Basic Electric Circuits
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Module 10 - Sinusoidal Steady State Analysis 2
Lecture 47 - Balanced Three phase connections

Namaskar, so in the last session we were discussing about the phase sequence of three phase circuit, so we will continue our discussion from that point onwards and let us see.

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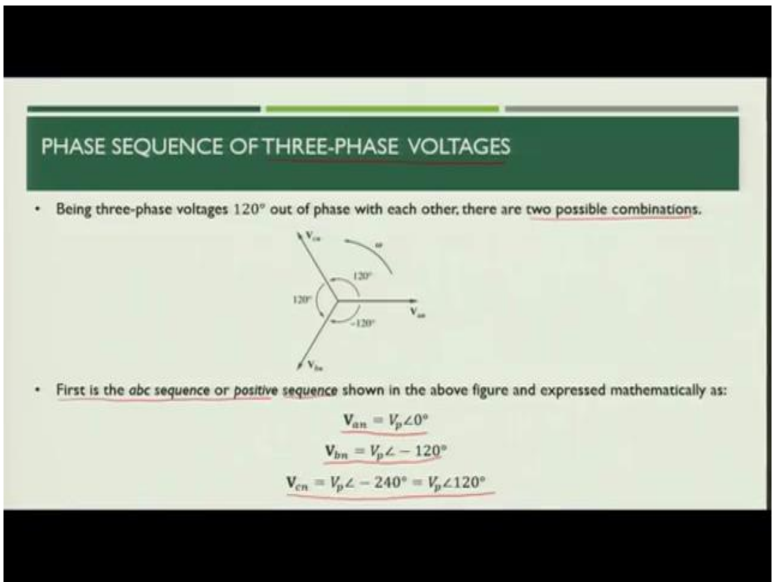

When we talk about the three-phase voltage source phase sequence we see there are two possible combination of phase sequencing.

In first sequence we discuss that we call it as abc or positive sequence where the reference voltage that is $\mathbf{V}_{an}$ is leading $\mathbf{V}_{bn}$ by 120 degree and $\mathbf{V}_{bn}$ is again leading $\mathbf{V}_{cn}$ and by 120 degree. So we can write,

$$\mathbf{V}_{an} = V_p\angle 0°$$

$$\mathbf{V}_{bn} = V_p\angle -120°$$

$$\mathbf{V}_{cn} = V_p\angle -240° = V_p\angle 120°$$

In that case if you see abc sequence is rotating in counter clockwise and it is in the direction of the rotation of the rotor of the generator.

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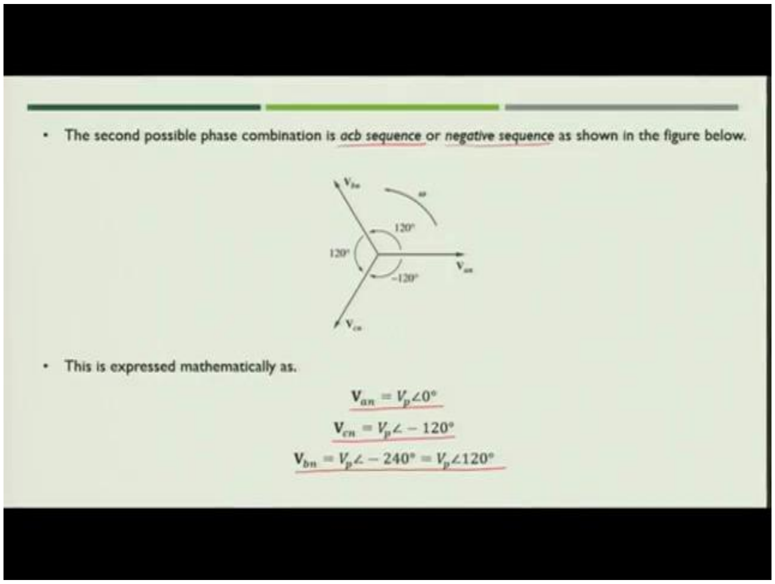

Now next possible combination is acb sequence which is called as a negative sequence, so in that case what happens that you are reference phasor that is $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ and $\mathbf{V}_{cn}$ leads $\mathbf{V}_{bn}$ by 120 degree, so what we write mathematically,

$$\mathbf{V}_{an} = V_p\angle 0°$$

$$\mathbf{V}_{cn} = V_p\angle -120°$$

$$\mathbf{V}_{bn} = V_p\angle -240° = V_p\angle 120°$$

So now in that case if you see the phase sequence abc will rotate in this direction which will be opposite to direction of the rotation of the rotor.

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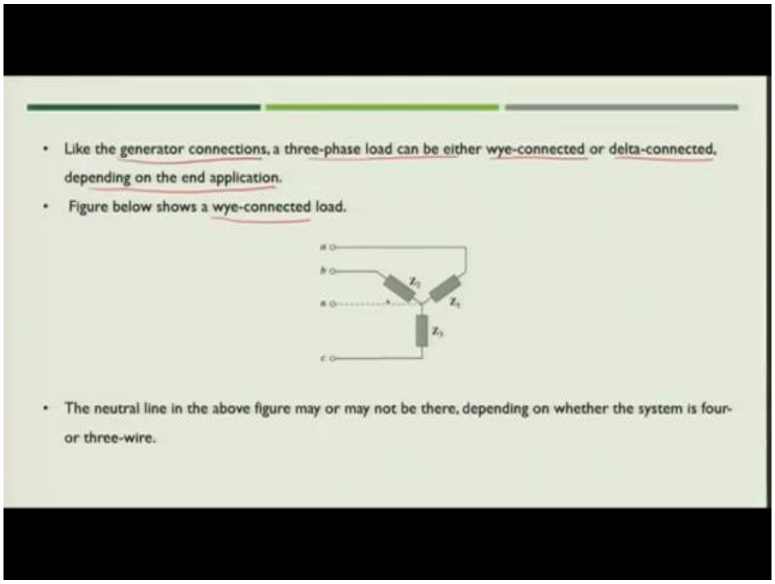

Now like the generator connection which we discussed previously three phase load can also be either your wye connected or you can say star connected or delta connected and it depends upon what type of end application you have related to that particular load, so in the case wye connected you will have the loads components connected in star combination with neutral it may be available or may not be available it is not necessary that you will always have neutral physically present and reachable for connections, so here you might have three phase three wire system for the load or you can have three phase four wire system for the load.

