**Basic Electric Circuit**
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**Lecture 41**
**Sinusoidal Steady State Analysis 1**
**Nodal and Mesh Analysis**

Namaskar. We are entering into the ninth week of our course, till now we discussed various circuit theorems, with respect to dc source, these 2 weeks, that is ninth and tenth week will devote more towards circuit analysis, with the ac sources available. We will say that the ac circuit analysis with the help of what we discussed till now in case of dc source.

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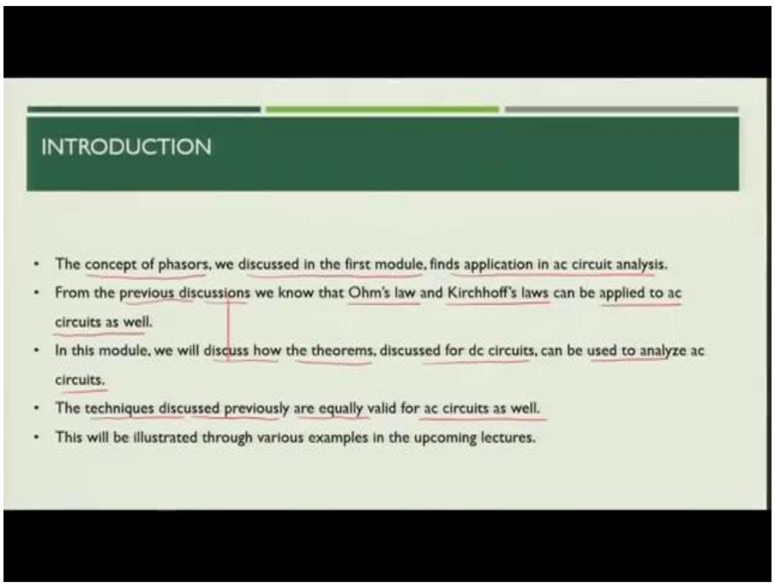


Let us start the discussion of the circuit analysis. We will use the sinusoidal steady state, state analysis concepts to understand the various phenomena of ac circuit now as we know that the concept of phasors was discussed in the first module, the concept of phasors we will use and the circuit analysis for the cases where we have the ac source available. The concept of phasor is very important in the case of ac circuit analysis.

We also know that the previous discussions we had based on basic laws like ohms law Kirchhoff's law, the same can be applied to ac circuit also. In this module we will discuss particularly on how the theorems which we discussed in case of dc circuit can be used to analyze the ac circuits. The techniques we used in previous lectures, we can equally use those techniques for our ac circuit analysts. We take the help of various examples to understand how the various theorems which we use in case of dc circuit can be utilized in ac circuit as well.

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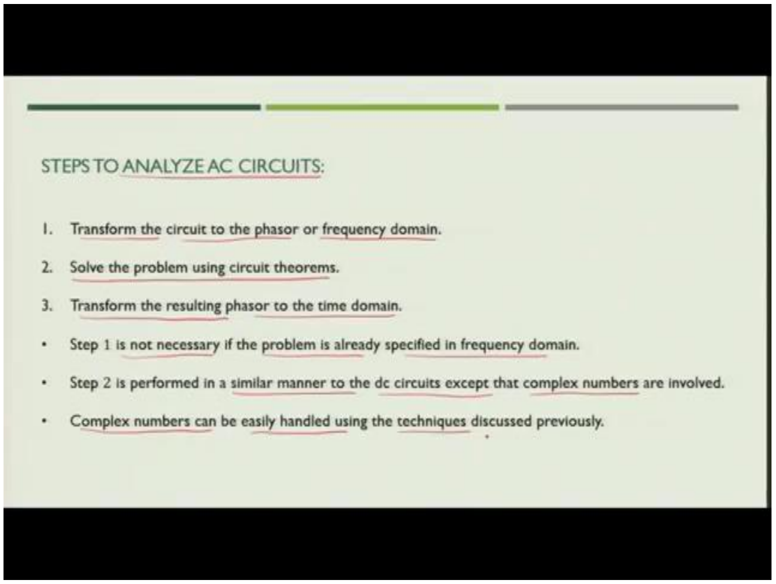


Now what are the steps to analyze the ac circuit? First, we will transform the circuit to the phasor or frequency domain. That means if the circuit is given in time domain will first convert the circuit into phasor or frequency domain. Then we will solve the problem using various circuit theorem which we discussed previously in case of dc circuits and then transform the resulting phasor to the time domain back.

