Basic Electric Circuits
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Module 9 Sinusoidal Steady State Analysis 1
Lecture 44
Magnetically Coupled Circuits

Namashkaar.

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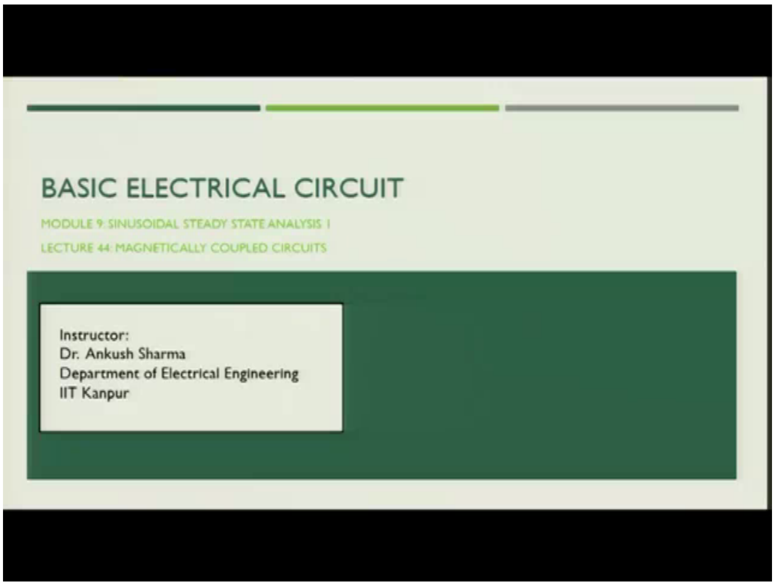

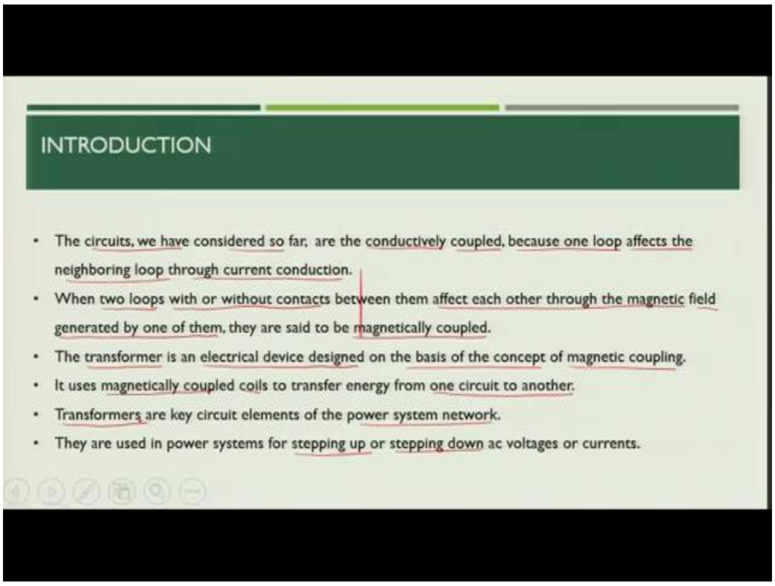

In today's session we will discuss about the magnetically coupled circuit. Let us start our discussion about the topic. Basically the circuits which we have discussed till now were conductively coupled that means that because of one loop the neighbourhood loop was getting affected through current conduction that means that both of the lops were joint together and current was flowing from one loop to other. Now in this case we will see when the two loops which may be or may not be connected electrically but they are coupled together through the magnetic field generated by one of them.

In those cases when we see those kind of circuit combinations, we call them as magnetically coupled circuit. The most common example for this is the transformer where the electrical device is design based on the concept of magnetic coupling now it uses magnetically coupled coils to transfer the energy from one circuit to the other. Transformers are the key circuit elements of power system network why because whenever we have to change the voltage level that means either you are stepping up or stepping down in the power system network you need transformer for those cases.

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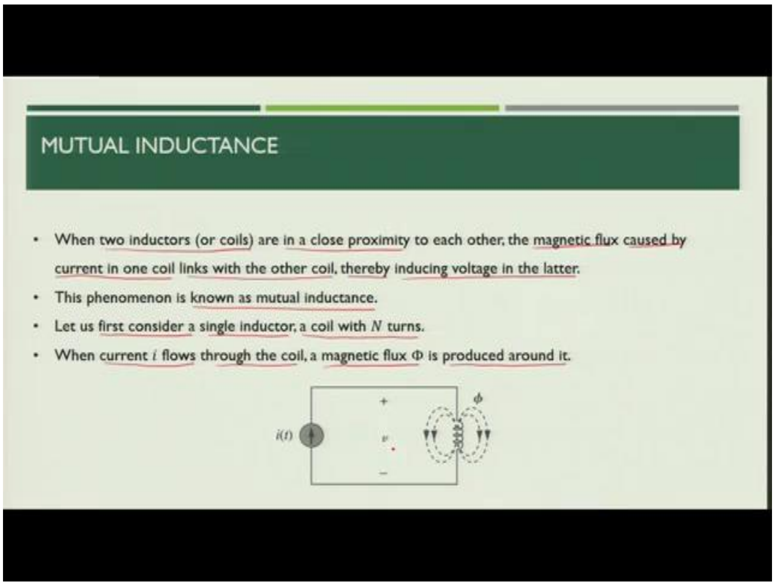

Now let us see first what is the mutual inductance? Now when two conductors or you can say when two coils are in a close proximity to each other the magnetics plugs caused by the current in one coil links with the other coil and thereby it induces the voltage in the second coil and this particular phenomenon is known as mutual inductance. Now let us first consider single inductor where the coil has N number of turns when the current flows the current, i, is flowing through the coil the magnetic flux phi will be produced around it if you see the figure the current, i is flowing in the inductor and flux phi is generated. So, because of that flux generated you will have the voltage v induced.

