Basic Electric Circuits
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Module-10
Sinusoidal Steady State Analysis 2
Lecture-46
Ideal Transformer and Introduction to Three Phase Circuits

Namashkar. In the last class we were discussing about the transformer polarity. Today we will continue our discussion.

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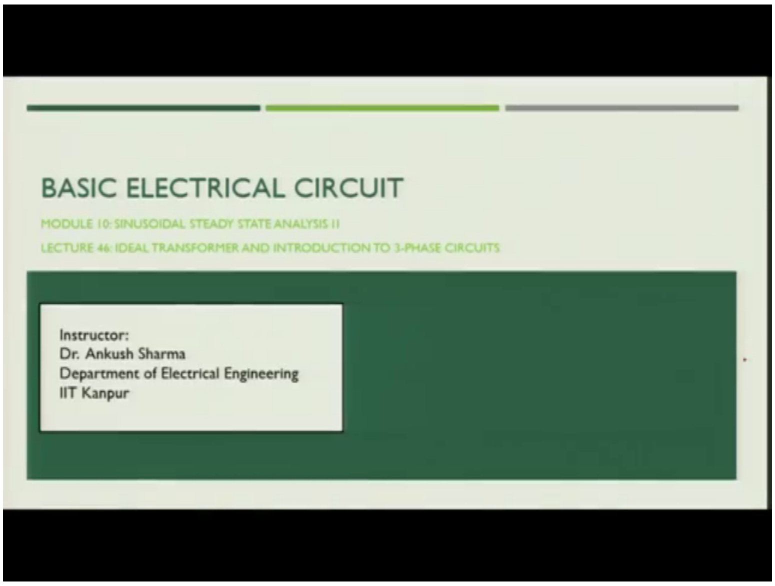

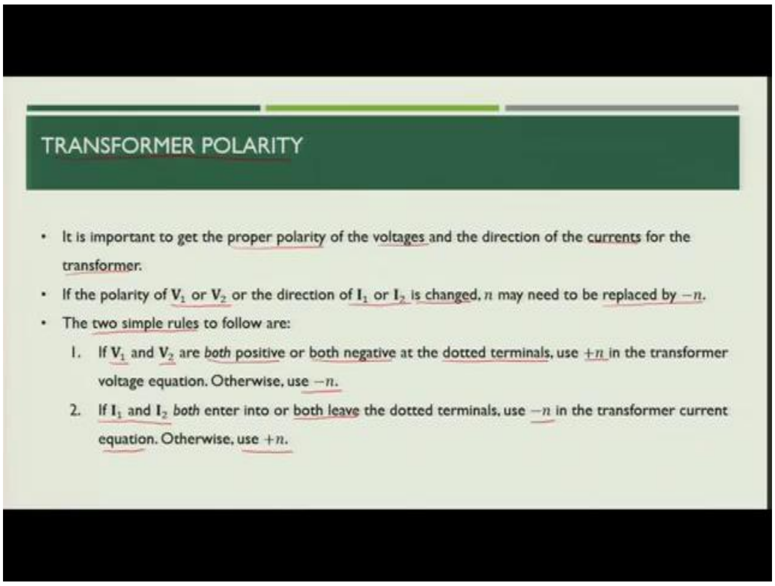

So, let us see, the transformer polarity which we discuss in the last class, we discuss that it is important to get the polarity of the voltage and the direction of the current for the transformer.

Now, the polarity of voltages that is on primary and secondary side of the transformer and the direction of current $\mathbf{I}_1$, $\mathbf{I}_2$ that is again primary current and the secondary current.

So, if the polarity of the direction of the current changes, the transformation ratio, that is n may need to be replaced by $-n$. How we will decide whether it will be n or $-n$, we will follow these 2 simple rules.

1. If $\mathbf{V}_1$ and $\mathbf{V}_2$ aare *both* positive or both negative at the dotted terminals, use $+n$ in the transformer voltage equation. Otherwise, use $-n$.

2. If $\mathbf{I}_1$ and $\mathbf{I}_2$ *both* enter into or both leave the dotted terminals, use $-n$ in the transformer current equation. Otherwise, use $+n$.

So, using these 2 rules, we can figure out that there are 4 possible combinations.

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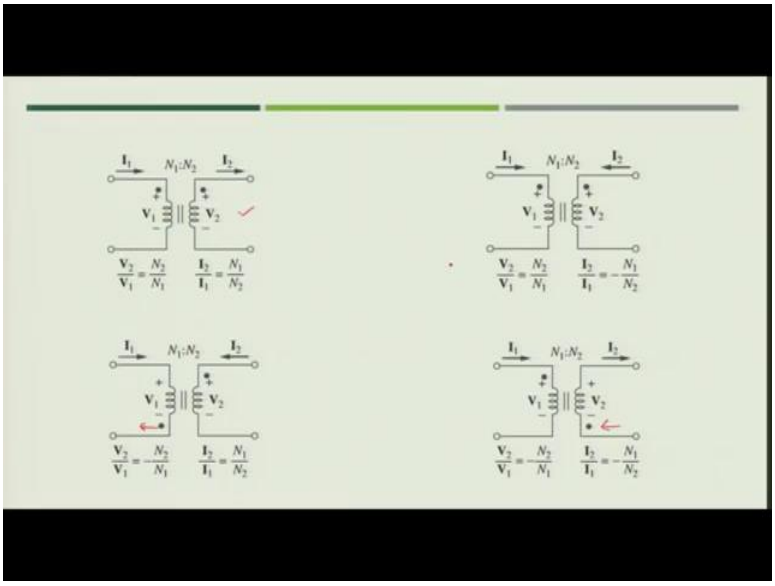

So, we discussed in the last class that these will be the 4 combinations. When the voltage polarity is same $\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$. Here, $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$ because the 1 is going inside the dotted terminal other is coming out from the dotted terminal. In the second case the transformer voltage polarity is same that is why $\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1}$. The direction of current is change that is why $\frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$. In the third figure here you see the polarity of voltages change that is why $\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1}$. Current will be $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$, why because 1 is going inside the dotted terminal, other is coming outside of the dotted terminal as shown in this figure. Now, in the 4th case, you will see the polarity for voltages change. So, that is why it will become $\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1}$. Current is also change because 1 is going inside the dotted terminal at this point also it is going inside the dotted terminal. So, $\frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$. So, this is what we discussed in the last class.

