Basic Electric Circuits
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Lecture 42
Sinusoidal Steady State Analysis I
Superposition Theorem and Source Transformation

Namaskar, So in last class we were discussing about the nodal and mesh analysis with the help of Kirchoff current law and voltage law. Today we will discuss about two more theorems which with respect to ac circuit analysis we will try to understand how we will implement those theorems in ac circuits.

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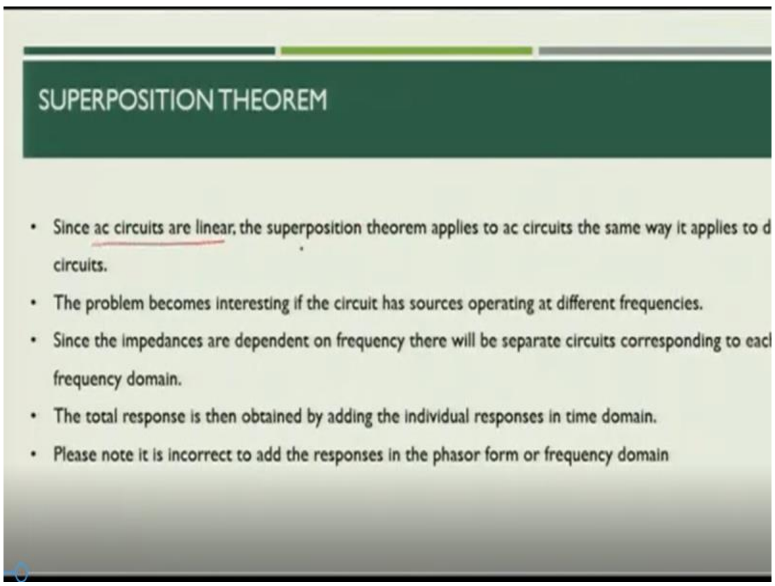

Let us understand the superposition and the source transformation theorem with respect to ac circuit analysis. Now as ac circuit is also linear you can utilize the superposition theorem in the same way as you did in case of dc circuits. Now, here the important thing which you have to keep in mind is that the processing of the superposition theorem is little bit different as we did in case of ac circuit because your source operating frequencies can be different.

If you have multiple sources connected in the circuit and those sources are having a different frequencies, then the problem will also become interesting and we will see how we will solve those kind of problems with the help of superposition theorem. Whenever we have different set, different voltage or current sources operating at different frequencies we will see that the value of the impedance offered by the inductance and capacitance will be different.

So when we will analyze the circuit with the help of superposition theorem the total response will be obtained by adding the individual responses which we get from the superposition theorem and we will convert the response in time domain for the final result. Now it is important to note here that the responses which we get with the help of superposition theorem cannot be mentioned in the form of phasor because the frequencies of different sources may be different. In that case, the phasor form cannot be used for representing the answer. So, rather we will use the time domain as a method to give the answer which we get finally after the circuit analysis.

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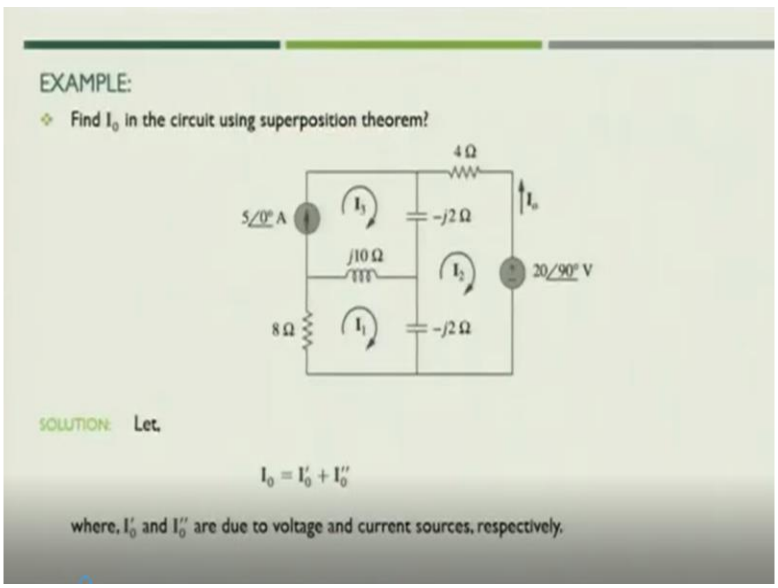

So, now let us understand the superposition theorem with the help of few examples under ac circuit. Now in this particular example, we need to find the value of $\mathbf{I}_0$ with the help of superposition theorem. So, $\mathbf{I}_0$ is the current which is flowing from voltage source. The voltage source is having magnitude as $20\angle 90°$ and we also have one current source that is $5\angle 0°$.

So let us assume that the total current is composed of two components $\mathbf{I}_0 = \mathbf{I}_0' + \mathbf{I}_0''$ where $\mathbf{I}_0'$ and $\mathbf{I}_0''$ are the currents because of voltage and current sources separately. So when you take one source at a time suppose if you take the voltage source at one time you will get $\mathbf{I}_0'$, then you take current source only then you get $\mathbf{I}_0''$. The total will be $\mathbf{I}_0 = \mathbf{I}_0' + \mathbf{I}_0''$. So lets us first take the voltage source. So, when you take the voltage source your current source will be open-circuited.