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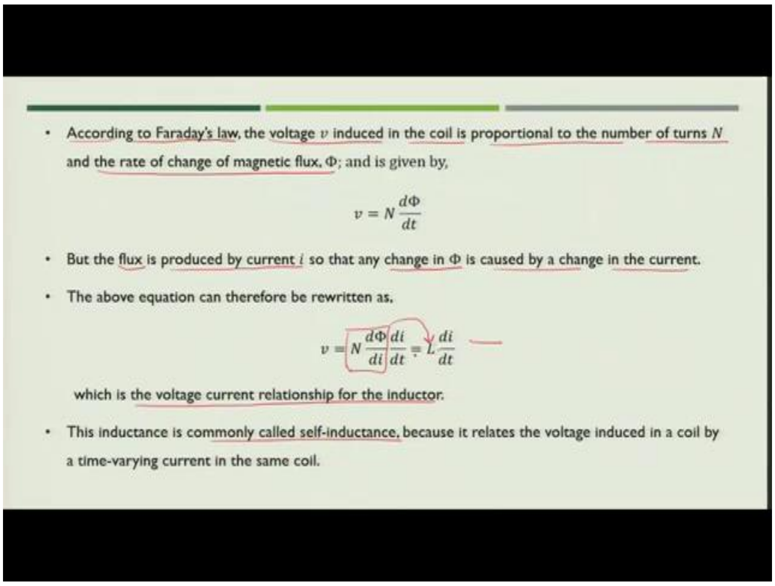

Now how the voltage will be induced because if you remember the Faraday's law it says that the voltage induced in the coil is proportional to the number of turns and the rate of change of magnetic flux. So how we can write the induced voltage $v = N\frac{d\Phi}{dt}$ Now the flux is produced by current i, so therefore we can also say that any change in Φ is cause by the change in current so this particular equation we can rearrange and write like,

$$v = N\frac{d\Phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

This equation now tells the voltage current relationship for the inductors. Now this inductance is commonly called as self-inductance because it relates the voltage induced in a coil by time varying current in the same coil.

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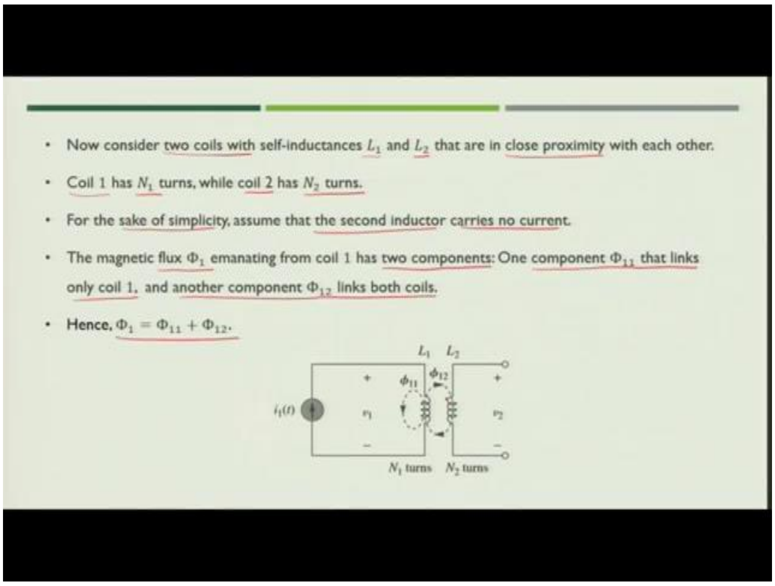

Now let us consider 2 coils and we assume that they have the self-inductances L_1 and L_2 and both coils are placed in a proximity with each other. Now if coil 1 has N_1 turns, while coil 2 N_2 has turns for this particular case. For the sake of simplicity let us assume that the second inductor carries no current. So now the magnetic flux Φ_1 which will be coming from the coil 1 will have 2 components one components that Links only the coil 1. If you see the magnetic flux generated in this coil has Φ_{11} which is link to only coil 1 and another component Φ_{12} which links the second coil also. So, what you can say you can say the total flux $\Phi_1 = \Phi_{11} + \Phi_{12}$.

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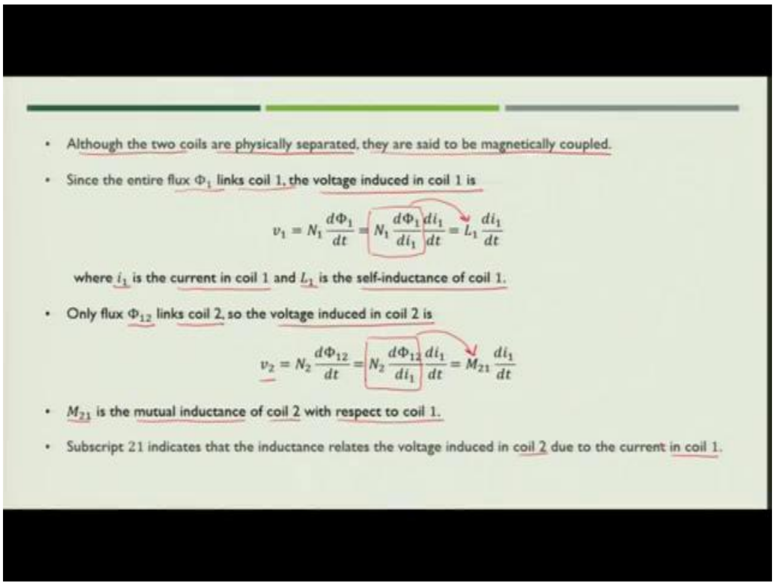