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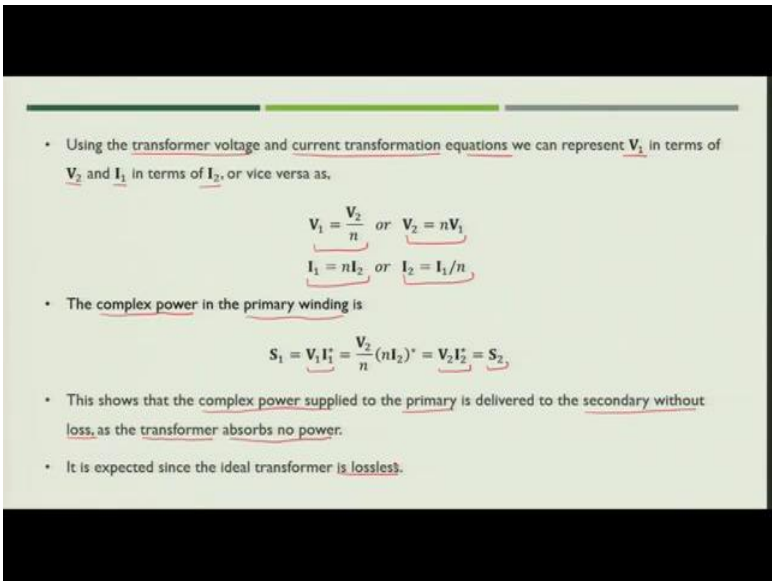

Now, let us see other aspects of the transformer using transformer voltage and current transformation equations, we can now represent voltage that is primary site voltage in terms of secondary site voltage and primary current in terms of secondary current or vice versa. So, what we can say, we can say

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{n} \quad or \quad \mathbf{V}_2 = n\mathbf{V}_1$$

$$\mathbf{I}_1 = n\mathbf{I}_2 \quad or \quad \mathbf{I}_2 = \mathbf{I}_1/n$$

Now, let us see complex power in the primary winding of the transformer. So, what will be the complex power, complex power will be,

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \frac{\mathbf{V}_2}{n}(n\mathbf{I}_2)^* = \mathbf{V}_2\mathbf{I}_2^* = \mathbf{S}_2$$

So, you can say that complex power supplied by the primary is delivered to the secondary winding without any loss because we have considered transformer as a lossless equipment and ideal. So, transformer will absorb no power. Now, this is any way expected because in case of ideal transformer, you will take transformer as a lossless equipment.

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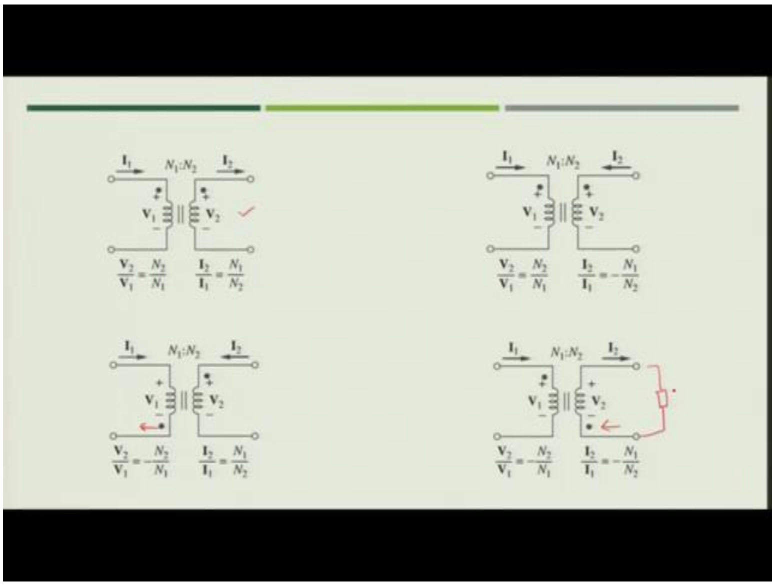

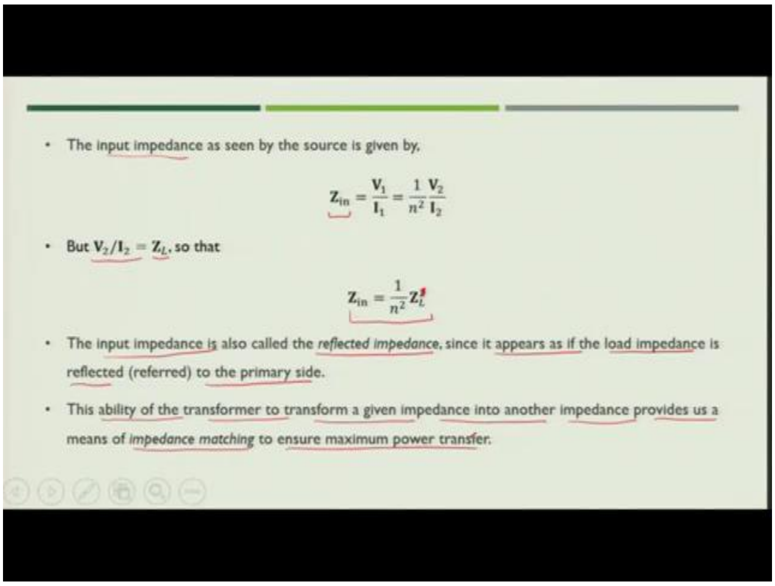

Now the input impedance, if you see that input impedance, if you see from the primary side that is,

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{n^2}\frac{\mathbf{V}_2}{\mathbf{I}_2}$$

Now, at the secondary side if you connect the load. So, if you connect load at the secondary side say $\mathbf{Z}_L$. So, what will happen in that case, your $\mathbf{V}_2/\mathbf{I}_2 = \mathbf{Z}_L$ and in that case you can say

$$\mathbf{Z}_{\text{in}} = \frac{1}{n^2}\mathbf{Z}_L^2$$

So, basically the input impedance is also called the reflected impedance, because it appears as if load impedance is reflected or referred to the primary side. Sometimes you will see in the literature it is written as the load impedance reflected to primary side and somewhere you will see that it is written as load impedance referred to the primary side. So, both are correct. In that case what you will do you will take the load impedance converted as if it is connected to the primary side. This ability of the transformer to transform a given impedance into another impedance it will provide us means of impedance matching which will ensure the maximum power transfer.

