Course Title
Basic Electric Circuits
Module 1
Basic Circuit Elements and Waveforms
Lecture -05
AC Power Analysis
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Namaskar. So yesterday in the class we discussed about the resistance, inductance and capacitance and we saw the response of those circuits when they are connected in series, parallel, combination. So today we will see particularly the capacitor and inductor Phasor relationship with respect to voltage and current.

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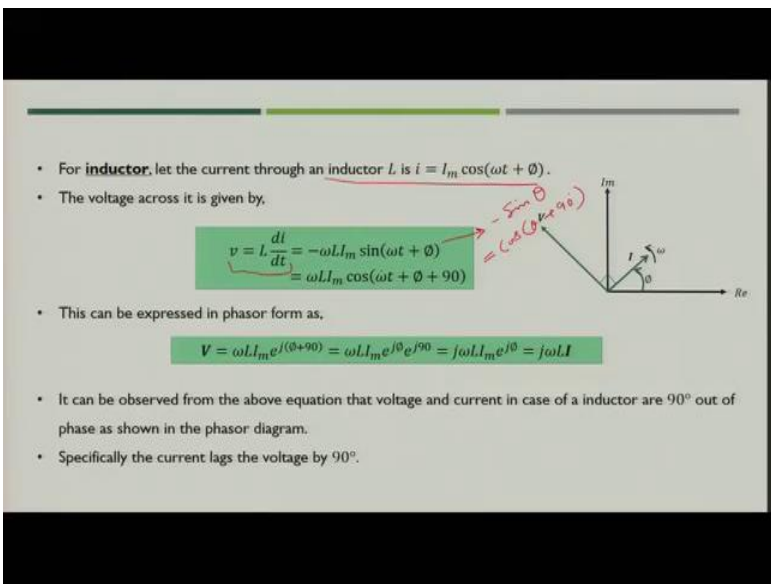

So, for inductor let us assume that the current is $i = I_m \cos(\omega t + \emptyset)$. Let us see the performance of inductor voltage and current with respect to Phasor in the inductor.

Let us say the current in the inductor is $i = I_m \cos(\omega t + \emptyset)$. As we know that the voltage across the inductor can be defined as

$$v = L\frac{di}{dt}$$

In this case the voltage would be, if you differentiate the current i with respect to time you will get

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \emptyset)$$

Now, we discussed couple of lectures back that if you have minus of sin theta you can represented in the form of cos as cos theta plus 90 degree. So the same we have utilized here and we converted the sin into cos and now we can write it as

$$v = \omega L I_m \cos(\omega t + \emptyset + 90)$$

So now how you will represent the voltage in phasor terms?

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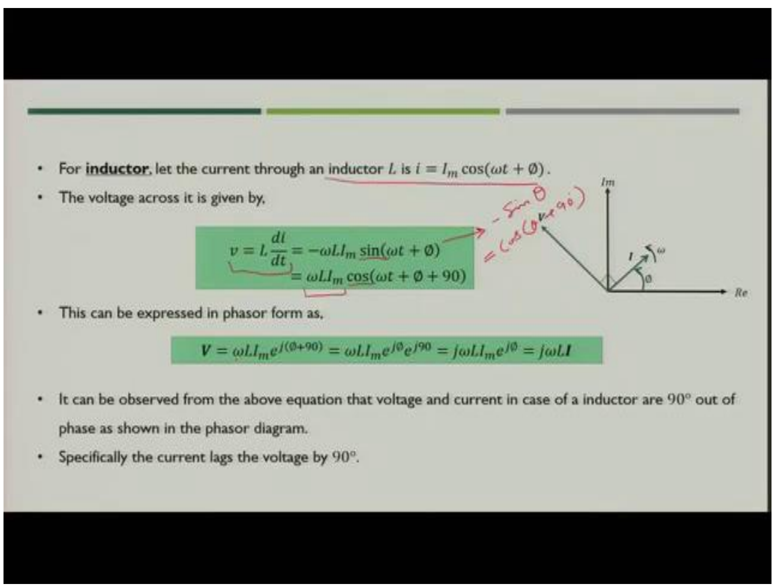

This term $\omega L I_m$ is constant. Then $\cos(\omega t + \emptyset + 90)$ can be represented in Euler form. So this can be written as

$$\boldsymbol{V} = \omega L I_m e^{j(\emptyset+90)} = \omega L I_m e^{j\emptyset} e^{j90}$$

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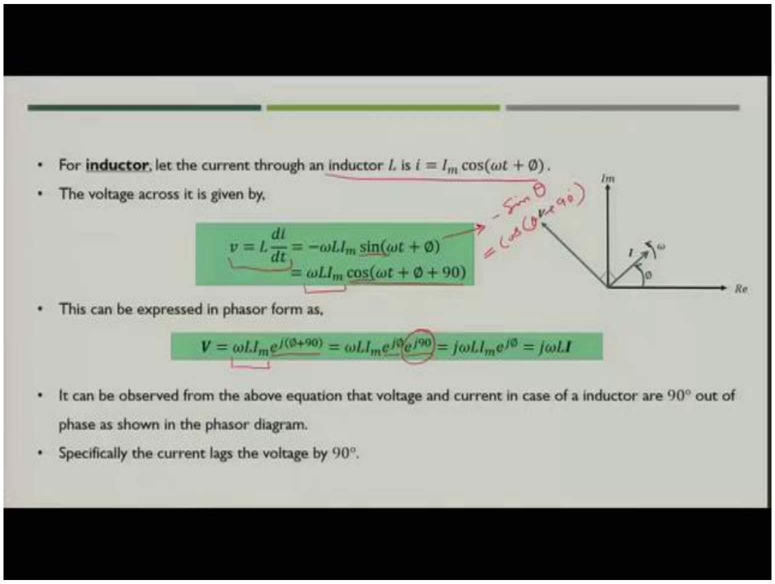

Now, if you see this term, what you will write? As you know that $e^{j\emptyset} = \cos\emptyset + j\sin\emptyset$. So instead of $\emptyset$ if you write 90 degree, so $e^{j90} = cos90 + jsin90 = j$ will become j because cos 90 will be zero.