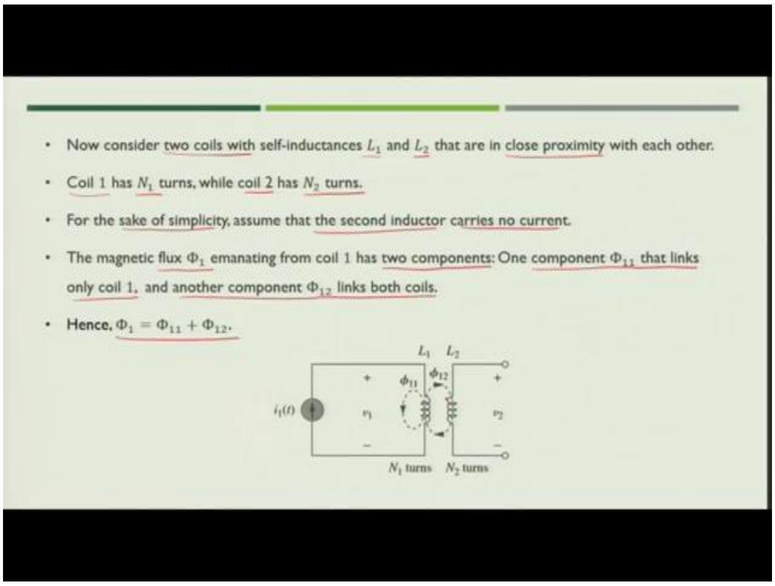

Now as we know that the two coils are physically separated means they are not electrically connected. So we can say that they are simply magnetically coupled. Now since flux Φ_1 links coil 1 because the if you see the figure flux of Φ_1 that is $\Phi_{11} + \Phi_{12}$ into is completely linking this particular coil one. So what we can write the voltage induced in coil1 is,

$$v_1 = N_1 \frac{d\Phi_1}{dt} = N_1 \frac{d\Phi_1}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

where i_1 is the current in coil 1 and L_1 is the self-inductance of coil 1.

Only flux Φ_{12} links coil 2, so the voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\Phi_{12}}{dt} = N_2 \frac{d\Phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

M_{21} will be the mutual inductance of coil 2 with respect to coil 1. Now the subscript that is 21 written in M21 it indicates that the inductance relates to voltage induced in coil 2 due to the current flowing in coil 1.

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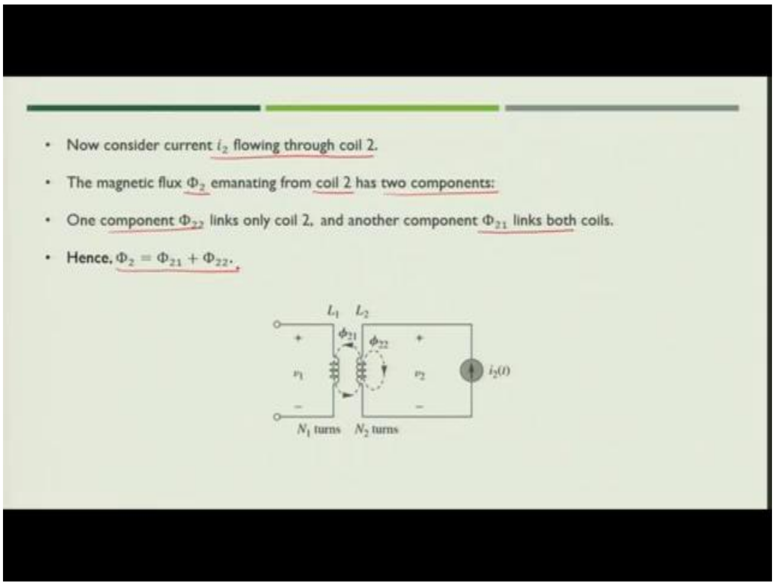

Now let us consider the current i_2 flows through coil 2. So, if you see the figure in the slide now i_2 is the current source connected to the coil 2. The flux Φ_2 which is generated by the current coil i_2 will again have the 2 components 1 component Φ_{22} which will link only the second coil while the Φ_{21} will link both coils. So, you can say $\Phi_2 = \Phi_{21} + \Phi_{22}$.

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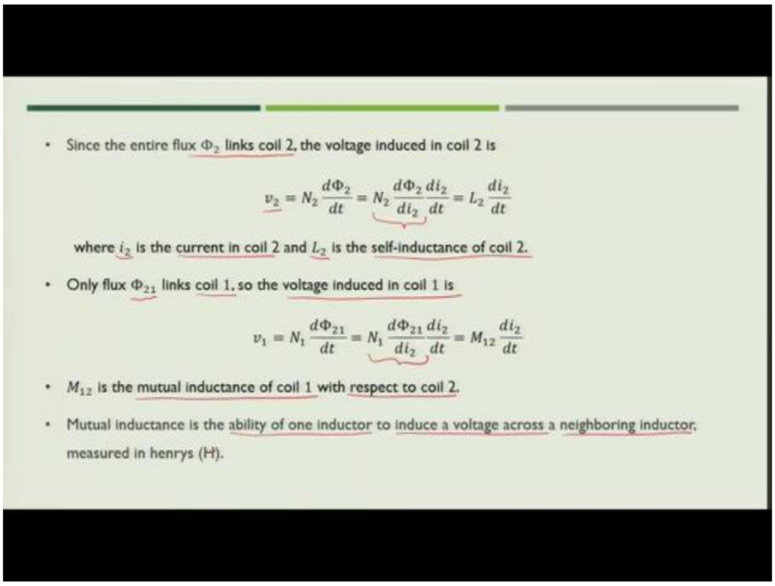

Now again as was in the case of coil 1 in this case also we will see Φ_2 will link completely to the coil 2. We can write,

$$v_2 = N_2 \frac{d\Phi_2}{dt} = N_2 \frac{d\Phi_2}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

where i_2 is the current in coil 2 and L_2 is the self-inductance of coil 2.

