Basic Electric Circuits
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Module 10 - Sinusoidal Steady State Analysis 2
Lecture No. 48 - Balanced Wye Delta and Delta Delta Connections

Namaskar, so in last session we started our discussion about the 3 phase circuits and last we discussed about the Y-Y balance circuit that means you can also say Y-Y balance circuit. So, in today's session we will discuss about Y-Δ as well as Δ-Δ circuits.

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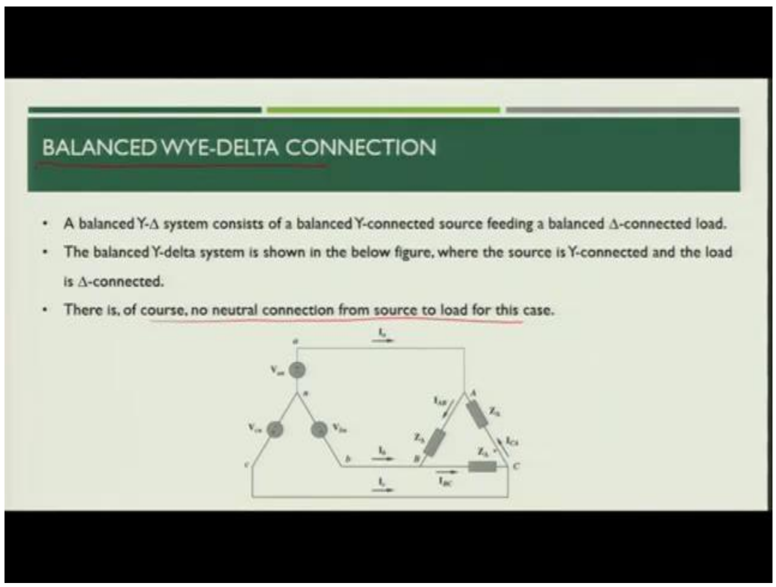

Let us start our discussion related to first we will discuss balanced Y-Δ connections. You will see there we have Y-Δ circuits where the source is connected in star or you can say Y and load is connected in delta form. You will see that the a phase of the source is connected to A of load, b is connected to B of load and then c is connected to C phase of the load. In this way you can say that the star source is feeding power to the delta connected load. Now if you see this arrangement, you can simply say that that we do not have neutral connection between the source as well as load.

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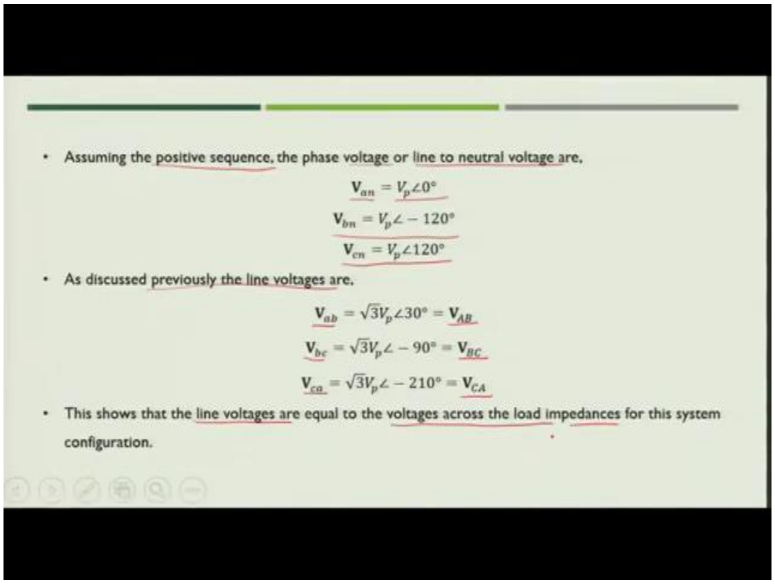

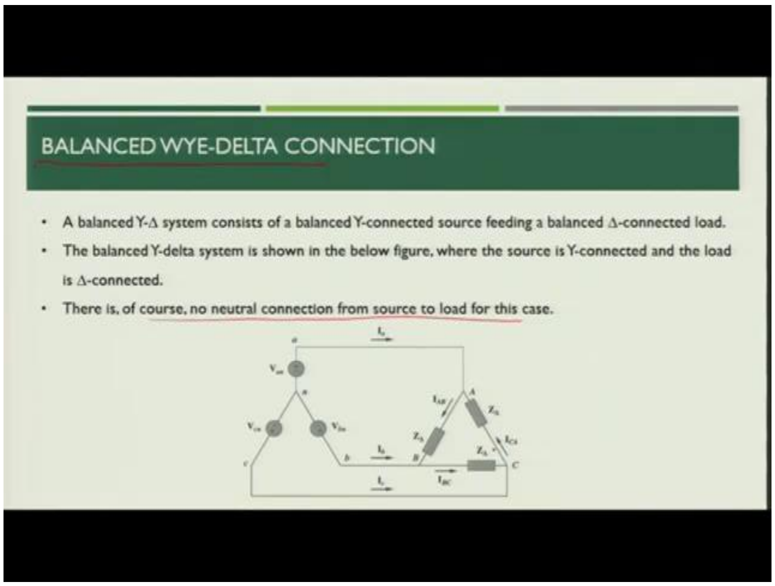

Now if we assume the positive sequence, the phase voltage or you can also say line to neutral voltage will be,

