Basic Electric Circuits
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Module-10
Sinusoidal Steady State Analysis 2
Lecture-49
Balanced Delta-Wye Connections and Power in Balanced Three Phase System

Namashkar, so in the last two sessions, we have discussed about three phase connections specially star star and then we discussed star delta connections and then in the last lecture we discussed about the Delta Delta connection. So today, we will start our session with delta star connection. You can also say Delta Wye connection where delta is the source and the star connected is the load and in the later part of the session, we will discuss about the energy for a three phased system. So, let us start our discussion.

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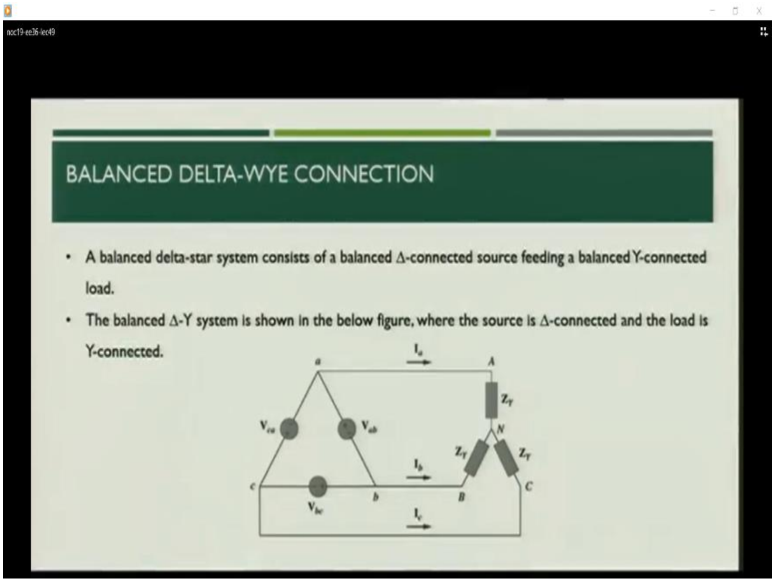

So basically, in case of Delta Wye connection or Delta Star connection, we have source delta connected and the load star connected. So, if you see in this figure, the source is connected in delta i.e. load is connected in star and the line currents are $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

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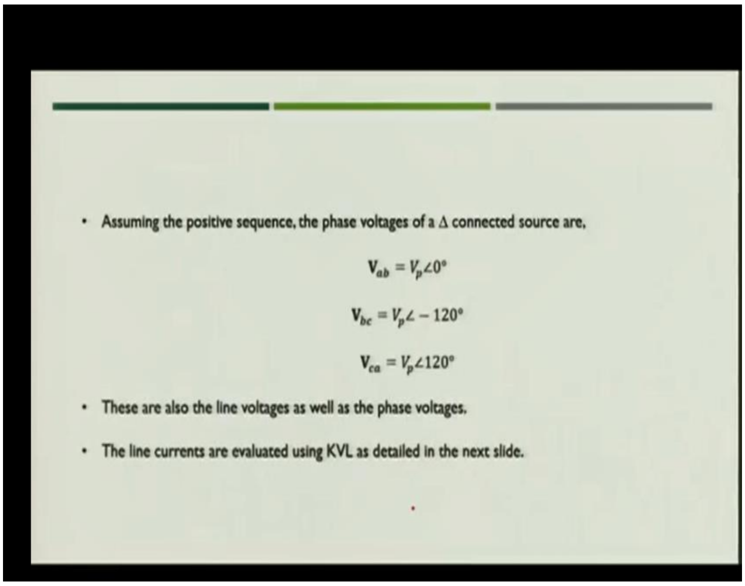

Now as we discussed in the previous cases, in this case also we assume that the positive sequence is considered for the source and the phase voltage of delta connected source can be given as,

$$\mathbf{V}_{ab} = V_p\angle 0°$$

$$\mathbf{V}_{bc} = V_p\angle -120°$$

$$\mathbf{V}_{ca} = V_p\angle 120°$$

As you know, in case of delta connected source, the line voltage and phase voltages are same. Now let us apply KVL to find out the line current.

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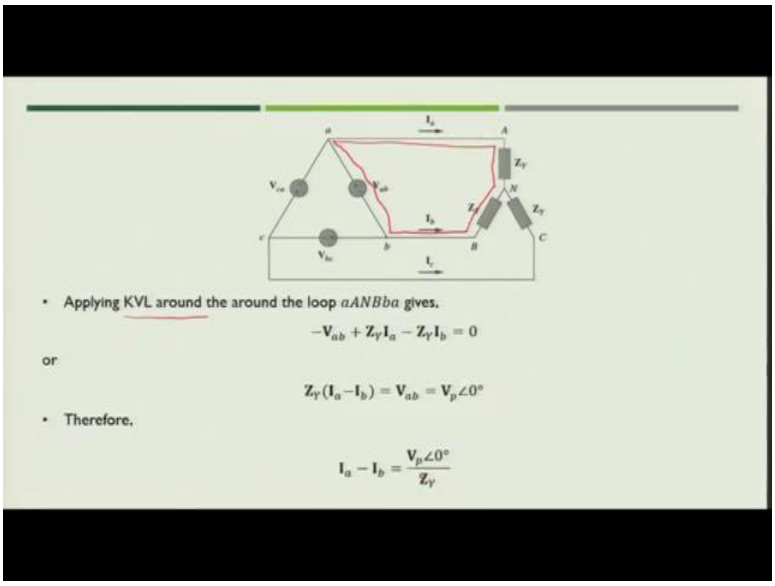

Let us consider the figure which we saw in the first slide. Now we will apply Kirchhoff voltage law around the loop $aANBba$. This will be the loop in which we will apply the Kirchhoff voltage law. So if we apply,

$$-\mathbf{V}_{ab} + \mathbf{Z}_Y\mathbf{I}_a - \mathbf{Z}_Y\mathbf{I}_b = 0$$

or

$$\mathbf{Z}_Y(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = \mathbf{V}_p\angle 0°$$

Therefore,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{\mathbf{V}_p\angle 0°}{\mathbf{Z}_Y}$$