Only flux Φ_{21} links coil 1, so the voltage induced in coil 1 is

$$v_1 = N_1 \frac{d\Phi_{21}}{dt} = N_1 \frac{d\Phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

M_{12} is the mutual inductance of coil 1 with respect to coil 2. Now this mutual inductance is the ability of one inductor to induce voltage across a neighbouring inductor. So, this will again be represented it will be measured in Henry's or in short you can write as capital H.

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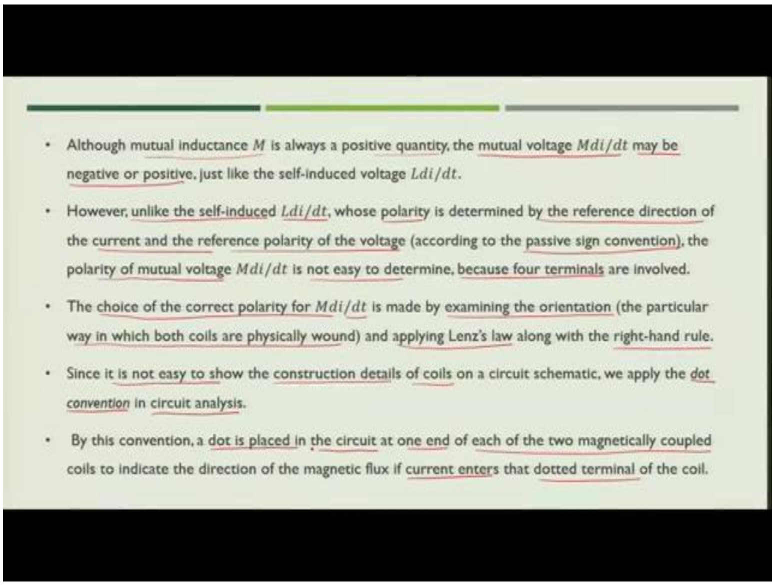

Now although the mutual inductance M is always a positive quantity the voltage that is mutual voltage represented as Mdi/dt may be negative or positive. So unlike the self-induced Ldi/dt because Ldi/dt you can find out the polarity how you will determine the polarity by the reference direction of the current and reference polarity of the voltage. If you remember we discussed the passive sign convention in our first week you can use that discussion to find out the polarity in case of you have self-induced voltage that is Ldi/dt.

Now for the polarity of mutual voltage that is Mdi/dt it is not that easy because it now involves four terminals. So two terminals of coil 1 and two terminal of coil 2. So since we have four terminals we cannot use the passive sign convention directly to find out the polarity of mutual voltage. Now the choice of correct polarity for Mdi/dt is made by examining the orientation of the coil that means the way in which both coils are physically bound and then applying the Lenz's law along with the right hand rule to find the correct polarity for mutual voltage.

Now since it is not easy to show the construction detail of the coil that means how both of the coil are physically bound, what we do? We generally use the dot convention for our circuit analysis. So by this convention a dot is placed in the circuit at one end of each of the 2 magnetically coupled coils to indicate the direction of the magnetic flux if current enters that doted terminal of the coil.

So we show when the current enters the doted terminal of the coil how the mutual voltage would be oriented.

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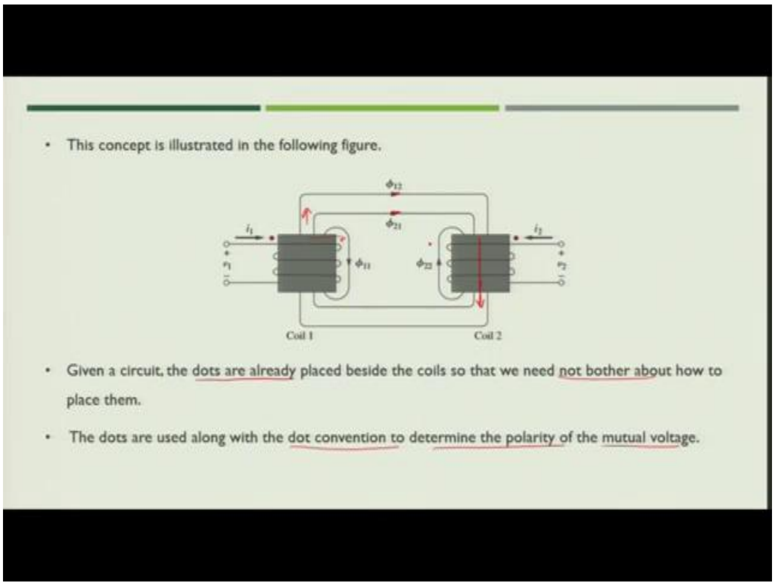

Now let us understand this concept with the help of this figure here if you see the current i_1 is entering the dot. So the orientation if you see the coil are the coil is bound like this if you place a hand palm in such a way that it is completely curling the side of the coil if you place your hand and curl your finger like this the coil is bound. So you will see when you place end like this your thumb will be in this direction.

So this thumb direction will show you how and in what direction you are flux is rotating. So here the current i_1 is going inside you are getting flux Φ_{12} linking in the clockwise direction to the coil 2. Similarly, in the coil 2 current is entering the dot and the positive polarity of the voltages when where the dot is connected here also the positive where the dot is connected.

So, when you place the right hand in the similar way as we did in this case your thumb will be in the downward direction. Again Φ_{21} will again link the first coil in the clock wise direction. So now in this particular case the dots are already placed. So, we will not bother how to place them the dots are used along the dot convention to determine the polarity of the mutual voltage.