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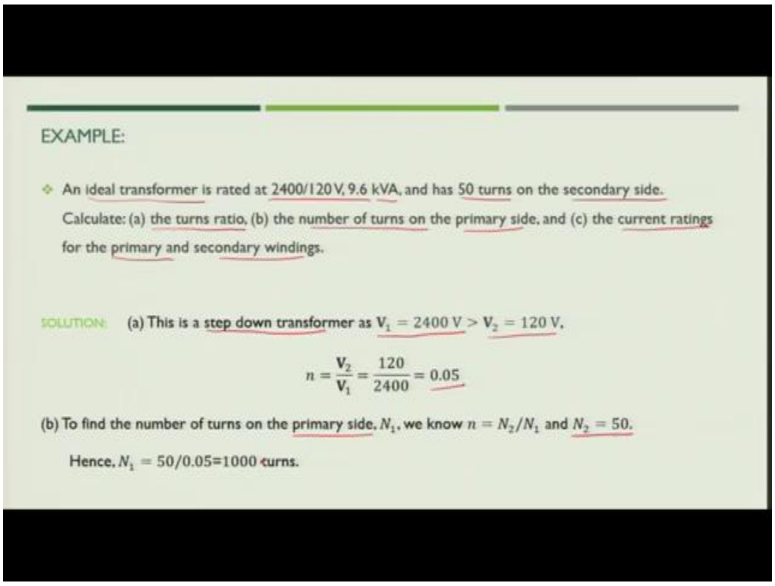

Now, let us take a couple of examples related to the ideal transformer to understand the concepts which we have discussed till now. Now, if an ideal transformer is rated as 2400/120 V, 9.6 kVA, has 50 turns on the secondary side, we need to calculate the turns ratio and the number of turns on the primary side and current ratings for primary and secondary windings. Now, since $\mathbf{V}_1 = 2400$ V $> \mathbf{V}_2 = 120$ V that means that we are talking about the step down transformer. So, you get turns ratio as

$$n = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{120}{2400} = 0.05$$

Now, based on this you can find out the value of number of turns on the primary side because $n = N_2/N_1$ and $N_2 = 50$.

Hence, $N_1 = 50/0.05 = 1000$ turns.

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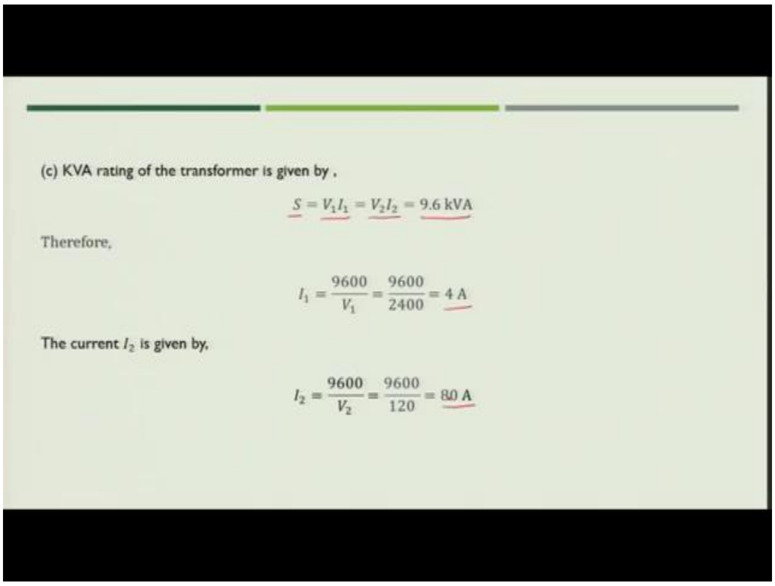

Now, we know the kVA ratings. So, as that is the rating of the transformer,

$$S = V_1 I_1 = V_2 I_2 = 9.6 \text{ kVA}$$

Therefore,

$$I_1 = \frac{9600}{V_1} = \frac{9600}{2400} = 4 \text{ A}$$

The current I_2 is given by,

$$I_2 = \frac{9600}{V_2} = \frac{9600}{120} = 80 \text{ A}$$

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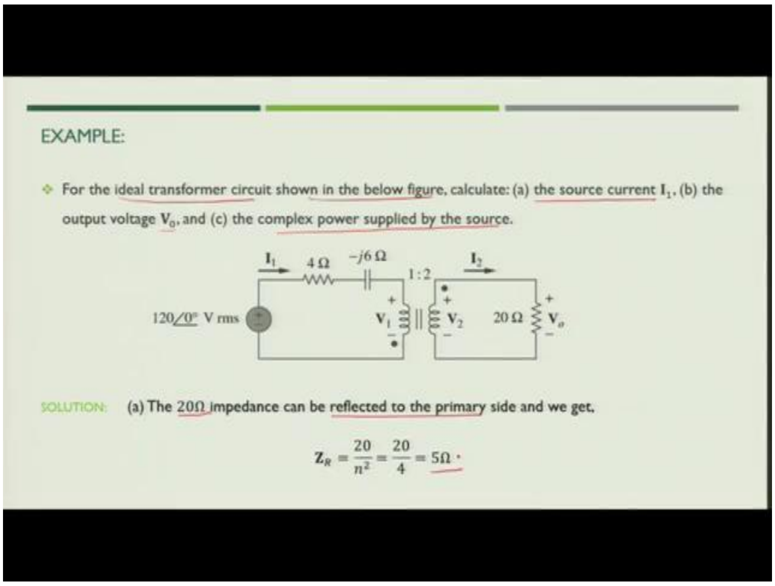

Now, suppose if the ideal transformer contains the circuit as shown in the figure below, we need to calculate the source current $\mathbf{I}_1$ and the output voltage $\mathbf{V}_0$ and complex power supplied by the source. So, in this case, you will see that the 20 ohm is the load impedance. So, to simplify what we first have to convert that load impedance in such a way that it is reflected to the primary side.

So, how we will do it? Suppose $\mathbf{Z}_R$ is the reflected impedance you can write,

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\Omega$$

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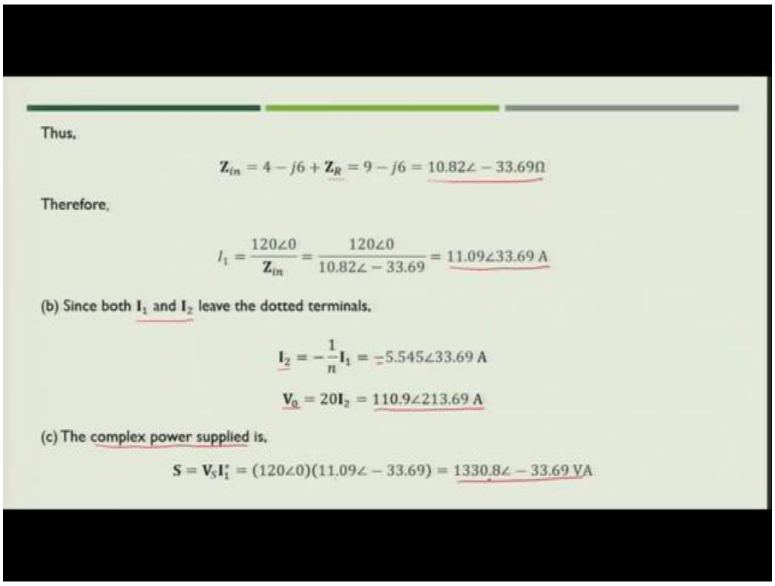