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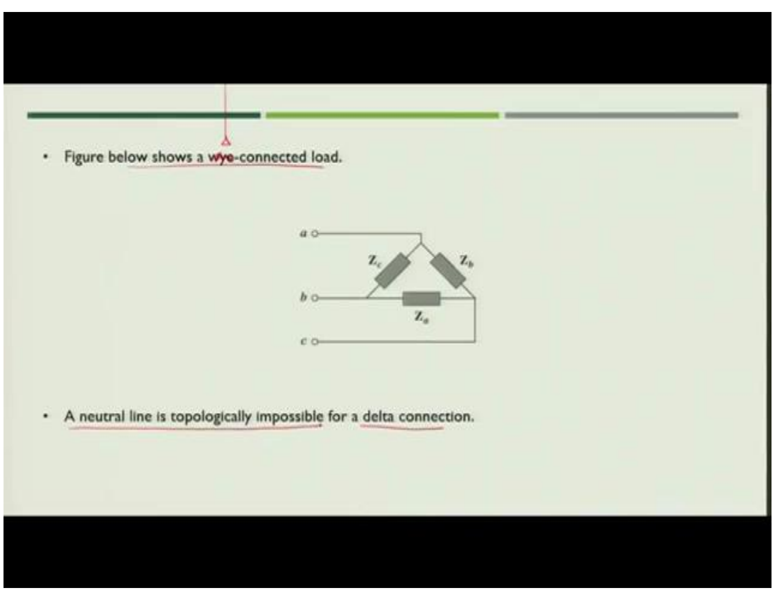

Now next combination is delta connected, so this is basically the delta connected configuration. Here you will see abc are connected in triangular fashion and your phase ABC is taken out from the load and if you see this arrangement in this particular arrangement you cannot take the neutral connection out because it is topologically impossible to take the neutral connection from the delta connected load.

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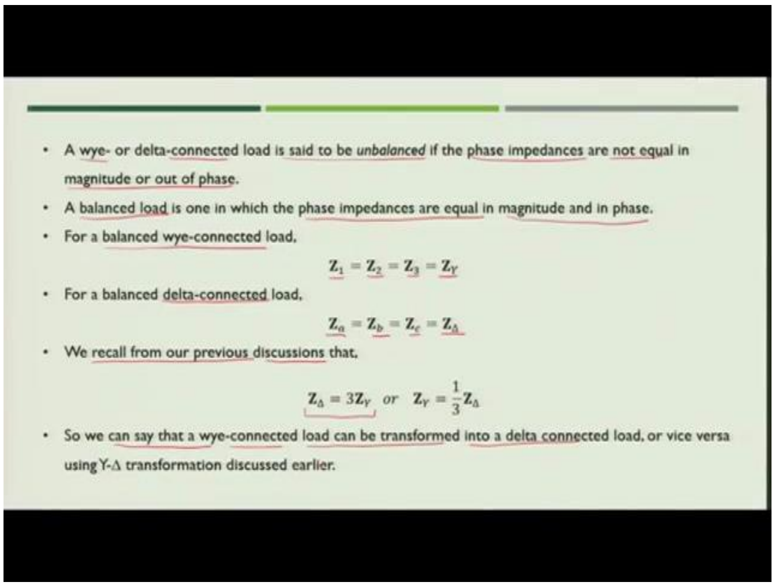

Now whether it is star connected or delta connected will say that if it is unbalanced then the phase impedance will not be equal, and the phase sequence will also be out of phase.

So, you will have the magnitude as well as phase both either of two will be out of phase or you can have magnitude as well as phase both out of phase, so when you say the load is balanced it means that phase impedances in all three legs of the load are equal and in phase, in phase means you will have equally connected with respect to phase. So, in case of balanced star connected or wye connected load you will have $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$ all same so you will have $\mathbf{Z}_Y$, so $\mathbf{Z}_Y$ is nothing but a phasor quantity so you will see the magnitude as well as phase both are same in all the three legs.

In case of balanced delta connection you will have $\boldsymbol{Z}_a = \boldsymbol{Z}_b = \boldsymbol{Z}_c = \boldsymbol{Z}_\Delta$, so for star connected we will specify individual legs impedance as $\mathbf{Z}_Y$. In case of delta connected we will specify as $\mathbf{Z}_\Delta$. Now we have already discuss the star delta transformation in our previous lecture, so taking reference of the star delta transformation we earlier proved in case of $\boldsymbol{Z}_\Delta = 3\boldsymbol{Z}_Y$ when you convert delta connected network into star connected network.

Similarly, $\mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$ if you connect if you convert a star connected load into delta connected load, so we can say that the star connected or wye connected load can be transformed into delta connected load, or vice versa with the help of the star delta transformation which we discussed earlier.