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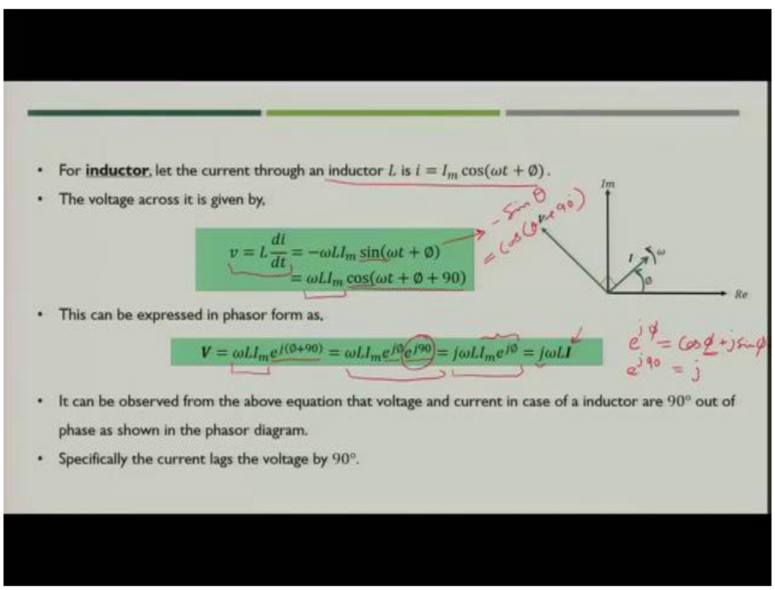

So, this can term can be simplified as $j\omega L I_m e^{j\emptyset}$. Now, if you see this particular term this is nothing but the phasor representation of current. You can write $\boldsymbol{I}$ in bold that shows that this is the phasor representation of current. So you can simplify the voltage Phasor as

$$V = j\omega L I$$

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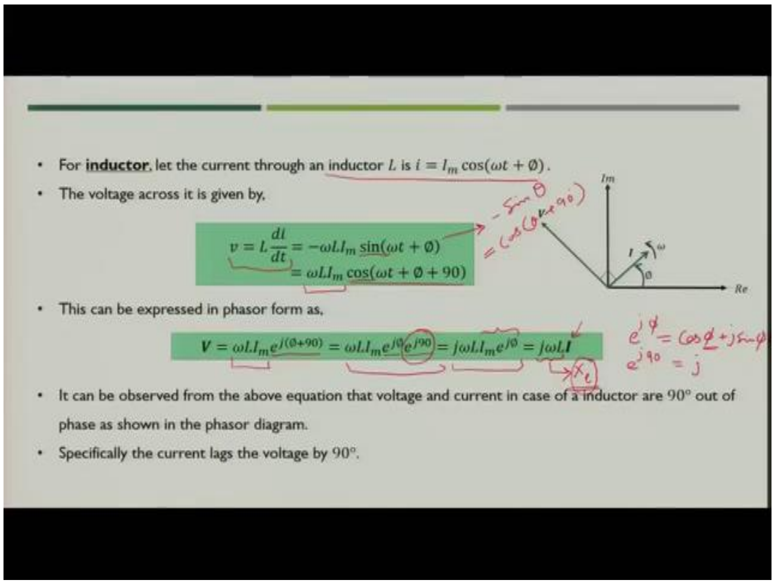

Now, what is $j\omega L$? This term is basically the inductive reactance. So, in simple term we represent it like X_L. So, what does it mean? If you correlate with the Ohm's law, you will see that voltage is represented as $v = iR$. So, if you correlate that Ohm's law with this particular equation, you can easily say that this is something which resist the flow and that is why it is called inductive reactance in case of inductor.

Now, if you plot the current and voltage Phasor on the plane. So, in the x-axis you will have real value and the y-axis you will have imaginary value and if you plot current so it will be Phasor will be in one particular direction making $\emptyset$ angle from real axis.

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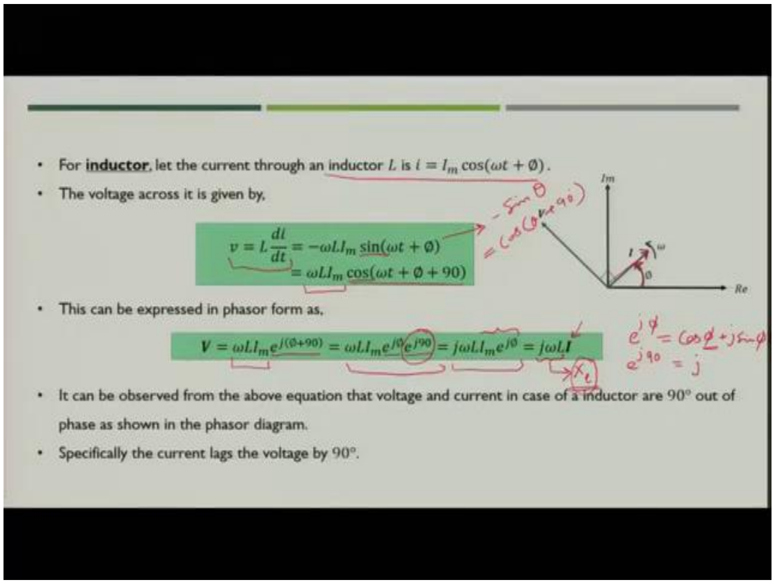

So this would be the Phasor of current I. Now if you see Phasor for voltage.

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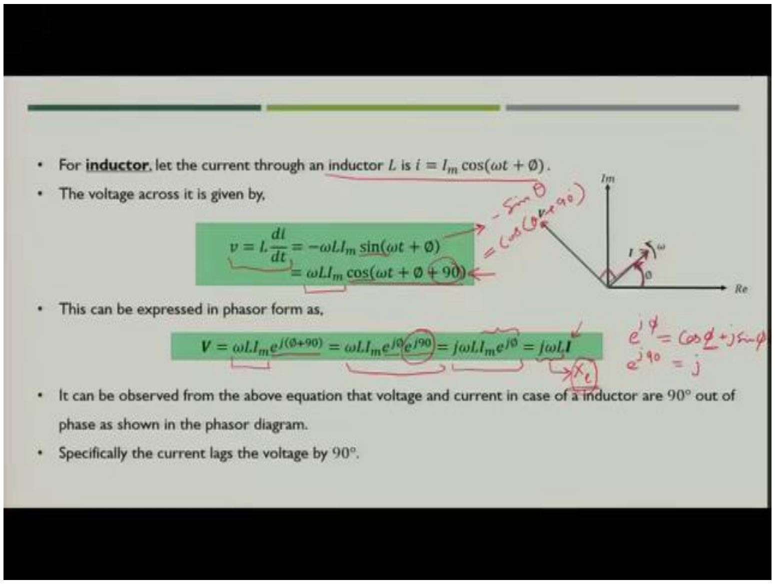

If you see the equation you will come to know that voltage is leading current by 90 degree. So, the voltage Phasor would be 90 degree leading with respect to current and it has some value. The value would be $\omega L I_m$. So that is what we have derived for voltage.

Here the value would be different but the current would be lagging with respect to voltage. If you see the phasor diagram it is clear that voltage angle is $\emptyset + 90$ with respect to real axis and it is having 90 degree leading with respect to current.

Now, let us talk about the capacitor. Suppose the voltage across capacitor is given as $v = V_m \cos(\omega t + \emptyset)$. Then how you will calculate the current?