We will see how if this is the condition how we can determine the induced mutual voltage polarity.

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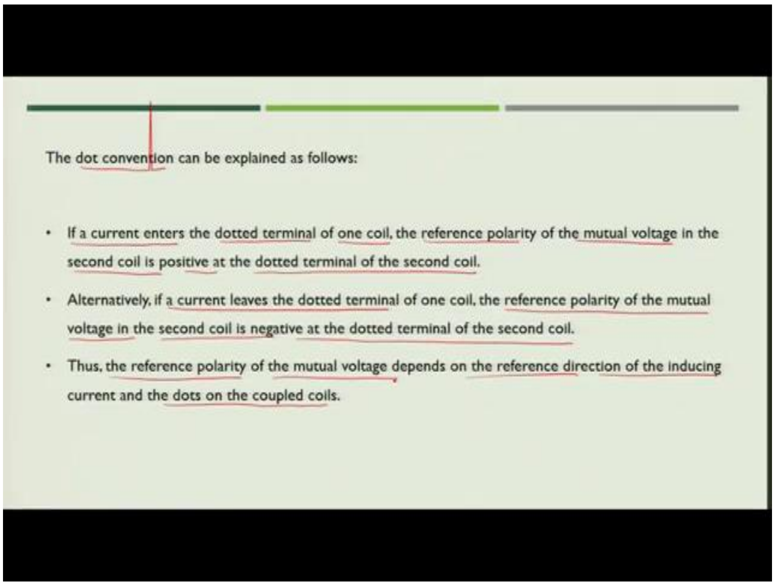

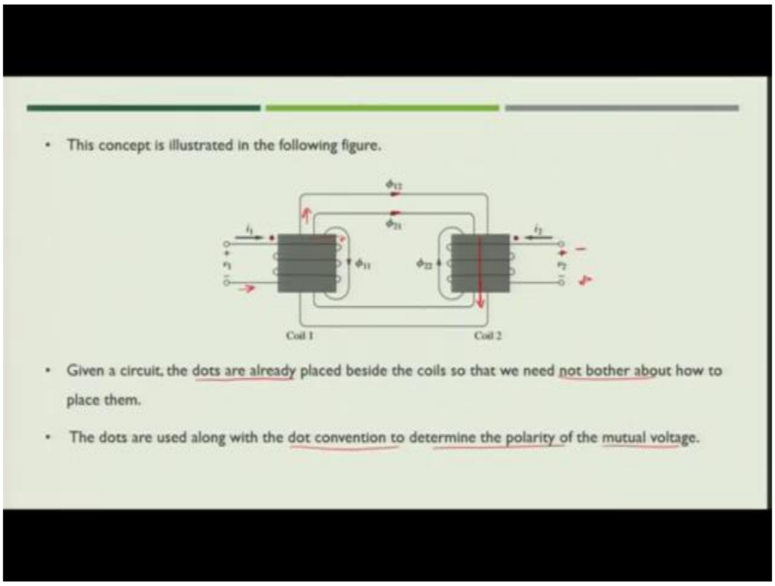

Now let us understand the dot convention first if a current enters the doted terminal of one coil the reference polarity of the mutual voltage in the second coil is positive at the doted terminal of the second coil. So if the current is entering here in mutual inductance will have the positive polarity in the doted side of the terminal.

Now if the current leaves the doted terminal of one coil the reference polarity of the mutual voltage in the second coil is negative at the doted terminal of the coil. So suppose if the direction of the current is opposite and current is flowing from this side in that case the polarity of the voltage that is induced mutual voltage would be like this. So that means that the second coil the mutual voltage in the second coil will be negative at the doted terminal of the second coil.

Thus the reference polarity of the mutual voltage depends upon the reference direction of inducing current and the dots on the couple coil. So this two things are important to find out the reference polarity of the mutual voltage.

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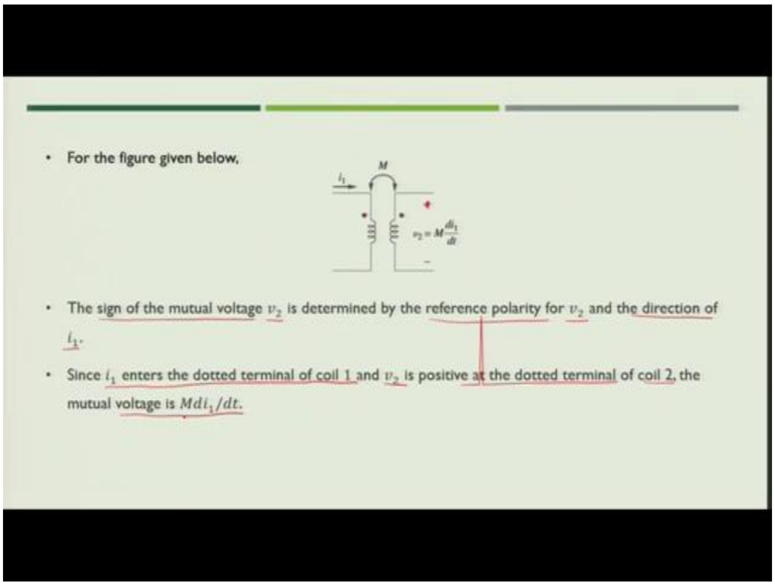

Let us understand this particular phenomenon with the help of couple of examples let us see this particular figure where the sign of mutual voltage v_2 is determine by the reference polarity for v_2 and the direction of i_1. So if you see the current i_1 is flowing inside the dot the reference polarity for v_2 will have positive at the doted side of the terminal on the secondary side.

Because i_1 enters the doted terminal of the coil 1 and v_2 is positive at the doted terminal of coil 2. The mutual voltage will be positive Mdi_1/dt.

