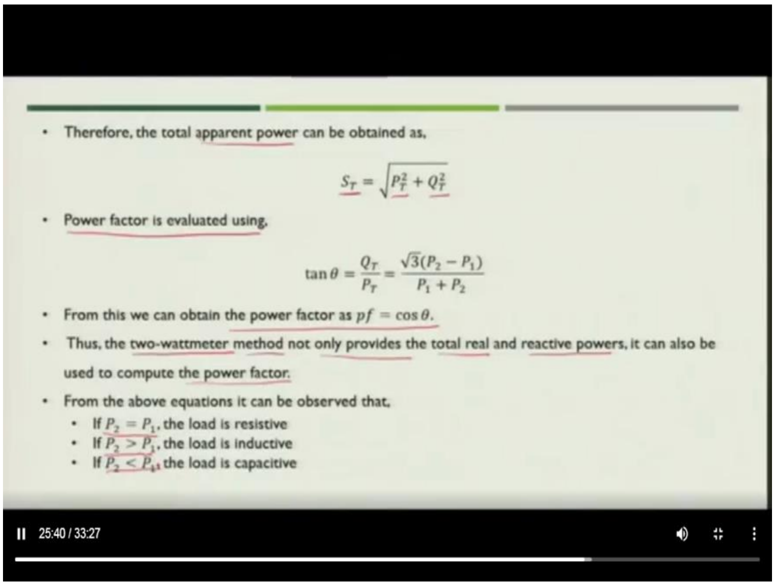

So the apparent power you can now write as,

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

Power factor is evaluated using,

$$\tan\theta = \frac{Q_T}{P_T} = \frac{P_2 - P_1}{P_1 + P_2}$$

From this we can obtain the power factor as $pf = \cos\theta$.

Thus, the two-wattmeter method not only provides the total real and reactive powers, it can also be used to compute the power factor.

From the above equations it can be observed that,

- If $P_2 = P_1$, the load is resistive
- If $P_2 > P_1$, the load is inductive
- If $P_2 < P_1$, the load is capacitive

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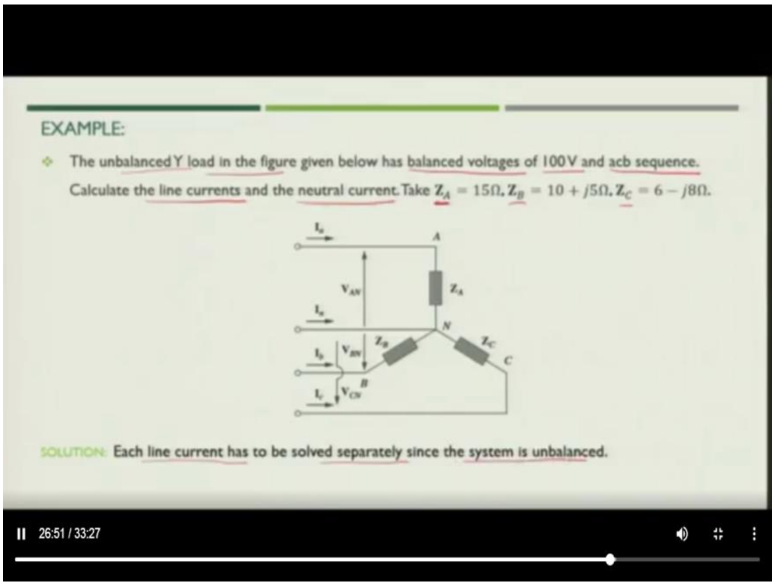

