Basic Electric Circuits
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Module 9 - Sinusoidal Steady State Analysis 1
Lecture 43 - Thevenin's, Nortons, and Maximum Power Transfer Theorems

Namaskar, so in the last class we discussed about Superposition Theorem and the source transformation. We will continue our discussion on various theorems with respect to ac analysis. In today's session we will discuss about Thevenin's, Norton's theorem and Maximum power transfer theorem.

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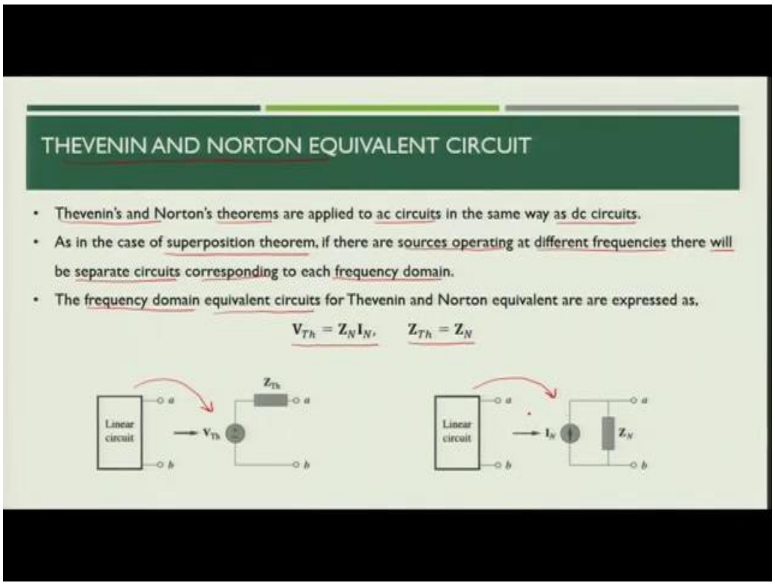

Let us see what those theorems with respect to the ac circuit analysis are. First, we will discuss about the Thevenin and Norton's theorem, we will try to find out what would be the Norton and Thevenin equivalence when we talk about the ac circuit analysis. So, in this case also the Thevenin's and Norton's theorems are applied in ac circuit in the same way as did in case of dc circuit. Now, if you see in case of superposition theorem, we discussed that if there are sources with different frequencies, we need to create the separate circuits corresponding to each frequency domain and then process the information.

In this case we will also create the different circuits for different frequencies and then finally we will use the superposition theorem to solve the circuit. Now, let us understand the Thevenin and Norton's theorem for one frequency domain by the equivalent circuit. Equivalent circuit is nothing but Thevenin's or Norton's equivalent, so how we will create Thevenin equivalent and

Norton equivalent? We will use the same technique which we used in case of dc circuit analysis where we will have one linear circuit.

This linear circuit will be represented with the help of Thevenin equivalent, we will have Thevenin voltage in series with Thevenin impedance. Similarly, in case of Norton's equivalent linear circuit will be converted into the Norton's equivalent where current source that is Norton's current source that is $\mathbf{I}_N$ will have the Norton's impedance in parallel. So, you can see that these two circuits that is Thevenin's equivalent and Norton's equivalent are dual of each other and the Thevenin voltage that is $\mathbf{V}_{Th} = \mathbf{Z}_N\mathbf{I}_N$. And in both cases your Thevenin's impedance and Norton's impedance will be same. In this way you can convert Thevenin's equivalent into Norton's equivalent.

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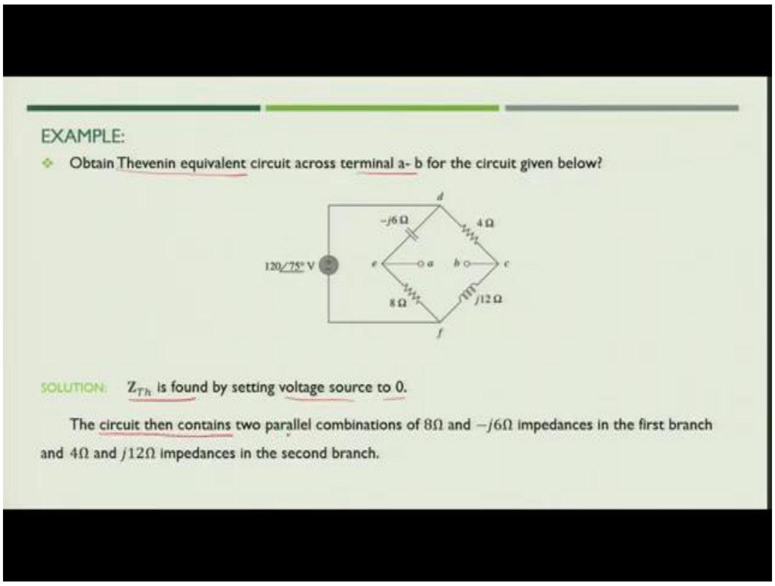

Now, let us understand these equations with the help of few examples. First, we will use the Thevenin's theorem to find the Thevenin equivalent across the terminal a b of the circuit given in the example. So here, first we will find the value of $\mathbf{Z}_{Th}$. $\mathbf{Z}_{Th}$ is nothing but the impedance seen from the terminal a b, while, setting the voltage source equal to 0. Setting voltage source equal to 0 means the voltage source will be short circuited and current source will be the open circuited.

In this case we are having the voltage source short circuited and when we will rearrange this particular circuit with the help of our knowledge on topological aspect of the circuit. We will come to know that this circuit will now contains two parallel combinations of 8 ohm resistance in parallel with $-j6\Omega$ impedance and then in series with the another parallel combination of 4Ω and $j12\Omega$ impedance.

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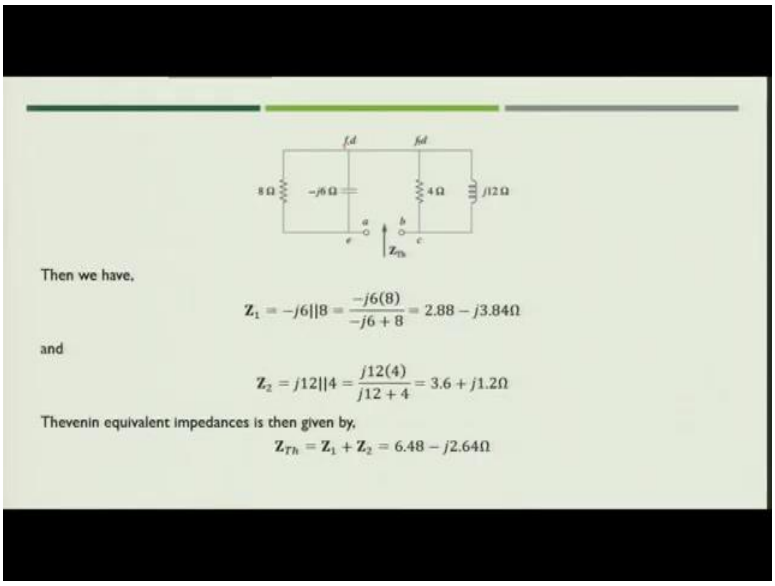

So, how our circuit will look like? The circuit will look like now as you can see in the figure f and d are short circuited, so they are clubbed together in the figure.

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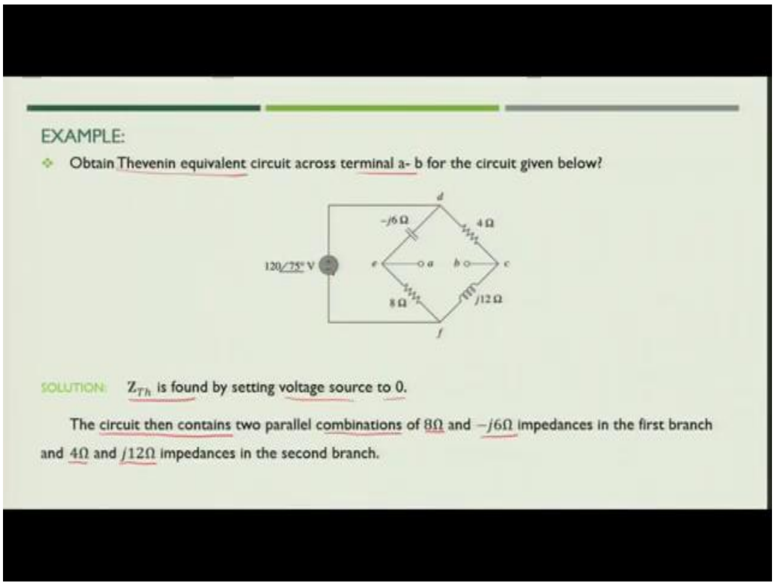

So, if you go back to the circuit f and d are not short circuited because you have put voltage source as a short circuit.