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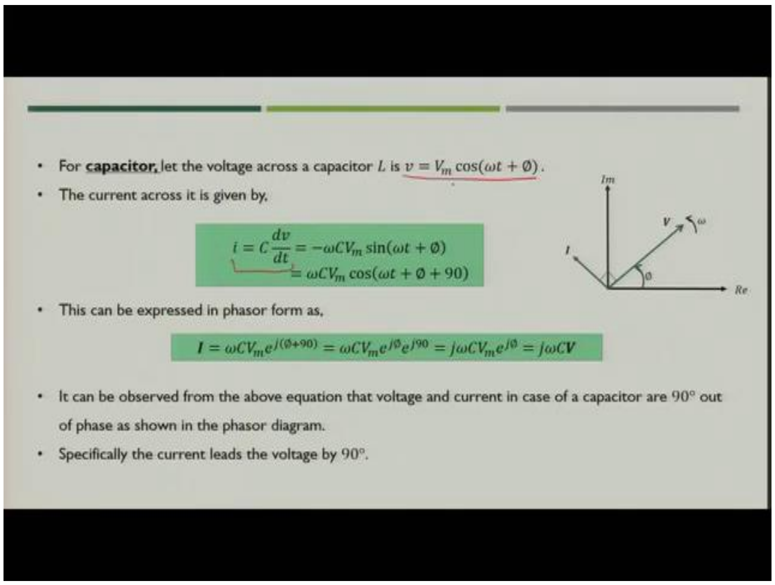

You will utilize the equation

$$i = C\frac{dv}{dt}$$

It means that you have to simply differentiate the voltage term and you will get

$$i = C\frac{dv}{dt} = -\omega C V_m \sin(\omega t + \emptyset)$$

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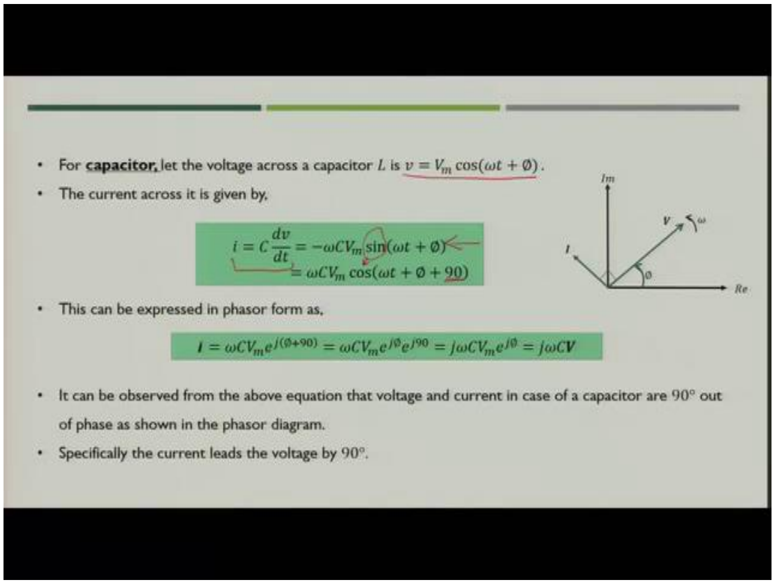

Now as we did in case of inductor we can replace the sin term with cos and V at 90 degree to make it positive because negative sign will be removed as

$$i = \omega C V_m \cos(\omega t + \emptyset + 90)$$

Phasor for current can be represented in Euler form as

$$\boldsymbol{I} = \omega C V_m e^{j(\emptyset+90)} = \omega C V_m e^{j\emptyset} e^{j90} = j\omega C V_m e^{j\emptyset} = j\omega C \boldsymbol{V}$$

So now if you write voltage $\boldsymbol{V} = \boldsymbol{I}/j\omega C$. The term $1/j\omega C$ is again similar to what we saw in case of inductor or what we saw in case of resistance.

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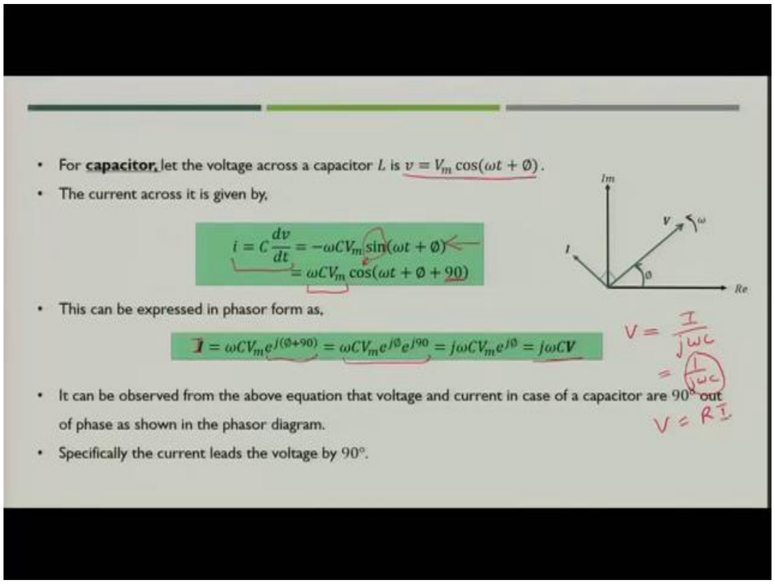

So, in case of resistance also the term was $v = iR$. If you correlate this will also have the property to resist the flow.

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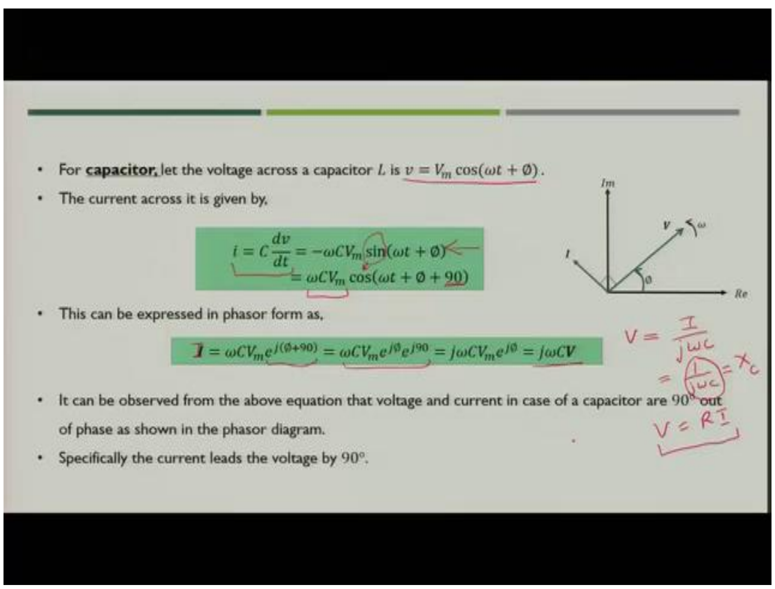

This can be written as Xc which is symbolized as capacitive reactance. So, you can see that both of the terms are similar to what we describe in case of Ohm's Law. Here, if you plot the voltage and current on the with real axis on x-axis and imaginary on the y-axis. If you plot voltage V, which will have some phase angle Phi with respect to real axis.