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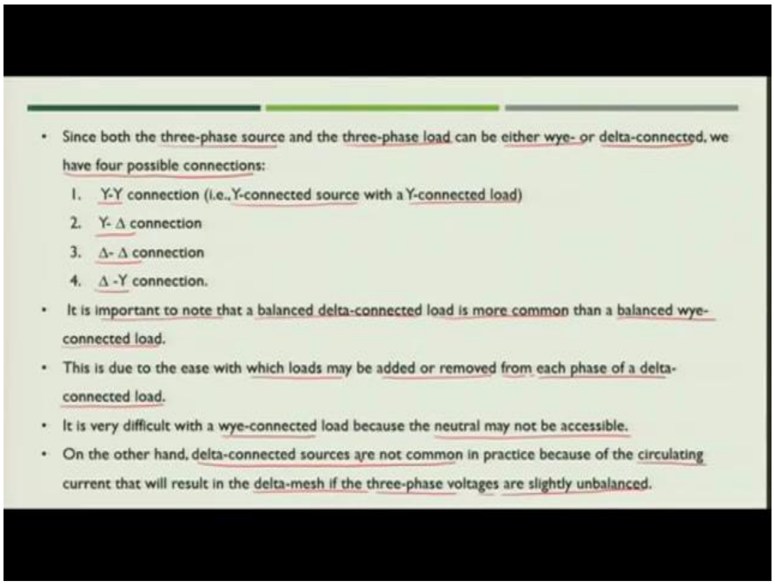

Now since you can have source and load both three phase connected in either star or delta connected fashion, so we can have now four possible combinations of the connections between source and load as,

1. Y-Y connection (i.e., Y-connected source with a Y-connected load)
2. Y- Δ connection
3. Δ- Δ connection
4. Δ -Y connection.

Now it is important to note here that the balanced delta connected load is more common in the network than the balanced Y connected load, because it is easy with the delta connected load that you can add or remove the load from each phase of the delta connected load.

It is difficult is case of wye connected or star connected because it is not always possible to have neutral, which is accessible from outside for the connection, so that is why the balanced delta connected load is more feasible for reconfiguring or taking any leg out or doing the changes in the delta connected load.

But on the other hand you will not see delta connected source as a common practice because if you have the delta connected source you will see the circulating current because delta will create a delta mesh which will cause a current to circulate within the delta network of the source and this will be because of any slight unbalance in the voltages of the three phases, so because of this you will have circulating current and this will cause the overheating of the coils of the generator, so that is why we generally avoid the delta connected source.

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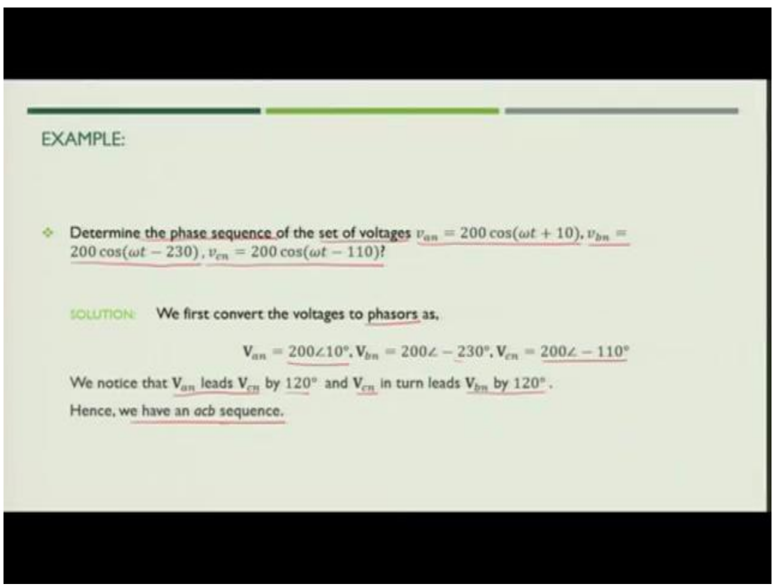

Now let us take one example in which we need to determine the phase sequence of the set of voltages like $v_{an} = 200\cos(\omega t + 10)$, $v_{bn} = 200\cos(\omega t - 230)$, $v_{cn} = 200\cos(\omega t - 110)$. What we have to do first, first we will convert it into phasors, so $\mathbf{V}_{an} = 200\angle 10°$, $\mathbf{V}_{bn} = 200\angle -230°$, $\mathbf{V}_{cn} = 200\angle -110°$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by $120°$ and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by $120°$.

Hence, we have an *acb* sequence.

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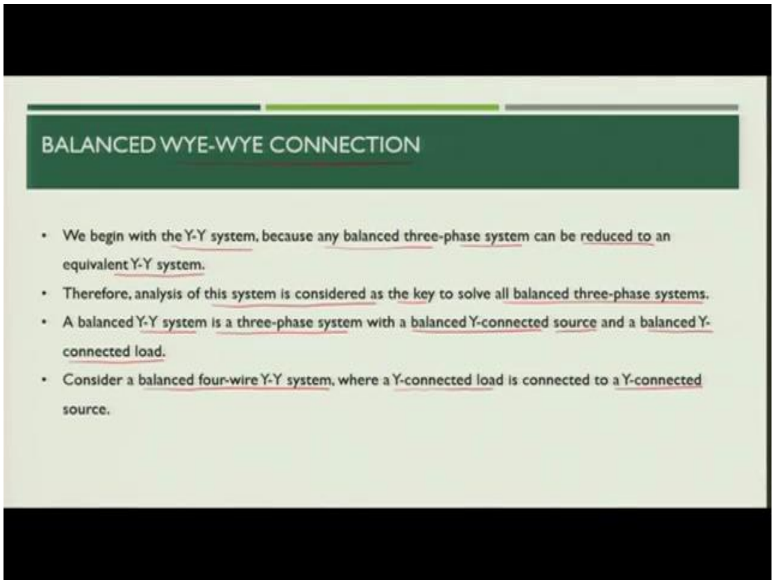

Now let us talk about the connections we saw that there are first four possible combinations that is Y-Y, Y delta, delta-delta and delta-Y connections between source and load, so we will first start with Y-Y connection for the source and load, why we are starting with Y-Y connection because any balanced three phase system can be reduced to a Y-Y system with the help of star delta transformation, so therefore this system is considered as a key to solve the all balance three phase system.