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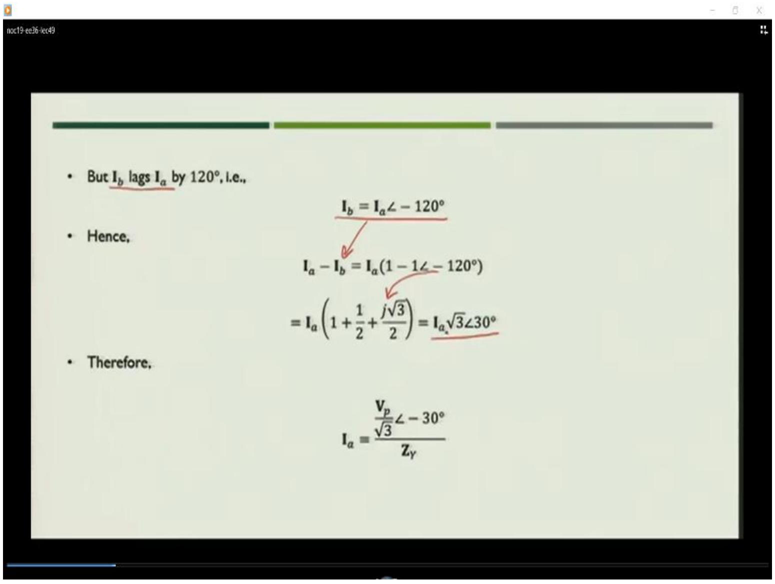

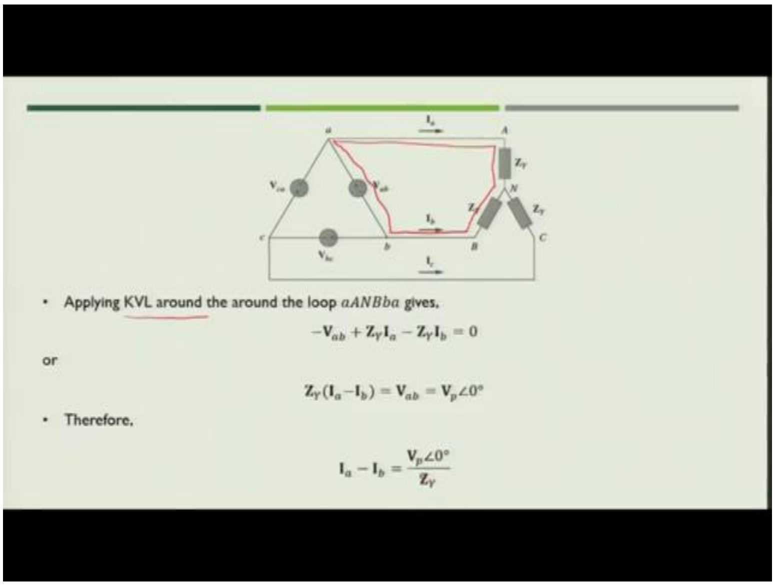

Now as we know,

$$\mathbf{I}_b = \mathbf{I}_a \angle -120°$$

Hence,

$$\mathbf{I}_a - \mathbf{I}_b = \mathbf{I}_a(1 - 1\angle -120°)$$

$$= \mathbf{I}_a\left(1 + \frac{1}{2} + \frac{j\sqrt{3}}{2}\right) = \mathbf{I}_a\sqrt{3}\angle 30°$$

Therefore,

$$\mathbf{I}_a = \frac{\frac{\mathbf{V}_p}{\sqrt{3}}\angle -30°}{\mathbf{Z}_Y}$$

In this way you can calculate the line current for delta star connected system.

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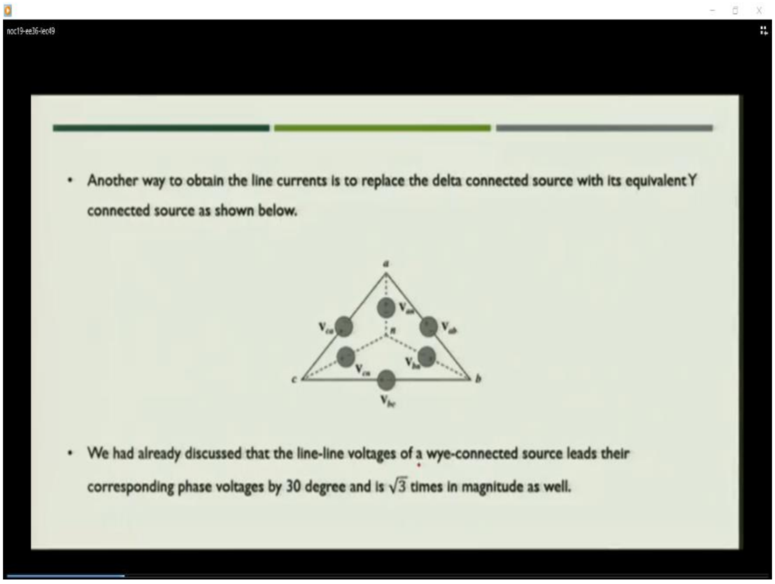

Now another way to obtain the line current is that you replace the delta connected source into its equivalent star connected or Y connected source. So when you convert, let us say that the phase voltages are Van, Vbn and Vcn and we have already discussed that line line voltage of Wye connected source leads their corresponding phase by 30 degree and root 3 times the magnitude.