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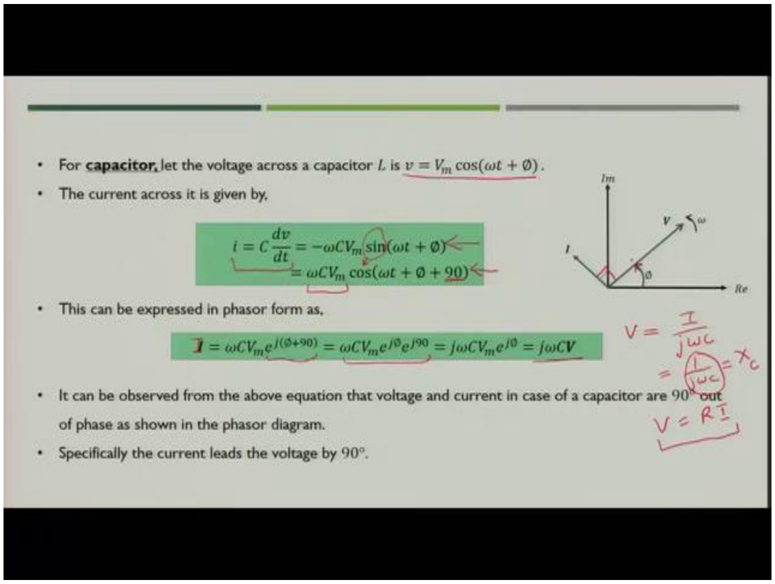

Then if you see this particular expression you will come to know that current I is leading with respect to voltage by 90 degree. So, what we can say that in this case current will lead voltage by 90 degree. Then both of the cases, whether it is inductor and capacitor voltage and current would be 90 degree out of phase. So, when you compare with the resistor, where voltage and current both are in phase. In this case capacitor and inductor both will be out of phase.

In case of inductor current will be lagging with respect to voltage by 90 degree, while in case of capacitor current would be leading by 90 degree with respect to voltage.

Now, let us take one example so that you can better understand the term of Phasor. Let us see that voltage suppose voltage $v(t) = 12\cos(60t + 45°)$ is applied to an inductor of 0.1H. You are asked to find the steady state current through inductor.

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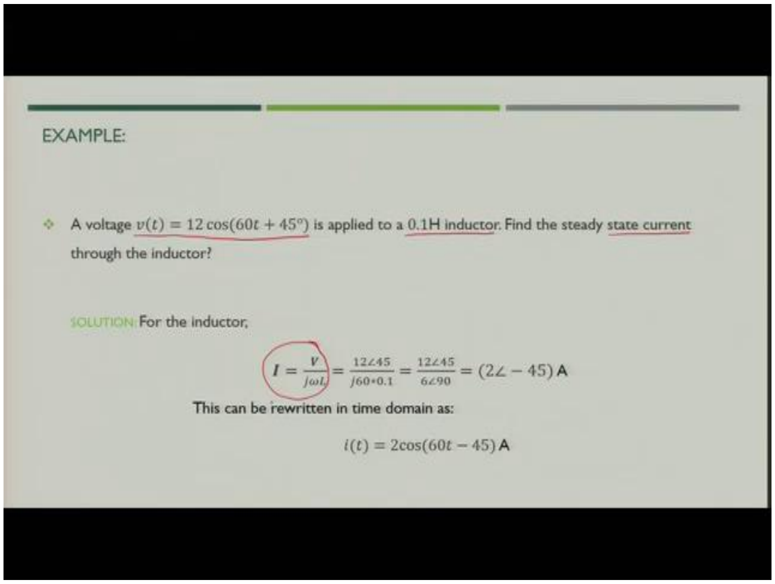

We will use the term which we just derived that is

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{j\omega L}$$

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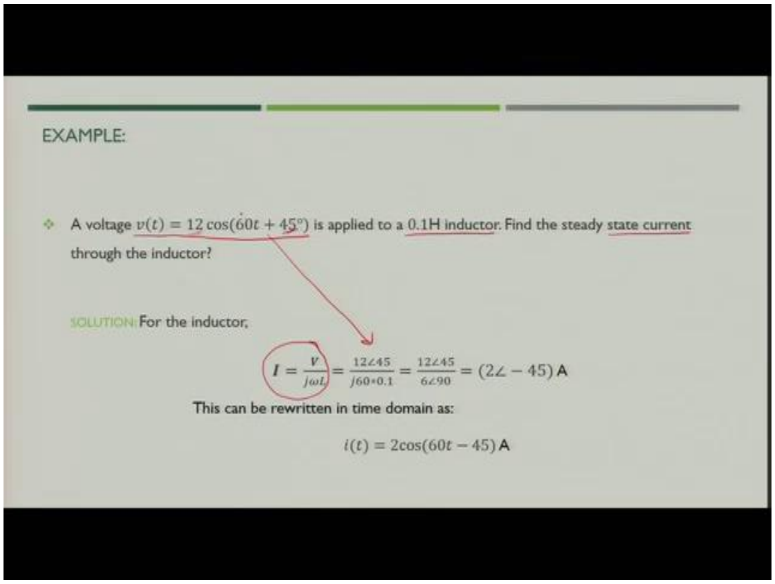

Now, voltage Phasor you can take from here. So, ω is 60 which you can see from here multiplied by value of inductor that is 0.1H. Therefore,

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{j\omega L} = \frac{12\angle 45}{j60 * 0.1} = \frac{12\angle 45}{6\angle 90} = (2\angle -45)\text{A}$$

So, here if you multiply j with one angle 90 degree you will get six angle 90 degree as a denominator and when you simplify, the 12 will be come 2 when you divided it by 6 and then 45 minus 90 degree would be the final phase angle that is minus 45-degree Ampere.

So, that would be the current value which we get from the inductor and if you are asked to write in the time domain, you can simply write as since we know that $i = I_m \cos(\omega t + \emptyset)$ is the time domain representation of the current.

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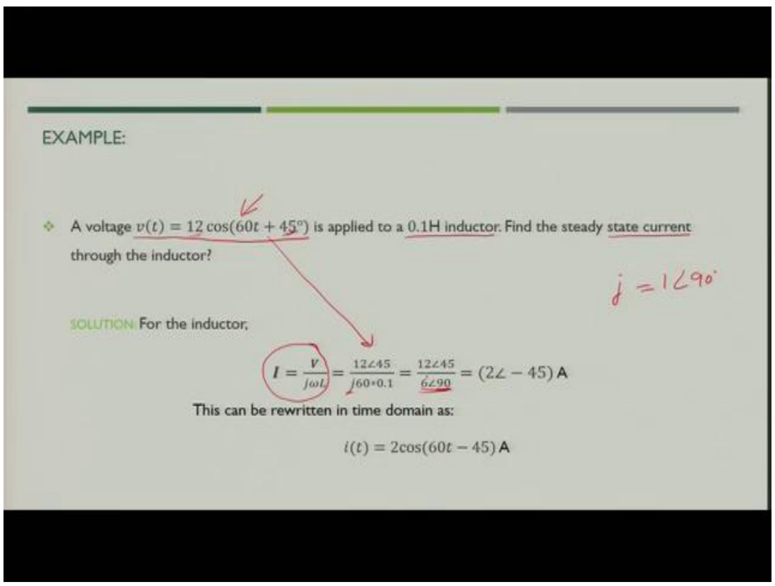

Here Im is nothing but 2 which we just calculated. Then cos Omega T is nothing but 60t over here and then Phi is minus 45 degree Ampere.

$$i(t) = 2\cos(60t - 45)$$

Now, let us talk about the Instantaneous power. Instantaneous power is the power at any instant of time consumed by an element or being supplied by any voltage or current source. So instantaneous power $p(t)$ absorbed by any element is the product of instantaneous voltage $v(t)$ and the instantaneous current $i(t)$, which is flowing through it. So in mathematical terms what we can write? We can write $p(t) = v(t)i(t)$, both are in time domain.