Now a balanced three phase Y-Y system is a three phase system with a balance Y connected source and a balanced Y connected load. Now let us consider three phase four wire Y-Y connected system where star or Y connected load is connected to star connected source.

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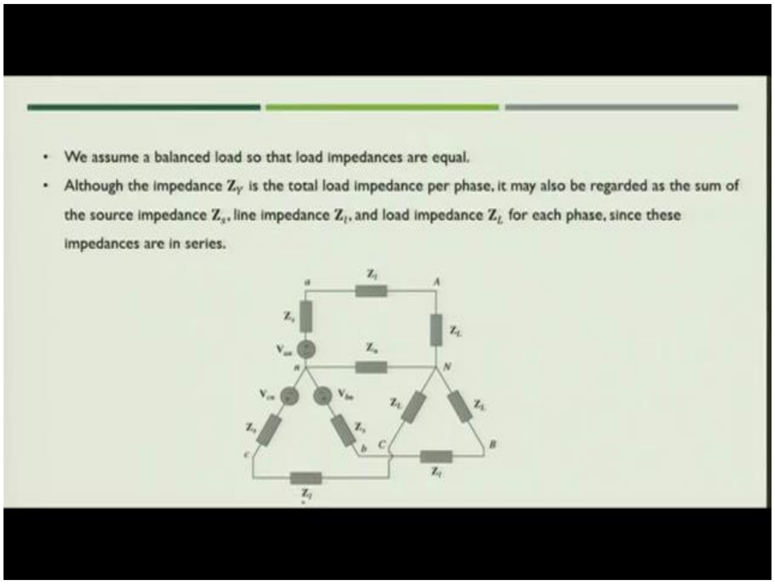

So how it will look like if you see this figure the source is Y connected $\mathbf{V}_{an}$ is connected to the a phase of the load through internal resistance or internal impedance of the source, so we will call it as source impedance and then you will have transmission line impedance and then the load impedance. So now the phase A of the source is connected to phase A of the load through transmission line, similarly, phase B of the source is connected to phase B of the load through the transmission line and then phase C of the load is connected to phase C of the source through the transmission line.

Now the neutrals of both source as well as load are connected through wire having impedance equal to $\mathbf{Z}_n$. So now you can see this combination is three phase four wire arrangement were the source impedance as well as the transmission line impedance is considered for the analysis.

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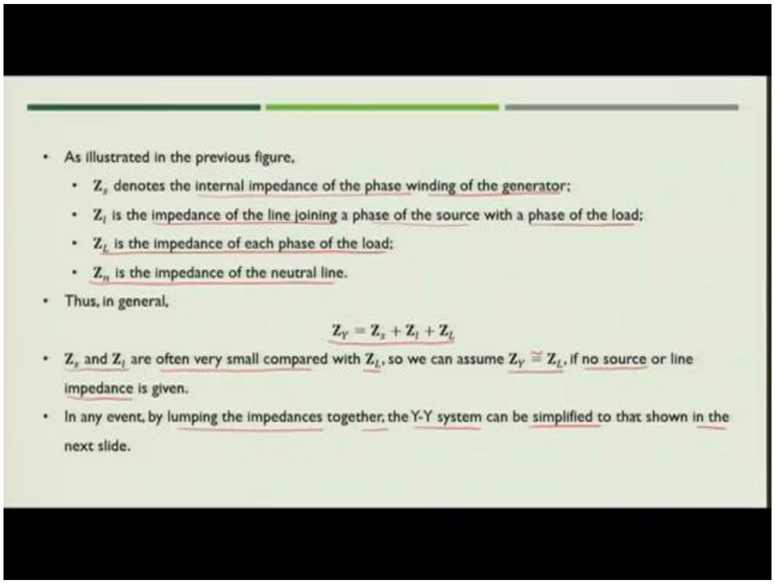

Now since this three are in series $\mathbf{Z}_s$, $\mathbf{Z}_l$ and be load impedance $\mathbf{Z}_L$ you can club all of them as,

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_l + \mathbf{Z}_L$$

Where,

- $\mathbf{Z}_s$ denotes the internal impedance of the phase winding of the generator
- $\mathbf{Z}_l$ is the impedance of the line joining a phase of the source with a phase of the load
- $\mathbf{Z}_L$ is the impedance of each phase of the load
- $\mathbf{Z}_n$ is the impedance of the neutral line.

Now in the power system network the $\mathbf{Z}_s$ and $\mathbf{Z}_l$ are often very small as compared to $\mathbf{Z}_L$, so we can assume $\mathbf{Z}_Y = \mathbf{Z}_L$ even if no source or line impedance is given in the question. In any event by lumping the impedance together, the Y-Y system can be simplified to that circuit which we shown in the next slide.

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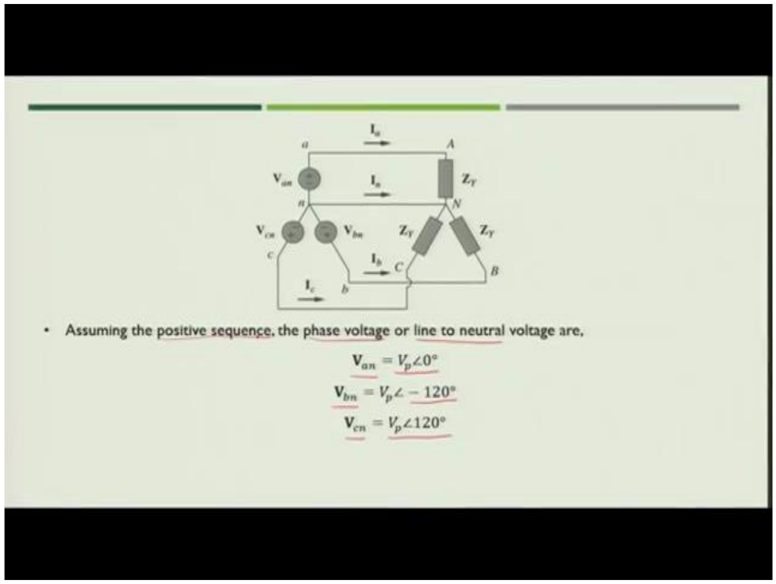