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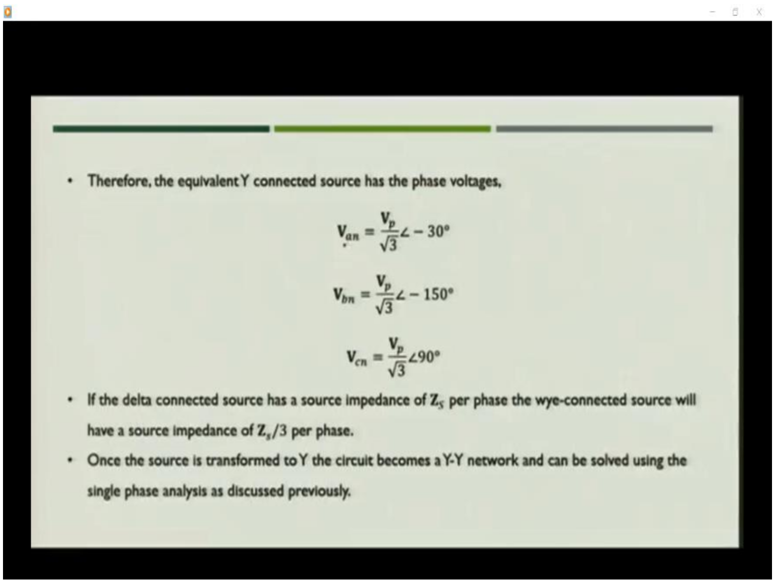

So, in that case what you can write,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_p}{\sqrt{3}}\angle -30°$$

$$\mathbf{V}_{bn} = \frac{\mathbf{V}_p}{\sqrt{3}}\angle -150°$$

$$\mathbf{V}_{cn} = \frac{\mathbf{V}_p}{\sqrt{3}}\angle 90°$$

Now incase if you have a delta connected source with some source impedance $\mathbf{Z}_S$, in that case you can apply star delta transformation and find out the per phase source impedance for equivalent star connected source. In that case your source impedance will be become $\mathbf{Z}_S/3$ as we have already seen when we were doing the star delta transformation. Now once the source is transformed into Y or star, these circuit becomes Y-Y network which can be easily solved using single phase analysis which we have already discussed.

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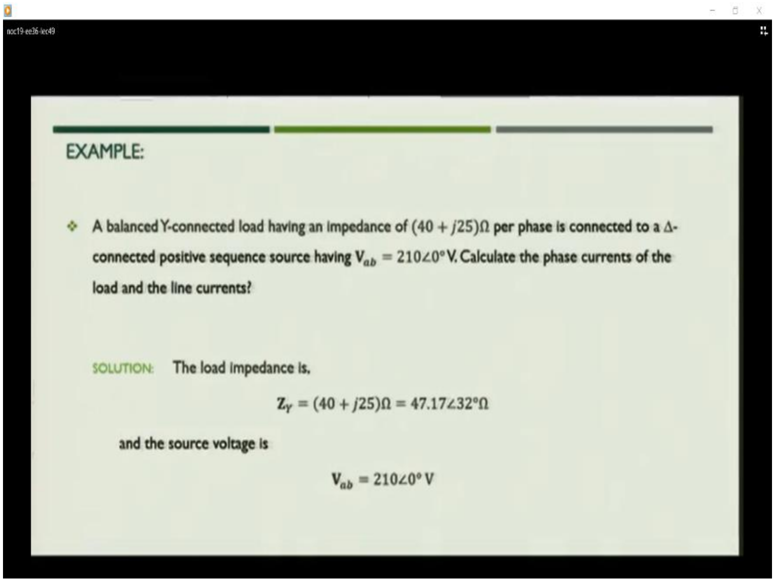

Now let us take one example. Suppose if balanced star connected load is having impedance of $(40 + j25)\Omega$ per phase is connected to delta connected positive sequence source having $\mathbf{V}_{ab} = 210\angle 0°$ V. We need to calculate the phase currents and line currents.

So, we have load impedance as given. We convert it to phasor format as,

$$\mathbf{Z}_Y = (40 + j25)\Omega = 47.17\angle 32°\Omega$$

and the source voltage is,

$$\mathbf{V}_{ab} = 210\angle 0° \text{ V}$$

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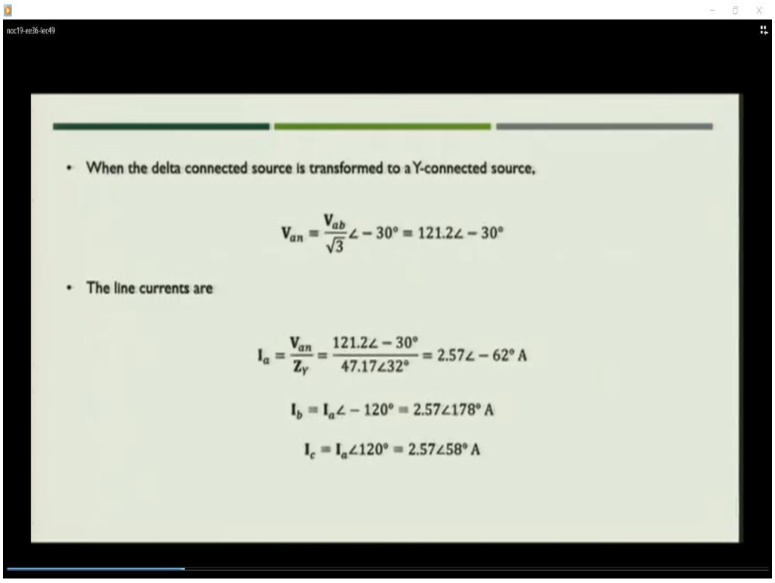

We will convert delta connected source into star connected. So in that case, per phase voltage for equivalent star connected source will be

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30° = 121.2\angle -30°$$

Now what will be the line currents? Line currents will be,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\angle -30°}{47.17\angle 32°} = 2.57\angle -62° \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = 2.57\angle 178° \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = 2.57\angle 58° \text{ A}$$

Since the load is balanced so the magnitude of currents i.e. line currents will remain the same. The only change will be the angle value.