Now, when you reflect this to the primary side, the total impedance of the circuit will become,

$$\mathbf{Z}_{in} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\angle -33.69\Omega$$

Therefore,

$$I_1 = \frac{120\angle 0}{\mathbf{Z}_{in}} = \frac{120\angle 0}{10.82\angle -33.69} = 11.09\angle 33.69 \text{ A}$$

Since both $\mathbf{I}_1$ and $\mathbf{I}_2$ leave the dotted terminals,

$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\angle 33.69 \text{ A}$$

$$\mathbf{V}_0 = 20\mathbf{I}_2 = 110.9\angle 213.69 \text{ A}$$

So, this is how you have got the angle value different from the what we have seen in case of $\mathbf{I}_2$ when minus n was present. Now, next is how much complex power is being supplied. So, source voltage we know current $\mathbf{I}_1$ we have calculated. So, for finding out the complex power will take the conjugate of the $\mathbf{I}_1$ as,

$$\mathbf{S} = \mathbf{V}_S\mathbf{I}_1^* = (120\angle 0)(11.09\angle -33.69) = 1330.8\angle -33.69 \text{ VA}$$

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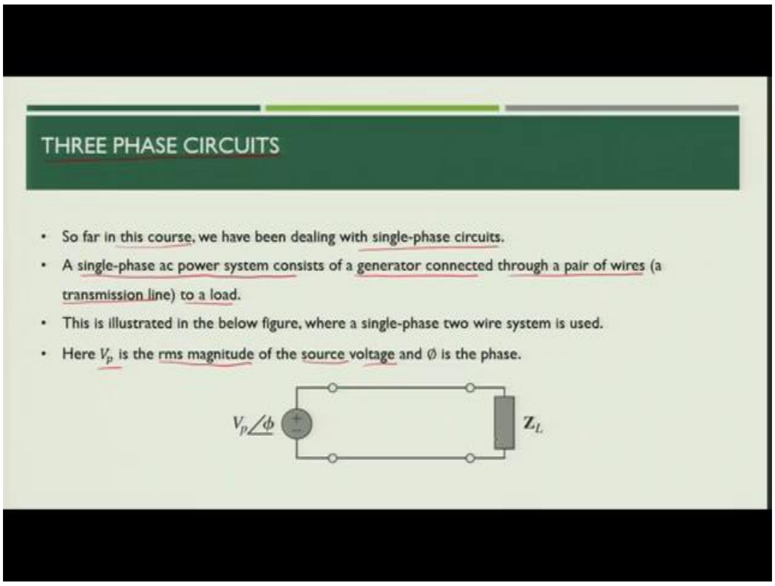

Now, let us start another topic, which is 3 phase circuit. So, till now what we have discussed in our course was basically related to single phase circuits. So, in case of single phase the power system we consist of 1 generator connected through a pair of wires or we call it as transmission line and this line is connected to load. So, that means, if you see this particular figure, this is a single phase ac power system where you have voltage source connected to the load with the help of the pair of wires. So, we call them as a transmission line. Now, here $\boldsymbol{V_p}$ is nothing but the RMS magnitude of the source voltage and $\emptyset$ is the phase angle and Z_L is the load impedance.

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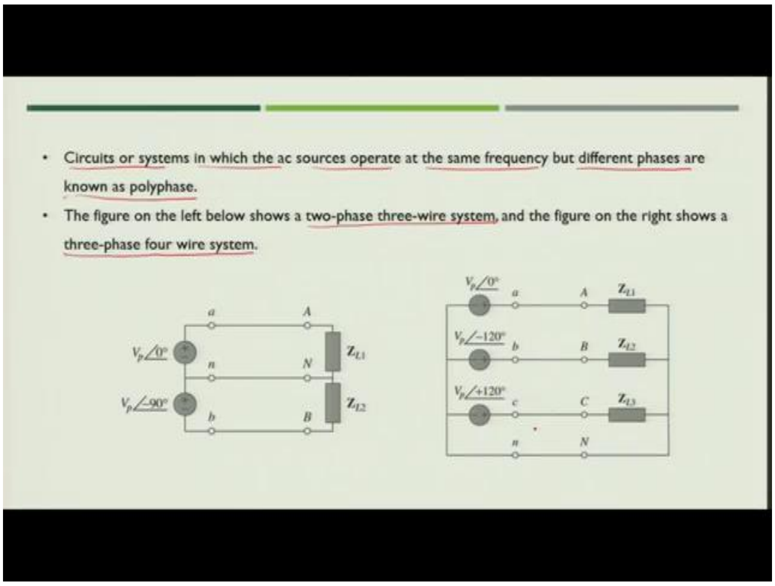

Now, there are circuits or systems in which ac source operate at the same frequency, but there may be different phases So, we call them as poly phase circuit. So, if you see the figure below, you will see on the left there are 2 voltage sources connected to the load with the help of 3 transmission lines and load Z_{L1} and Z_{L2} are connected. So, if you see these 2 sources, 1 is $V_p\angle 0$ another is $V_p\angle -90$.

So, both are operating at same frequency and also the RMS value of the voltage magnitude is same, but angle is different. So, this is basically our 2 phase 3 wire system where we see the phases or 2 in the quantity. So, 2 different phases are connected to these circuit and in this figure which is on the right, you will see there are 4 wires. So, we say them ABC and n, so, the these we can say as A phase, B phase and C phase and then the neutral wire.

Now, these 4 wires are connecting the 3 voltage sources to the 3 loads. So, loads are Z_{L1}, Z_{L2}, and Z_{L3} voltage sources are having same frequency and same RMS voltage, but the phase values are different. So, in this case you will see phase value is 0 degree in phase B, you will have phase value minus 120 degree and in C you will have phase value as plus 120 degree, and you can say phase value is minus 240 degree. So, basically here you will see that the 3 sources are having 3 different phase value. So, this particular set of circuit is called 3 phase 4 wire system because we have 3 phase output, and 4 transmission lines are required to connect to the load. In these cases you will see that the courses are more than 1 and number of wires are also more than 2 to connect to the load.