$$\boldsymbol{V}_{an} = V_p\angle 0°$$

$$\boldsymbol{V}_{bn} = V_p\angle -120°$$

$$\boldsymbol{V}_{cn} = V_p\angle 120°$$

Now when we discussed about the Y-Y connection, we discussed about the phase voltage as well as line voltage so in this case also the line voltage will be $\mathbf{V}_{ab}$. So, since the source is your star connected, so you will see that the line voltage will be $\mathbf{V}_{ab}$ and we derived that the value of $\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$. This is what we derive when we discussed about the Y-Y connection.

Similarly, $\mathbf{V}_{bc} = \sqrt{3}V_p\angle -90°$, $\mathbf{V}_{ca} = \sqrt{3}V_p\angle -210°$. Now if you see this arrangement, the line voltage of the source will be equal to phase voltage of the load because line voltage that is $\mathbf{V}_{ab}$ is coming across the phase AB of the delta. So what we can write that $\mathbf{V}_{ab} = \mathbf{V}_{AB}$ that is the load phase voltage, $\mathbf{V}_{bc} = \mathbf{V}_{BC}$ and $\mathbf{V}_{ca} = \mathbf{V}_{CA}$ of the load.

So, in this arrangement, you will come to know that in case of delta connected load or in case of delta connect source the phase voltage will be equal to line voltage. So, whenever we see the delta connected source or load, we will say that the phase voltage of the delta will be equal to line voltage of the delta whether it is source or load. So, we can say that line voltage are equal to voltage across the load impedances for this system configuration. That means line voltage will be equal to phase voltage coming across the load impedance.

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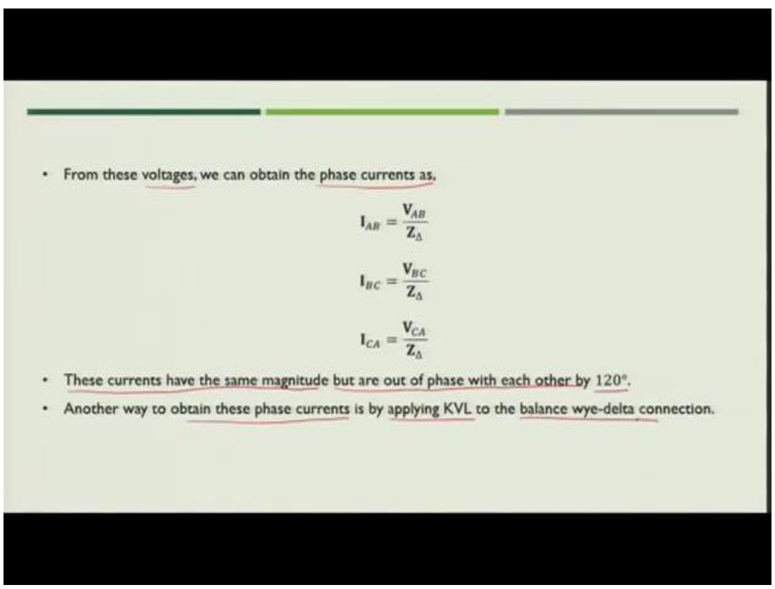

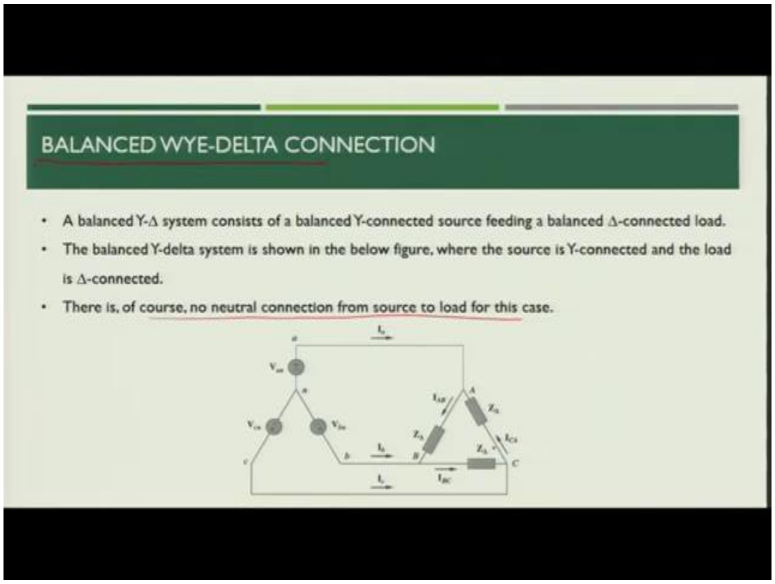

Now from the voltage values which we got, we can find out the value of the phase current. So, phase current will be,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{\Delta}}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{\Delta}}$$

So now here you can say that these currents have the same magnitude but these will be out of phase with each other by 120 degree because these voltages are out of phase with each other by 120 degree. Now we have another way to obtain these phase currents. How? We will simply apply Kirchoffs Voltage Law to the balanced Y- delta connection.