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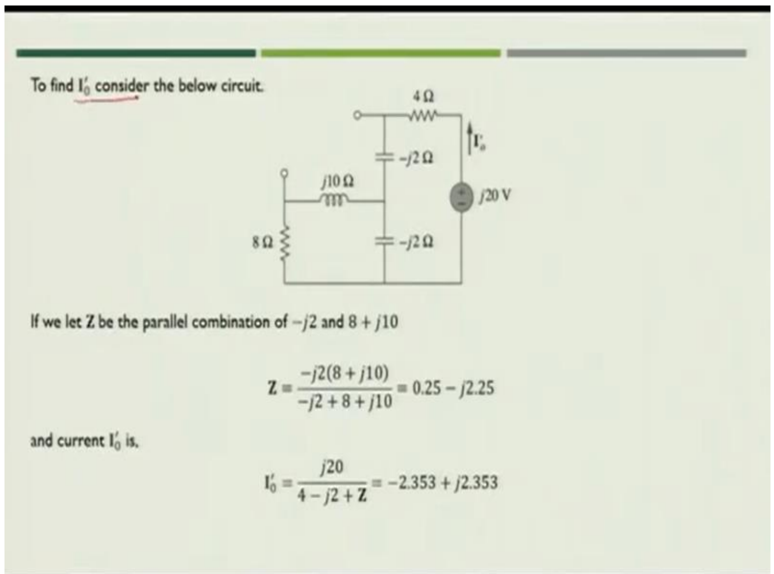

So in that case the value that we need to find out is $\mathbf{I}_0'$ and the modified circuit will be as shown in the figure. Now the voltage source is $20\angle 90°$ so you can conveniently write it as $j20$ in complex form. Now you have the circuit as shown in the figure in which the capacitor is connected in parallel with the series combination of 8 ohm, and 8-ohm resistance, and j 10-ohm impedance.

Now, what we will do. We will first try to find out the total impedance of this particular block. So, let **Z** be the parallel combination of $-j2$ and $8 + j10$. So when you solve it

$$\mathbf{Z} = \frac{-j2(8+j10)}{-j2+8+j10} = 0.25 - j2.25$$

and current $\mathbf{I}_0'$ is,

$$\mathbf{I}_0' = \frac{j20}{4 - j2 + \mathbf{Z}} = -2.353 + j2.353$$

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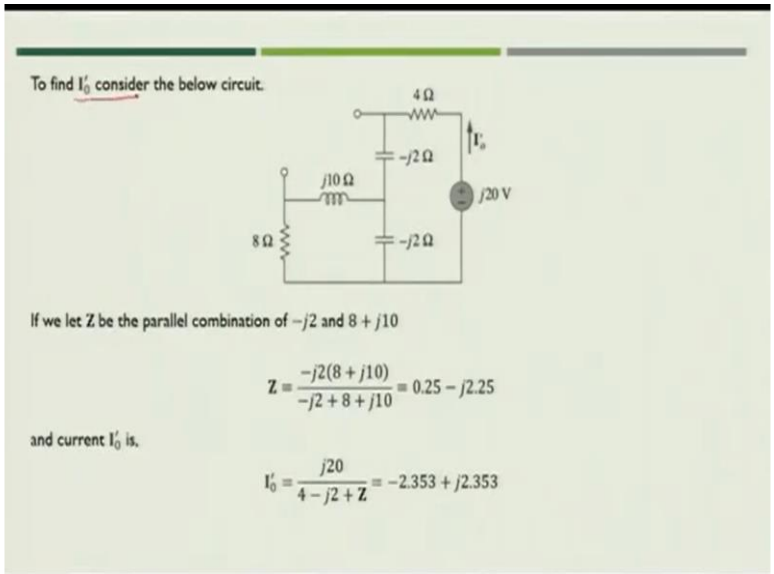

Now next is you consider the current source and short circuit the voltage source.

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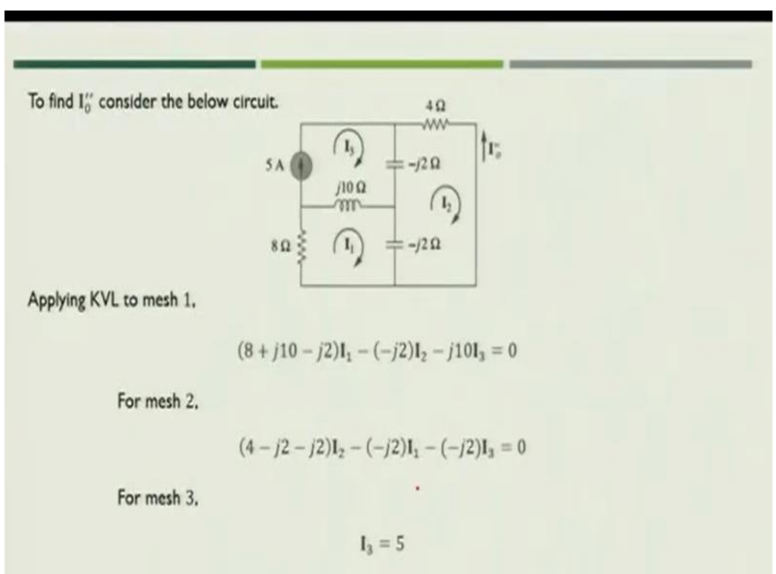

So, your modified circuit will now be as shown in the figure. Here you have 5 ampere current source back in the circuit and the voltage source is short-circuited. Now, in this case, we see there are three loops. So what we will do. We will utilize the Kirchoff's voltage law to find out the loop current.