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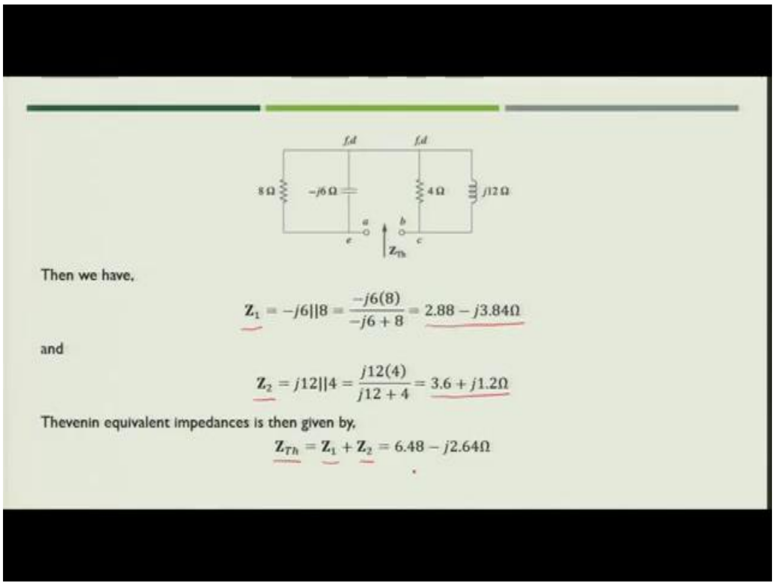

Now the updated circuit is as shown in the figure $\mathbf{Z}_1$,

$$\mathbf{Z}_1 = -j6||8 = \frac{-j6(8)}{-j6+8} = 2.88 - j3.84\Omega$$

and

$$\mathbf{Z}_2 = j12||4 = \frac{j12(4)}{j12+4} = 3.6 + j1.2\Omega$$

Thevenin equivalent resistance is then given by,

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\Omega$$

So now you have got the Thevenin's equivalent of the circuit.

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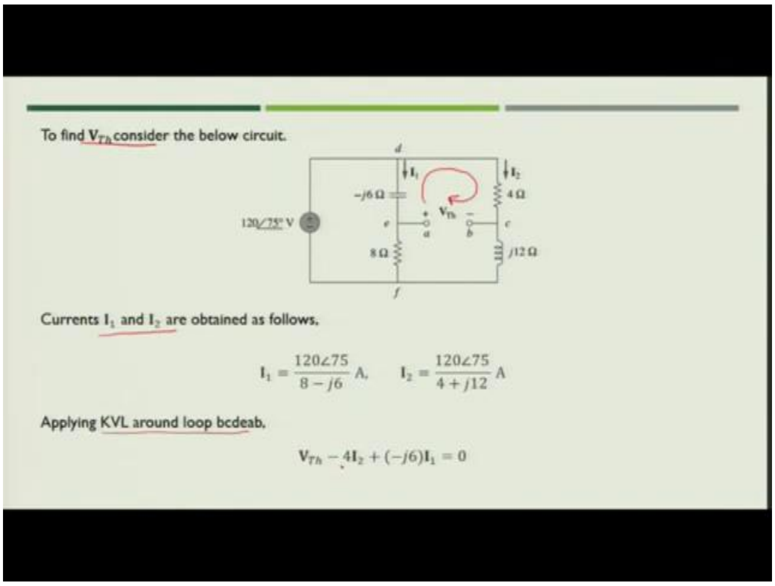

Next task is, to find out the Thevenin voltage. So Thevenin voltage is nothing but the voltage across the terminal a b. So, how to solve, we will first try to find out the currents I1 and I2. I1 is flowing in the d f segment and I2 is flowing in the right side of the segment. So

$$\mathbf{I}_1 = \frac{120\angle 75}{8 - j6} \text{ A}, \ \mathbf{I}_2 = \frac{120\angle 75}{4 + j12} \text{ A}$$

Applying KVL around loop bcdeab,

$$\mathbf{V}_{Th} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

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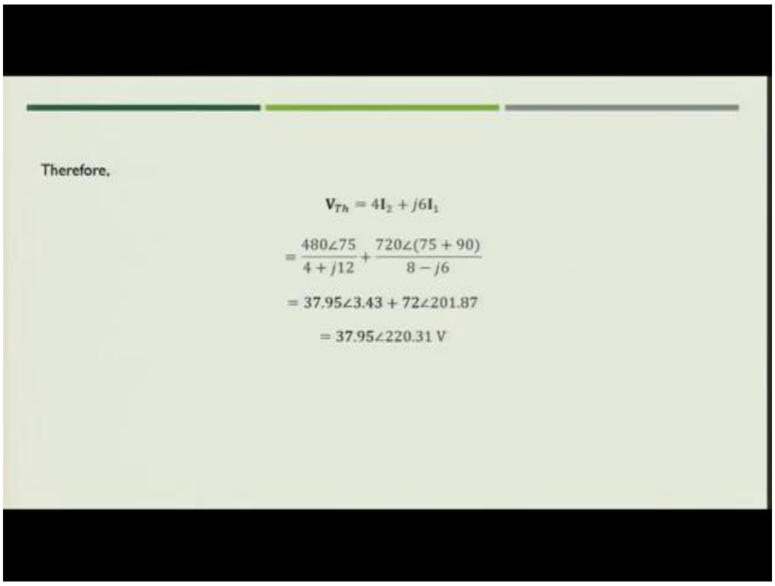

And then if you solve,

$$\mathbf{V}_{Th} = 4\mathbf{I}_2 + j6\mathbf{I}_1 =$$

$$= \frac{480\angle 75}{4 + j12} + \frac{720\angle(75 + 90)}{8 - j6} =$$

$$= 37.95\angle 3.43 + 72\angle 201.87 =$$

$$= 37.95\angle 220.31 \text{ V}$$

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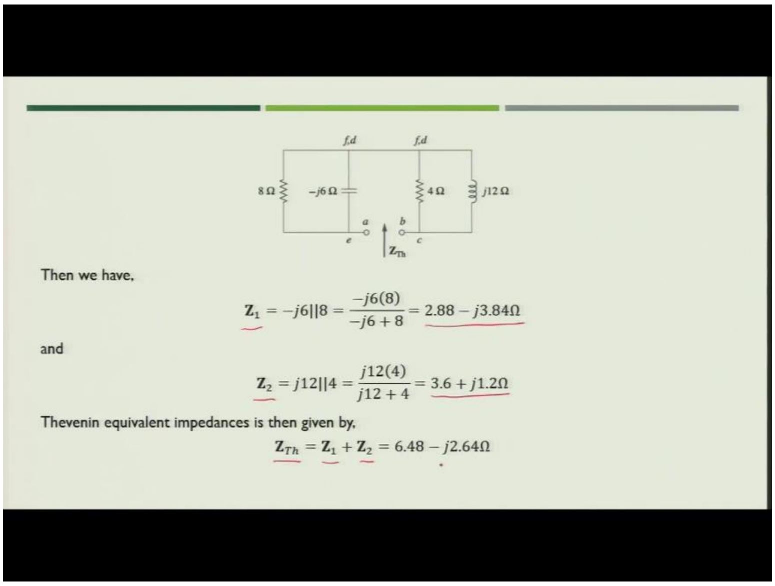

So now you have got the value of the Thevenin impedance that is the Zth.