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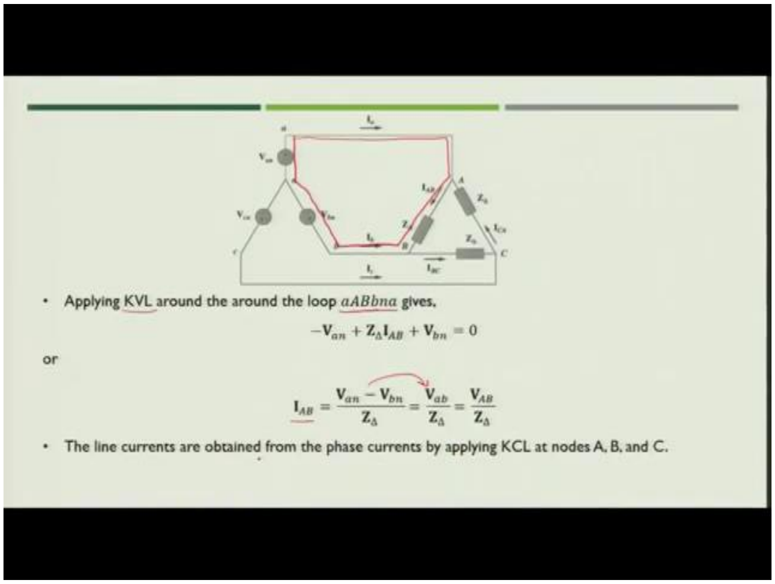

Let us see this particular figure and to apply KVL we will consider the loop that is $aABbna$ to get,

$$-\mathbf{V}_{an} + \mathbf{Z}_\Delta \mathbf{I}_{AB} + \mathbf{V}_{bn} = 0$$

or

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{an} - \mathbf{V}_{bn}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}$$

In this way also you can find the value which is same. Now to find out the line current what we have to do? We have to take the phase current which we derived and apply KCL at the nodes A, B, and C. So now if you apply KCL at node A,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 0$$

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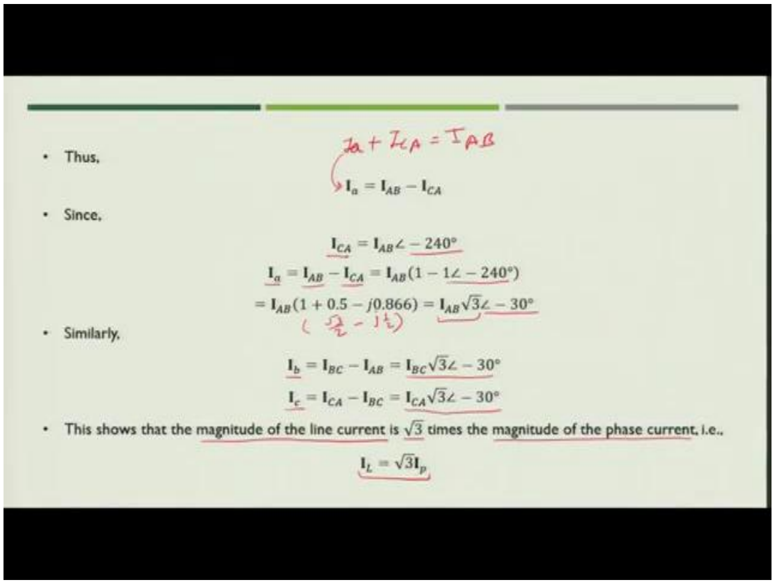

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle -240°$$

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240°)$$

$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30°$$

Similarly,

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_{BC}\sqrt{3}\angle -30°$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \mathbf{I}_{CA}\sqrt{3}\angle -30°$$

This shows that the magnitude of the line current is $\sqrt{3}$ times the magnitude of the phase current, i.e.,

$$\mathbf{I}_L = \sqrt{3}\mathbf{I}_p$$

From the line currents you can simply say that the magnitude of the line current is root 3 times the magnitude of the phase current in delta connected load. For the delta connection the line current will be equal to root 3 times of the phase current.

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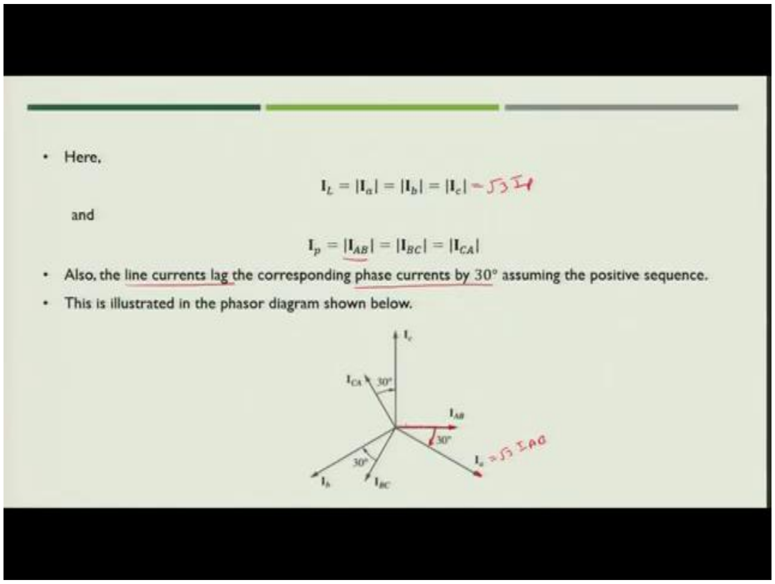