Applying KVL to mesh 1,

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 = 0$$

For mesh 3,

$$\mathbf{I}_3 = 5$$

So what next you have to do. You put the value of I 3 equations of mesh 1 and 2 we get,

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j10$$

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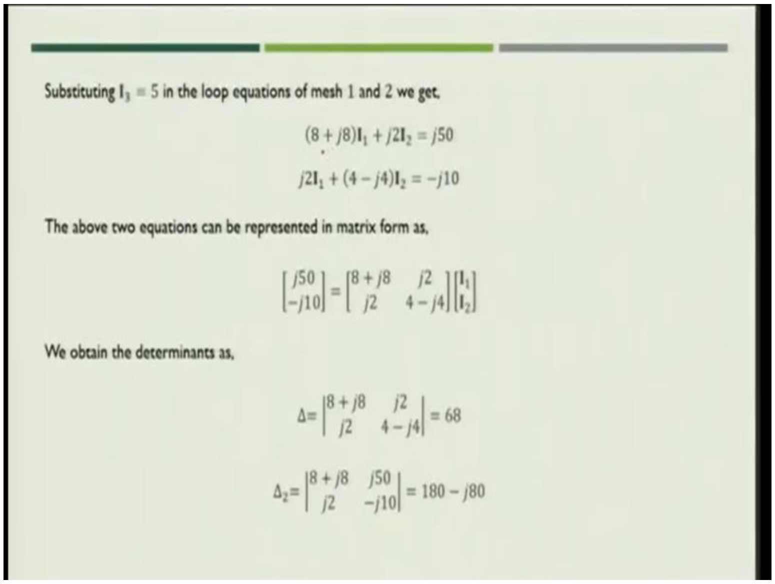

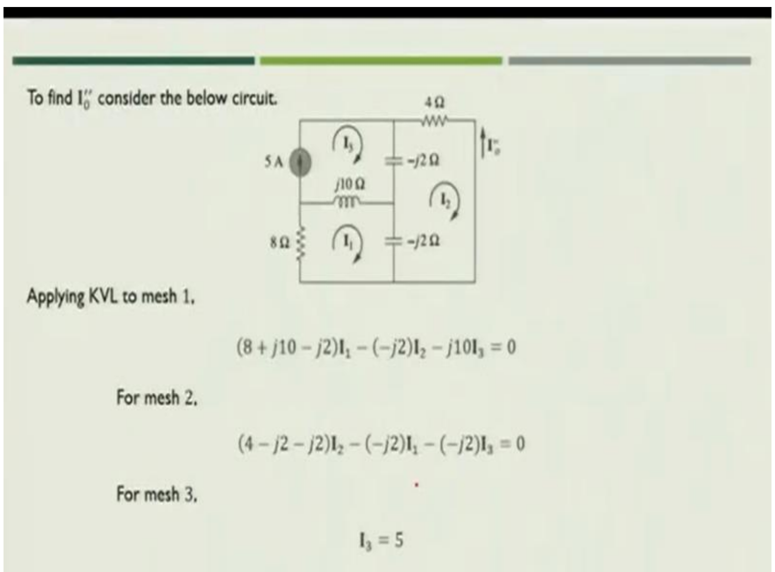

So, now you get two equations and two unknowns I 1 and I 2. You can represent this equation in matrix form.

$$\begin{bmatrix} j50 \\ -j10 \end{bmatrix} = \begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

We obtain the determinants as,

$$\Delta = \begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix} = 68$$

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j10 \end{vmatrix} = 180 - j80$$

Now, to solve it the objective for finding out the solution for this particular equation is to find the value of $\mathbf{I}_0'' = -\mathbf{I}_2$. But,

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{180 - j80}{68} = 2.647 - j1.176 \text{ A}$$

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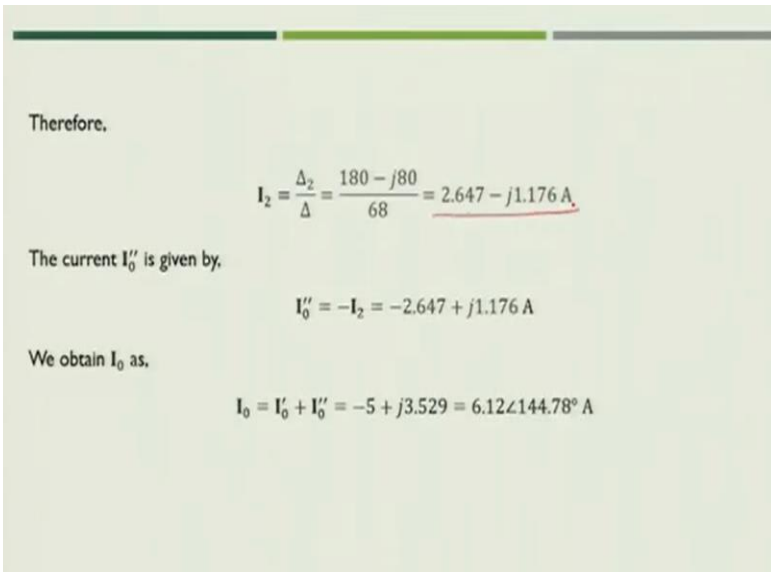

So, when you simplify you get,

The current $\mathbf{I}_0''$ is given by,

$$\mathbf{I}_0'' = -\mathbf{I}_2 = -2.647 + j1.176 \text{ A}$$

We obtain $\mathbf{I}_0$ as,

$$\mathbf{I}_0 = \mathbf{I}_0' + \mathbf{I}_0'' = -5 + j3.529 = 6.12\angle 144.78° \text{ A}$$