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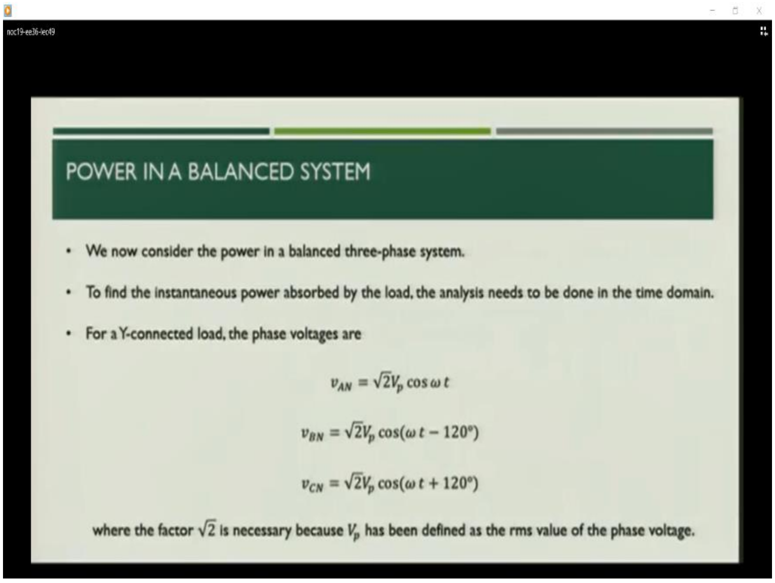

Now let us discuss the power in the balance three phase system. To find the instantaneous power absorbed by any load in case of the three phased system, we will do the analysis in the time domain. So, what we will do we will assume that we have the load as star connected. The phase voltage in that case, you can say

$$v_{AN} = \sqrt{2}V_p \cos \omega t$$

$$v_{BN} = \sqrt{2}V_p \cos(\omega t - 120°)$$

$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120°)$$

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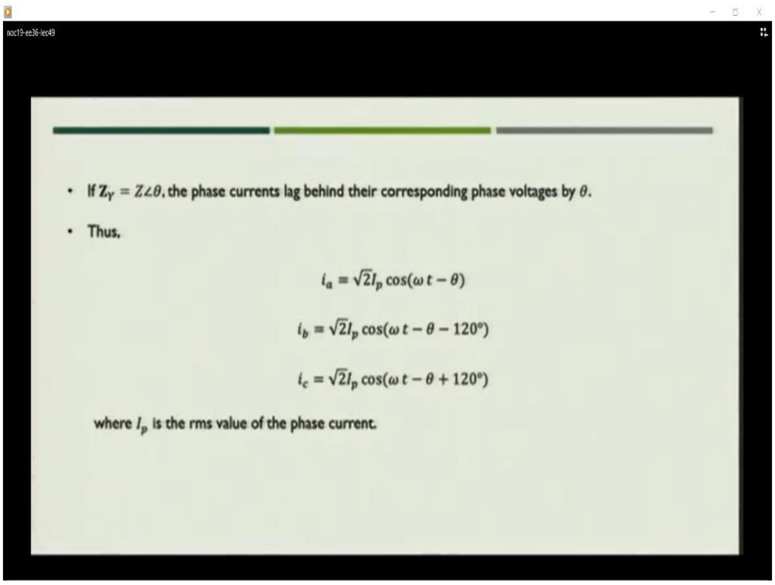

Now, if load is $\mathbf{Z}_Y = Z\angle\theta$. The phase currents will lag their corresponding phase voltages in that case by angle theta. We can write the currents as,

$$i_a = \sqrt{2}I_p(\cos\omega\, t - \theta)$$

$$i_b = \sqrt{2}I_p\cos(\omega\, t - \theta - 120°)$$

$$i_c = \sqrt{2}I_p\cos(\omega\, t - \theta + 120°)$$

where I_p is the rms value of the phase current.

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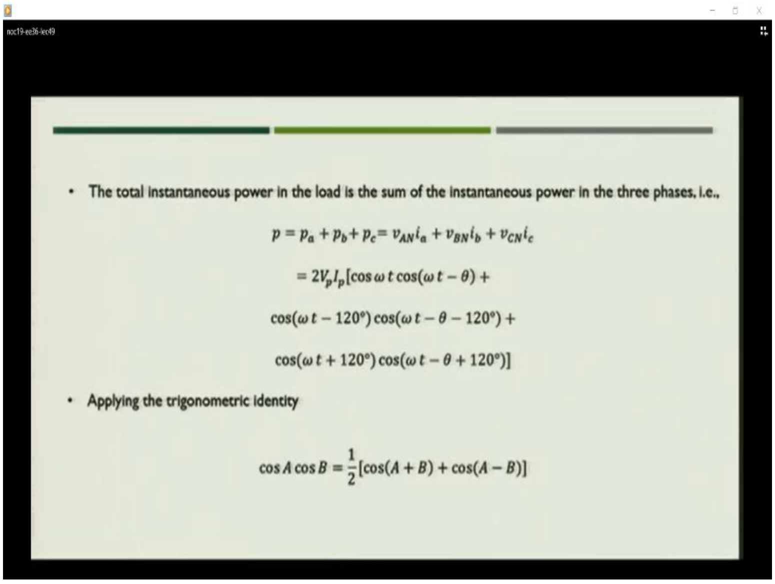

Now total instantaneous power in the load, you can say the total instantaneous power will be the sum of individual phase instantaneous powers. So, we can write total instantaneous power,

$$p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c$$

$$= 2V_pI_p[\cos\omega\, t\cos(\omega\, t - \theta) +$$

$$\cos(\omega\, t - 120°)\cos(\omega\, t - \theta - 120°) +$$

$$\cos(\omega\, t + 120°)\cos(\omega\, t - \theta + 120°)]$$

Applying the trigonometric identity

$$\cos A\cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

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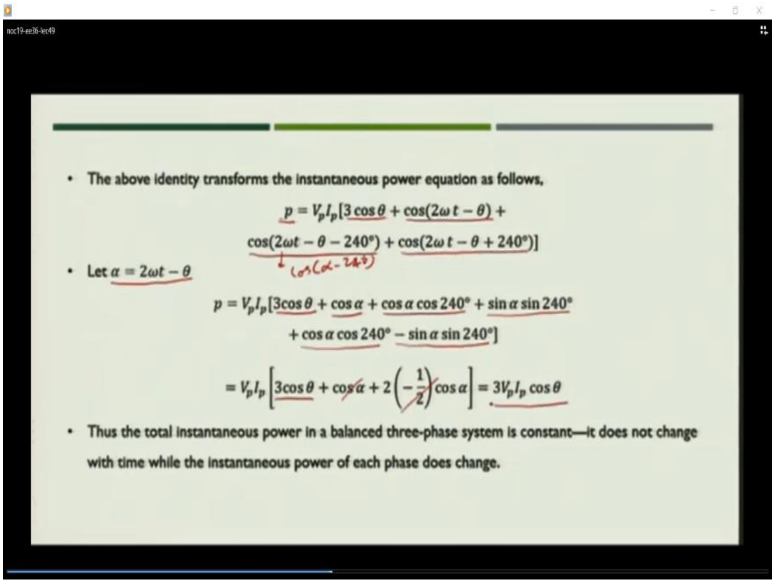