Now step 1 is not necessary if the problem is already specified in frequency domain. So that means if you already have the problem given in phasor or frequency domain, we need not to follow the first step. We can straight away start from second step. The second step is performed in a similar manner to what we did in case of dc circuits.

The only changes that here instead of real numbers, as we used in case of dc circuit here, we will use complex numbers for ac circuit analysts. Now the complex numbers we can easily handle because we have already discussed how to deal with the complex numbers when we were discussing about the phasors and the other fundamental theorems of the circuit analysis.

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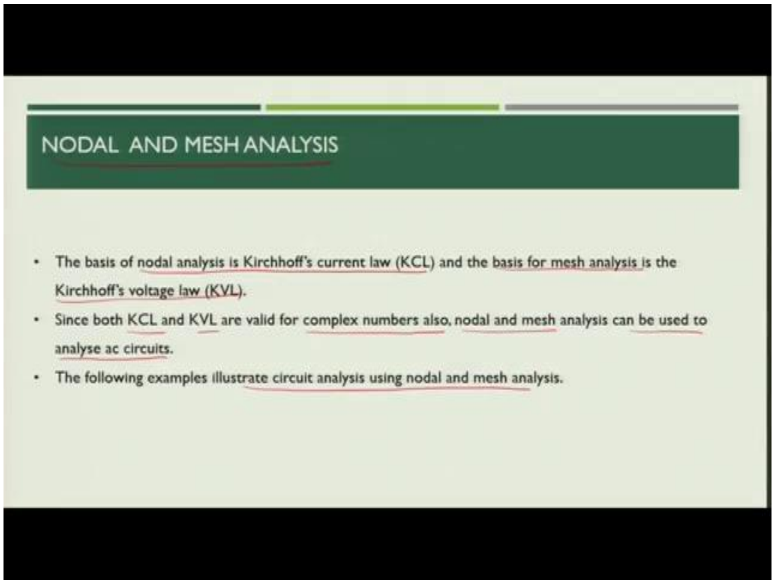


Let us try to understand the nodal and mesh analysis first in case of ac circuit. The basis of nodal analysis is Kirchhoff current law and the basis for mesh analysis that is also called as loop analysis is based on Kirchhoff voltage law. Now, since KCL and KVL, both are valid for complex number. Also, the nodal and mesh analysis can be used in case of ac circuits. Let us see a few of the examples so that we can understand how we will utilize KCL and KVL when we are asked to find the various circuit parameters in case of ac circuit.

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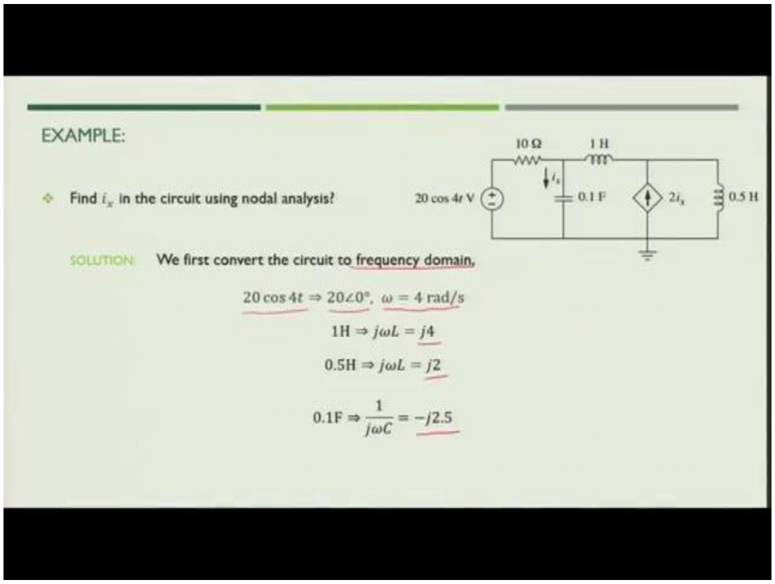