So, how will calculate the value of power? Suppose we have voltage $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$. Now, here Vm and Im are the amplitudes and θ_v and θ_i are the phase angles for the voltage and current respectively.

Now, if you put these value in the equation

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

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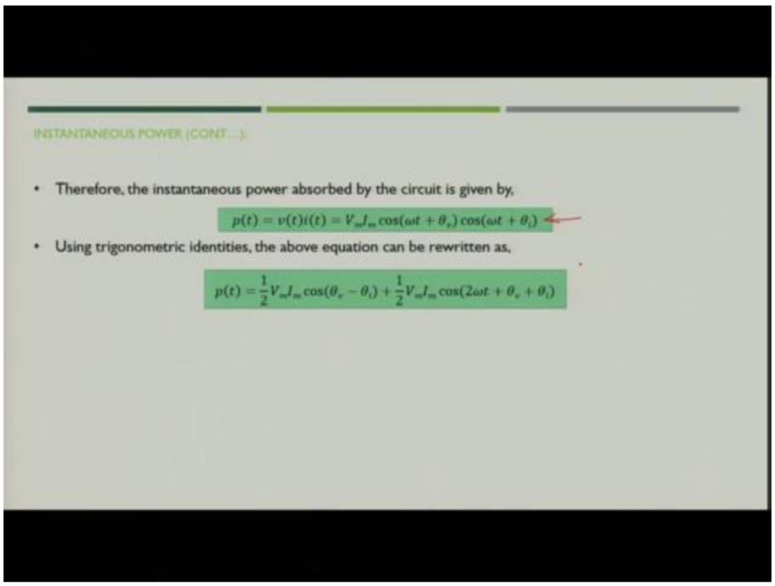

Now, if you do some simple trigonometric operation you will get

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

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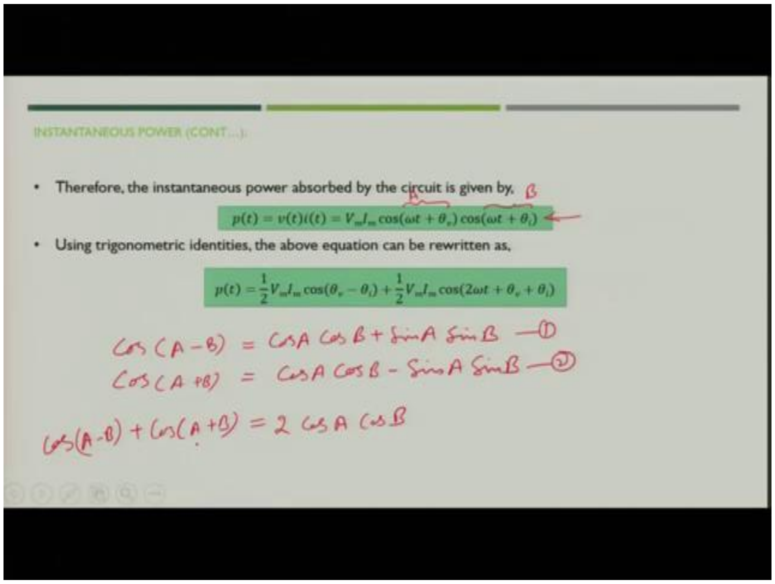

Now, let us assume that this angle is A and this angle is B. So if you recollect the previous lectures we discuss cos A minus B is nothing but cos A cos B plus sin A sin B. Similarly, cos A plus B is nothing but cos A cos B minus sin A sin B. So if you sum up these two equations what you will get, two times cos A cos B is nothing but cos A minus B plus Cos A plus B.

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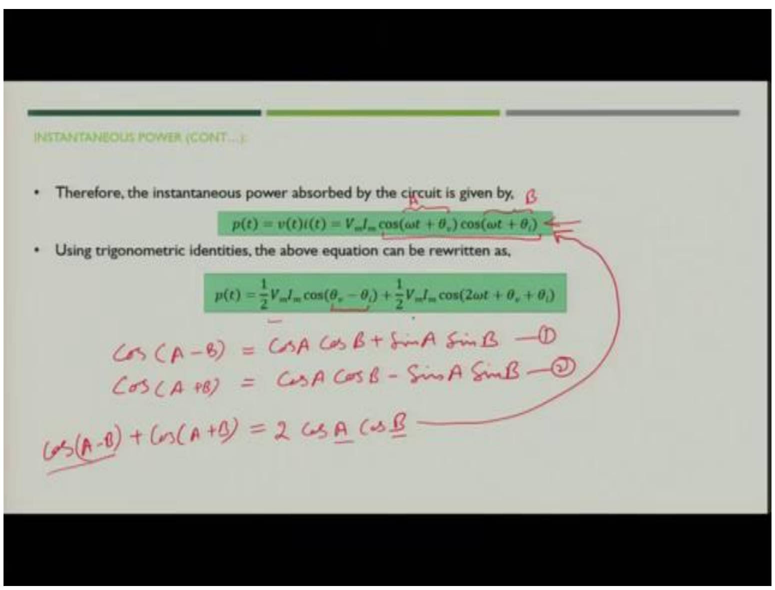

So now if you see correlate this equation with the term of cos A cos B because A we have assumed that $\omega t + \theta_v$ and Cos and B we have assumed $\omega t + \theta_\mathrm{i}$.

So now if you correlate and substitute the term of A and B, you will get the equation

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

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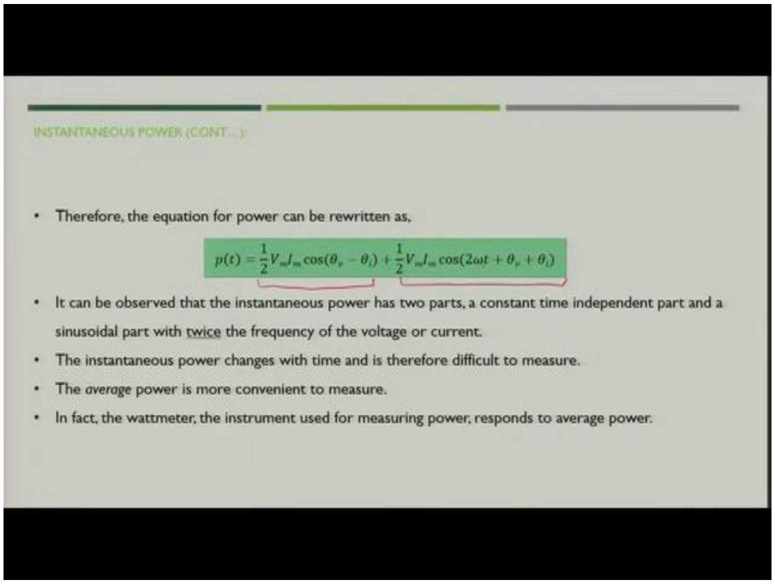

So, in this way we can represent the instantaneous power as the summation of two terms. Now, if you carefully see these two terms the first term is constant, because there is no time dependent portion available in this particular term. While if you see the second part, it has the sinusoidal part which has a frequency of twice the frequency of voltage or current. So, these two terms define the instantaneous power.