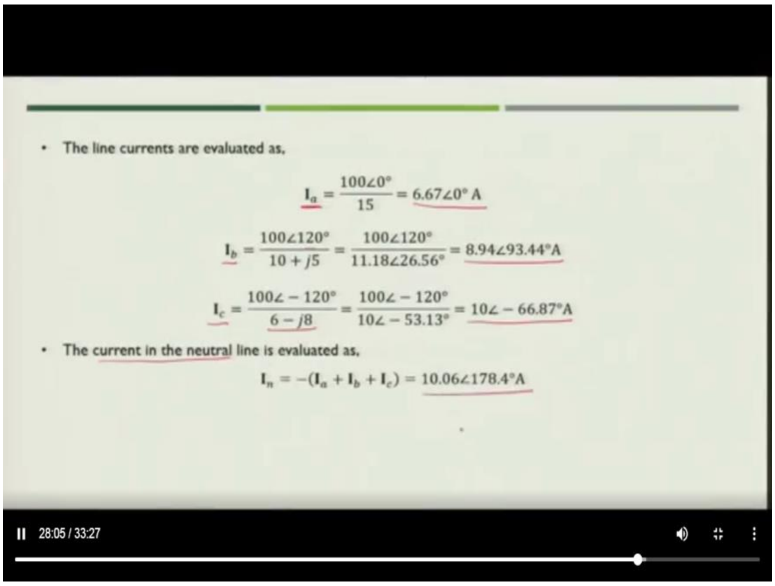

Now, let us take a couple of examples to understand what we have discussed till now. Let us see if we have unbalanced star connected load as shown in the figure is connected to balanced voltage of 100 V and the phase sequence is acb. We need to calculate the line currents and the neutral current for this arrangement where the $\mathbf{Z}_A = 15\Omega$, $\mathbf{Z}_B = 10 + j5\Omega$, $\mathbf{Z}_C = 6 - j8\Omega$. So, if you see this particular star connected load, $\mathbf{Z}_A$, $\mathbf{Z}_B$, $\mathbf{Z}_C$ are not equal.

So that means we have to solve for line current separately because the line currents are not equal, and this system is unbalanced. We calculate,

$$\mathbf{I}_a = \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ \text{ A}$$

$$\mathbf{I}_b = \frac{100\angle 120°}{10 + j5} = \frac{100\angle 120°}{11.18\angle 26.56°} = 8.94\angle 93.44°\text{A}$$

$$\mathbf{I}_c = \frac{100\angle -120°}{6 - j8} = \frac{100\angle -120°}{10\angle -53.13°} = 10\angle -66.87°\text{A}$$

The current in the neutral line is evaluated as,

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 10.06\angle 178.4°\text{A}$$

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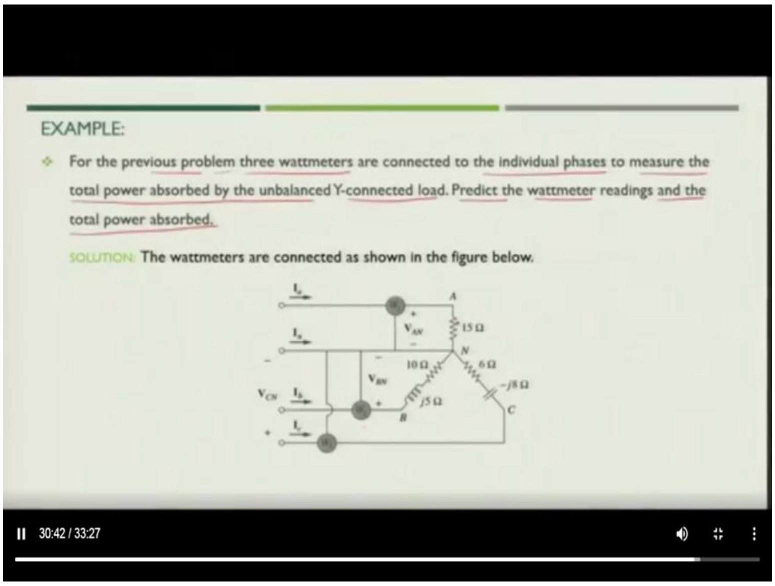