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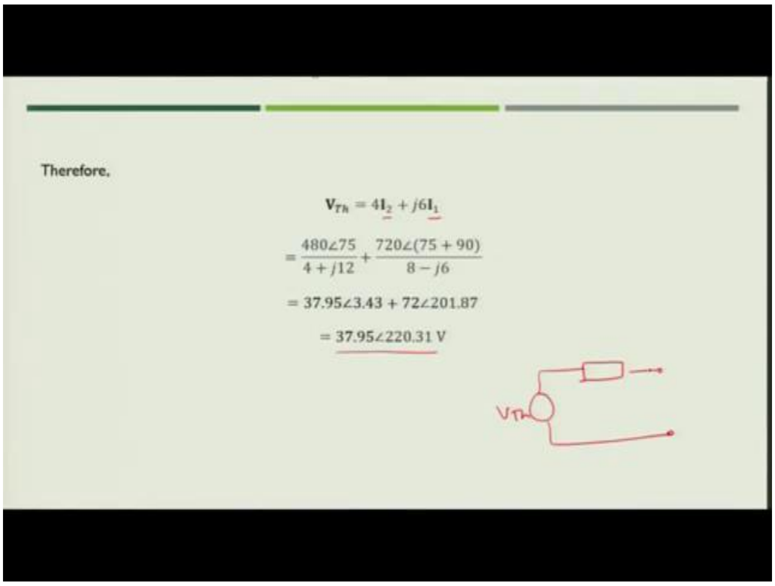

And then you have found the value of Vth so you can simply draw the Thevenin equivalent so you will get Vth and in series you will have Zth which you have calculated.

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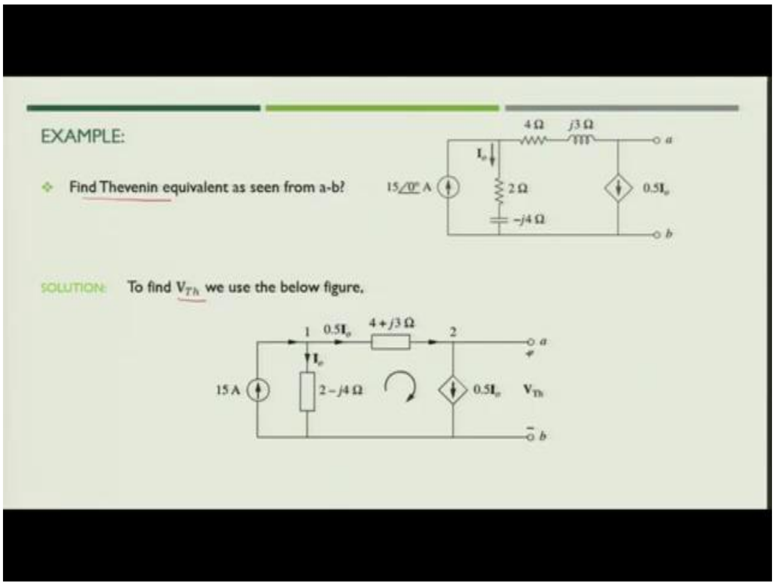

Now, let us see another example where we need to find again the Thevenin equivalent for the circuit. Where, you see the dependant current source. Now to find the $\boldsymbol{V_{Th}}$, what we will do will find the value of voltage across the terminal a b. so, how to find? If you see this figure the value of current which is flowing through the current source is equal to the I naught plus 0.5 I naught. So, if you applied KCL apply to this node.

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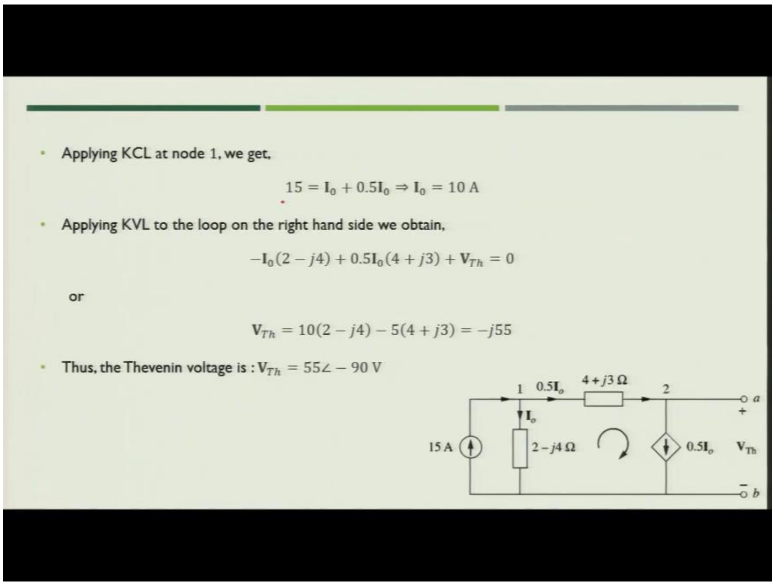

So, what you can write? You can write,

$$15 = \mathbf{I}_0 + 0.5\mathbf{I}_0 \Rightarrow \mathbf{I}_0 = 10 \text{ A}$$

Now, you need to apply KVL to the loop on right hand side. So, if you apply KVL you can simply write,

$$-\mathbf{I}_0(2 - j4) + 0.5\mathbf{I}_0(4 + j3) + \mathbf{V}_{Th} = 0$$

or

$$\mathbf{V}_{Th} = 10(2 - j4) - 5(4 + j3) = -j55$$

Thus, the Thevenin voltage is

$$\mathbf{V}_{Th} = 55\angle -90 \text{ V}$$

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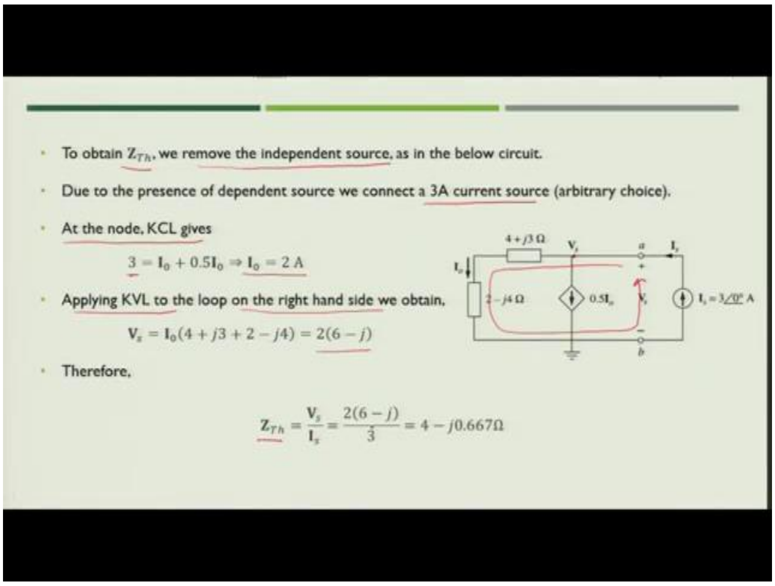

Next task is you need to find Zth. So, to find the Zth we remove the independent source. So, independent source in this case is the current source.

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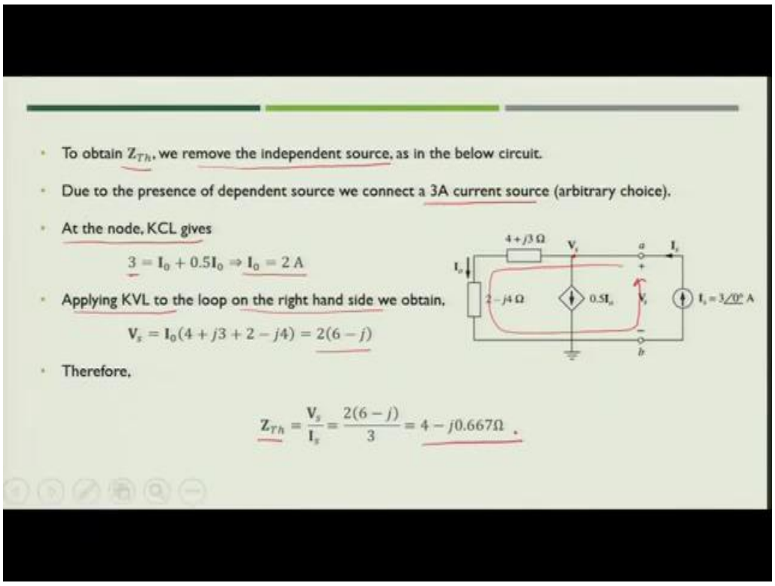