So if you see in this case what we have done, we have simplified the arrangement which we saw in the previous slide and we lump the $\mathbf{Z}_S$, $\mathbf{Z}_l$ for transmission line and the load impedance through a impedance called $\mathbf{Z}_Y$, so now it is much simplified and easy to understand now you say that both phases of source and load are connected, A is connected A, B is connected to B of the load, C is connected to C of the load. Now let us assume that the current which is flowing in the line connecting phase A of the source to load is $\mathbf{I}_a$, similarly the line connecting phase B of the source to phase B of the load is $\mathbf{I}_b$ and a line connecting C phase of the source to C phase of the load is $\mathbf{I}_c$.

We will also assume there is some current $\mathbf{I}_n$ which is flowing in the neutral wire connecting neutral of the source to neutral of the load. So, the values of the voltage source, since we have assumed positive sequence will be,

$$\mathbf{V}_{an} = V_p\angle 0°$$

$$\mathbf{V}_{bn} = V_p\angle -120°$$

$$\mathbf{V}_{cn} = V_p\angle 120°$$

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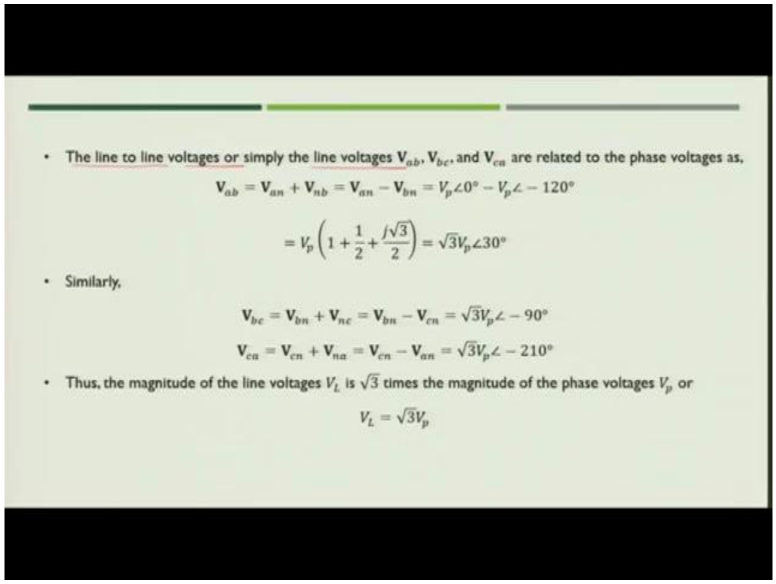

Now if you are asked to find line to line voltage or simply the line voltage, line voltage would be $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ or $\mathbf{V}_{ca}$.

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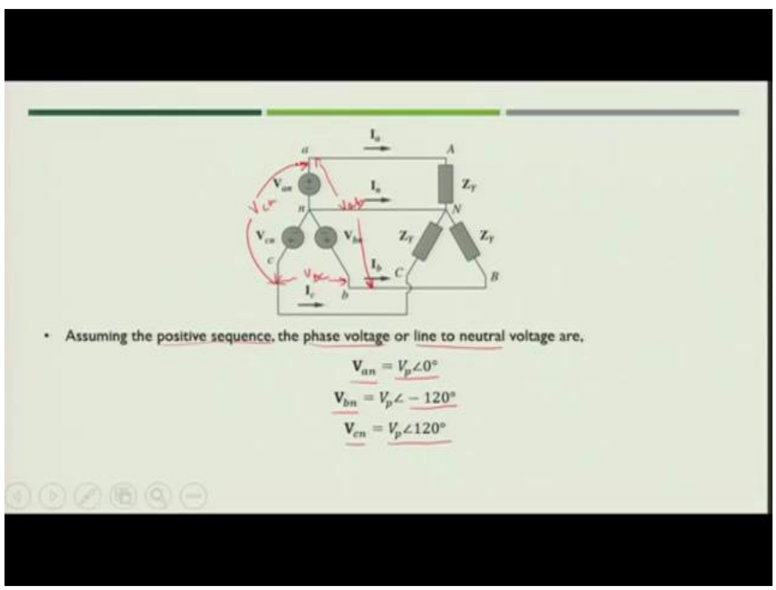

That means the voltage between a phase and b phase, so this will be your $\mathbf{V}_{ab}$ then similarly the voltage between b and c is the $\mathbf{V}_{bc}$ and similarly again between c and a will be $\mathbf{V}_{ca}$.

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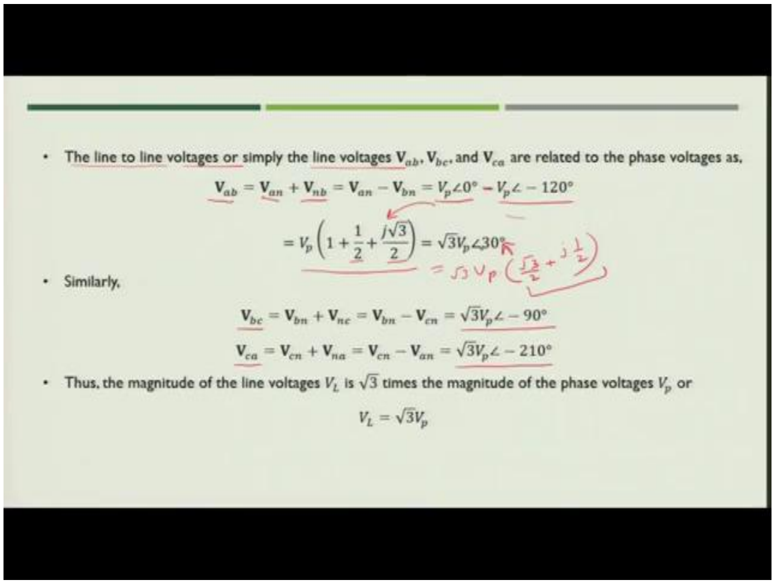