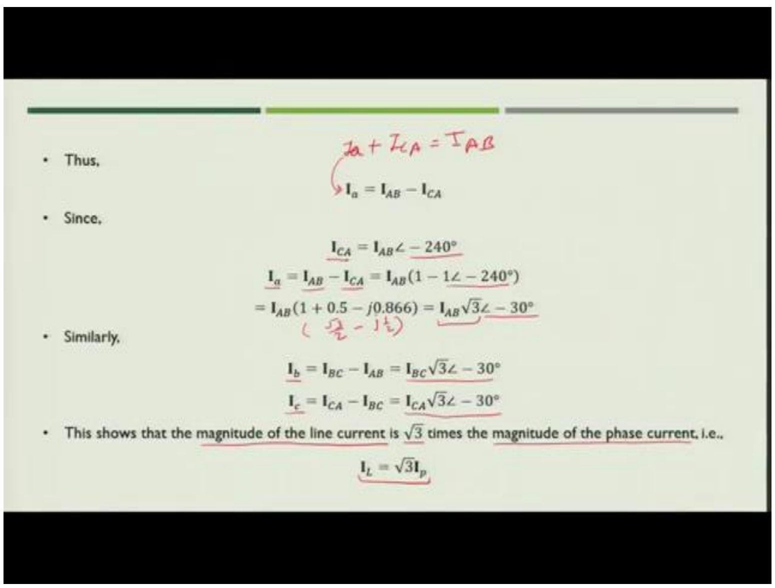

Now what you can write, you can write,

$$\mathbf{I}_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

and

$$\mathbf{I}_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

The important thing which we have to keep in mind is that the line current in case of delta connected load will lag the corresponding phase current by 30 degree. So, this you can see from the current expressions that the line current is lagging the phase current by 30 degree.

If you represent line current as well as phase current on the phasor diagram, if you take $\mathbf{I}_{AB}$ as a reference, $\mathbf{I}_a$ that is the line current will be 30 degree lagging from $\mathbf{I}_{AB}$. Similarly, $\mathbf{I}_b$ will be lagging by 30 degree from $\mathbf{I}_{BC}$ and $\mathbf{I}_c$ will be lagging 30 degree from $\mathbf{I}_{CA}$. So, this will give you an idea that how the current in case of delta connected will be having relationship with respect to the phase currents. Now important thing you need to remember that the phase current is having value smaller than $\mathbf{I}_a$ that is why it is represented in the phasor diagram $\mathbf{I}_a$ larger than $\mathbf{I}_{AB}$.

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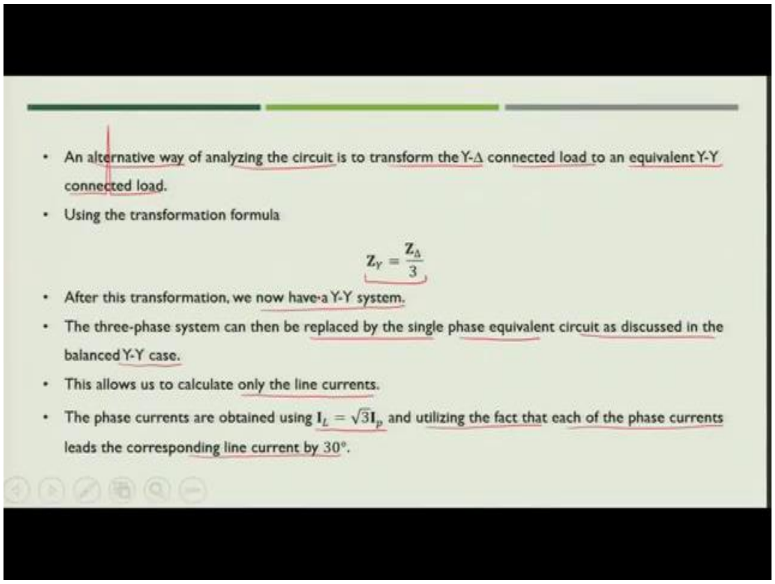

Now there is an alternate way also to analyze the circuit to transform star delta connected load to equivalent star star connection. In that case what you have to do, you have to convert delta connected load into equivalent star connected load which we can do with the help of star delta transformation. In that case you can say $\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$ and when you put you can analyze the system as we did in case of star star connection.

Now the 3 phase system here can be replaced by single phase equivalent circuit which we discuss in case of star star case. This will allow us to only find the line current then we have to use our recent discussion that $\mathbf{I}_L = \sqrt{3}\mathbf{I}_p$. So, phase current you can obtain with the help of this relationship and we utilize the fact that each of the phase currents leads the corresponding line current by 30 degree. This will help you to analyze the star delta circuit while you convert star delta into star star combination.