If you remove the current source the updated circuit will be like shown in the figure and now, across terminal a b we apply a current source having value 3 ampere. We have taken 3 ampere as an arbitrary choice you can assume any value. The Thevenin impedance $\mathbf{Z}_{Th} = \frac{\mathbf{V}_s}{\mathbf{I}_s}$. If you apply KCL at the node,

$$3 = \mathbf{I}_0 + 0.5\mathbf{I}_0 \Rightarrow \mathbf{I}_0 = 2 \text{ A}$$

Now, let us apply the KVL to the loop on the right-hand side that is the loop which you see here. So, you apply KVL, to get,

$$\mathbf{V}_s = \mathbf{I}_0(4 + j3 + 2 - j4) = 2(6 - j)$$

Therefore,

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6-j)}{3} = 4 - j0.667\Omega$$

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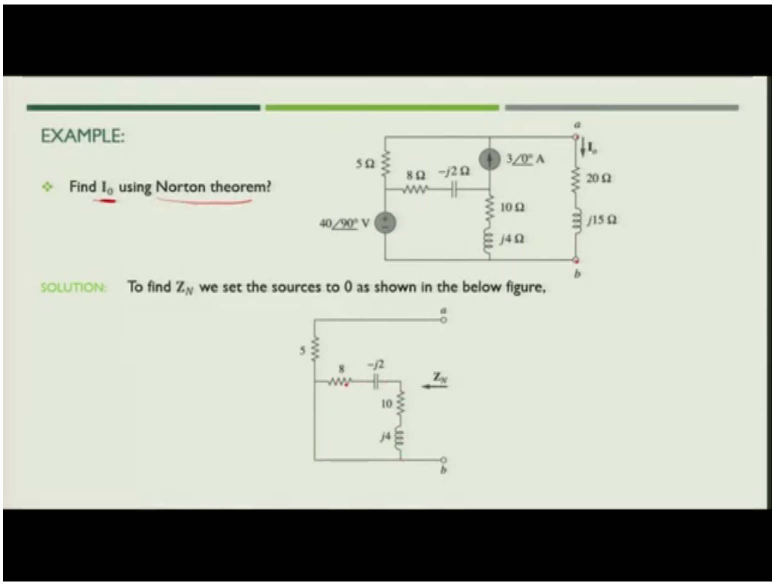

Now, in this particular example we need to find out the value of $\mathbf{I}_0$ that is the current flowing through the segment a b using Norton's theorem. So, to find out the Norton's equivalent first we need to remove the load from terminal a b. Now what we need to find out? We need to find out value of Norton's equivalent impedance that is $\mathbf{Z}_N$. When you are ask to find the value of $\mathbf{Z}_N$ first what you will do? You will short circuit the voltage source, and open circuit the current source.

You will get the updated circuit as shown in this figure. Now, if you see this particular segment that is 8-ohm resistance in series with $-j2$ ohm reactance offered by capacitance and then 10-ohm resistance and $j4$ reactance offered by the inductor. So, this will be sorted because you have a current voltage source connected which was short circuited. This leg will be completely short circuited then and that case the $\mathbf{Z}_N = 5\ \Omega$.

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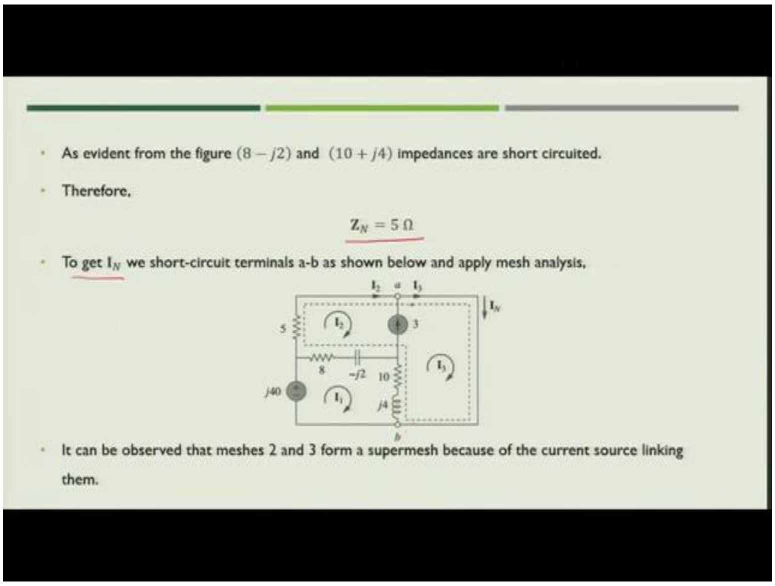

So, what you can say? Your Norton's equivalent impedance is 5 ohm. Next task is you need to find out the value of $\mathbf{I}_N$, to find the value of $\mathbf{I}_N$ we short circuit the terminals a b and then we solve for the value of $\mathbf{I}_N$. Now, from this figure you can see the current source which is there in the figure is coming between two loops. So, what you can create? You can create super mesh because the current source is common to both meshes. We will create one super mesh containing mesh 2 and 3 and then we will write the mesh equation.

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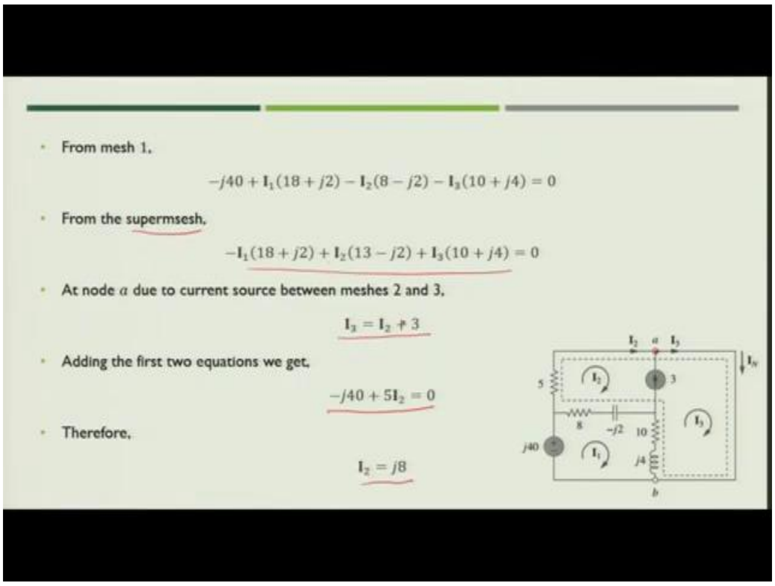

From mesh 1,

$$-j40 + \mathbf{I}_1(18 + j2) - \mathbf{I}_2(8 - j2) + \mathbf{I}_3(10 + j4) = 0$$

From the supermsesh,

$$-\mathbf{I}_1(18 + j2) - \mathbf{I}_2(13 - j2) + \mathbf{I}_3(10 + j4) = 0$$

At node a due to current source between meshes 2 and 3,

$$\mathbf{I}_3 = \mathbf{I}_2 + 3$$

Adding the first two equations we get,

$$-j40 + 5\mathbf{I}_2 = 0$$

Therefore,

$$\mathbf{I}_2 = j8$$

And then you can put the value of I2 in this equation.