So now we need to find out the value of the line to line voltage or we simply say the line voltage VAB you can write as VAN plus VNB

$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0° - V_p\angle -120°$$

$$= V_p\left(1 + \frac{1}{2} + \frac{j\sqrt{3}}{2}\right) = \sqrt{3}V_p\angle 30°$$

Similarly,

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} + \mathbf{V}_{nc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90°$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} + \mathbf{V}_{na} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210°$$

Thus, the magnitude of the line voltages V_L is $\sqrt{3}$ times the magnitude of the phase voltages V_p or

$$V_L = \sqrt{3}V_p$$

Now if you see $\mathbf{V}_{ab}$ leads $\mathbf{V}_{bc}$ by 120° and $\mathbf{V}_{bc}$ leads $\mathbf{V}_{ca}$ by 120° so that the line voltages sum up to zero as do the phase voltages.

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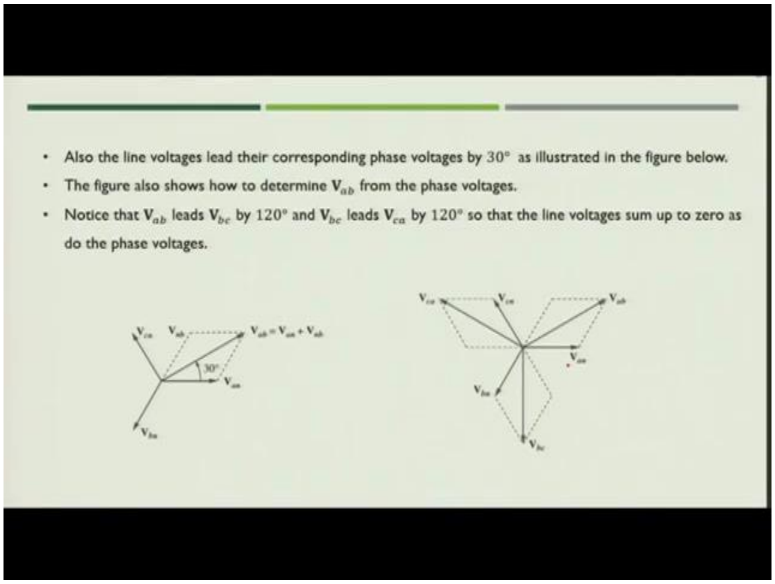

So, in the line voltage you will see that these are leading with respect to their phase voltages by 30 degree, so we also see that the line voltage is now root 3 times of the phase voltage in magnitude.

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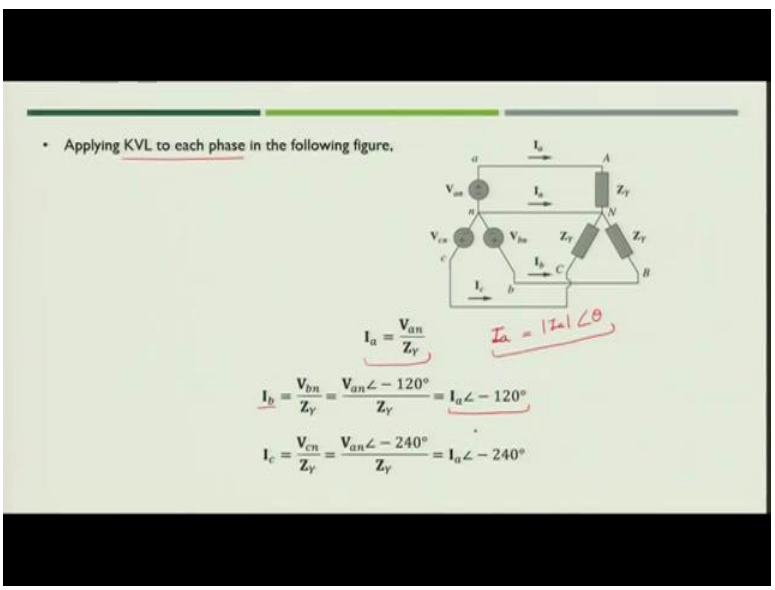

Now if you apply KVL to each phase what you can write the value of

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -120°}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -120°$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -240°}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -240°$$

Now here $\mathbf{I}_a$ is the phasor quantity, so the angle of current $\mathbf{I}_a$ with respect to $\mathbf{V}_a$ will be depending upon the phasor value of $\mathbf{Z}_Y$. So in the analysis we have to be very careful about the phasor value of $\mathbf{Z}_Y$ so that we get the proper value of $\mathbf{I}_a$ in phasor terms with respect to $\mathbf{V}_a$.