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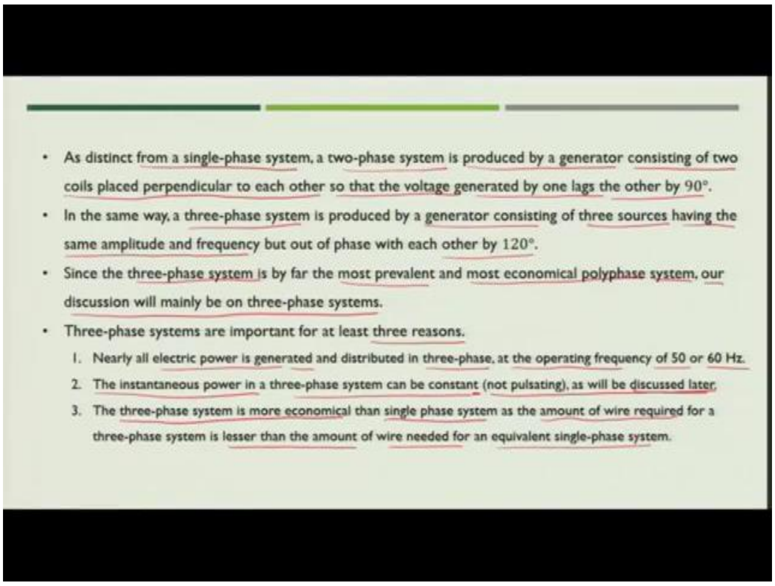

Now, as distinct from the single phase system, the 2 phase system which we saw in the left side of the previous slide is produced by generator consisting of 2 coils placed perpendicular to each other. The coils will be arranged in the generator will be 90 degrees out of phase. Those will be perpendicular to each other that the voltage generated by 1 coil lags the other by 90 degree. The same way in case of 3 phase system the generator will have 3 coils. So, you can say that the there are 3 different sources having the same amplitude and frequency, but they will be out of phase by 120 degree.

Now, since 3 phase system is most prevalent in and most economical polyphase system, so, that is why you will see in the normal power system network, most of the time the power is being supplied through 3 phase system. In our discussion, we will concentrate more towards the 3-phase system. Now, 3 phase system is very important because, firstly, the electric power which we get in our houses or in our industrial establishments are mainly the 3-phase power. In houses you will have single phase output of this 3-phase power. But in industrial establishment, you will directly see that 3-phase power supply is given to run the big loads and all these operating either at 50 degree or 50 hertz or 60 hertz. Now, the instantaneous power in a 3-phase system can be constant that means that when you power the voltage or the power is being generated from the generator the value of power p that is average power will always be constant. This particular phenomenon will discuss in later our course that how we can say that the power which is generated from the 3 phase system is constant. Now, 3 phase system is more economical than the single-phase system because the amount of wire which is required for 3 phase system is lesser than the amount of wire needed for an equivalent 3 single phase systems.

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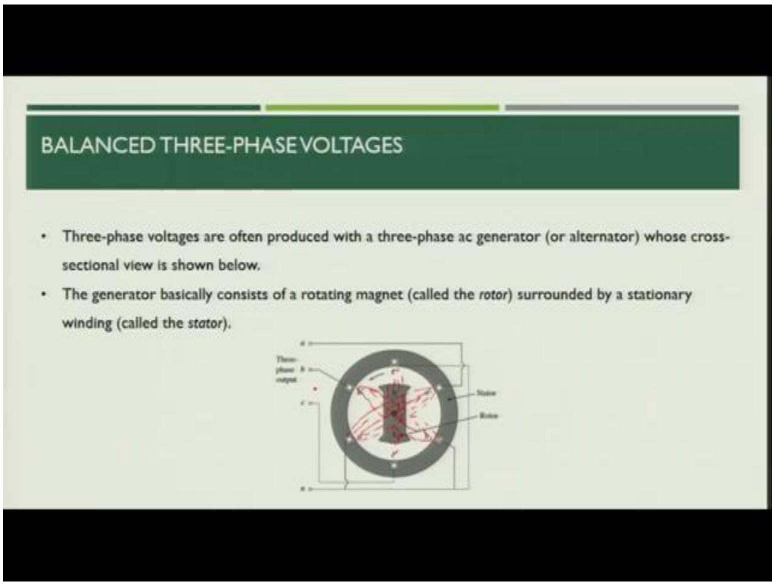

Let us see how we generate balance 3 phase voltages. If you see the cross section of the generator, how you generate the 3-phase power. There are 3 set of windings are arranged in the stator side of the generator. Basically, you will have 3 sets of coils which are 120 degree apart and when the magnet which is rotating in the generator will cut the coils you will get the voltage induced in these coils. Generally, these rotating magnets are either permanent magnet or you arrange the coils in such a way that you get the rotating magnet in the generator. Since these all 3 windings are 120 degree apart you will get 3 phase power as output.

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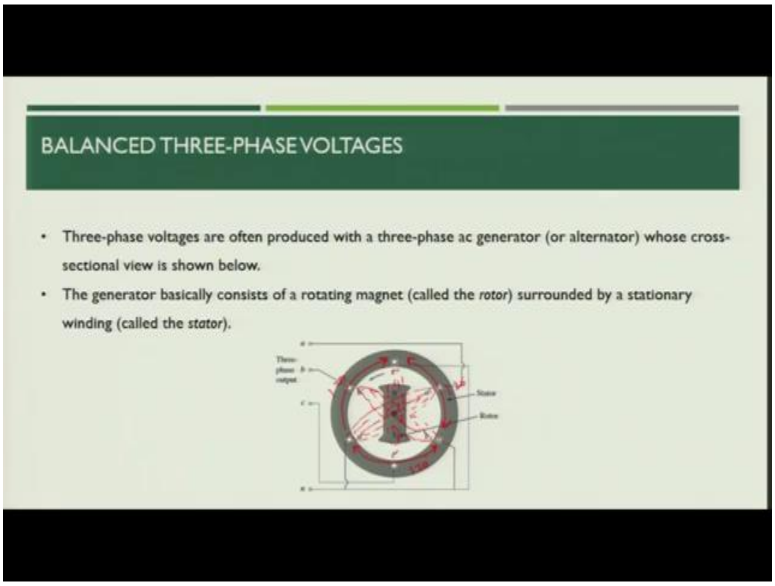

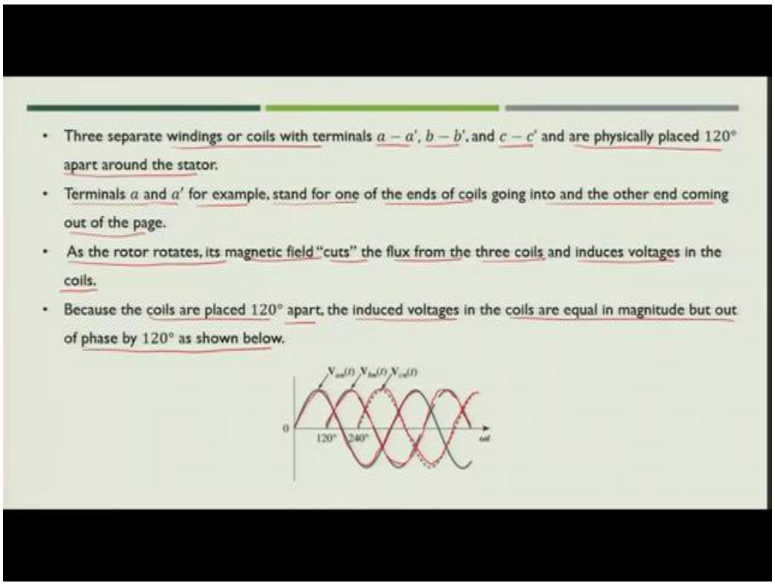

Now, these separate winding or coils which you see as $a - a'$, $b - b'$, and $c - c'$ in the figure. Basically, we arranged the coils in such a way that it completes 1 set of arrangement for 1 particular phase. They are physically 120 degree apart around the stator. So, if you see A and B, so, the angle difference will be physically 120 degree. Similarly, between B and C and between A and C will be also 120 degree.