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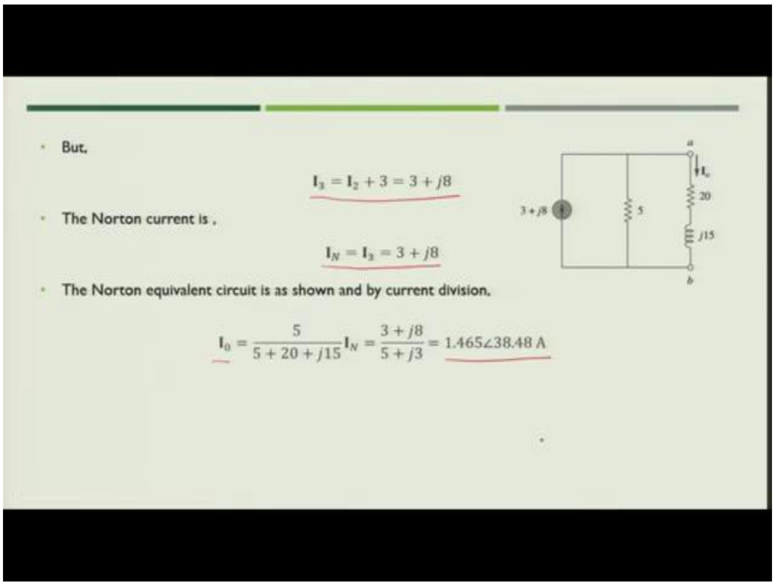

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$

The Norton current is ,

$$\mathbf{I}_N = \mathbf{I}_3 = 3 + j8$$

The Norton equivalent circuit is as shown and by current division,

$$\mathbf{I}_0 = \frac{5}{5+20+j15}\mathbf{I}_N = \frac{3+j8}{5+j3} = 1.465\angle 38.48 \text{ A}$$

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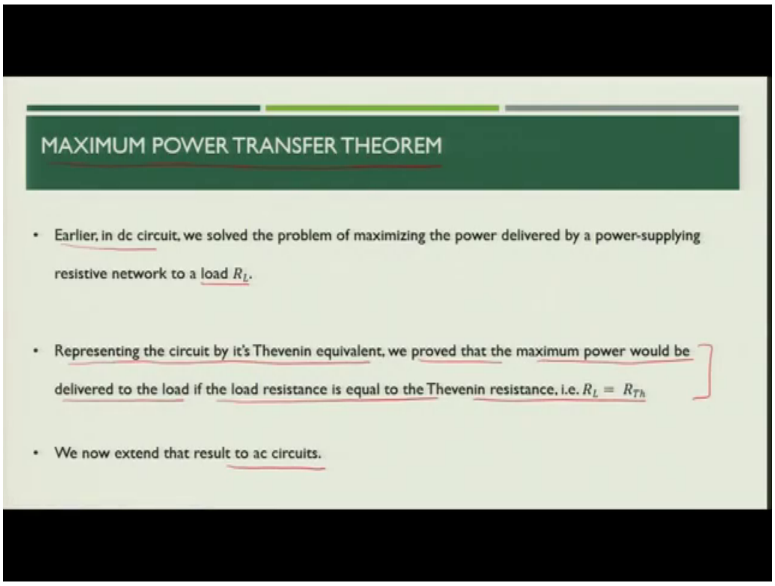

Now, let us come to the, another theorem called Maximum power transfer theorem. So, in case of dc circuit we solve the problem of maximizing the power delivered by the power supplying network to load RL. So, how we solve? We solved mathematically and saw that the first the circuit is represented by its Thevenin equivalent and then mathematically we prove that maximum power would be deliver to the load, if load resistance is equal to Thevenin resistance.

So, this is what we proved in case of dc circuit. Now the same concept will extend for the ac circuit and we will try to find out the condition under which the maximum power would be transferred to the load if ac source are connected to the circuit.

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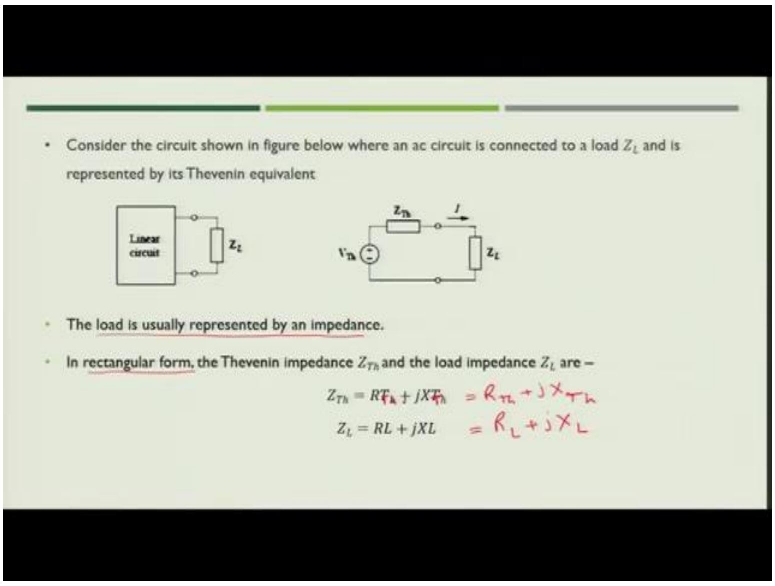

So, now again let us consider the circuit we consider a linear circuit. We have a load ZL is connected. So this linear circuit will first convert into Thevenin equivalent. So, we will have Vth in series with Zth and then load ZL will be connected across the Thevenin equivalent terminal. Now, the load is generally represented by an impedance. So, that is why here we are representing load with ZL. Now, let us represent ZL and Zth in rectangular form. So, what you can write? You can write Zth is nothing but that is Rth plus j Xth. So, ZL you can write as RL plus jXL. So, basically this is nothing but Rth plus j XTh and ZL is equal to RL plus j XL. So, now Zth and ZL you can divide into the resistance and the reactance component.

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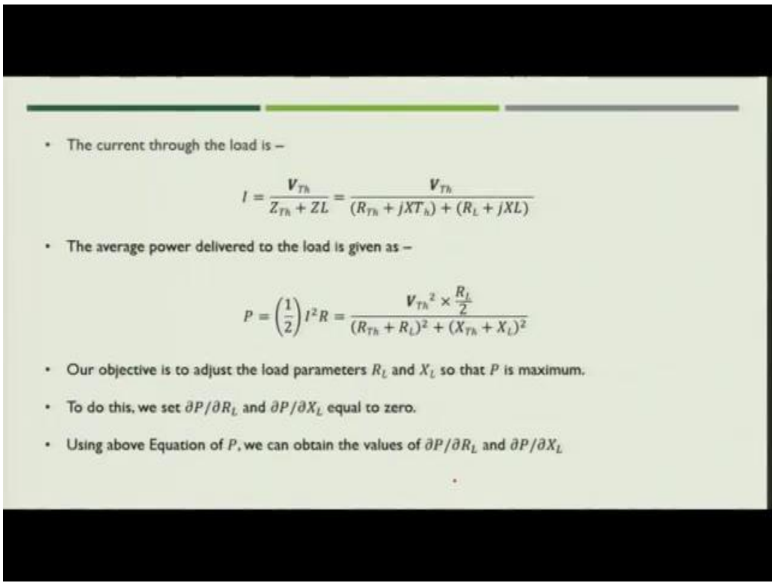

Now, next is you need to find out the current which is flowing through the load. So what is the value of current? Current is nothing but Vth divided by the Zth plus ZL. So, that is Zth plus ZL. Now, you simply replace the values with the rectangular form of the Zth and ZL.

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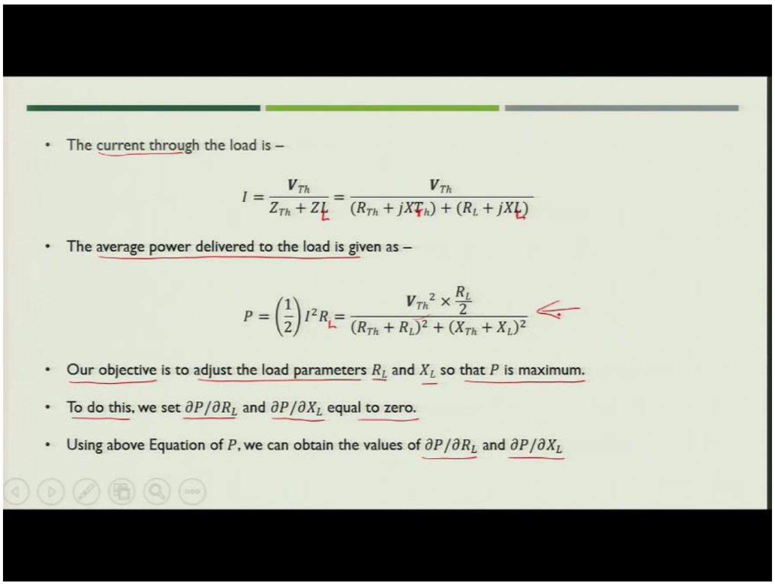

So, you can simply write as Rth plus j XTh plus RL plus j XL. So now you got the value of I. What is the average power delivered to the load? The average power delivered to the load can be written as P is equal to 1 by 2 I square R. Why 1 by 2 term is coming? Because, this is not the RMS value. So if it is an peak value you will get the value P as 1 by 2 I square R. So, if I is RMS then you can write I RMS square into R that is the value of average power delivered to the load. So here in this case, P will be 1 by to 2 I square R.