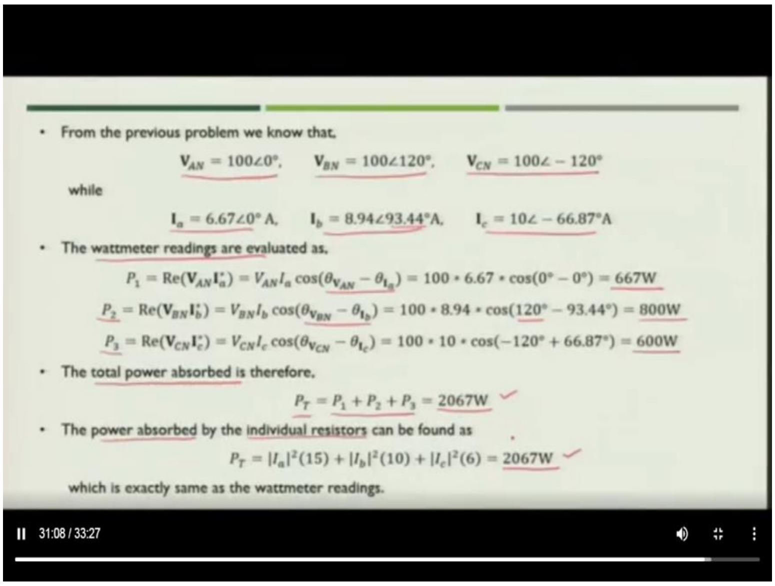

Now let us take another example. So, in the previous problem what we saw we will use three watt meter method where the watt meters are connected to individual phases to measure the total power absorbed by the unbalanced Y connected load. What we need to find out? We need to find out the watt meter readings and the total power absorbed.

So in this case we know $\mathbf{V}_{AN} = 100\angle 0°$, $\mathbf{V}_{BN} = 100\angle 120°$, $\mathbf{V}_{CN} = 100\angle -120°$ while $\mathbf{I}_a = 6.67\angle 0°$ A, $\mathbf{I}_b = 8.94\angle 93.44°$A, $\mathbf{I}_c = 10\angle -66.87°$A. So, the watt meter readings you can simply find out the value as

$$P_1 = \mathrm{Re}(\mathbf{V}_{AN}\mathbf{I}_a^*) = V_{AN}I_a\cos(\theta_{\mathbf{V}_{AN}} - \theta_{\mathbf{I}_a}) = 100 * 6.67 * \cos(0° - 0°) = 667\mathrm{W}$$

$$P_2 = \mathrm{Re}(\mathbf{V}_{BN}\mathbf{I}_b^*) = V_{BN}I_b\cos(\theta_{\mathbf{V}_{BN}} - \theta_{\mathbf{I}_b}) = 100 * 8.94 * \cos(120° - 93.44°) = 800\mathrm{W}$$

$$P_3 = \mathrm{Re}(\mathbf{V}_{CN}\mathbf{I}_c^*) = V_{CN}I_c\cos(\theta_{\mathbf{V}_{CN}} - \theta_{\mathbf{I}_c}) = 100 * 10 * \cos(-120° + 66.87°) = 600\mathrm{W}$$

The total power absorbed is therefore,

$$P_T = P_1 + P_2 + P_3 = 2067\mathrm{W}$$

The power absorbed by the individual resistors can be found as

$$P_T = |I_a|^2(15) + |I_b|^2(10) + |I_c|^2(6) = 2067\mathrm{W}$$

which is exactly same as the wattmeter readings.

Now if you want to cross verify whether you have arrived at the correct value of P, what we can do we can simply find the power absorbed by the individual resistors because the circuit contains three registers and real power will only be absorbed by the resistors. So 15, 6 ohm and 10 ohm are the resistors. So what we can do? We can just calculate the value of $\mathbf{I}^2R$ for each and individual phases and when we sum up we get the value of total power absorbed by the load is equal to 2067 watt. So this says that we have arrived at the correct value of real power.