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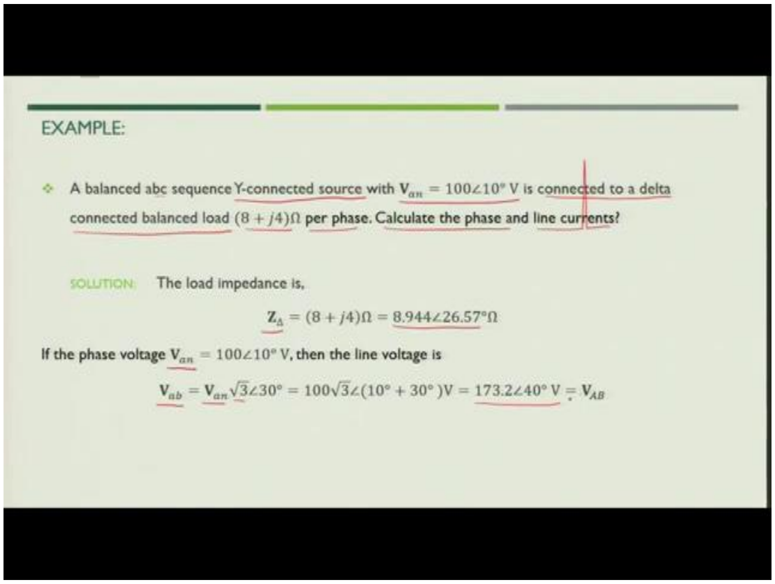

Now let us take 1 example. Suppose there is A, B, C phase sequence star connected source with the phase voltage $\mathbf{V}_{an} = 100\angle 10°$ V and it is connected to a delta connected balanced load with impedance $(8 + j4)\Omega$ per phase. We need to calculate the phase and line currents. So, the load impedance you can convert it into phasor form as,

$$\mathbf{Z}_\Delta = (8 + j4)\Omega = 8.944\angle 26.57°\Omega$$

Now the phase voltage $\mathbf{V}_{an} = 100\angle 10°$ V.

Now what will be the line voltage, line voltage,

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\angle 30° = 100\sqrt{3}\angle 10° + 30°\text{ V} = 173.2\angle 40°\text{ V} = \mathbf{V}_{AB}$$

Now in case of star delta combination, you know that the line voltage of the source side will be equal to phase voltage at the delta side.

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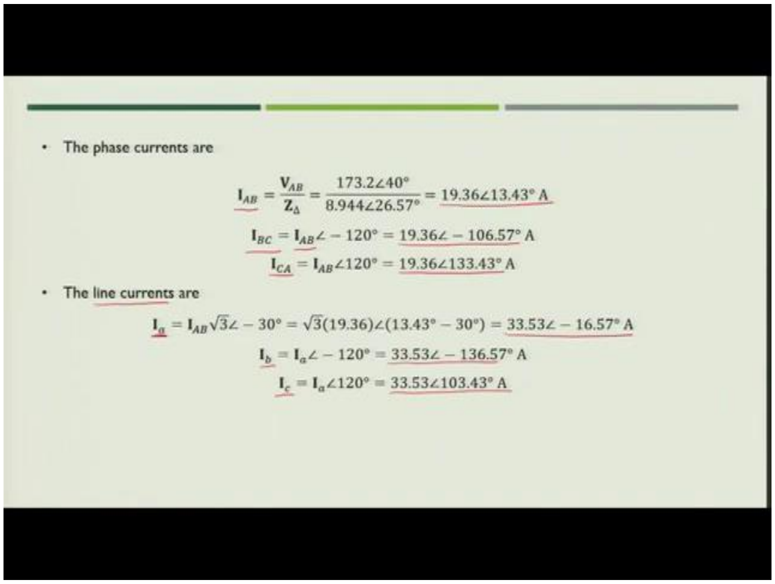

So, you can easily find out the value of phase currents as,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{173.2\angle 40°}{8.944\angle 26.57°} = 19.36\angle 13.43° \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = 19.36\angle -106.57° \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = 19.36\angle 133.43° \text{ A}$$

Now what will be the line currents in case of the delta connected load. Line current,

$$\mathbf{I}_{a} = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \sqrt{3}(19.36)\angle(13.43° - 30°) = 33.53\angle -16.57° \text{ A}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle -120° = 33.53\angle -136.57° \text{ A}$$

$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120° = 33.53\angle 103.43° \text{ A}$$

In this way you can analyze the circuit if it is star delta if it is in the star delta combination.