That means Vth square into RL by 2. So here, R means RL the power delivered to the load. And then we write the value of I in terms of Vth and the Zth and ZL. So we can write power P in terms of Vth, RL and Rth like shown in the slide. Now, what is the next task? Our objective is to adjust the load parameter, that is RL and XL. Show that the value of P is maximum. So, how to get the value of maximum P? To do this we said del P by del RL and del P by del XL equal to 0. So, if you obtain the value of del P by del RL and del P by del XL using this equation. What you can write?

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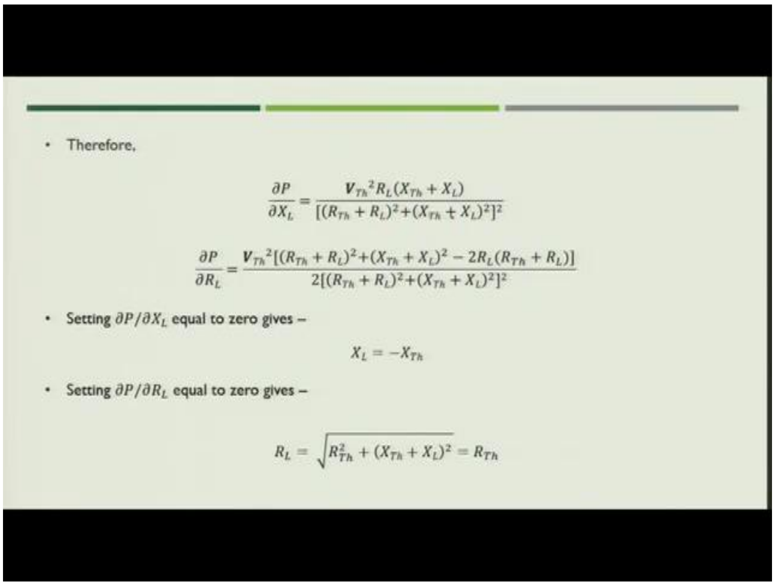

Del P by del XL is nothing but Vth square into RL into Xth plus XL divided by Rth plus RL square plus Xth plus XL square whole square.

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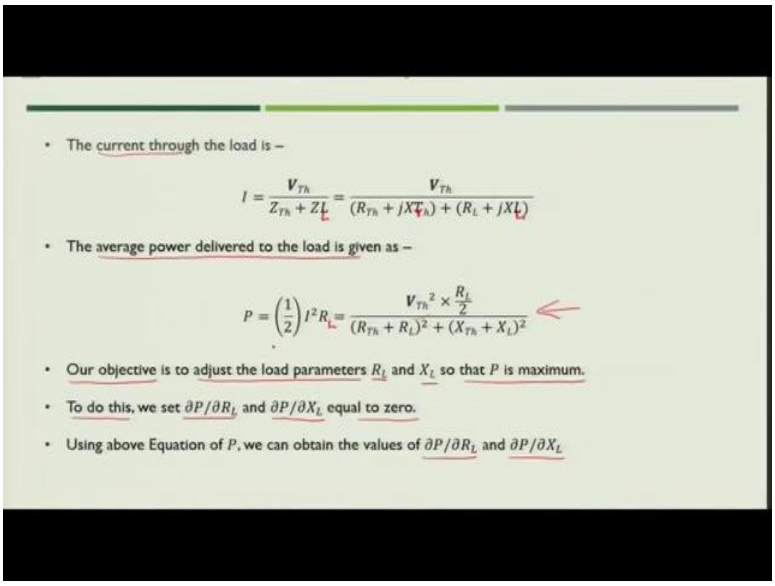

So, this is what you get when you differentiate this is the equation with respect to XL.

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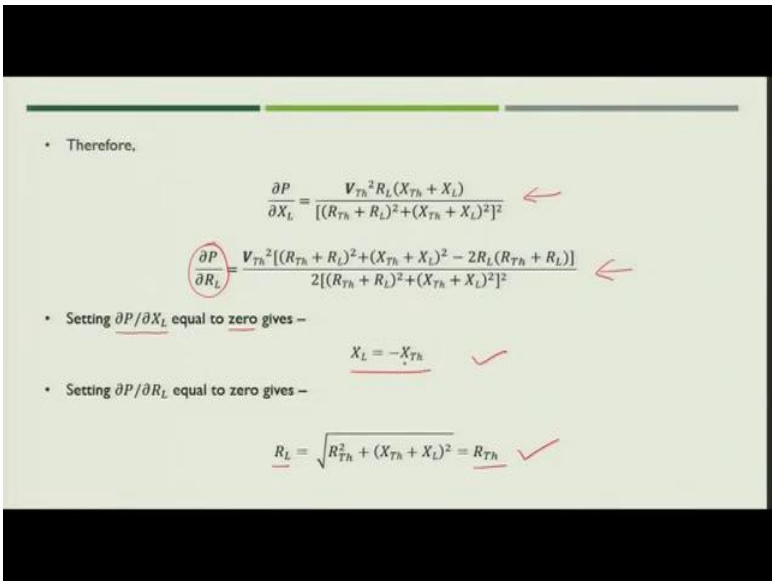

Similarly, when you differentiate power P with respect to RL you get the expression as shown in the slide. Now if you, when you set del P by del XL is equal to 0. What will happen? So this cannot be 0, this is again cannot be. So, what can be 0 is Xth plus XL. So, when you set Xth plus XL equal to 0, you get the condition that XL should be equal to minus of Xth. Similarly, when you set del P by del RL equal to 0. And solve the this particular, the part of the expression then you get RL equal to under root of Rth square plus Xth plus XL square.

Now, if you put the value of XL is equal to minus Xth in this expression. You can simplify it and you get RL is equal to Rth. So, now for maximum power transfer condition you got to expression for the condition. That is XL should be equal to minus of Xth. And RL should be equal to Rth.

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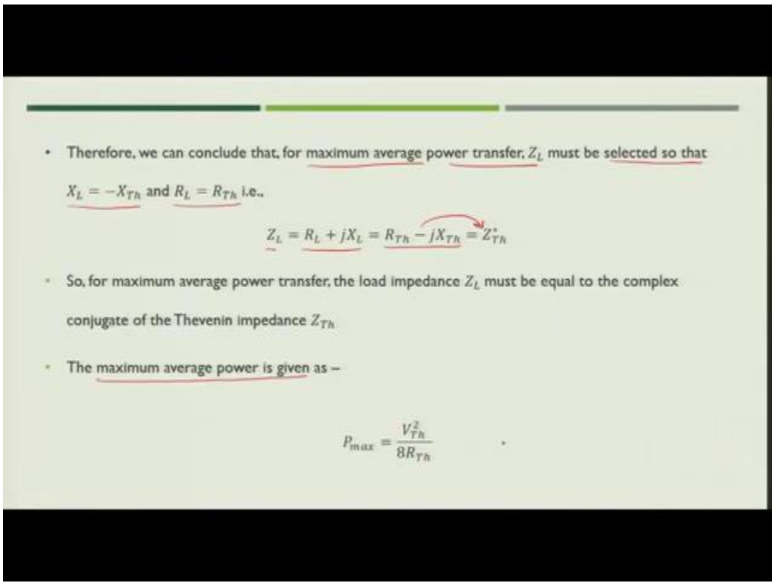

So, what you can write now that for maximum average power transfer ZL should be selected in such a way, that XL should be equal to the minus Xth and RL should be equal to Rth. Or you can write ZL is equal to RL plus jXL is equal to Rth minus jXth. What is this? This is nothing but the complex conjugate of Zth. So, you can easily say that ZL is equal to complex conjugate of the Thevenin impedance. So, now what would be the maximum power to be transferred? In this case P max the maximum power which would be transferred would be equal to Vth square by 8 into Rth. How you got?