So, your windings are now physically apart by 120-degree. Let us take first $a - a'$, which stands for 1 end of the coil. So, what we can see, we, we have wound as I shown that we have found in such a way that the coil goes inside and then comes out and goes inside and so, so, you can take multiple number of turns based on the voltage output which you want from the generator.

Now, when the rotor rotates, as we saw that when you rotate the rotor the magnetic field will cut the flux from these coils and the voltage will be inducing these coils. Now, since the coils are placed 120 degree apart, the induce voltage in the coils are also equal in magnitude but will be out of phase by 120 degree. So, when the moment you see the, the magnet is cutting phase a coil, so, that is $a - a'$ coil we will see that the voltage is being building up in the coil A.

So, when, when you see in this figure if you start from A then if you move to B that means that after 120 degree the voltage will start building up in the coil. So, you will see now the voltage is started building up in B coil. Similarly, after another 120 degree you will see voltage is now being building up in the C coil. So, now, all these 3 phases are 120 degree apart and the output which you will see in the individual coil is almost sinusoidal.

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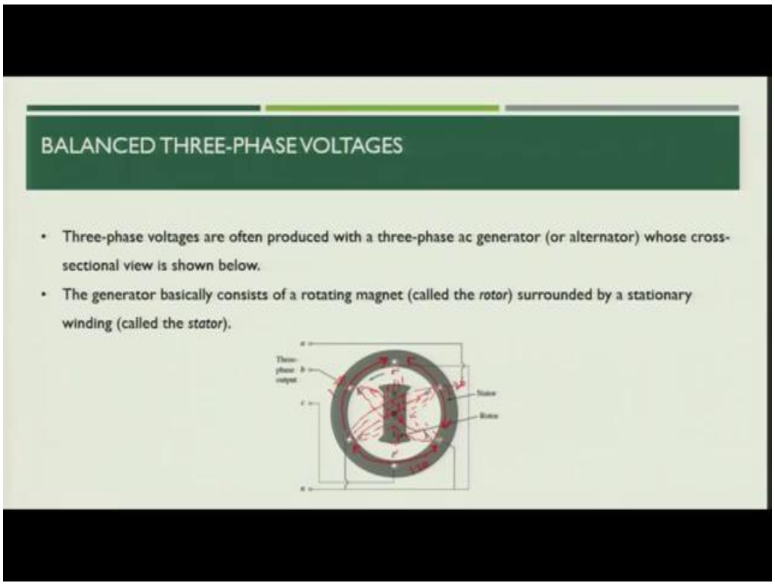

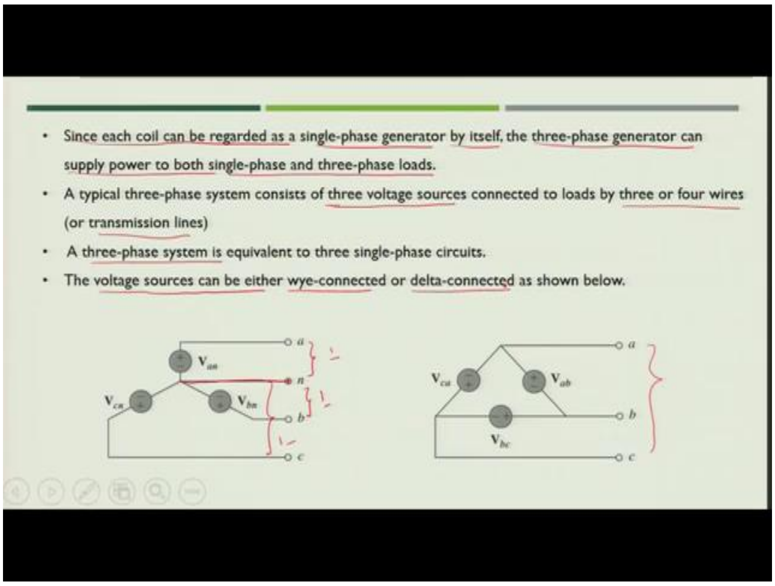

Now since both coils can be arranged as a single-phase generator by itself, the 3 phase generator can supply power to both single phase and 3 phase loads. So, if you see this figure, so, these 3 coils are independent coils, so, you can consider them as 3 independent sources. So, what you can do you can arrange them in such a way that it looks like there are 3 different voltage sources. So, if you see this figure the generator is wound in such a way that it is star connected. In that case you will see that phase A phase, B phase and C can be considered as 3 independent voltage sources and if you have neutral coming out of the generator, so, you can have 3 single phase created out of 3 phase arrangement. So, neutral and A will create 1 phase B and N will again create another phase and C and N will create, create another phase. So, now, you will have 3 phase system which will give you also 3 independent single-phase systems.

This is typically visible in case of you have 3 phase 3 phase and 4 wire systems. Now, if you have 3 phase 3 wire system, you will arrange the sources in such a way that it is looking like a delta connected arrangement.

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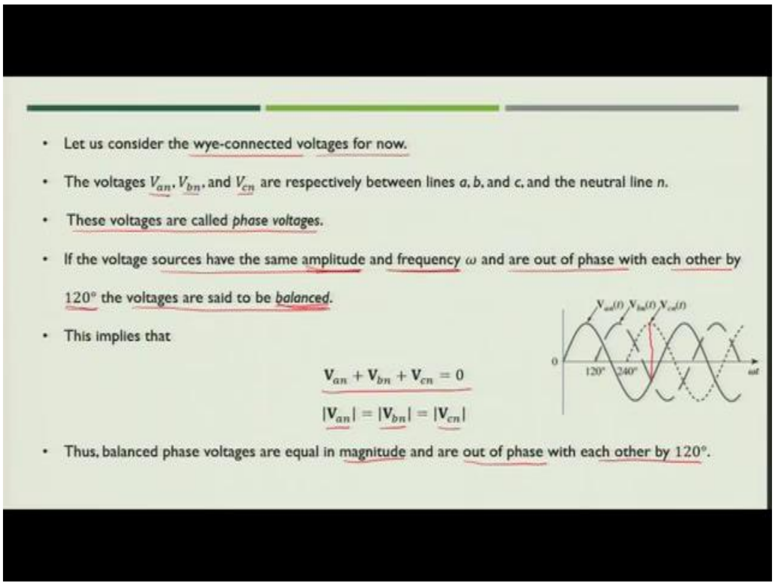