So if we use this in the above expression, what we can write,

$$p = V_p I_p[3\cos\theta + \cos(2\omega\, t - \theta) +$$

$$\cos(2\omega t - \theta - 240°) + \cos(2\omega\, t - \theta + 240°)]$$

Let $\alpha = 2\omega t - \theta$

$$p = V_p I_p[3\cos\theta + \cos\alpha + \cos\alpha\cos 240° + \sin\alpha\sin 240°$$

$$+ \cos\alpha\cos 240° - \sin\alpha\sin 240°$$

$$= V_p I_p\left[3\cos\theta + \cos\alpha + 2\left(-\frac{1}{2}\right)\cos\alpha\right] = 3V_p I_p\cos\theta$$

Now if you will see this expression, what you can observe, you can observe that the total instantaneous power in a balanced three phase system is constant. That means that there is no time factor coming in this expression. So even your instantaneous power of each phase will change with respect to time. The three phased power will remain constant.

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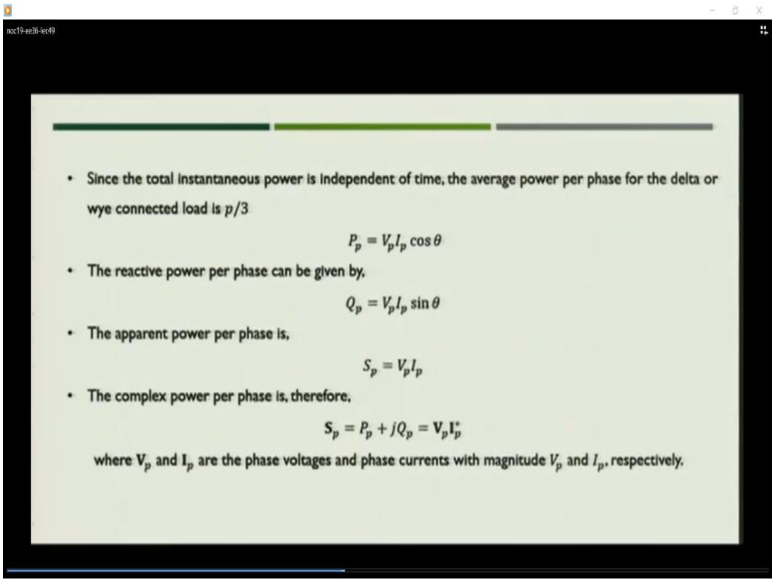

Now since the total instantaneous power is independent of time, you can say the average power per phase for the delta or star connected load will be $p/3$. So per phase, real power you will get

$$P_p = V_p I_p \cos\theta$$

The reactive power per phase is given by,

$$Q_p = V_p I_p \sin\theta$$

The apparent power per phase is given by,

$$S_p = V_p I_p$$

The complex power per phase is given by,

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$

where $\mathbf{V}_p$ and $\mathbf{I}_p$ are the phase voltages and phase currents for with magnitude V_p and I_p, respectively.

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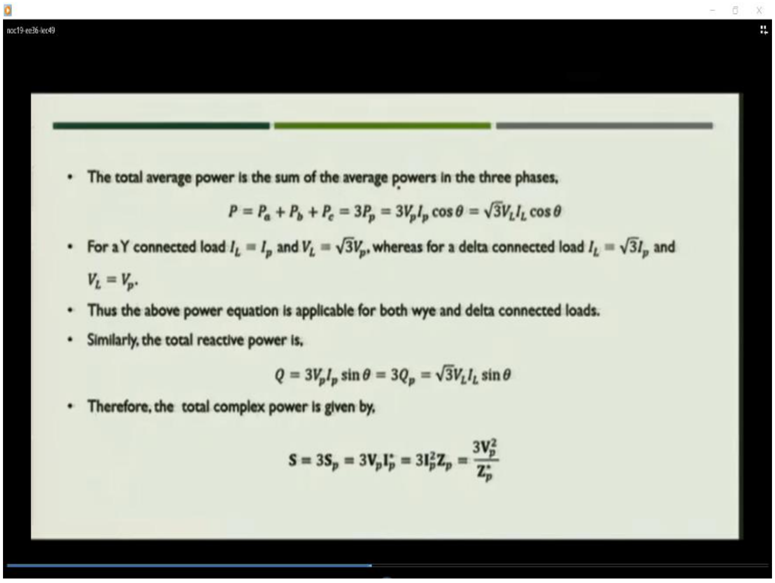

Now for three phase system, total power,

$$P = P_a + P_b + P_c = 3P_p = 3V_pI_p\cos\theta = \sqrt{3}V_LI_L\cos\theta$$

Now in case you have star connected load, $I_L = I_p$ and $V_L = \sqrt{3}V_p$ whereas for delta connected load, $I_L = \sqrt{3}I_p$ and $V_L = V_p$. So what we can say, irrespective of the load, whether it is star connected or delta connected, your total power will remain equal to $\sqrt{3}V_LI_L\cos\theta$.