Let us see one example. In this case you will see that we have one sinusoidal voltage source or given by $20\cos 4t$ and we have the inductor of 1H. We also have the dependent current source and the 0.1F capacitor, 0.5H inductor. The first task, what we must do is that we have to convert this particular circuit into frequency domain.

That means we will convert the sources into phasor and the elements which we see in the circuit will be converted into the frequency domain. So now let us see this voltage source,

$$20\cos 4t \Rightarrow 20\angle 0°,\ \ \omega = 4\ \text{rad/s}$$

Now this $\omega$ will help us to transform the various circuit elements into frequency domain.

The 1H inductor which we see in the figure will be converted into the frequency domain. It means that the inductor will be converted as $1\text{H} \Rightarrow j\omega L = j4$. In case of 0.5 Henry inductor, you will get $0.5\text{H} \Rightarrow j\omega L = j2$. For the capacitor it is, $0.1\text{F} \Rightarrow \frac{1}{j\omega C} = -j2.5$

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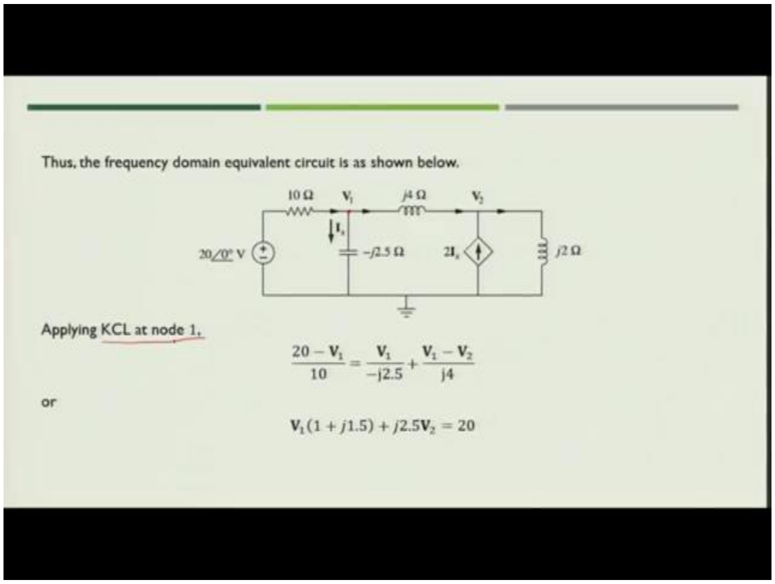


Your voltage source will be $20\angle 0°$, there will be no change in the value of resistance. Then we will put the value of the inductors and capacitor, in frequency domain. Now we will apply the KCL and node 1. If you apply KCL at node 1, what you can write, the current direction is in this site which is going inside the node and other 2 currents are going outside the node So we can write,

$$\frac{20-\mathbf{V}_1}{10}=\frac{\mathbf{V}_1}{-j2.5}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j4}$$