Now, if you see this term you can say that instantaneous power changes with time therefore it will be difficult to measure. So, what do we measure in general? We measure, in general, the average power, because it is more convenient to measure. And in fact, if you see the wattmeter, it measures the average power then the instantaneous power.

So, let us see how we will calculate the average power from the previous equation which we derived. So average power we can represent in watts would be the average of instantaneous power over one particular time period. So how you will represent it mathematically? You will say that the average power P is

$$P = \frac{1}{T}\int_0^T p(t)dt$$

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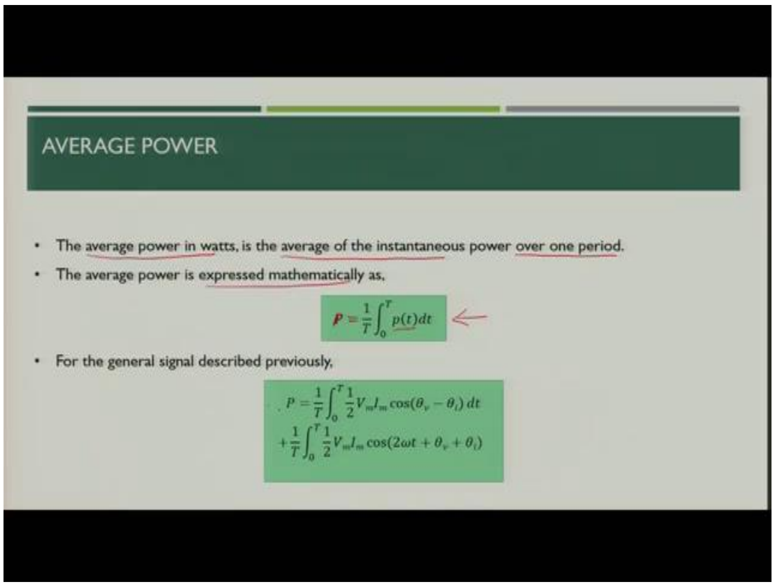

So, we will use the same term we derived before for $p(t)$. We can write average power is nothing but

$$P = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Now, you need not to worry about how to integrate these two terms. Looking at these two term, you can identify what would be the value of average power. How? Let us see the first term. First term is anyway constant, so the average of the constant will always remain constant.

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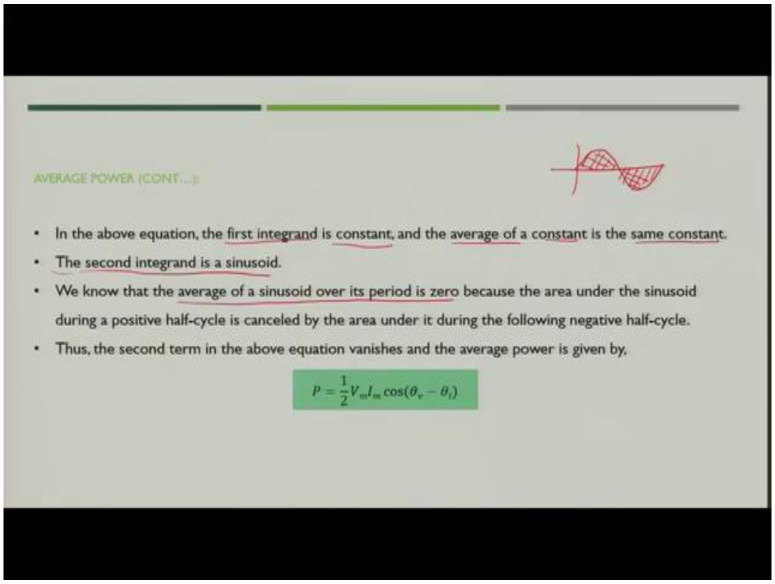

So, this term will remain same as it is after integration but if you see the second term second term is sinusoid and as we know that the average of sinusoid over a time period is zero.

How? Let us see if you see a particular sinusoid, the area under the positive side would be equal for area below the x-axis. So, if you take the average over a particular time period T the positive cancel out negative, so eventually the average of this sinusoid over a time period would remain zero. So, we can say that average of sinusoid over a particular time period is zero.

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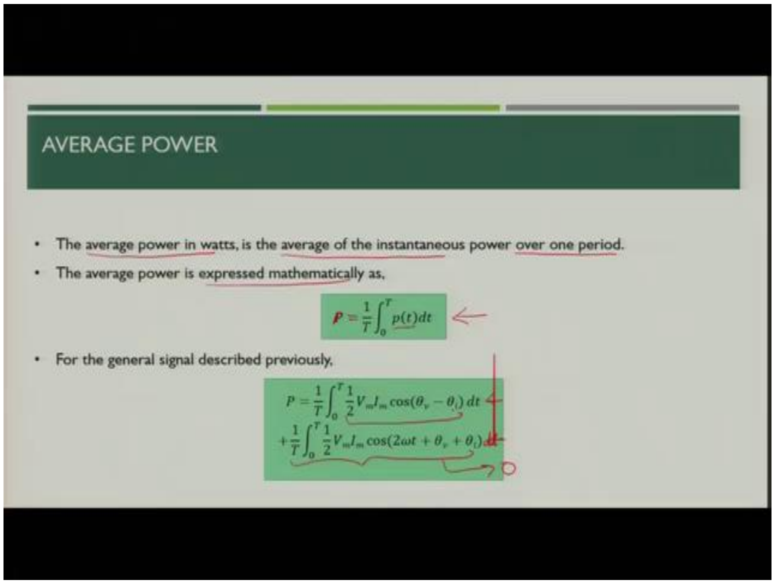

The second term will become zero and finally what we left with is

$$P = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Hence, we can say that average power is

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

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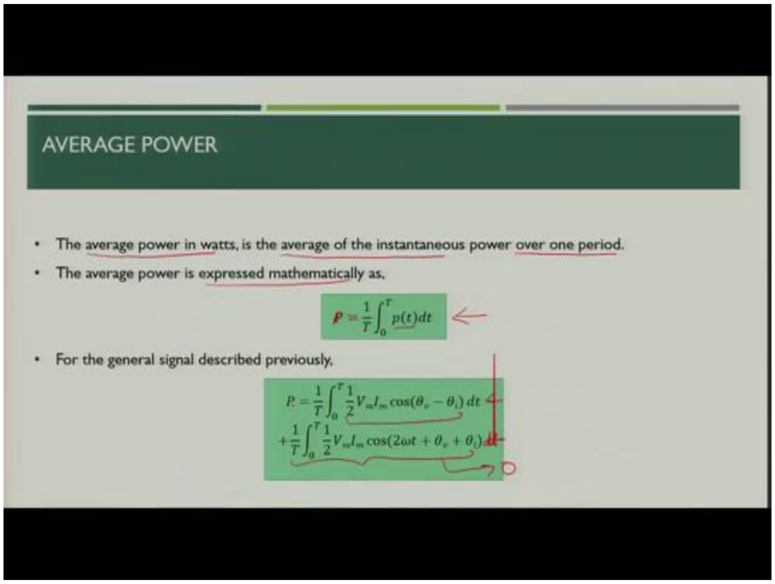

This is nothing but the constant term which is left from the instantaneous power expression. So, we can say that the average power P is nothing but the constant term which we have got from the instantaneous power.