Similarly, the reactive power will also be,

$$Q = 3V_pI_p\sin\theta = 3Q_p = \sqrt{3}V_LI_L\cos\theta$$

The total complex power is given by,

$$\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p\mathbf{I}_p^* = 3\mathbf{I}_p^2\mathbf{Z}_p = \frac{3\mathbf{V}_p^2}{\mathbf{Z}_p^*}$$

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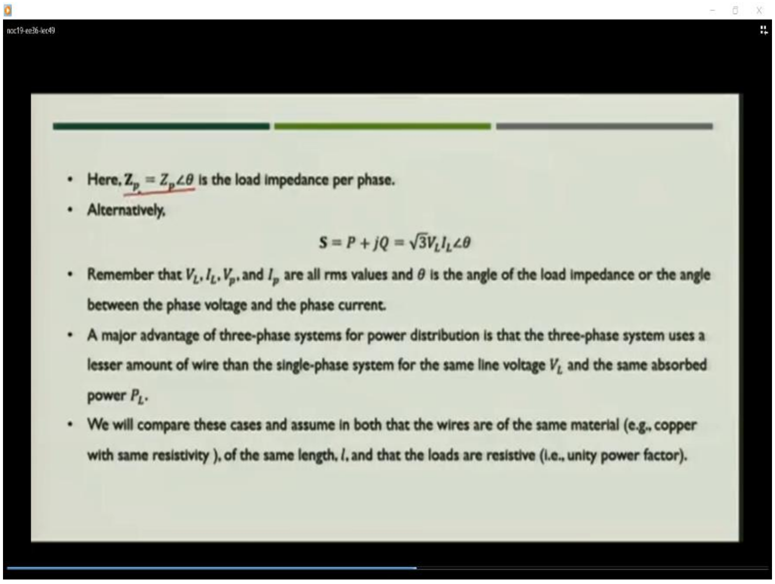

Now if $\mathbf{Z}_p = Z_p\angle\theta$, you can simply represent the complex power as,

$$\mathbf{S} = P + jQ = \sqrt{3}V_L I_L\angle\theta$$

The major advantages of three phase system for power distribution is that the three-phase system uses lesser amount of wire than the single-phase system for same line voltage V_L and the same absorbed power P_L. How? Let us try to find out and justify that the three-phase system for power distribution will use less amount of wire when we compare with single phase system which is having the same line voltage and the same power being fed to the load.

So, what we will do? We will first assume that in both systems one is three phase and the other is single phase system, in both of these systems we are using the same material that means copper with the same resistivity and same length of the wire is being used, let us say that the length of the wire is l and for simplicity we will assume that the loads are resistive that means the load is having unity power factor.

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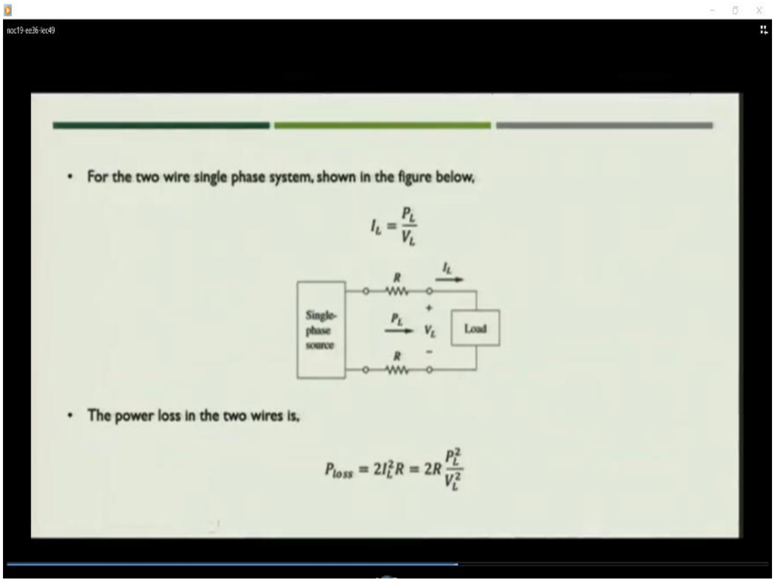

Now, in case of single-phase system what will happen we will have two wires to create the single-phase system where one single phase source will be supplying power to the load. Let us say that the power being supplied to the load is P_L and the voltage across the load is V_L and the current which is flowing in the load is I_L, R is the resistance of the transmission wire. So what will happen? The power loss in two wires because we are having 2 wires to create this single-phase circuit. Loss in the transmission will be,

$$P_{loss} = 2I_L^2 R = 2R\frac{P_L^2}{V_L^2}$$

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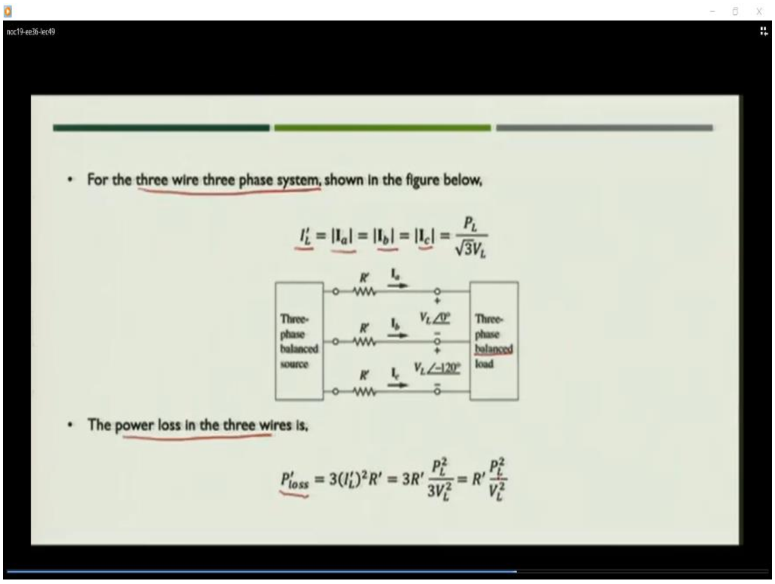

Now let us take the 3 wire three phase system. In this let us assume that I_L' is the current which is going to the load. Here the amount that is magnitude of all line currents will be we are considering as same and because we are considering that the load is balanced.

$$I_L' = |\mathbf{I}_a| = |\mathbf{I}_a| = |\mathbf{I}_a| = \frac{P_L}{\sqrt{3}V_L}$$