or

$$\mathbf{V}_1(1+j1.5)+j2.5\mathbf{V}_2=20$$

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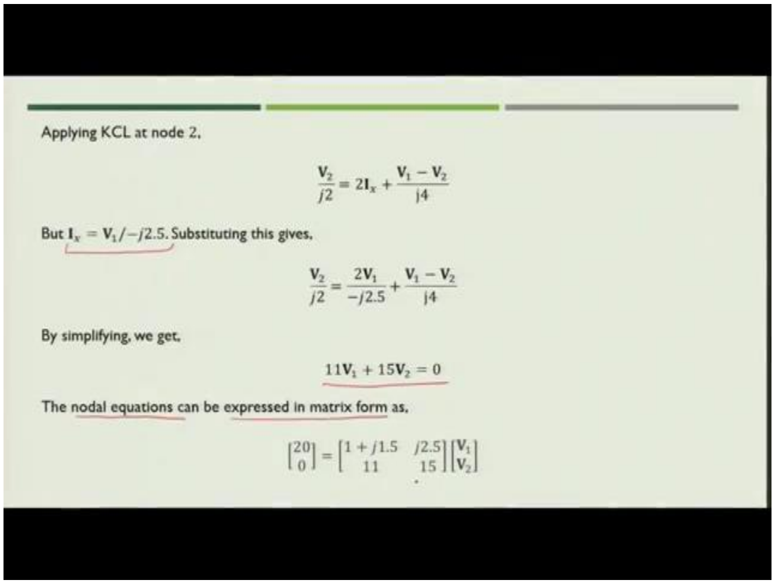


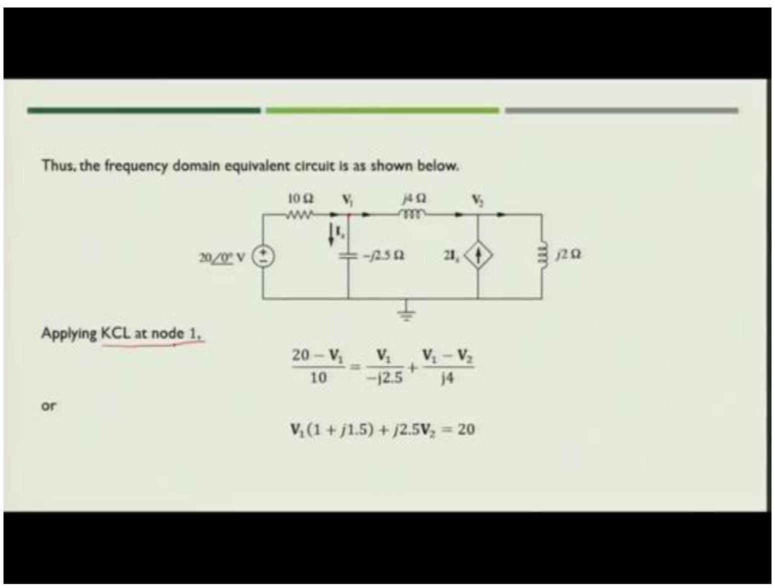


Similarly, if you apply KCL at node 2, you get,

$$\frac{\mathbf{V}_2}{j2} = 2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathrm{j}4}$$

Now we also know that $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Replacing this in the above equation gives

$$\frac{\mathbf{V}_2}{j2} = \frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathrm{j}4}$$

By simplifying, we get,

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

These 2 nodal equations we can express in the form of matrix as,

$$\begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

So now you have compiled these 2 equations in matrix form. Why we are compiling in matrix form because solving equation in matrix form is easier. How? Let us first take the determinant value of this metrics.

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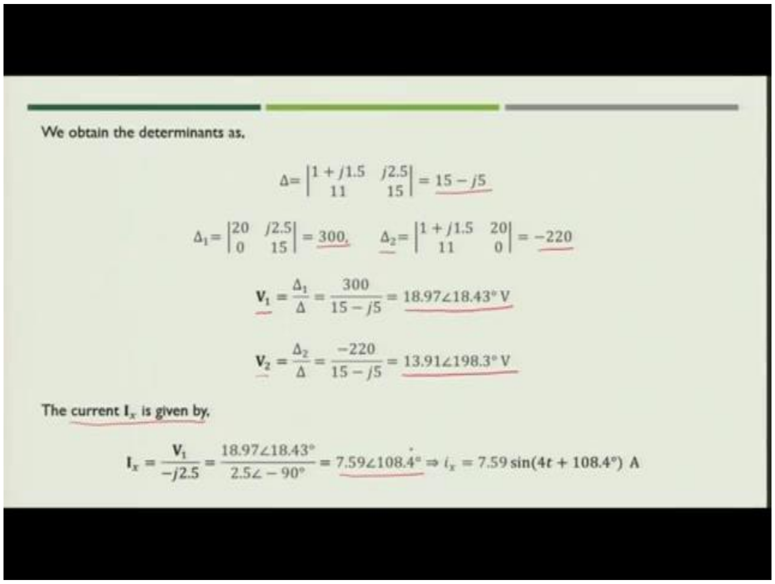