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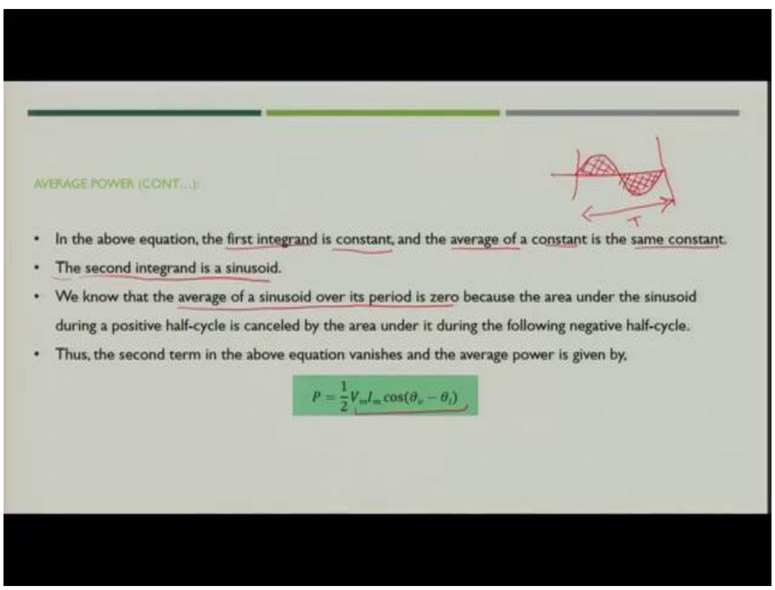

Now, if you see that particular expression that is average power P is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

If you put the value $\theta_v = \theta_i$ it means both voltage and current are in phase. So, if both voltage and current are in phase it means that the circuit is purely resistive.

Now, if the circuit is purely resistive the average power can be given by

$$P = \frac{1}{2} V_m I_m$$

because $\cos(\theta_v - \theta_i)$ is one and we can simply write average power across the resistive element would be

$$P = \frac{1}{2} VmIm = \frac{1}{2} Im2R$$

In case of resistive load, you can say that the circuit will always absorb power. In second case when suppose $\theta v - \theta i = \pm 90°$, that may be positive or negative. It means that the circuit is purely reactive that means that the circuit is having only inductive or capacitive elements.

In that case what will happen to average power? Average power P would

$$P = \frac{1}{2} VmIm \cos 90 = 0$$

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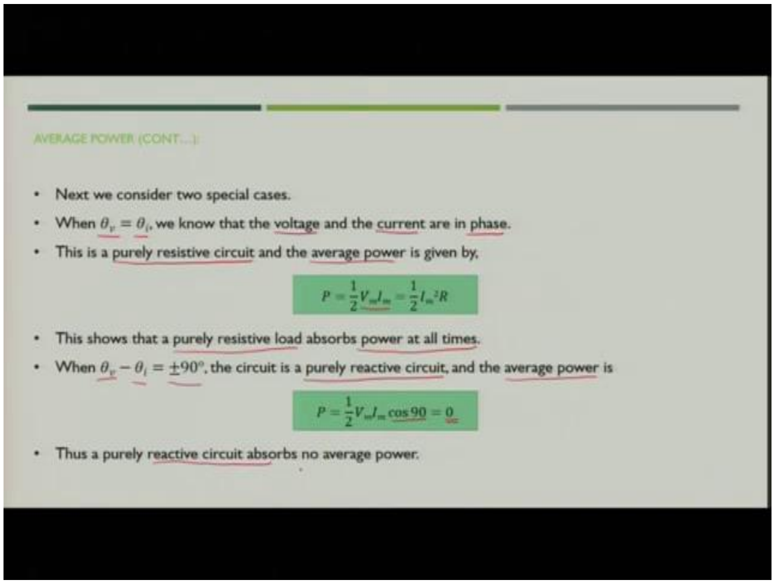

So, in this case, what you can say that in case of purely reactive circuit, the circuit will not absorb any average power. These are the two important boundary conditions for this particular average power calculation, where you can demonstrate that one is for purely resistive and second is for purely reactive.

Now, let us take one example so that you can understand what we discussed till now. Now, if you are asked to find the instantaneous and average power absorbed by a passive linear circuit and if it is excited by some voltage $v(t) = 120\cos(377t + 45°)$.

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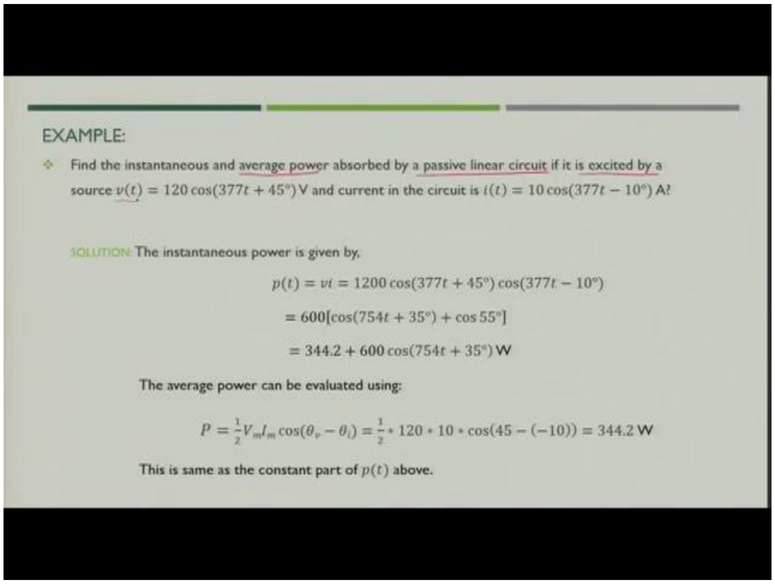

The current in the element is $v(t) = 120\cos(377t + 45°)$ ampere. So, voltage and current both are given, what we have to do we have to find out the value of instantaneous as well as average power. So instantaneous power p(t) can be given as multiplication of instantaneous voltage and current.