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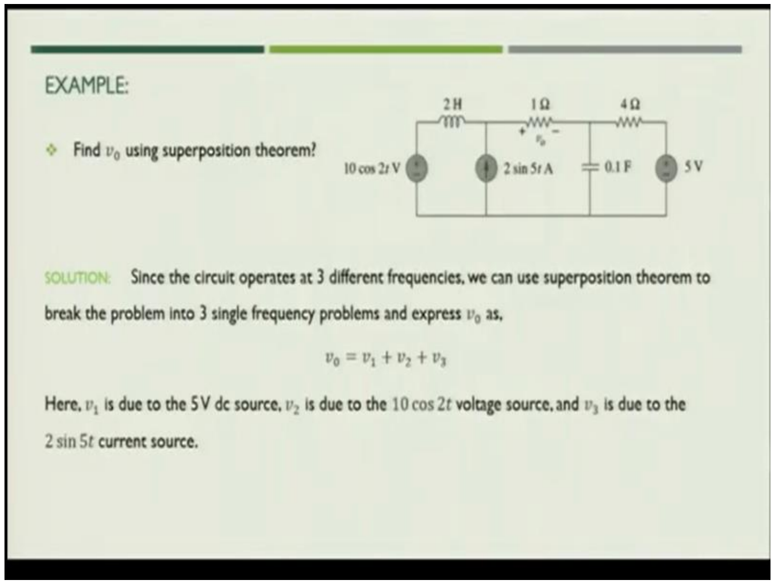

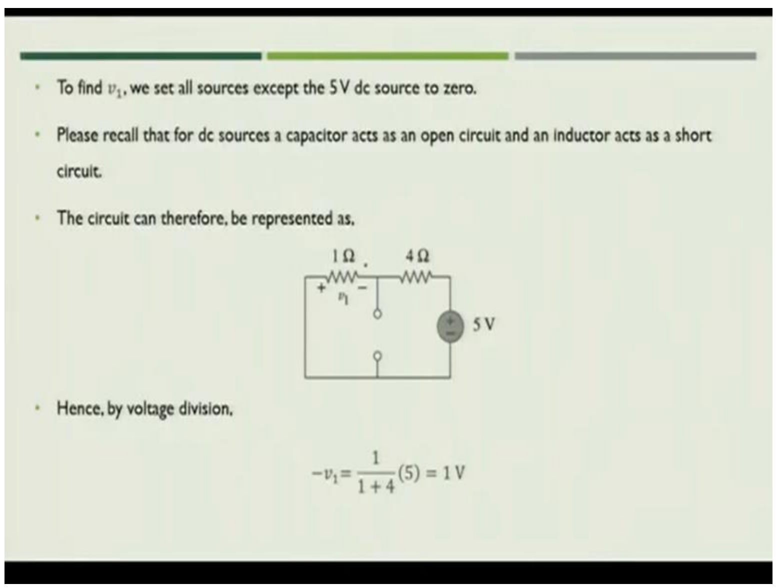

Now, let us take another example. This example will be really interesting because here we need to find out the value of v_0 using superposition theorem and if you see this particular circuit you will have three sources and all three are operating at a different frequency. So this is a dc voltage source. So it means that your frequency is 0. Here you have current source $\omega = 5$, here you have voltage source where $\omega = 2$. So when you have different sources operating at different frequencies we will utilize the superposition theorem by taking one source at a time and solve the circuit and finally we sum up the output so that we get the final output of the circuit. So, in this case, we need to find out the value of v_0.

So let us assume that $v_1,\ v_2,\ v_3$ are the individual voltage responses, where, v_1 is due to the 5 V dc source, v_2 is due to the $10\cos 2t$ voltage source, and v_3 is due to the $2\sin 5t$ current source.. So the final voltage will be $v_0 = v_1 + v_2 + v_3$. Now let us take one voltage at a time. First, let us take the dc voltage 5 V so this will be open-circuited, this will be short-circuited.

Now you have to keep in mind since we are talking about the dc voltage source it means that inductor will also be short-circuited and capacitor will be open-circuited. So in that case what will happen your modified circuit will look like as shown in the figure. You will left only with the resistances that is 1-ohm resistance and 4-ohm resistance.

So 1 ohm resistance you got from this particular section and 4 ohm is from this section where you have capacitor in between but capacitor will be open-circuited and this is short circuit, this is short circuit so the complete left 2 ohm resistance will also be short-circuited, so you will left with only 1 ohm and 4 ohm resistances and 5 volt voltage source.

Now, what we have to do, we need to find out the voltage across 1-ohm resistance. So what we can do, we can utilize the voltage division. So,

$$-v_1 = \frac{1}{1+4}(5) = 1\ \text{V}$$

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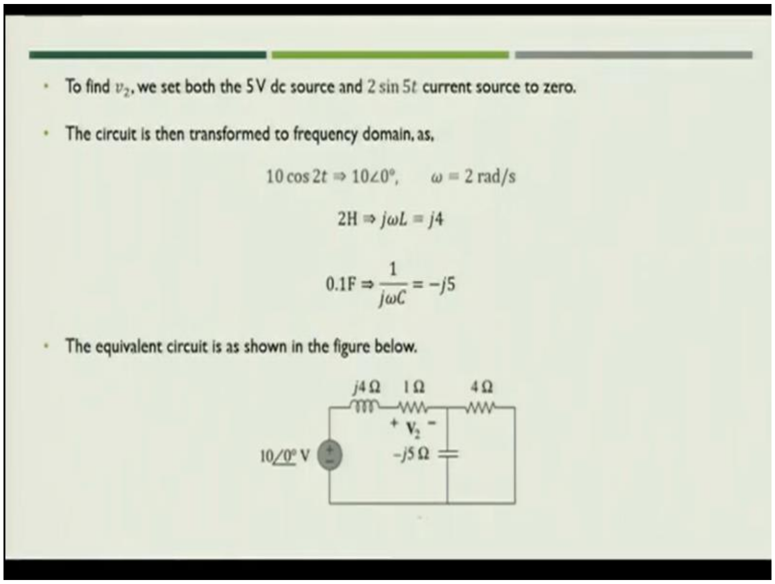