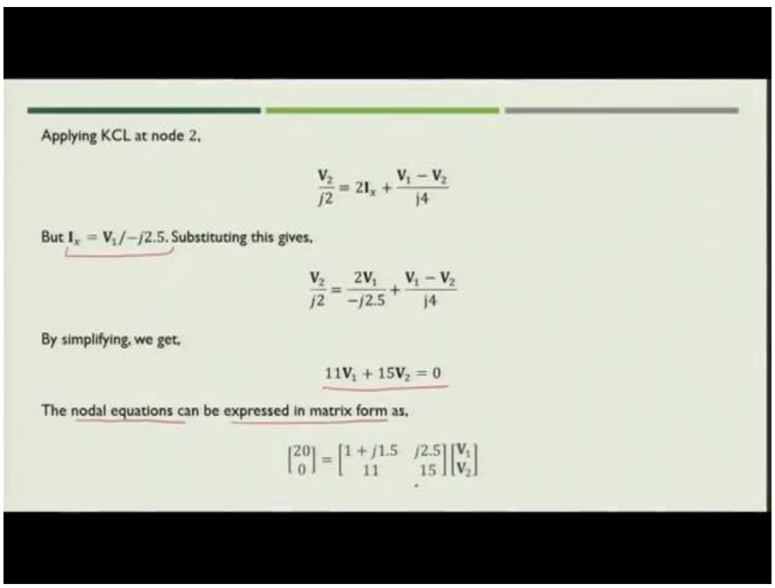


So,

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \quad \Delta_2 = \begin{vmatrix} 1+j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

Now the value of $\mathbf{V}_1$,

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\angle 18.43° \text{ V}$$

Similarly, in case of $\mathbf{V}_2$, you will find out,

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\angle 198.3° \text{ V}$$

Now the value of current $\mathbf{I}_x$,

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43°}{2.5\angle -90°} = 7.59\angle 108.4° \Rightarrow i_x = 7.59 \sin(4t + 108.4°) \text{ A}$$

With this method you can easily find out the value $\mathbf{I}_x$which was asked in the question.

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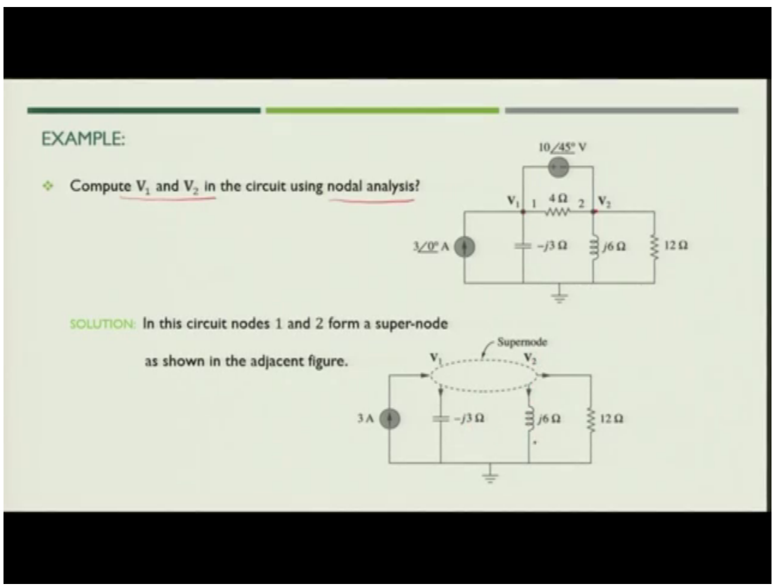


Now let us take another example where we are asked to find out the value of $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit. We will again use the nodal analysis. Now if you see this figure, the voltage or node, $\mathbf{V}_1$ and $\mathbf{V}_2$ are connected through one voltage source. So, we can create 1 super node, which includes $\mathbf{V}_1$ and $\mathbf{V}_2$, because $\mathbf{V}_1$ and $\mathbf{V}_2$are connected through some voltage source. We create one super node which includes $\mathbf{V}_1$ and $\mathbf{V}_2$ and we will apply KCL for this super node.