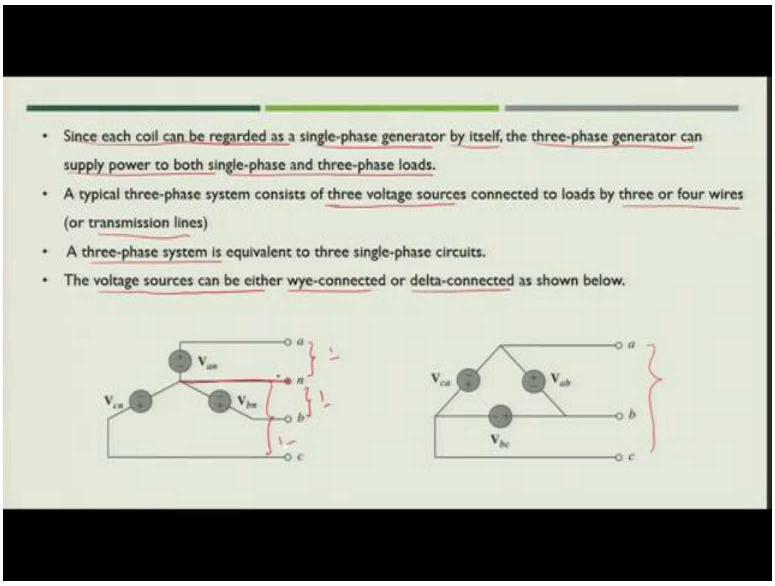

Now, let us consider the star connected that is wye connected voltage for now. So, here we will have phase voltages as V_{an}, V_{bn}, and V_{cn}. So, these are respectively the voltages between line abc and the neutral. So, we have seen that $\mathbf{V}_{an}$ is nothing but what is between a and n and lines $\mathbf{V}_{bn}$ is between b and n and $\mathbf{V}_{cn}$ is between c and n. So, now, these voltages are called phase voltages because you have taken them with respect to neutral.

So, if the voltage source have the same amplitude and frequency and are out of phase with each other by 120 degree, the voltage are said to be balanced. So, the criteria for balanced voltage output is that the voltage magnitudes should be same frequency should be same and all should be 120 degree apart from each other. So, if you see this figure, all our voltages are 120 degree from each other. So, now, when you see this arrangement, you can simply say that,

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

So, if you take voltage at any point say if you see at this point, you will see that the total sum of voltage will be 0. So, you can see at any point you will get the value of

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

and this is again the criteria for a balanced system. This is in phasor form, but in magnitude form, since, we have same amplitude will get

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

We can say now, the balanced phase voltages are equal in magnitude, but are out of phase with each other by 120 degree.

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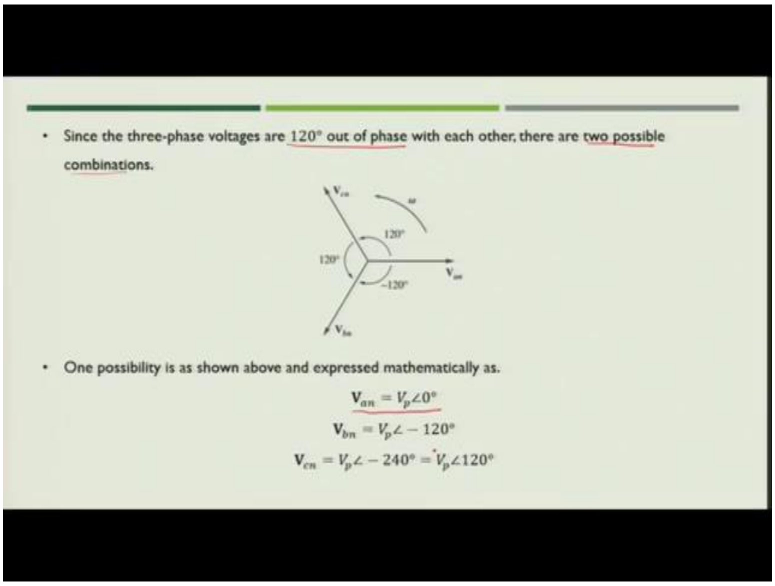

So, now, since the voltages are 120 degree out of phase, there are 2 possible combinations for the arrangement of these phases. So in 1 such arrangement is shown in this slide, where $\mathbf{V}_{an}$ is leading $\mathbf{V}_{bn}$ by 120 degree and $\mathbf{V}_{bn}$ is again leading $\mathbf{V}_{cn}$ and by 120 degree. So, what you can write,

$$\mathbf{V}_{an} = V_p\angle 0°$$

$$\mathbf{V}_{bn} = V_p\angle -120°$$

$$\mathbf{V}_{cn} = V_p\angle -240° = V_p\angle 120°$$

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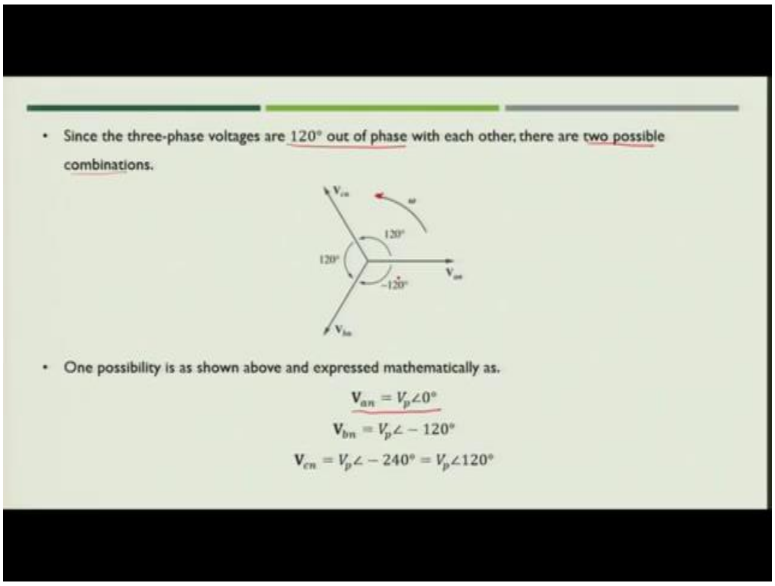