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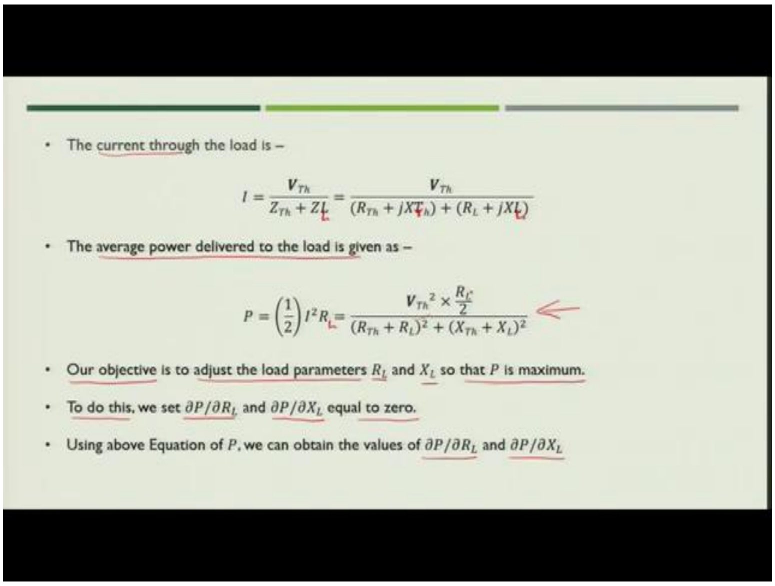

You just simply put the value of RL as Rth and XL is equal to minus Xth. So, this will be anyway become zero. This will be equal to 4Rth square and then in numerator you will get Vth square into Rth by 2. So, 1 Rth will be canceled out and finally you get Vth square divided by 4 into 2 that is 8 Rth.

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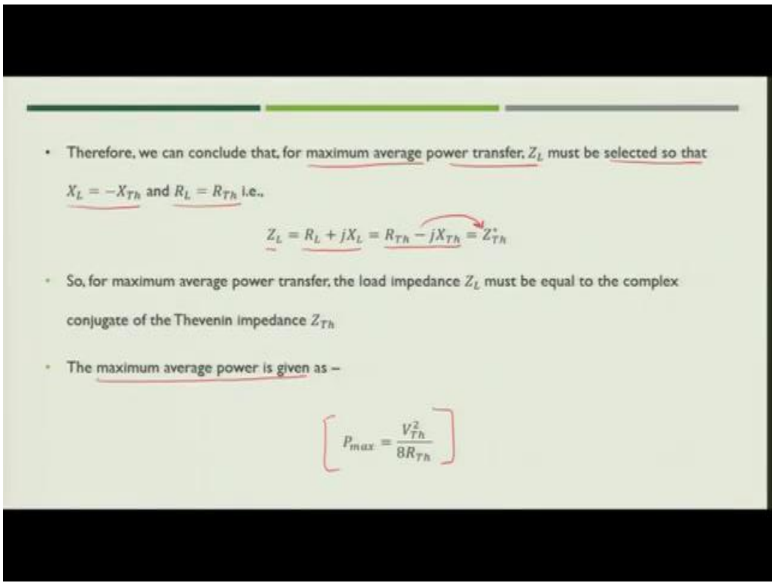

So, this is what you get under the maximum power transfer condition. So, if you compare from the dc circuit this is slightly different because of the introduction of reactive component in the impedance.

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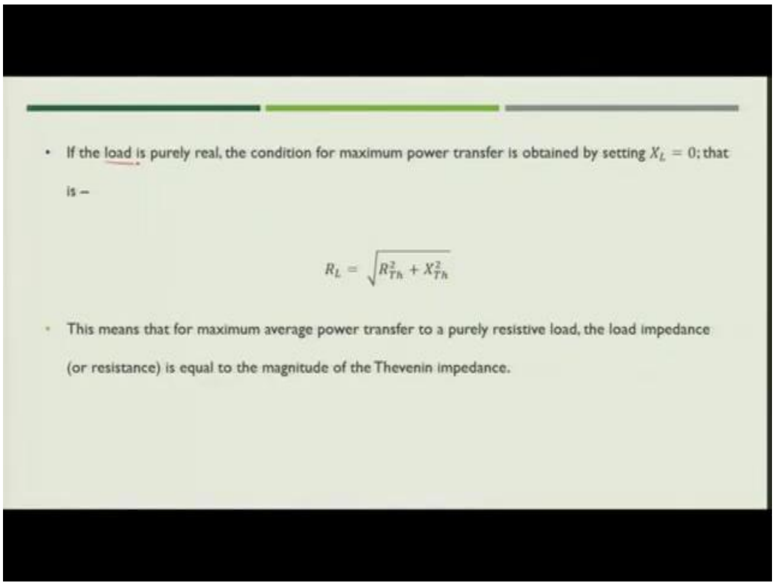

Now, in case of ac if your load is purely real what would be the condition? The condition for maximum power transfer will be obtained by setting XL is equal to 0.

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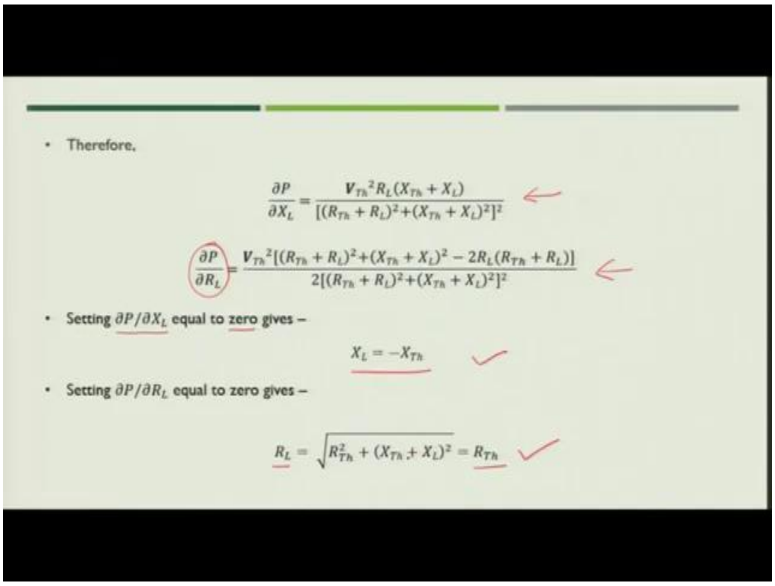

So, where you will put the value? You set the value of XL is equal to 0 here. What you get? RL would be equal to under root of Rth square plus Xth square.

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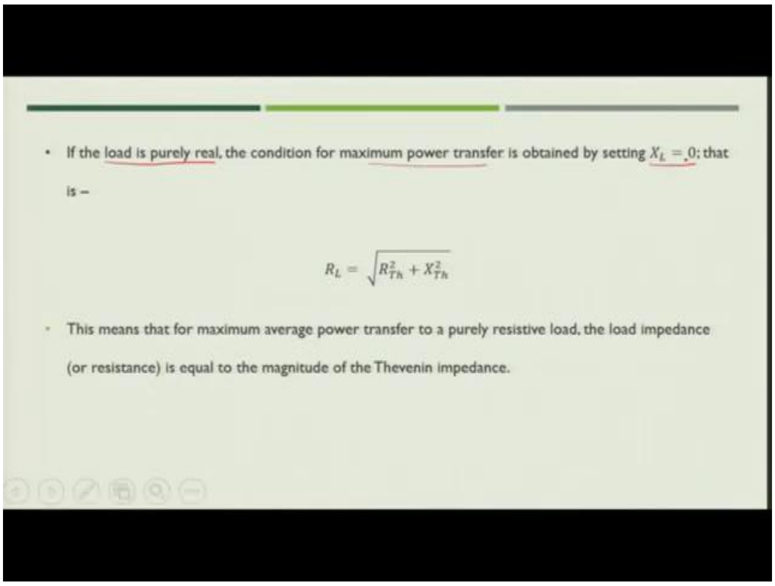

So, in case of dc, the maximum power transfer was obtained by setting RL is equal to Rth, but in case of ac the value of RL would be equal to under root of Rth square plus Xth square. So, what does it mean?Tthat means that the maximum power transfer to a purely resistive load the load impedance that is the resistance in this case will be equal to magnitude of the Thevenin impedance.