If you apply KCL at the super node, so what you get, you get ampere as a current going inside and other 3 currents are going out. We can write,

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j3} + \frac{\mathbf{V}_1}{12}$$

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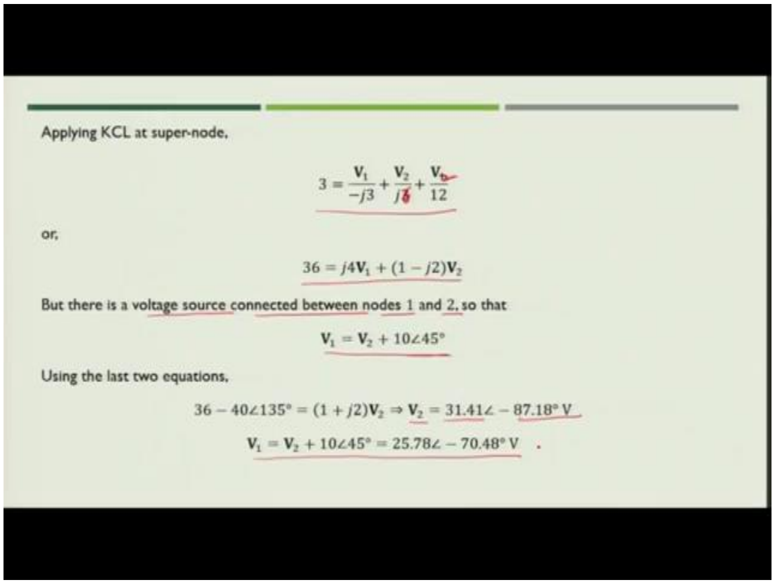


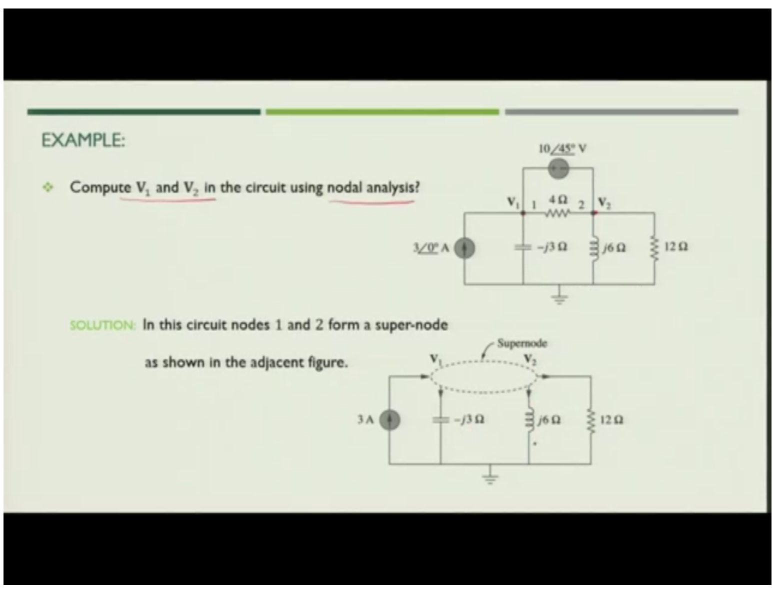


We will simplify this and when we simplify, we get the equation as

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2$$

Now we also know that the voltage source is connected between nodes 1 and 2,

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45°$$

Using the last two equations,

$$36 - 40\angle 135° = (1 + j2)\mathbf{V}_2 \Rightarrow \mathbf{V}_2 = 31.41\angle -87.18° \text{ V}$$

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45° = 25.78\angle -70.48° \text{ V}$$

Using nodal analysis you can easily find out the value of $\mathbf{V}_1$ and $\mathbf{V}_2$.