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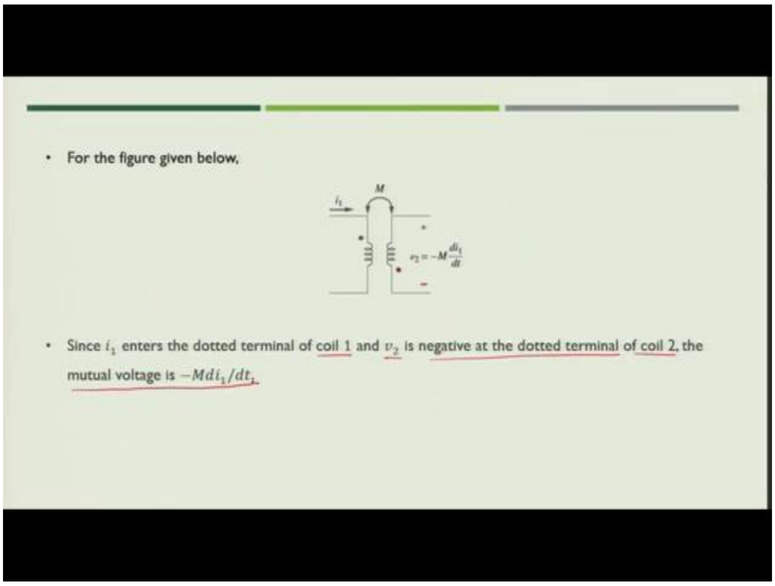

Now in this case now i_1 is entering the doted terminal of coil 1 but the v_2 is negative at the doted terminal of coil 2. Because the polarity of v_2 is opposite the doted terminal is having the negative. So the mutual voltage will be $-Mdi_1/dt$.

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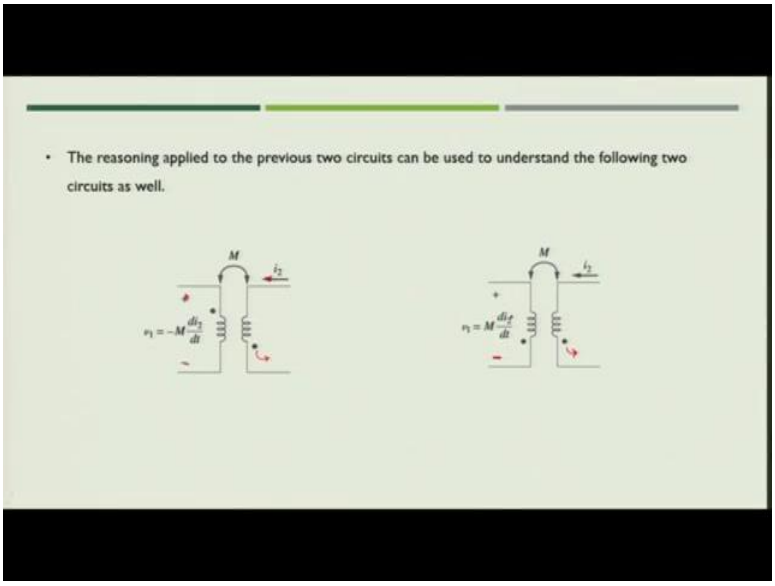

Now similarly in this case when the current is flowing in second coil. So the current is flowing i_2 is flowing in the second coil that means that i_2 is leaving the dot. So that means if d it is leaving the dot the value of mutual induced voltage will be negative at the downward terminal and positive at the doted terminal. Similarly in case of i_2 when i_2 is exiting the dot but the voltage polarity at the second will have minus at the doted terminal in that case your induced mutual voltage will $-Mdi_2/dt$.

Since here you are exiting the dot and the polarity of the voltage at the dot is negative it means that your mutual induced would be Mdi_2/dt which is positive.

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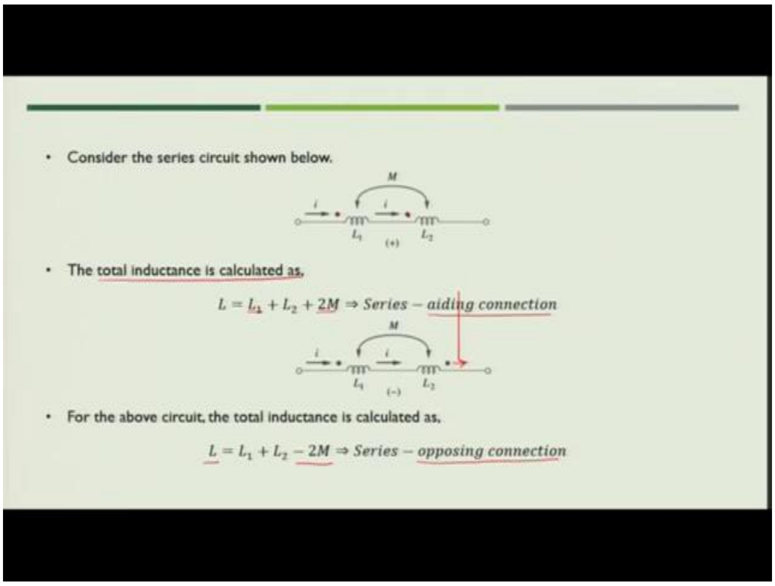

Now let us consider the circuit where the coils are electrically connected also. So if you see that coil L_1 is connected to L_2 physically as well as magnetically. Now in this case if the current is going inside the dot and in this case again the current is going inside the dot the total inductance will be the adding connection. That means

$$L = L_1 + L_2 + 2M \Rightarrow Series - aiding\ connection$$

Similarly, in the second case where current i is entering the dot and here current i is again leaving the dot that means that the mutual inductance will be opposite direction and the total inductance you will get as $L = L_1 + L_2 - 2M$. Because this is called opposing connection current is going inside the dot from one direction but exiting the dot from other side.