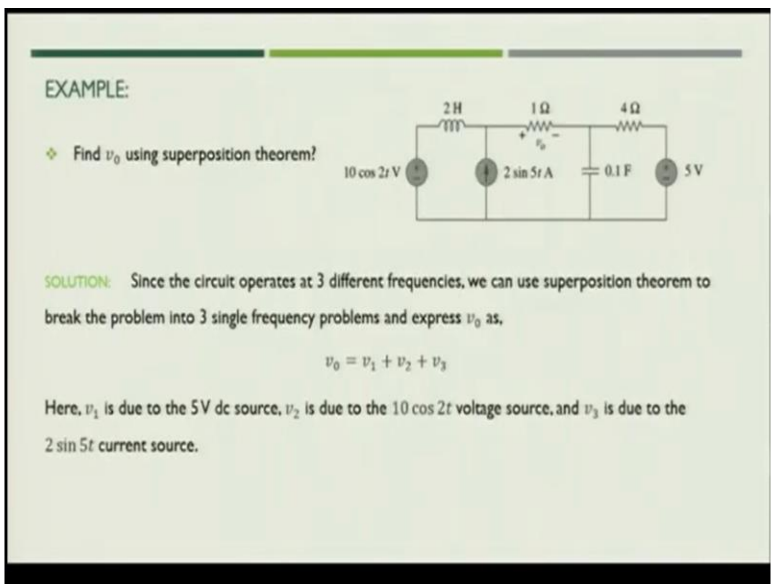

Now the second case, is that in case of v_2 we will set 5-volt dc source and the $2\sin 5t$ current source 0. So that means if you see the figure this is 0 and this is again 0. So now this will be short-circuited, this will be open-circuited. So in that case what will be the modified circuit.

You will have 4-ohm resistance in parallel now with the capacitor and the inductor and resistance will be in series that is 1-ohm resistance in series with the inductance. So the next task what we have to do. If you remember in case of the nodal and mesh analysis we discussed that first the elements need to be converted in frequency domain. So to analyze this particular circuit we will first transform the circuit into frequency domain.

So,

$$10\cos 2t \Rightarrow 10\angle 0°,\ \ \omega = 2\ \mathrm{rad/s}$$

$$2\mathrm{H} \Rightarrow j\omega L = j4$$

$$0.1\mathrm{F} \Rightarrow \frac{1}{j\omega C} = -j5$$

Now your circuit is ready for the analysis. We need to find out the voltage $\mathbf{V}_2$ across 1-ohm resistance. Now since these two elements are in parallel, so you can take the parallel combination of these two elements and you can find out the impedance. So what is the impedance Z?

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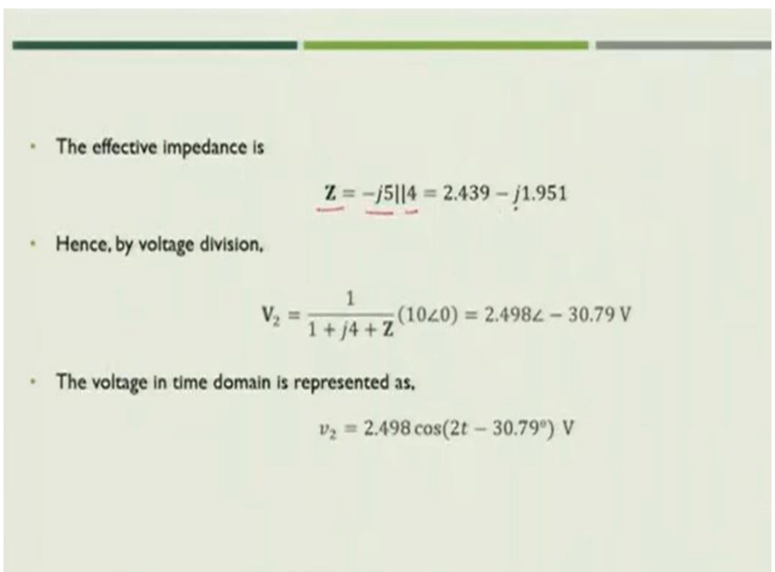

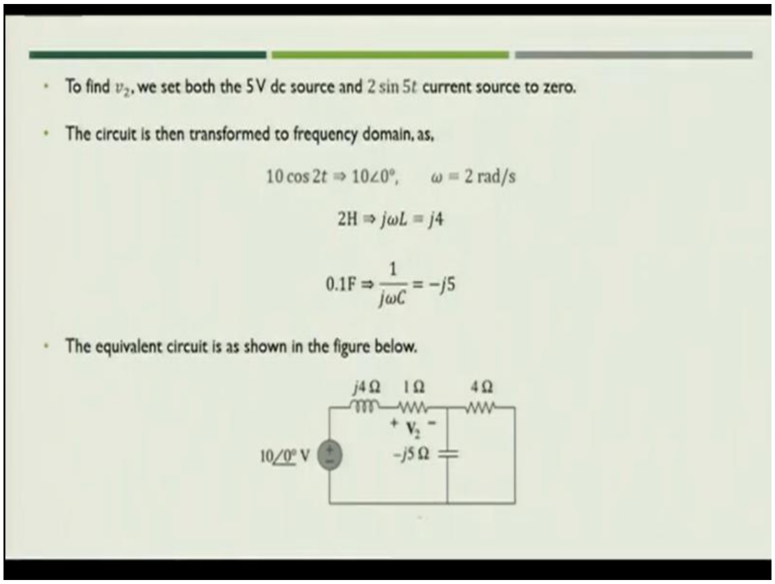

So in this case,

$$\mathbf{Z} = -j5||4 = 2.439 - j1.951$$

Hence, by voltage division,

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\angle 0) = 2.498\angle - 30.79 \text{ V}$$