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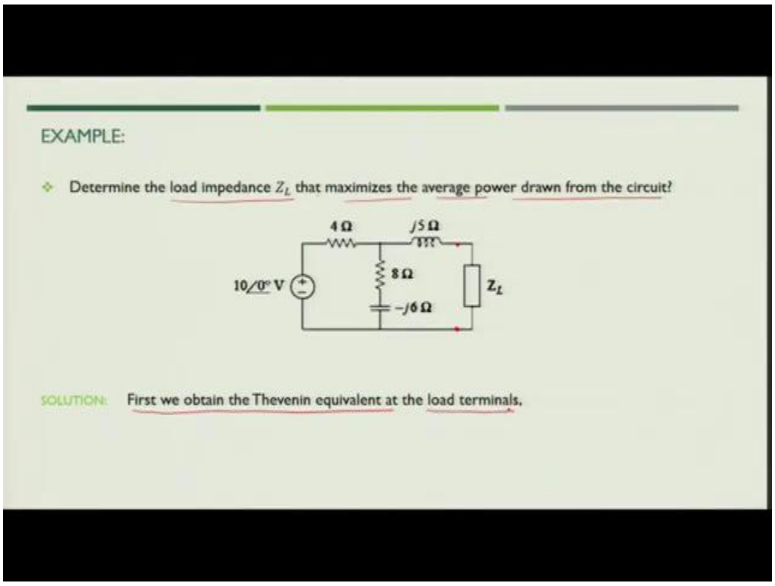

So, now we will use the maximum power transfer concept to determine the load impedance ZL that maximizes the average power drawn from the circuit. Now, if you see the circuit. You will have 10 angle 0 as a voltage applied and then resistance 4 ohm in series with the voltage source, in parallel you have 8 ohm in series with minus j 6 ohm as a reactance offered by this capacitor and then j 5 ohm reactance offered by the inductor. Now, the load ZL is connected across the terminal. So first task what we have to do is that first you need to obtain the Thevenin equivalent at the load terminals. So, first we will remove the load from the circuit.

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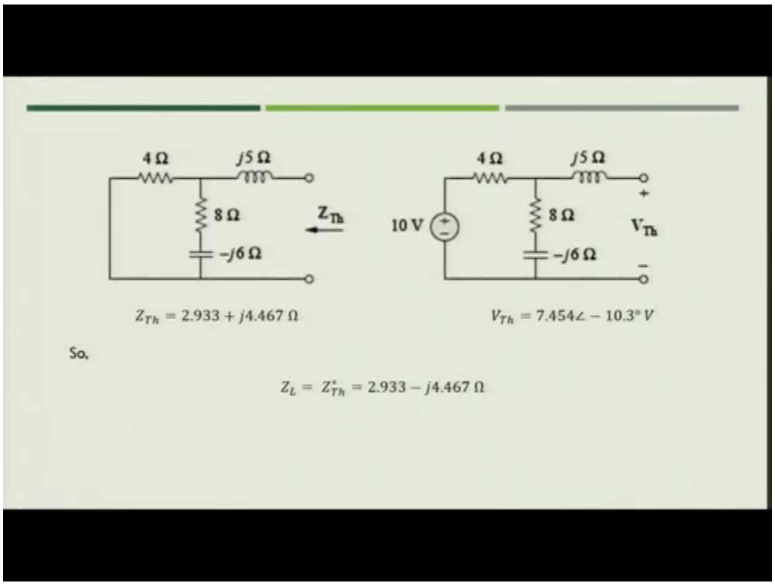

When, you remove you get the circuit like this.

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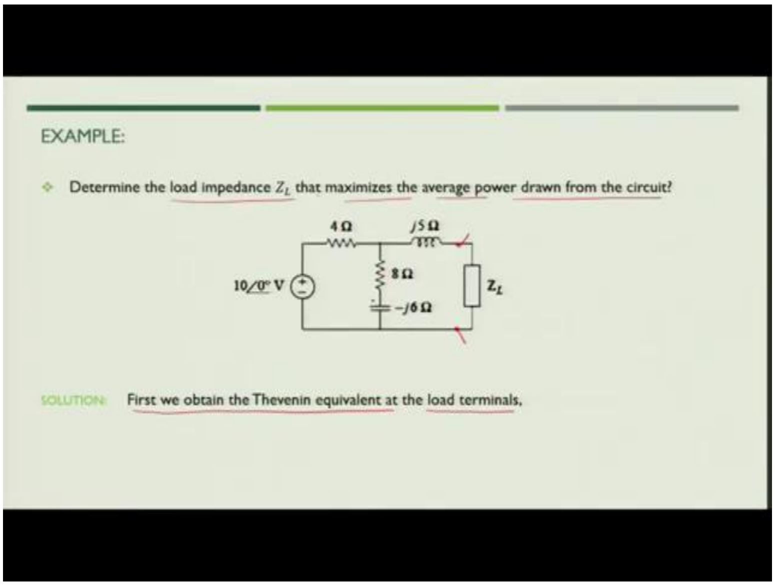

Wwhere you have 10 volt, 4 ohm,8 ohm and minus j 6 and j 5 ohm connected.

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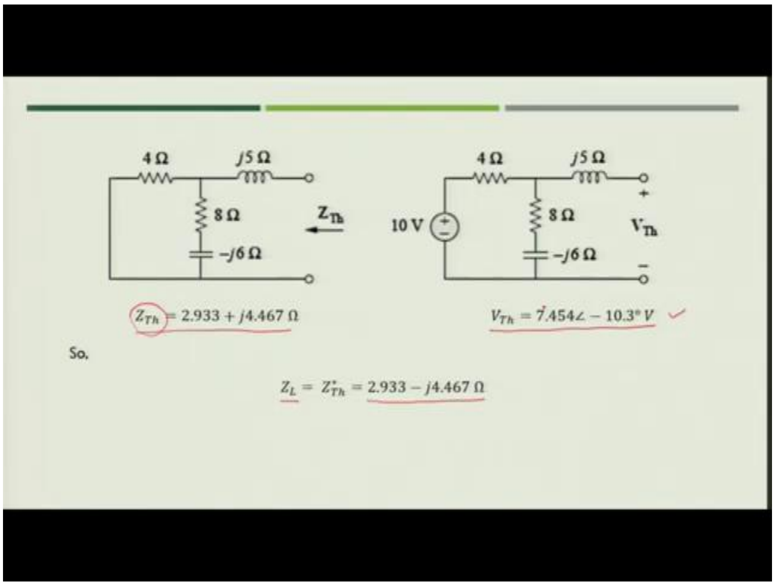

So, you will get the circuit like this. To find out the Thevenin impedance you will short circuit this. So your circuit will become like this. And this 4 ohm resistance will now be in parallel with 8 ohm resistance in series with minus j 6 ohm reactant. So, if you solve this, the parallel combination of this and then the result is added with j 5 ohm reactance offered by the inductance you will get Zth as 2.933 plus j 4.467 ohm.

So by solving this you will get the value of Zth. Now, you need to find the value of Vth. So, how will you able to find out the Vth? This is simple circuit, where you can utilize the voltage division concept and find out the value of Vth. So, Vth will be nothing but 8 plus minus j 6 divided by 8 minus j 6 plus 4 into 10. So, when you solve, you get Vth as 7.454 angle minus 10.3 degree volt. Now, you got Vth and Zth. Next task is you need to find out the value of ZL, which is the load connected across the terminal of the Thevenin equivalent.

So, what will be the value of ZL maximum transfer? ZL will be equal to complex conjugate of Zth. So, what will be the value of ZL? ZL you can simply find out by taking the complex conjugate of Zth that is 2.933 minus j 4.467 ohm. So, this you can see that the load impedance, you can easily find out by creating the Thevenin equivalent. So with this we can close our today's session. In this session we discussed about the maximum power transfer theorem, Norton's and Thevenin theorem with respect to our ac circuit analysis. So in the next lecture we will discuss about the magnetically coupled circuits. Thank You.