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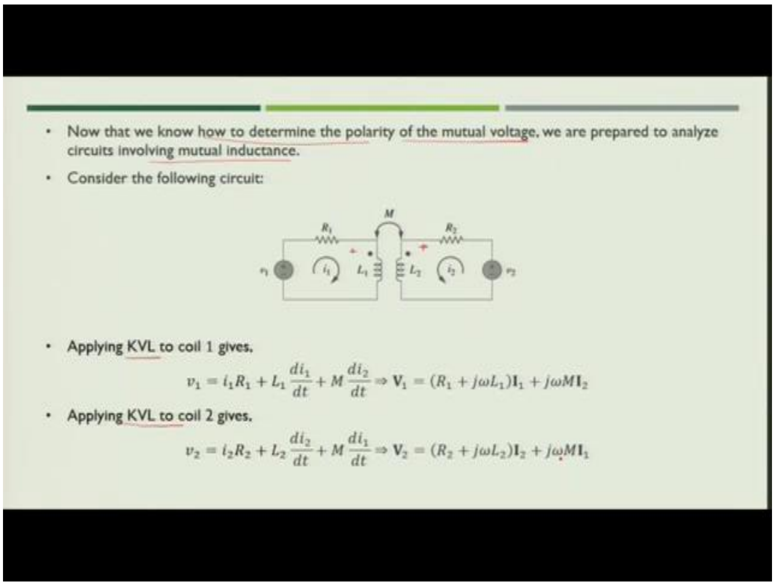

So now we can say that we know how to determine the polarity of the mutual voltage. Now let solve few of the circuits so that we can understand how you can involve the mutual inductance in our circuit analyses. Let us see this figure where we have v_1 and v_2 2 voltages applied across the 2 coils which are placed nearby, so these two are magnetically coupled.

So, we can use Kirchhoffs voltage law to write the equations for both measures. Let us say that the current i_1 in this particular match is in this direction i_2 is in this direction. So for coil 1,

$$v_1 = i_1R_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \Rightarrow \mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M\mathbf{I}_2$$

Now here in both cases your current is going inside the dot for reference polarity is positive in both of the sides where the dot is connected it means that the mutual inductance mutual voltage which you get because of the magnetic coupling of these two coils will be positive.

Now for the second coil what you can write,

$$v_2 = i_2R_2 + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \Rightarrow \mathbf{V}_2 = (R_2 + j\omega L_2)\mathbf{I}_2 + j\omega M\mathbf{I}_1$$

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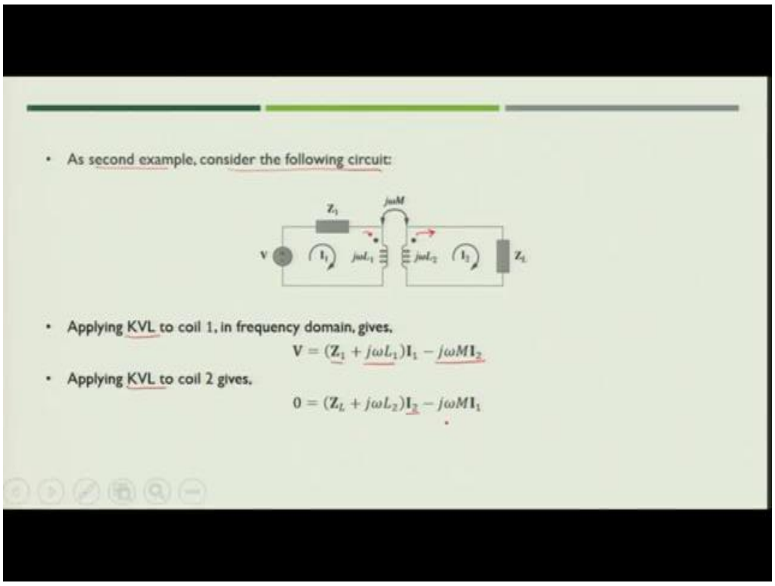

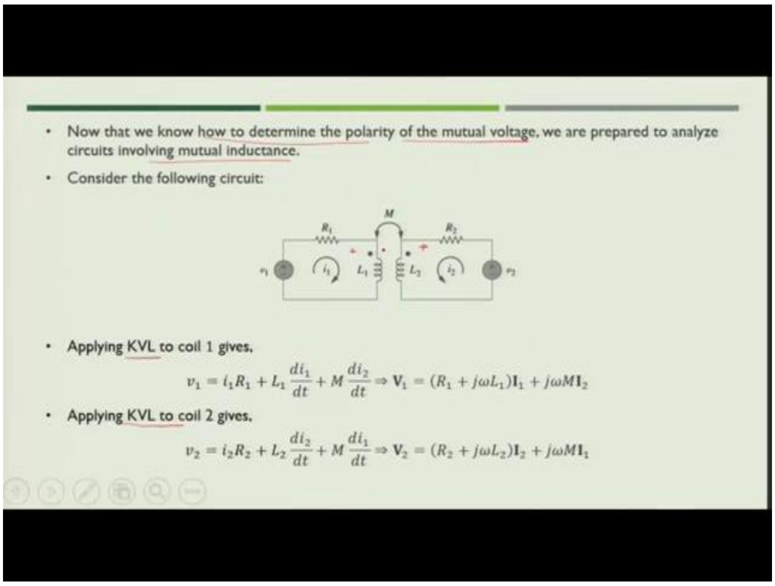

So now let us take another example where we consider this circuit as shown in the figure here in the first part of the circuit we have voltage V connected while in the second part we have load connected. Now let us apply KVL to coil 1,