The voltage in time domain is represented as,

$$v_2 = 2.498\cos(2t - 30.79°)\ \text{V}$$

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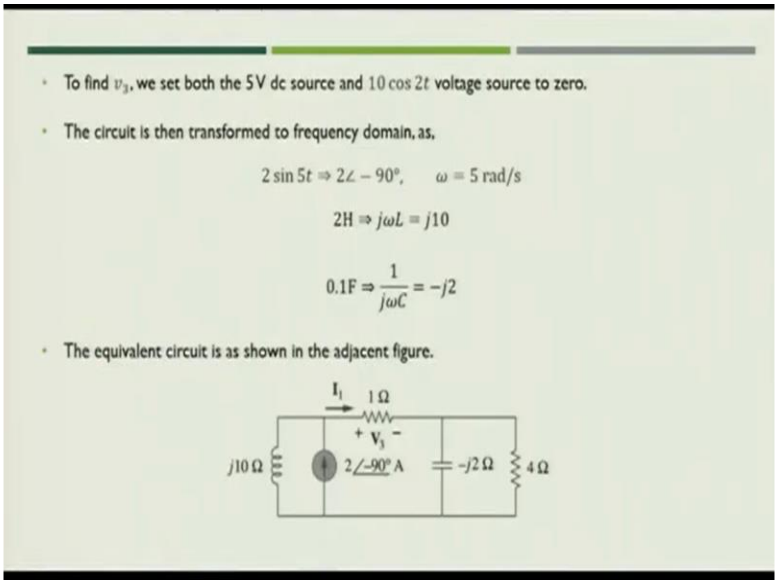

Now, the third one is the current source. So, we need to find out the value of v_3 when the 5 V dc source and $10\cos 2t$ voltage source to zero. It means that both of them will be short-circuited. So your modified circuit will look like as shown in the figure where the current source now will be in parallel with the inductance. Then we have 1-ohm resistance and in series with parallel combination of minus j 2 impedance offered by the capacitance and 4-ohm resistance.

Now first task is that again we have to transform this into frequency domain. So we will convert it into frequency domain as,

$$2\sin 5t \Rightarrow 2\angle-90°,\ \ \omega = 5\ \text{rad/s}$$

$$2\text{H} \Rightarrow j\omega L = j10$$

$$0.1\text{F} \Rightarrow \frac{1}{j\omega C} = -j2$$

Then what we have to find is the voltage across 1-ohm resistance so that is $\mathbf{V}_3$ which we have to find. So first what we have to do? So first we have to take these two the parallel combinations and simplify it.

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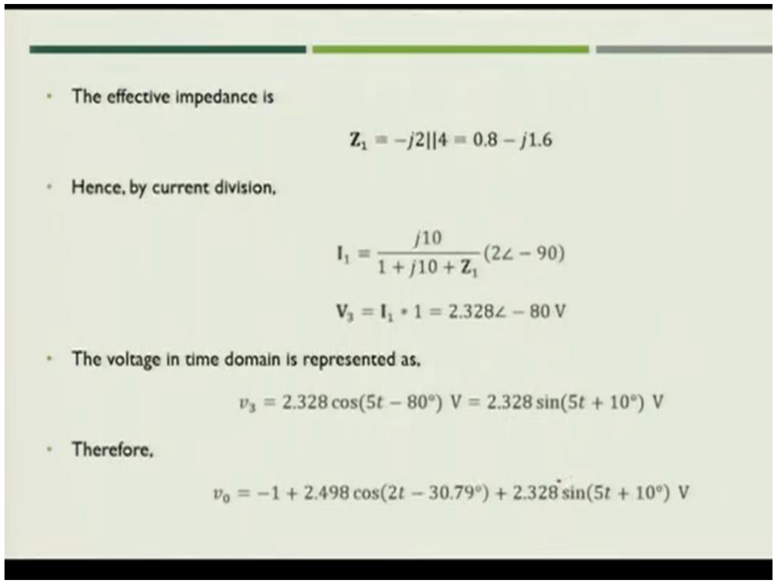