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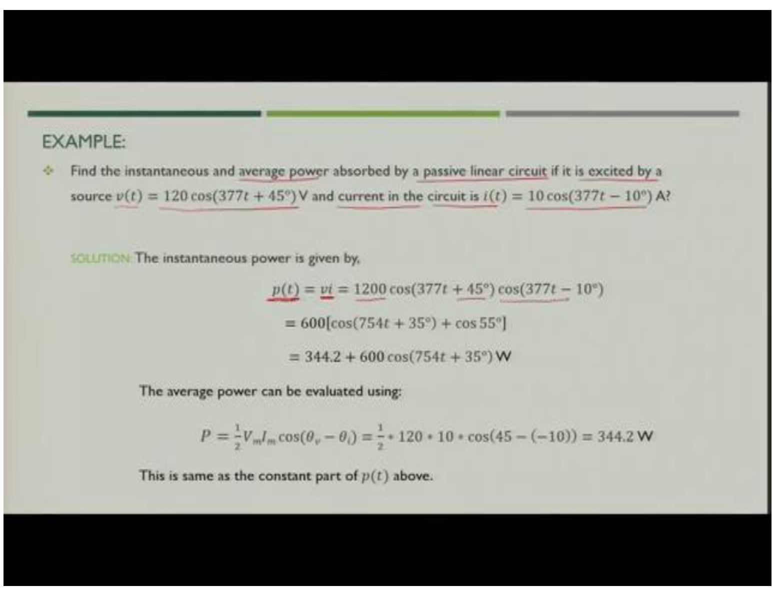

So here you can simply put the value of V and I. What you will get?

$$p(t) = vi = 1200\cos(377t + 45°)\cos(377t - 10°)$$

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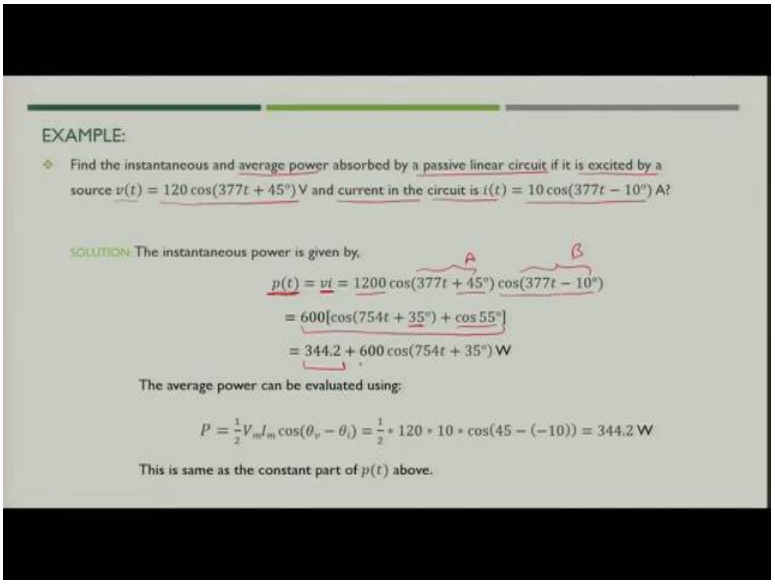

You can say that this is A and this is B. You can utilize cos A cos B value that is nothing but 1 by 2 cos A plus B plus cos A minus B. When you simplify, what you will get? You will get

$$600[\cos(754t + 35°) + \cos 55°] = 344.2 + 600\cos(754t + 35°)$$

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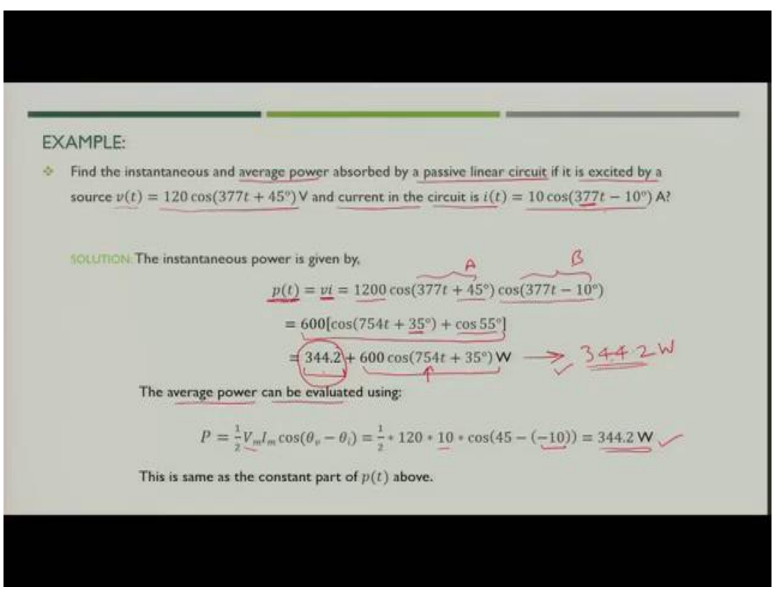

You have an average term and one sinusoidally varying term, here this will be twice the value of original frequency. So, you got one constant term plus one sinusoidally varying term for instantaneous power. Now, how you will calculate average power? For calculation of average power, we came to know that the constant term which we get from the instantaneous power is nothing but the total average power. So, from here itself you can say that the average power is 344.2 watt.

But how you will verify whether it is correct or not? Let us calculate the average power with the equation which we just derived for average power that is

$$P = \frac{1}{2} VmIm \cos(\theta v - \theta i) = \frac{1}{2} * 120 * 10 * cos\big(45 - (-10)\big) = 344.2$$

If you simplify again we will get the same value which we just saw in instantaneous power calculation. We can say that the average power which we calculate from the formula or average power we calculate from the instantaneous power constant term of instantaneous power both are same.

Using this you can verify the values. So, with this we finish our today's class and next class we will discuss about the one of the most important term called RMS values because whenever you analyze the electrical circuit, you will always get the Vm and Im as represented in terms of RMS value. Because in real scenario you will never get peak values as given in the equation or as defined in some experiment. You will always encounter the RMS value.

We will see how we can derive The RMS value with the help of the peak values and sinusoidal termed.

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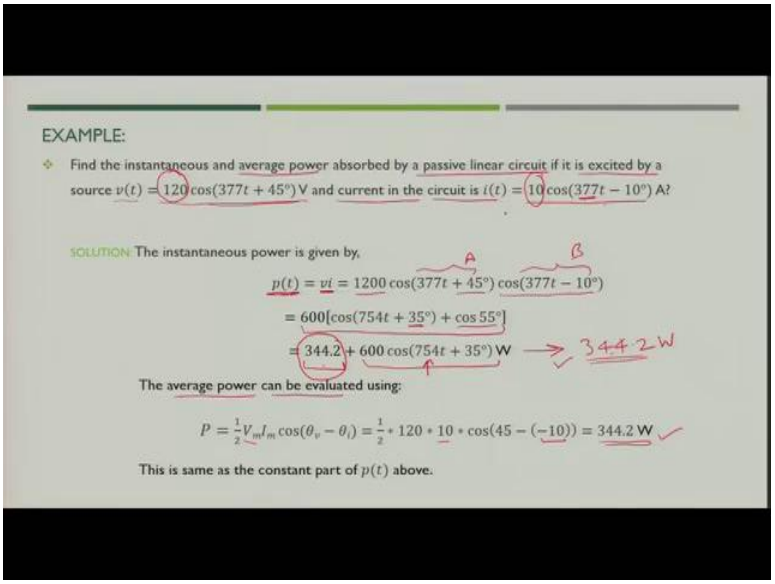

As for example if you see here, you will always get peak value of voltage and you will always get peak value of current. But suppose if you are asked to find out the RMS value how you will find that we will discuss in the next class and then we will continue with our discussion on Mesh and Nodal analysis. Thank you.