$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

Applying KVL to coil 2 gives,

$$0 = (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2 - j\omega M\mathbf{I}_1$$

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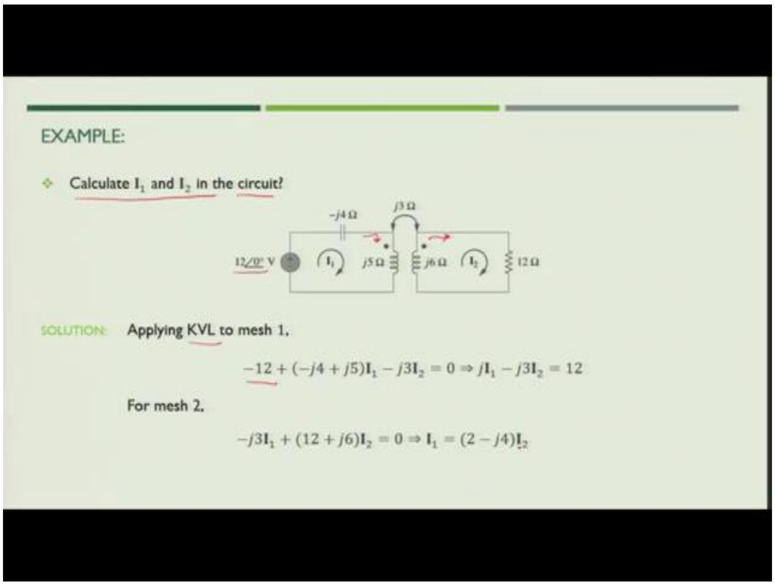

Now let us take 1 example to calculate the value of $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit. So, in this case we have applied voltage that is $12\angle 0°$ and in the second coil we have applied 12 ohm as a resistance. We will again use the KVL, to get,

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0 \Rightarrow j\mathbf{I}_1 - j3\mathbf{I}_2 = 12$$

For mesh 2,

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0 \Rightarrow \mathbf{I}_1 = (2 - j4)\mathbf{I}_2$$

Using mesh 2 equation in mesh ,

$$(j2 + 4 - j3)\mathbf{I}_2 = 12$$

Therefore,

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\angle 14.04° \text{ A}$$

The current $\mathbf{I}_1$ is given by,

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = 13.01\angle - 49.39° \text{ A}$$

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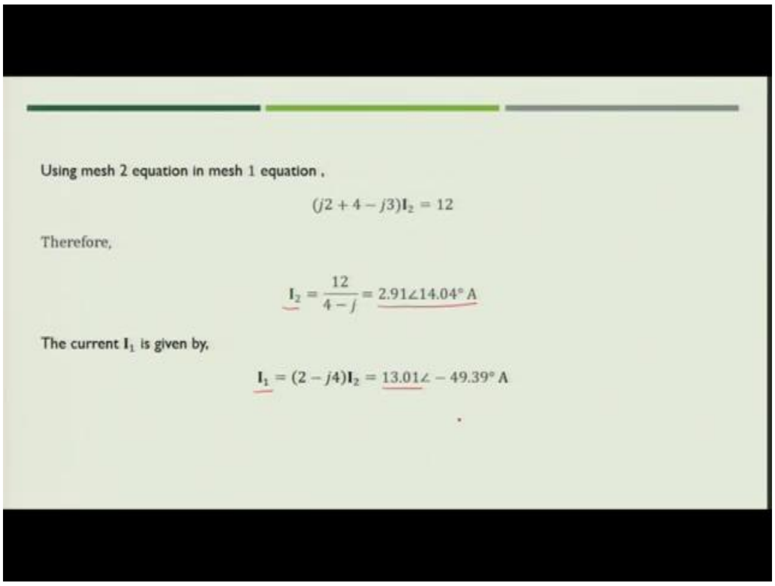

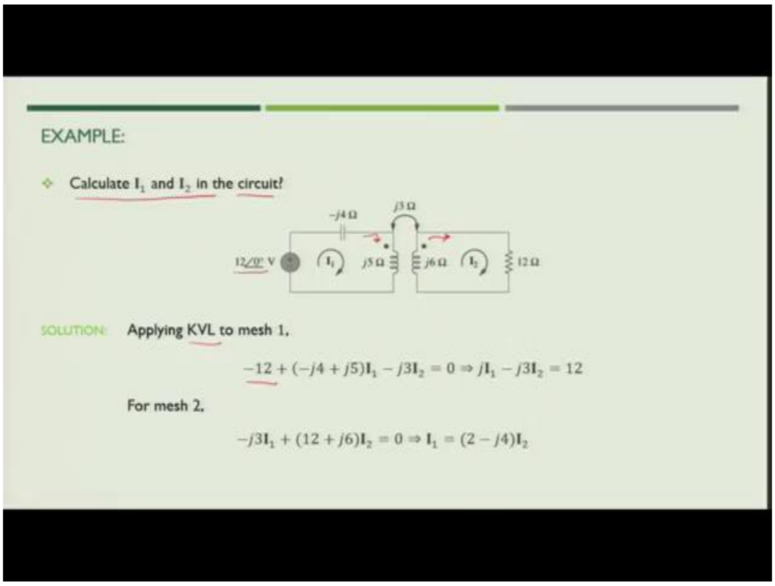

So using KVL again you can solve the circuits where you see these kind of magnetically coupled coils. So the important thing which you have to remember is that how you will find out the polarity of mutual voltage.

So with this we can close our today's session where we discussed about the magnetically coupled circuit we will discuss, we will continue our discussion on the magnetically coupled circuits in the next session also where we will discuss about the energy stored in the magnetically coupled circuit and then we will also see the ideal transformer and its various equations thank you.