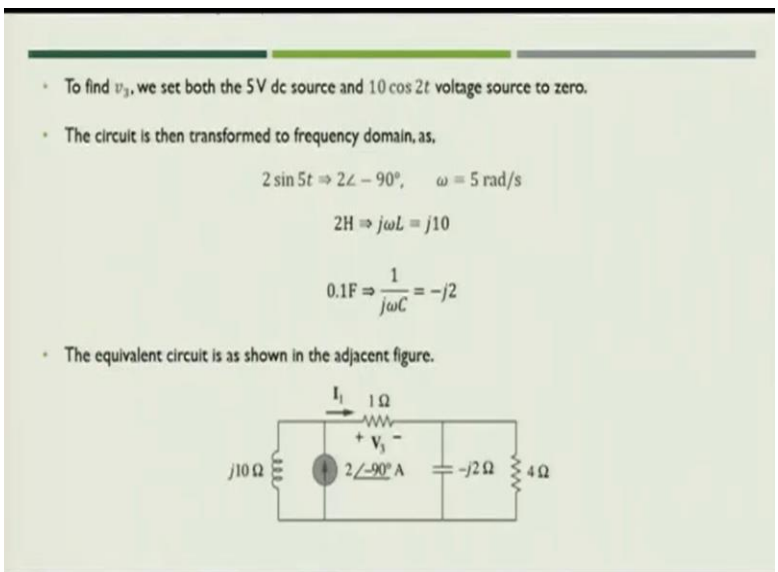

So, when you take these two, let us say the effective impedance is,

$$\mathbf{Z}_1 = -j2||4 = 0.8 - j1.6$$

Hence, by current division,

$$\mathbf{I}_1 = \frac{j10}{1 + j10 + \mathbf{Z}_1}(2\angle - 90)$$

$$\mathbf{V}_3 = \mathbf{I}_1 * 1 = 2.328\angle - 80 \text{ V}$$

The voltage in time domain is represented as,

$$v_3 = 2.328\cos(5t - 80°)\ \mathrm{V} = 2.328\sin(5t + 10°)\ \mathrm{V}$$

Now the total response,

$$v_0 = -1 + 2.498\cos(2t - 30.79°) + 2.328\sin(5t + 10°)\ \mathrm{V}$$

So now you can see that superposition theorem works well when you have voltage or current sources connected to the circuit and having the different frequencies.

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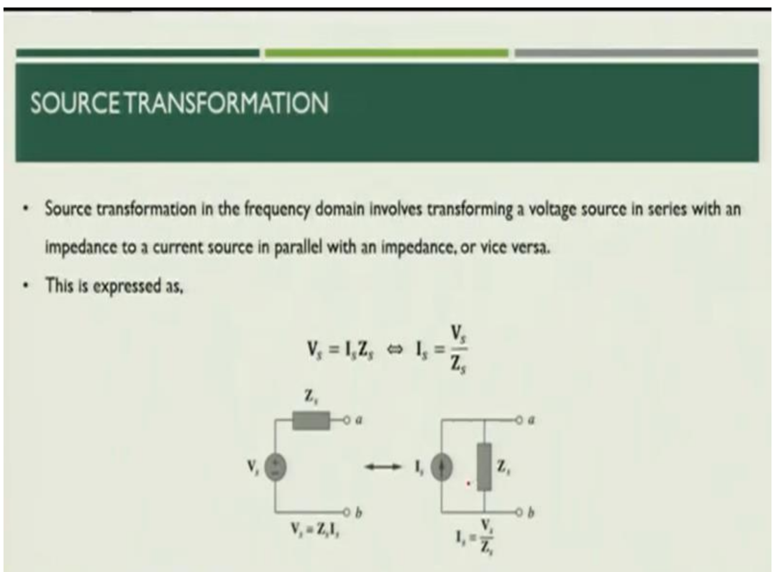

Now let us talk about the source transformation. Now in case of source transformation the, in frequency domain the source transformation involves the transforming a voltage source in series with impedance to a current source in parallel with impedance or vice versa that means if you have current source in parallel with impedance can be converted into voltage source in series with an impedance.

How we will express that? We will express it as $\mathbf{V}_S$, so this is voltage source in series with $\mathbf{Z}_S$ that is impedance. So, $\mathbf{V}_S = \mathbf{I}_S\mathbf{Z}_S \Leftrightarrow \mathbf{I}_S = \frac{\mathbf{V}_S}{\mathbf{Z}_S}$ and then in parallel with impedance Zs, So you can transform the voltage source in series with impedance to a current source in parallel with impedance as shown in the figure. So we will utilize this concept to solve the circuit.

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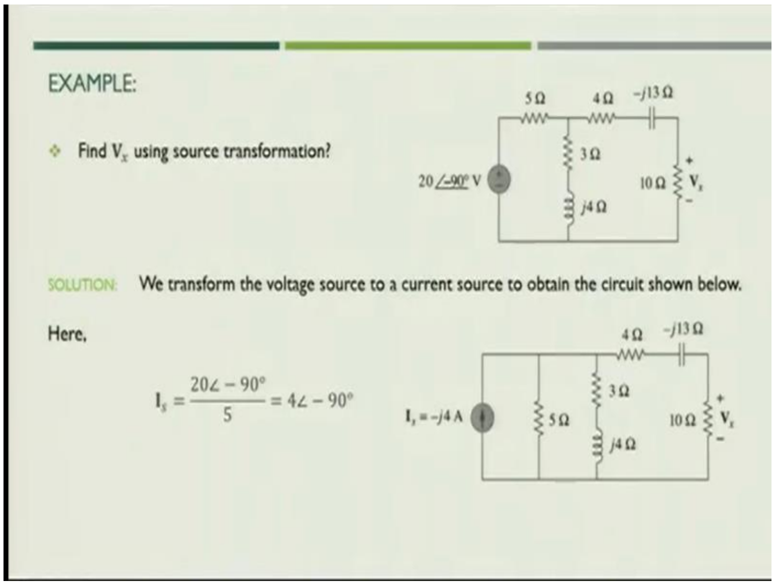

So let us take one example. we need to find out the value of $\mathbf{V}_x$ using source transformation. So in this case, how we will carry out the source transformation. You see that you have voltage source $20\angle -90°$ connected in series with 5 ohm. So this particular segment you can utilize to convert into equivalent current source in parallel with resistance. This can be converted into a current source as,

$$\mathbf{I}_s = \frac{20\angle -90°}{5} = 4\angle -90°$$

This current source will be in parallel with 5-ohm resistance. Now if you see the circuit these two legs are in parallel so you can further simplify then