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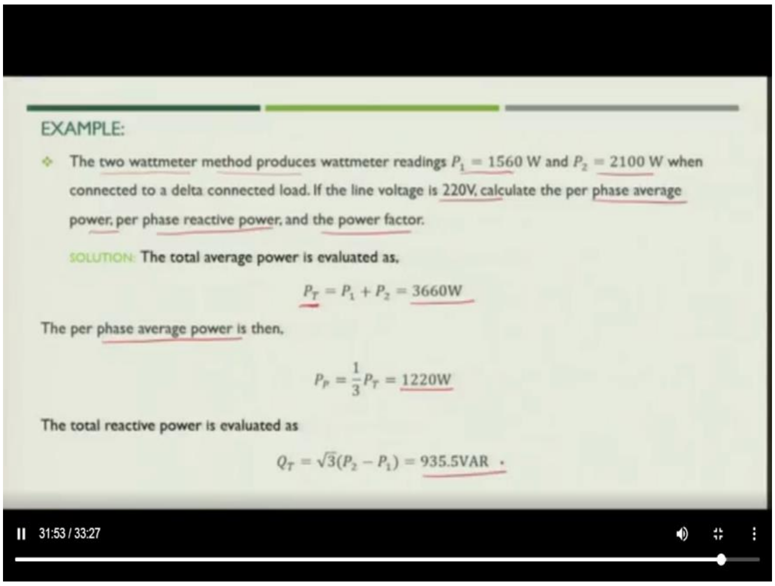

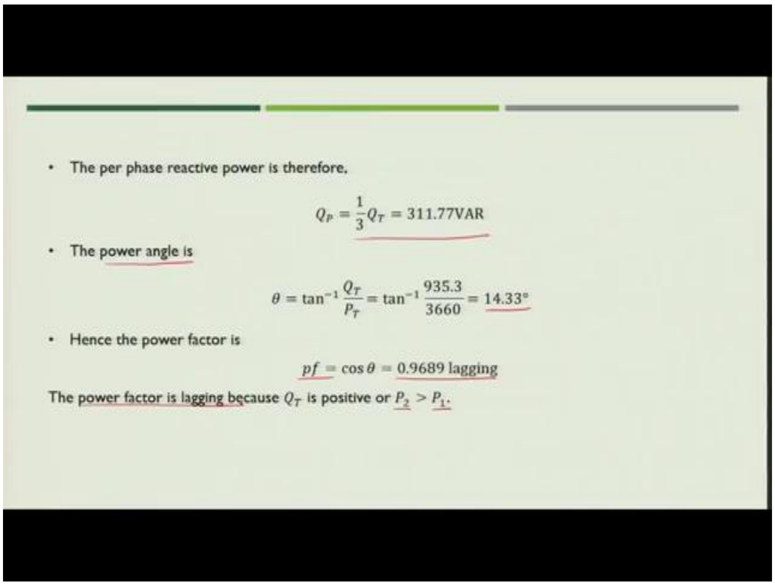

Now, let us see quickly the third method also, third example also where we need to use the two watt meter method to find the value of per phase average power, per phase reactive power and the power factor $P_1 = 1560$ W and $P_2 = 2100$ W is given and the line voltages is 220 volt. Since P 1 and P 2 is given total power average power you will get as

$$P_T = P_1 + P_2 = 3660\text{W}$$

The per phase average power is then,

$$P_P = \frac{1}{3}P_T = 1220\text{W}$$

The total reactive power is evaluated as

$$Q_T = \sqrt{3}(P_2 - P_1) = 935.5\text{VAR}.$$

So, per phase reactive power is,

$$Q_P = \frac{1}{3} Q_T = 311.77\text{VAR}$$

The power angle is,

$$\theta = \tan^{-1}\frac{Q_T}{P_T} = \tan^{-1}\frac{935.3}{3660} = 14.33°$$

Hence the power factor is,

$$pf = \cos\theta = 0.9689 \text{ lagging}$$

The power factor is lagging because Q_T is positive or $P_2 > P_1$.

So, with this we can close our today's session in which we discussed about the unbalanced three phase system. In next week we will start our topic related to state variable analysis. Thank you.