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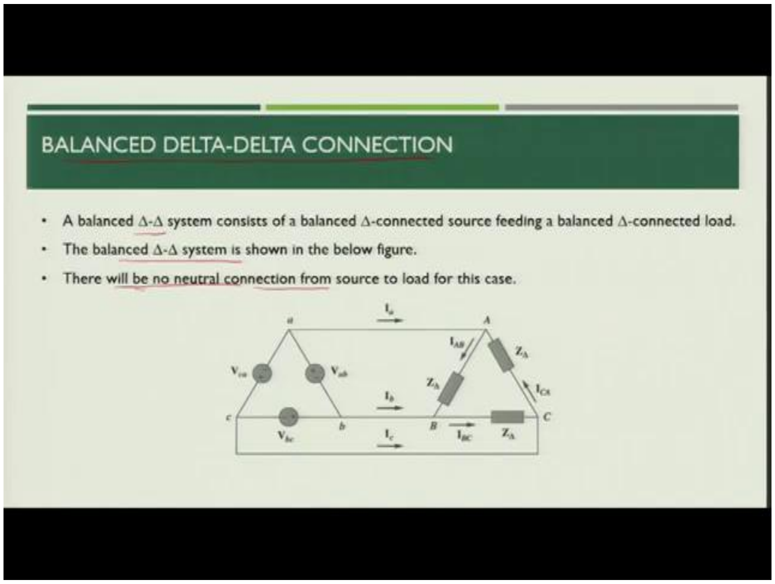

Now let us talk about the balanced Δ-Δ connection. In this case if you see this particular circuit, the circuit is Δ-Δ connected here the source is delta connected as well as the load is delta connected. So, the source voltage that is $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$ will be applied across the load impedances also. So, if the line current is $\mathbf{I}_{a}$, $\mathbf{I}_{b}$, $\mathbf{I}_{c}$ in the load we consider phase current as $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$ and $\mathbf{I}_{CA}$. Now since both are balanced delta-delta connections we will not be able to put neutral from source to load because we cannot identify neutral in case of delta-delta connection.

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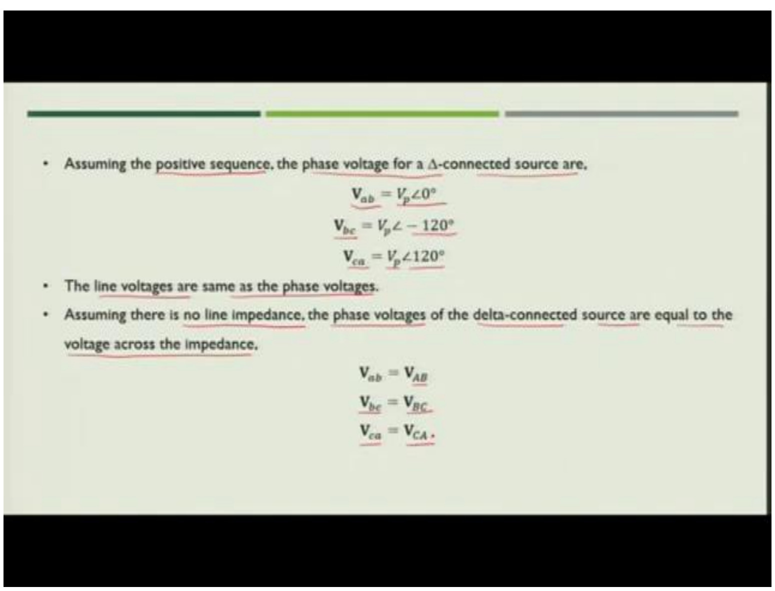

Now if we assume positive sequence that is the sequence is A, B, C the phase voltage for a delta connected source will be,

$$\mathbf{V}_{ab} = V_p\angle 0°$$

$$\mathbf{V}_{bc} = V_p\angle -120°$$

$$\mathbf{V}_{ca} = V_p\angle 120°$$

Now since the line voltages are same as the phase voltages in case of delta connection delta connected source as well as load.

If we assume there is no line impedance means there is no impedance for the transmission line. Transmission line is loss less. Then, the phase voltage of the delta connected source will be equal to the voltage across the impedance. So, in that case

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{CA}$$

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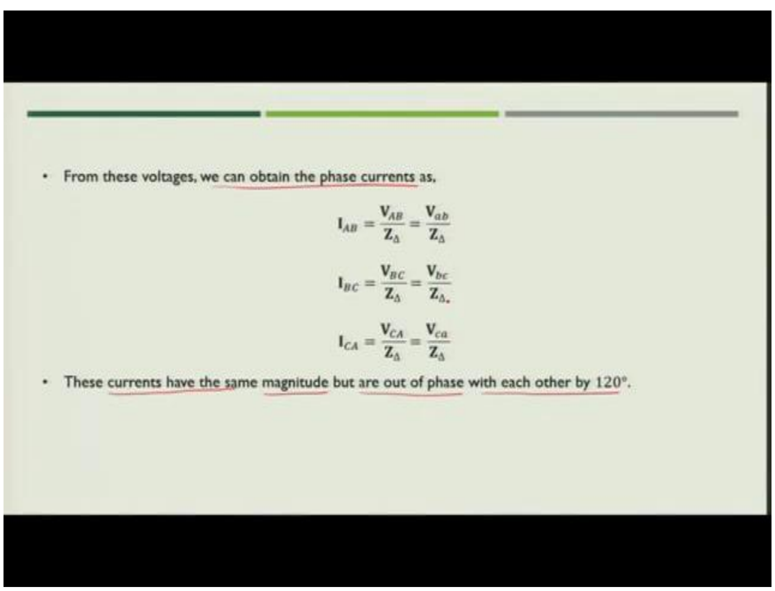

Now from the voltages what we can do, we can obtain the phase currents. In this case the phase current

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{\Delta}}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{\Delta}}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{\Delta}}$$

You can see from the current equations that the currents have the same magnitude but these will again be out of phase with each other by 120 degree because these voltages are out of phase with each other by 120 degree.