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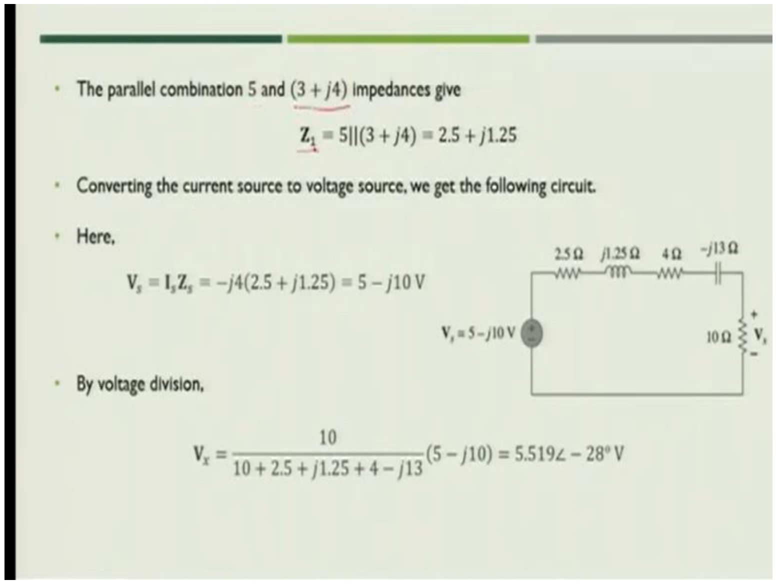

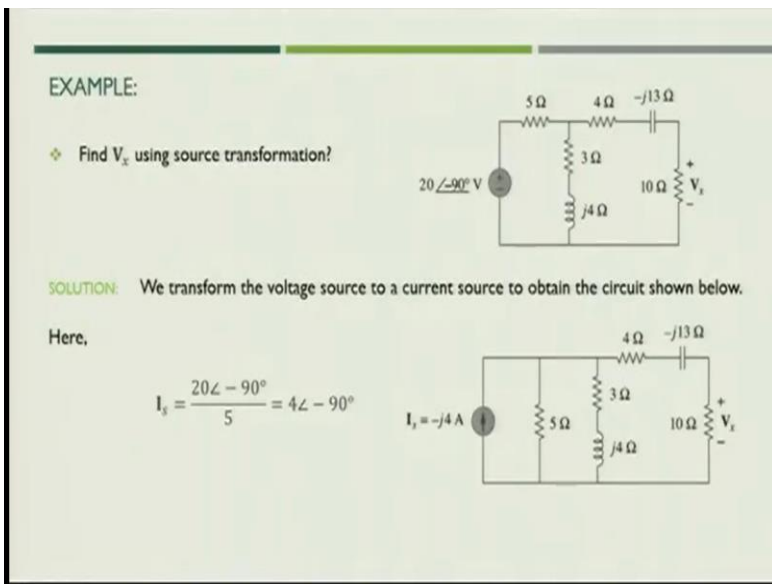

The parallel combination of 5 and $(3 + j4)$ will be simplified to,

$$\mathbf{Z}_1 = 5||(3 + j4) = 2.5 + j1.25$$

So if you see that the equivalent impedance is Z1 connected in parallel. So you can say again the current source is in parallel with impedance. So you can convert it back into a voltage source in series with the impedance. So what will be the value of voltage source in that case? The value of voltage source will be,

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_s = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$

- By voltage division,

$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\angle - 28° \text{ V}$$

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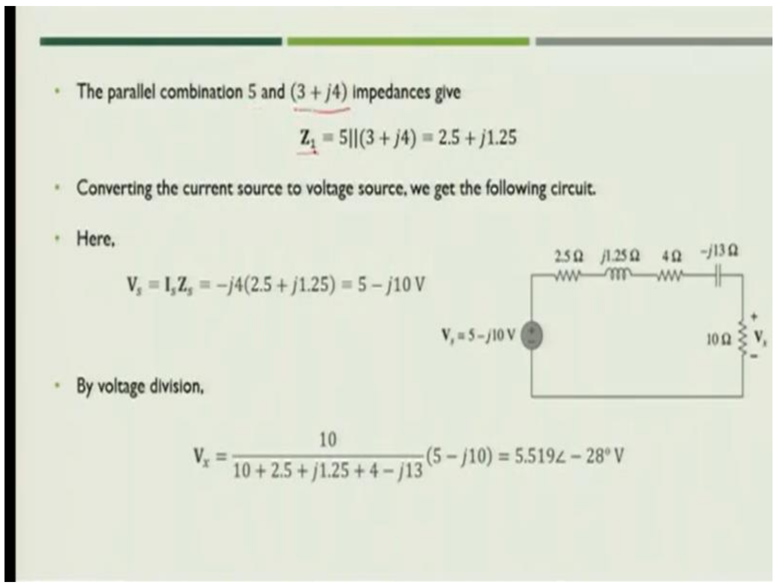

Now you can say that by successive implementation of source transformation in the circuit, you can simplify the network and find the value of $\mathbf{V}_x$ very easily.

So with this, we can close our today's session in which we discussed about the source transformation as well as superposition theorem. In next session, we will discuss about two more important theorems which we discuss in case of dc circuit that are your Thevenin's and Norton's theorem. We will discuss when we consider the ac circuit for the analysis. Thank you.