Now if you sum up,

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

So that,

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0$$

or

$$\mathbf{V}_{nN} = \mathbf{Z}_n\mathbf{I}_n = 0$$

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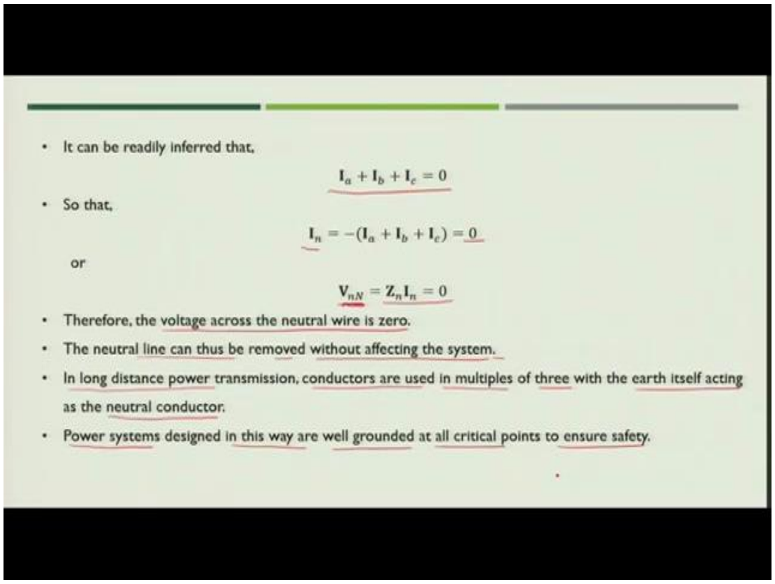

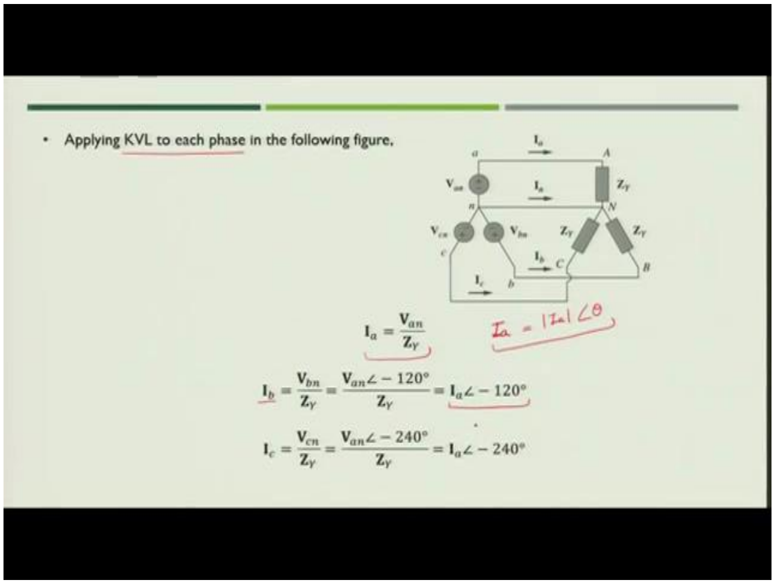

Now since $\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0$, the neutral current will also be zero in that case the voltage difference between source neutral to load neutral will be equal to zero because there is no current flowing, so therefore the voltage across the neutral wire will be zero. So neutral line can thus be removed because since, there is no current flowing you can remove the neutral wire and this is economically very important to understand that if the system is completely balanced the neutral wire is not required and we save cost of laying the neutral wire from source to load.

So, in a long-distance power transmission the conductors we use are in multiple of three because we are talking about the three phase system, so the conductors will be in multiple of three and earth will act as neutral conductor.

Now power system is designed in the way that the system is well grounded at all critical points why because we want to make ensure the safety of the system as well as the person who work on the power system network.

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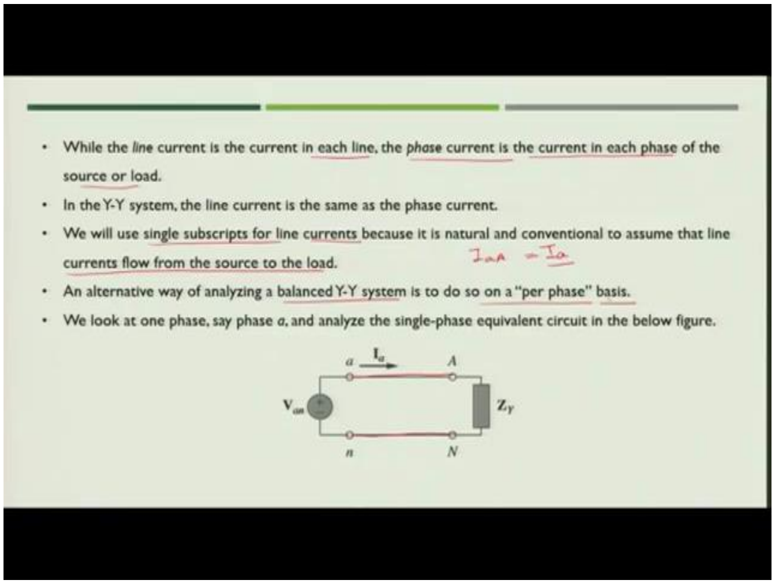

Now while the line current is the current in each line, the phase current is also same in this case because phase current is the current in each phase of source or load and when we talk about the Y-Y connected system we will see that the line current is same as the phase current. So, what we will do we will use single subscript for line current we will not write as $\mathbf{I}_{aa}$ we will simply write as $\mathbf{I}_a$ for the line current because it is understood that we are talking about the current which is flowing from source to load, so that is why we drop the second subscript and we simply write as $\mathbf{I}_a$.

Now alternative way of analyzing the balance star-star system is to convert it into per phase because the system is completely balanced you can convert the three system to its equivalent single phase system, so when you convert the three phase system which we are discussing then you get the equivalent circuit as shown in this slide here the $\boldsymbol{V}_{an}$ that is phase voltage is connected to the load ZY through the line, so one is line connecting between phase A to A of the source and load and then another line is for connection between neutral of the source and load.

So this will be single phase equivalent of the three phase Y-Y connected system which we discussed.