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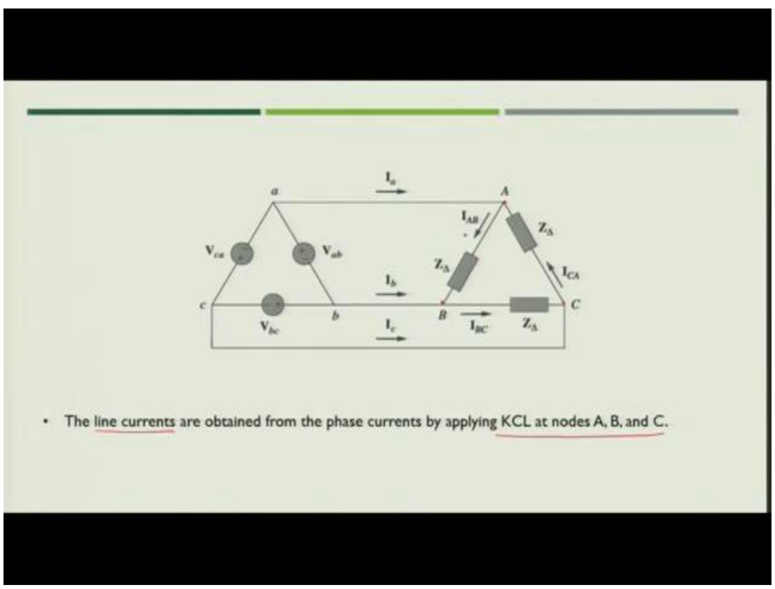

So now you have the phase current information. Next task is you need to find out the line current. So how you can find out the line current, you can simply apply KCL at the nodes either A or B or C you can apply the KCL. So here if you apply the KCL, you will get

$$\mathbf{I}_{a} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 0$$

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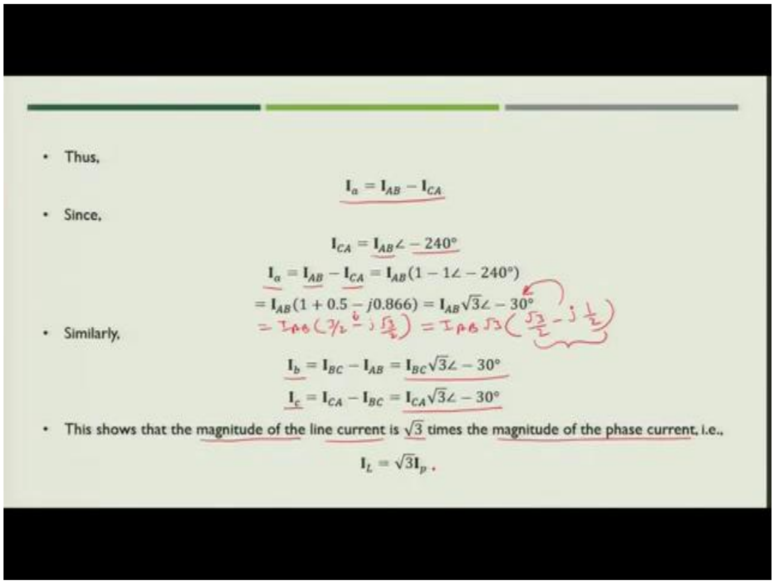

Now,

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle -240°$$

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240°)$$

$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30°$$

Similarly,

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_{BC}\sqrt{3}\angle -30°$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \mathbf{I}_{CA}\sqrt{3}\angle -30°$$

This shows that the magnitude of the line current is $\sqrt{3}$ times the magnitude of the phase current, i.e.,

$$\mathbf{I}_L = \sqrt{3}\mathbf{I}_p$$

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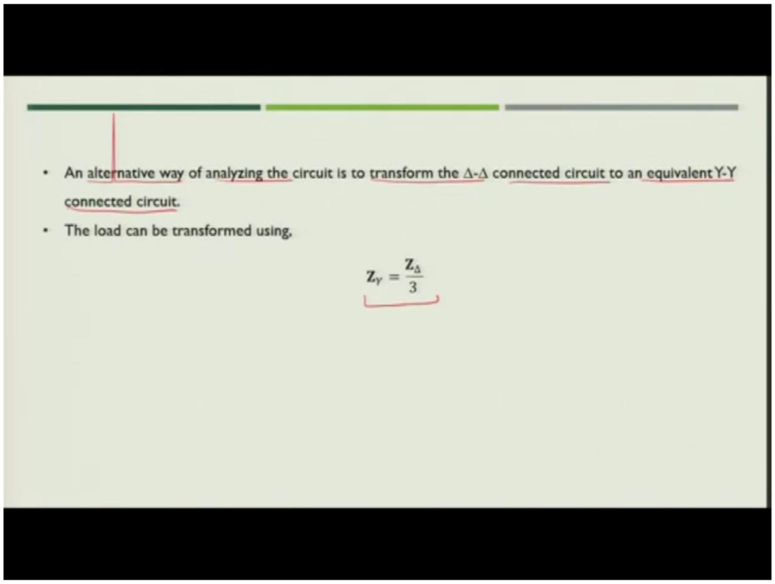