So, in that case the power loss in three wires because for the three-phase system will have three wires. Let us assume that the resistance of each wire is R' so the power loss in the case of three phase system will be,

$$P_{loss}' = 3(I_L')^2 R' = 3R' \frac{P_L^2}{3V_L^2} = R' \frac{P_L^2}{V_L^2}$$

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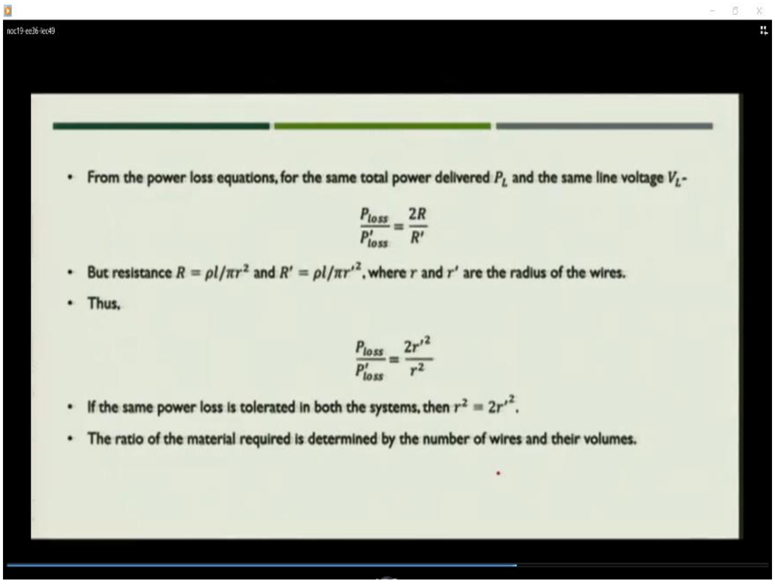

Now from these 2 equations, we can say,

$$\frac{P_{loss}}{P_{loss}'} = \frac{2R}{R'}$$

P_{loss} is for single phase P_{loss}' is for three phase system. Now resistance of the wire in case of single-phase system is $R = \frac{\rho l}{\pi r^2}$. Similarly, $R' = \rho l / \pi {r'}^2$ where r and r' are the radii of the wires. So, if you put these values in this, you will get,

$$\frac{P_{loss}}{P_{loss}'} = \frac{2{r'}^2}{r^2}$$

Now, if the same power loss is being tolerated in both of the systems means whether it is single phase or three phase system, the total loss in this system is same that means P_{loss} will be equal to P'_{loss} which means $r^2 = 2r'^2$.

Now the ratio of material required is determined by the number of wires and their volumes. So what we will try to find out. We will try to find out how much material is required to create the single-phase system as well as three phase system.

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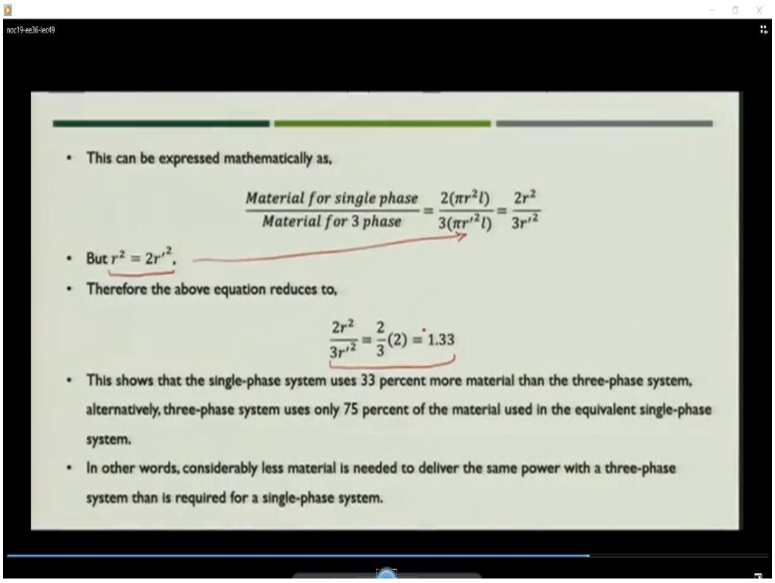

So, material for single phase means since we have 2 wires in the single phase, you can write 2 times of the wire, we are having wires as cylindrical in shape so the total volume will be,

$$\frac{Material\ for\ single\ phase}{Material\ for\ 3\ phase} = \frac{2(\pi r^2 l)}{3(\pi r'^2 l)} = \frac{2r^2}{3r'^2}$$

But $r^2 = 2r'^2$,

Therefore, the above equation reduces to,

$$\frac{2r^2}{3r'^2} = \frac{2}{3}(2) = 1.33$$

You can observe that the single phase system will use 33 percent more material than the three phase system to supply the same amount of power and to incur the same loss in the transmission system or you can write alternatively that the three phase system will use only 75 percent of the material as compared with the equivalent single phase system.

This shows that incase of three phase system, we need considerably less material to deliver the same power when we compare with the three-phase system and the single-phase system.

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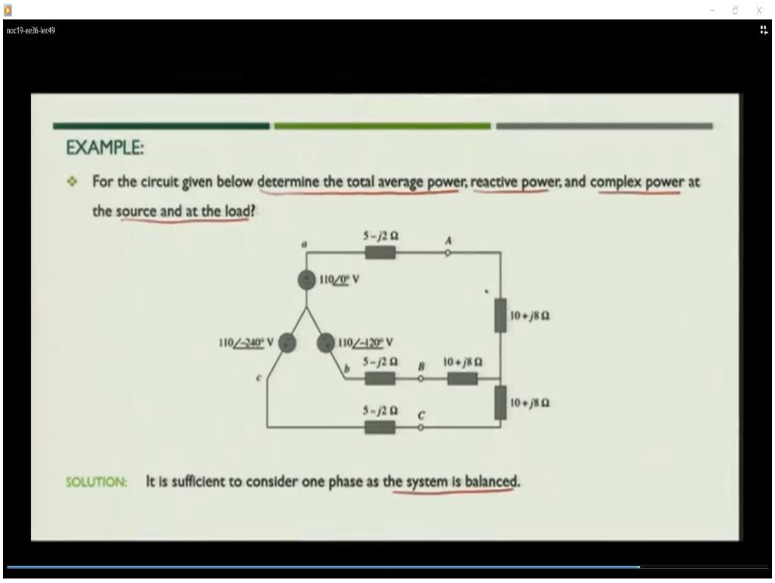