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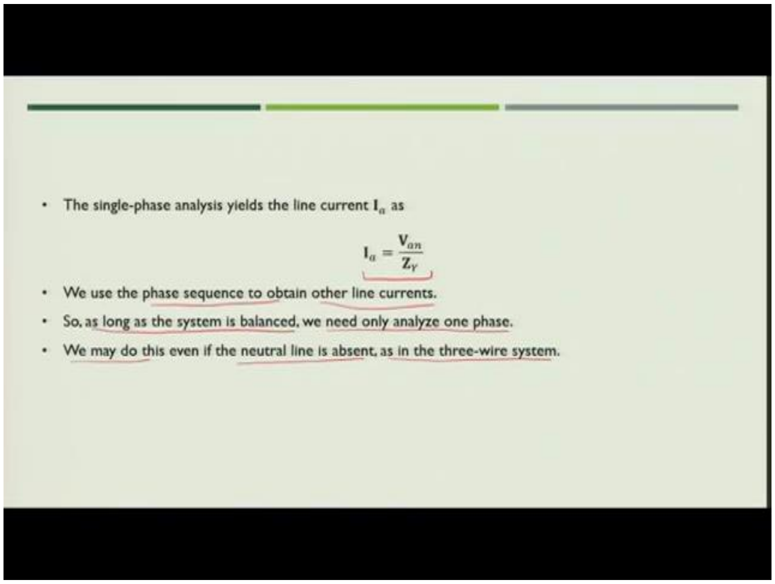

So in this case again you can see from the figure that $\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$. So we use phase sequence to obtain the other line currents because we know that the phase sequence is ABC, so we can easily find out the values of $\mathbf{I}_b$ and $\mathbf{I}_c$ with help of current $\mathbf{I}_a$ which we got from its single phase equivalent so as long as the system is balanced we need only analysis in case of single phase equivalent, so we can all we may do so even if the neutral line is absent because you can say that neutral is virtually connected for creating the single phase equivalent.

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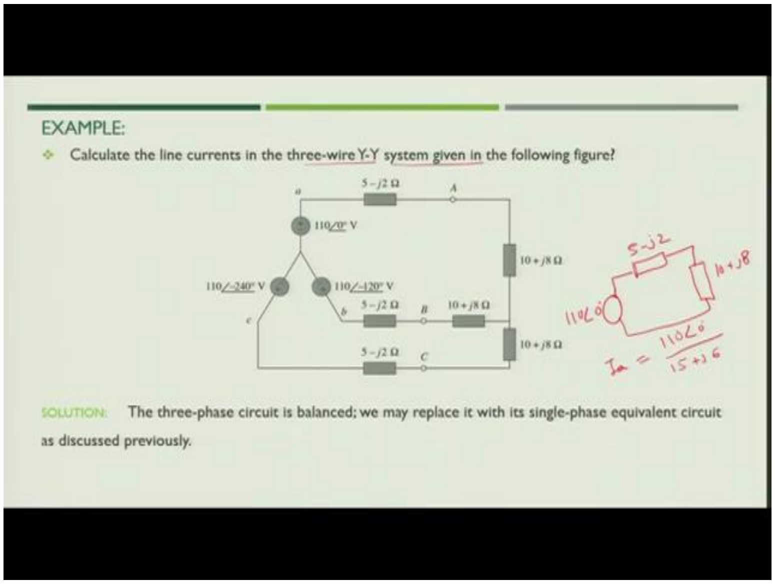

Now let us take one example before closing our today's we have three wire star-star connected system which is given in the figure so we have source completely balanced $110\angle 0°$, $110\angle -120°$, $110\angle -240°$ volt as the phase voltages line impedance is $5 - j2$ in all the three phases and load impedance is $10 + j8$ ohm, so you will see that this load is also star connected and it is connected to the start connected source we have we are not connecting the neutral because system is balanced.

So what we will do we will first create its single phase equivalent so when you create single phase equivalent what you get you will get the voltage source $110\angle 0°$ and then you will have impedance $5 - j2$ and then again the load impedance $10 + j8$, so this will be your single phase equivalent of this circuit. Here, $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8$. So,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{110\angle 0°}{16.155\angle 21.8°} = 6.81\angle -21.8°\text{A}$$

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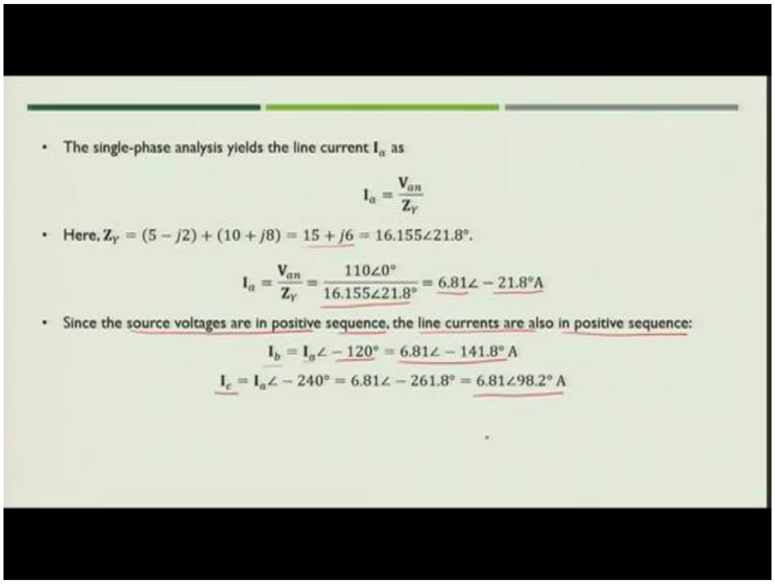

Now since the voltage sources are in positive sequence the line currents will also be in positive sequence, so

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = 6.81\angle -141.8° \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240° = 6.81\angle -261.8° = 6.81\angle 98.2° \text{ A}$$

With this we can close our today's session in which we discussed mostly about the Y-Y connected balanced system, so in next session we will discuss about star delta or you can say Y delta connected system thank you.