Now here also you can use the alternate way of analyzing the circuit, you can transform the delta-delta connected circuit to an equivalent star star connected circuit. So, in that case, you have to use star delta transformation on both sides, convert them into star star combination. In that case your load will be $\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$. So, this is what we can get from the star delta transformation. Using that when you convert delta-delta connected circuit into star star connected circuit, you can simply analyze as we discuss in case of star star connected circuit in the last lecture.

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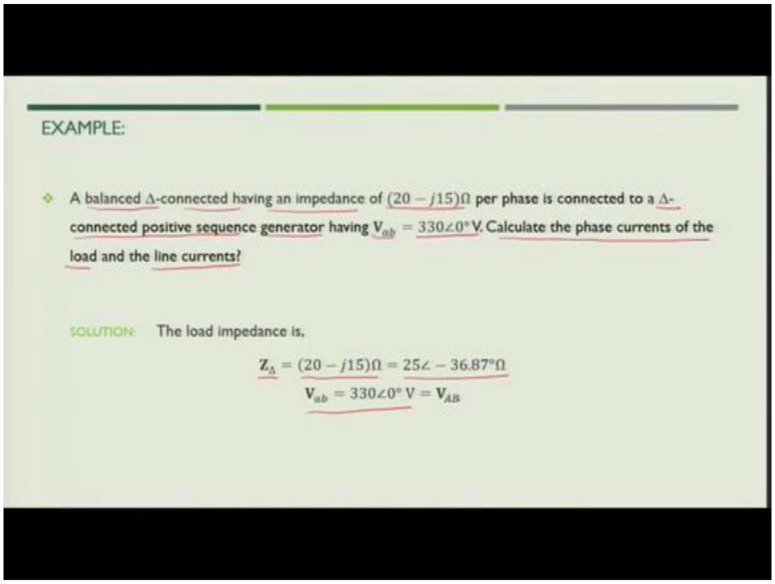

Now let us take the example related to our balanced delta-delta connection. In this case the balanced delta connected load having an impedance of $(20 - j15)\Omega$ is connected to a delta connected positive sequence generator having $\mathbf{V}_{ab} = 330\angle 0°$ V. What we must find out, we have to find out the phase current of the load and the line currents of the circuit. We can convert it into phasor as,

$$\mathbf{Z}_{\Delta} = (20 - j15)\Omega = 25\angle -36.87°\Omega$$

$$\mathbf{V}_{ab} = 330\angle 0°\text{ V} = \mathbf{V}_{AB}$$

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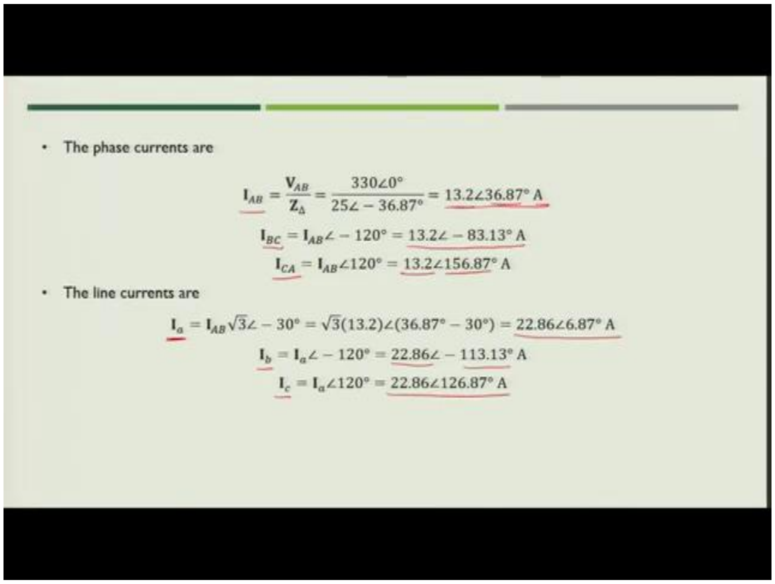

So,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{330\angle 0°}{25\angle -36.87°} = 13.2\angle 36.87° \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = 13.2\angle -83.13° \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = 13.2\angle 156.87° \text{ A}$$

The line currents are,

$$\mathbf{I}_{a} = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \sqrt{3}(13.2)\angle(36.87° - 30°) = 22.86\angle 6.87° \text{ A}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle -120° = 22.86\angle -113.13° \text{ A}$$

$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120° = 22.86\angle 126.87° \text{ A}$$

So, with this we can close our today’s session. In this session we discussed about the Wye Delta that is popularly known as star delta and then delta-delta connected circuits. In the next lecture we will start our discussion with the delta Wye connected circuit or delta star connected circuit. Thank you.