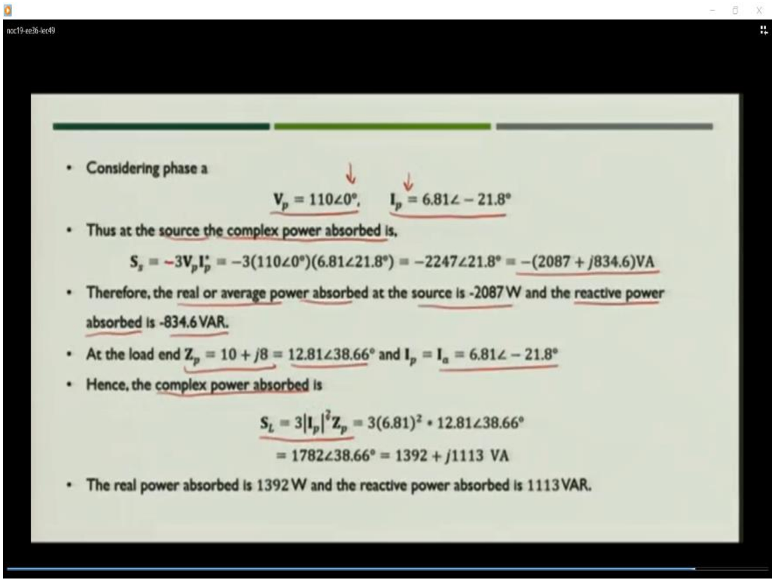

Now, let us take one example to understand what we have discussed in relationship with the power. Let us see there is one circuit as we see in the figure. We need to determine the total average power, reactive power and complex power at the source and at the load. So here if you see, the source is star connected, the per phase voltage is $110\angle 0°$ for Phase A, for phase B it is $110\angle -120°$ and for phase C it is $110\angle -240°$. $5 - j2$ ohm is the impedance of the transmission line and it is the same in all the three phases. Load we can see that it is star connected again where the per phase impedance of the load is $10 + j8$ ohm. Now from this figure, you can observe that the system is balanced.

So, in that case, if calculate the value of the current Ip, what you can do, you can simply use Kirchhoff Voltage Law in the loop and find out the value of current Ip.

So when you solve it, you will get this, anyways it is given in the question that $\mathbf{V}_p = 110\angle 0°$, $\mathbf{I}_p = 6.81\angle -21.8°$ This will be your per phase current.

Now when you have Vp and Ip calculated, you can find out the complex power absorbed at the source. Now since you know that source generally supply power to the load so if you are asked to find out the power absorbed that means that you will have negative sign in that. So total complex power absorbed by the source will be equal to,

$$\mathbf{S}_s = -3\mathbf{V}_p\mathbf{I}_p^* = -3(110\angle 0°)(6.81\angle 21.8°) = -2247\angle 21.8° = -(2087 + j834.6)\text{VA}$$

Now at the receiving end means at the load end $\mathbf{Z}_p = 10 + j8 = 12.81\angle 38.66°$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81\angle -21.8°$. Ip is also the line current also because in case of star connected phase current is equal to line current.

So complex power absorbed by the load, you can simply calculate by this particular formula

$$\mathbf{S}_L = 3\left|\mathbf{I}_p\right|^2\mathbf{Z}_p = 3(6.81)^2 * 12.81\angle 38.66°$$

$$= 1782\angle 38.66° = 1392 + j1113 \text{ VA}$$

So what you can write the real power absorbed by the load is 1392 Watts and the reactive power absorbed by the load is 1113 VAR.

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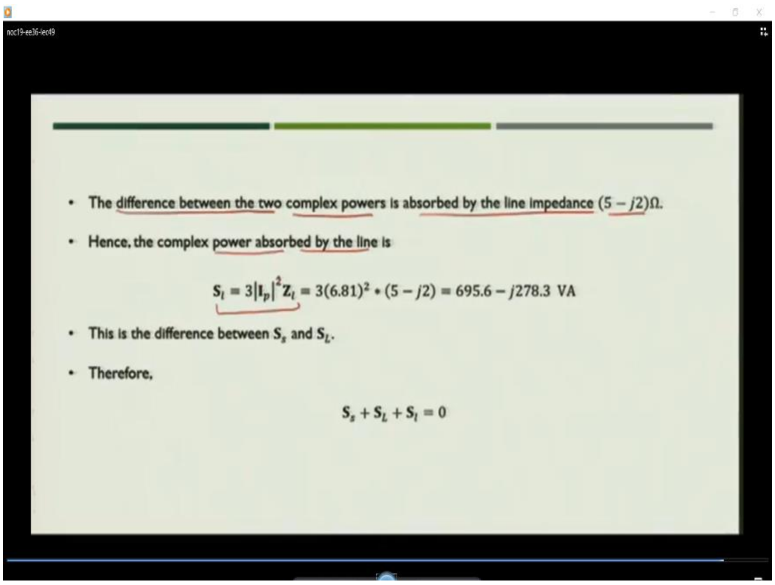

Now the difference between the two complex powers is absorbed by the line impedance that means the power supplied by the source minus the power absorbed by the load. So what we will do? We will first find out the value of power absorbed by the line with the help of this formula that is

$$\mathbf{S}_l = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_l = 3(6.81)^2 * (5 - j2) = 695.6 - j278.3 \text{ VA}$$

This will be the amount of power lost in the transmission which will be nothing but the difference between the source power minus the power absorbed by the load.

In a nutshell, what you can say that sum of the total power that is source power plus load power plus power absorbed by the transmission line will be equal to 0.

So, with this, we can close our today's session. In this session we discussed about the last combination that is delta star combination for the three-phase circuit and also discussed about the balanced three phase power. In the next session, we will discuss unbalanced three phase system and the calculation of power for the unbalanced three phase system. Thank you.