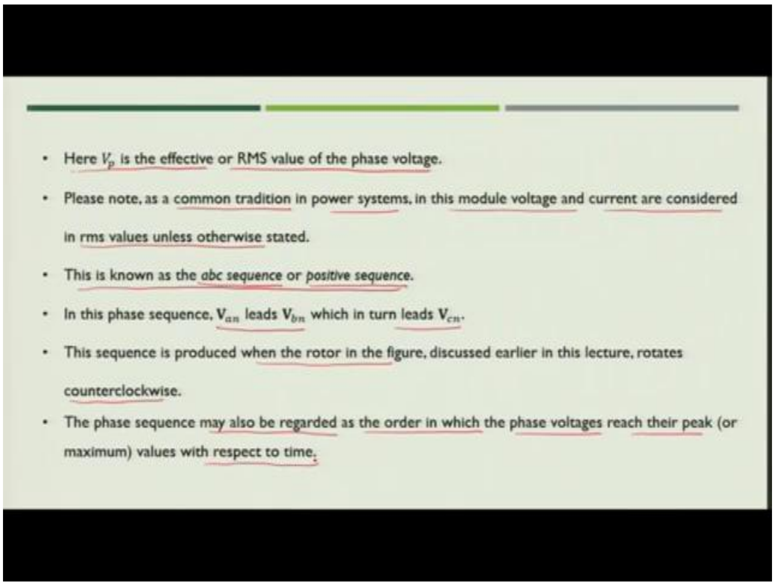

So, here V_p is the effective or RMS value of the phase voltage. Now, here the common tradition in the power system is, is the use of RMS values. So, in this module also the voltage and current we will consider in RMS values or otherwise stated in the discussion. So, what we get here we get abc sequence or the positive sequence of the phase arrangement. So, here if you see $\mathbf{V}_a$ $\mathbf{V}_b$ $\mathbf{V}_c$ are arranged in, in the direction of the rotation of the air gap flux. So, in that way if you see basically, the rotor and stator will create 1 flux which will cause the rotation of these phases. So, $\mathbf{V}_a$ $\mathbf{V}_b$ $\mathbf{V}_c$ you will see will be arranged in such a way that it is counterclockwise. So, in that way you can say that the $\mathbf{V}_a$ $\mathbf{V}_b$ $\mathbf{V}_c$ arrangement is called as abc sequence of the phase arrangement or you can call it as a positive sequence arrangement of the phases.

So, here we have seen that $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$ which in turn leads $\mathbf{V}_{cn}$ in this sequence is produced when rotor we have just seen that it is rotating in the counterclockwise as we have seen from the figure. Now, this phase sequence may also be regarded as the order in which phase voltage reach their peak value with respect to time. So, if the arrangement is like abc. So, will see first A will reach its peak value, then B will reach its peak value and then C will reach its peak value in the in this phase arrangement.

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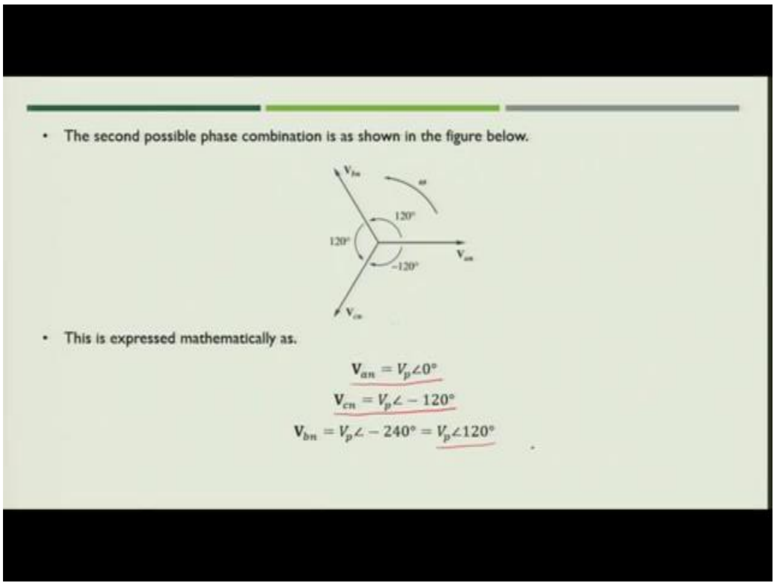

Now, there is another phase combination where $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ which in turn leads $\mathbf{V}_{bn}$. So, in that case what you can write, you can write,

$$\mathbf{V}_{an} = V_p\angle 0°$$

$$\mathbf{V}_{cn} = V_p\angle -120°$$

$$\mathbf{V}_{bn} = V_p\angle -240° = V_p\angle 120°$$

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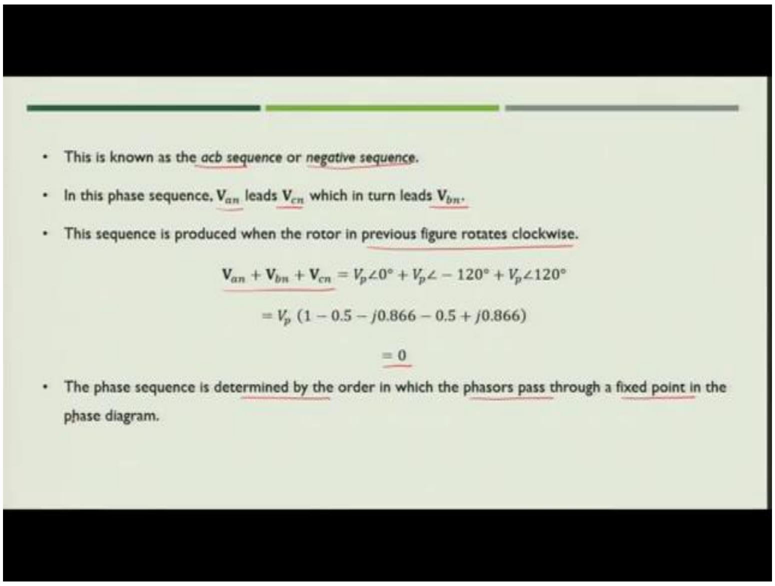

So, here now the sequences acb sequence all you generally call it as a negative sequence. So, in this case $\mathbf{V}_{an}$ leaves $\mathbf{V}_{cn}$ and $\mathbf{V}_{cn}$ leads $\mathbf{V}_{bn}$ by 120 degree. Now, so, sequence is produced when rotor is rotate in the rotor is rotating in the clockwise direction. So, here now, if you calculate,

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = V_p\angle 0° + V_p\angle - 120° + V_p\angle 120°$$

$$= V_p(1 - 0.5 - j0.866 - 0.5 + j0.866)$$

$$= 0$$

So, the phase sequence is determined by the order in which phasor pass through the fixed point in the phase diagram. abc is considered as a positive sequence and acb is considered as negative phase sequence. So, this closesour today's discussion at this point will continue our discussion related to this 3 phase network in our next session also. Thank you.