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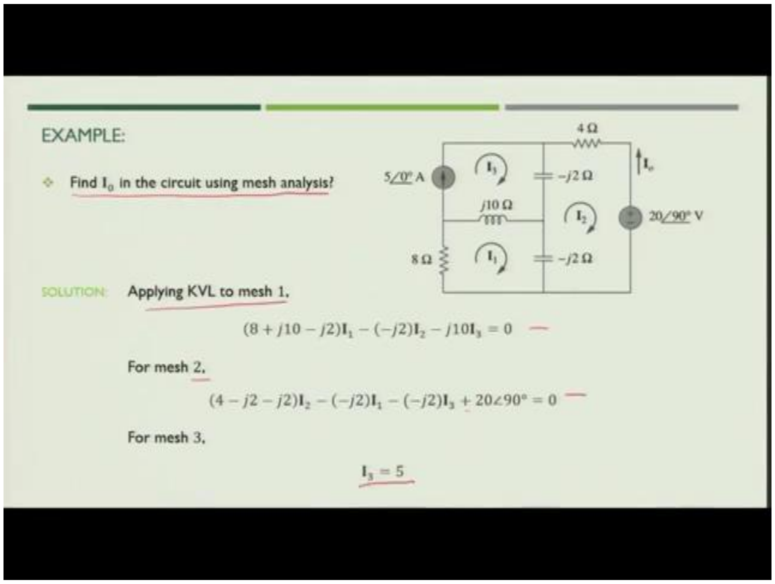


Next, we see how we will utilize the concept of mesh analysis. In the previous two examples, we used nodal analysis. In this example, we will try to understand how we will apply the mesh analysis. So now if you see this figure in this figure, we see there are 3 loops. We give the value of current as $\mathbf{I}_1$ for this loop $\mathbf{I}_2$ for this loop and $\mathbf{I}_3$ for the top loop. Now what we have to do, we have to apply the KVL. So if you have apply KVL to mesh 1,

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

Similarly, for mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90° = 0$$

For loop 3,

$$\mathbf{I}_3 = 5$$

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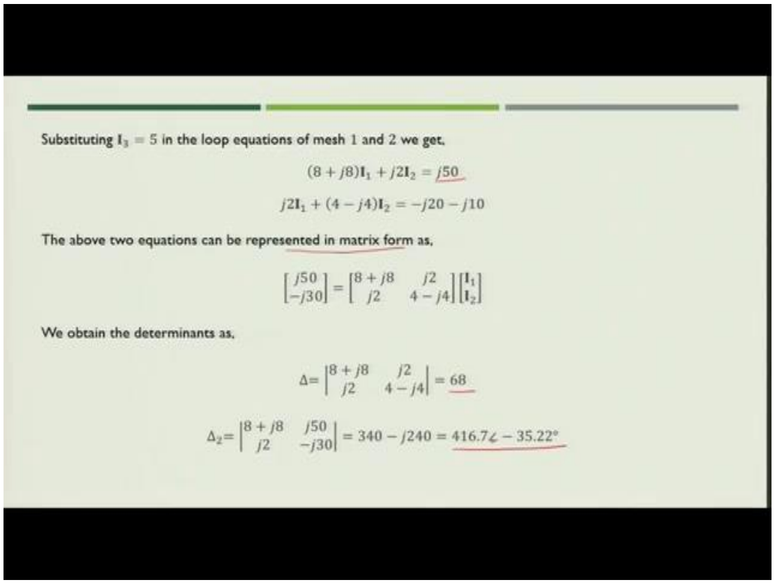


So when you simplify you will get

$$(8+j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50$$

$$j2\mathbf{I}_1 + (4-j4)\mathbf{I}_2 = -j20 - j10$$

Now again, what we can do, we can compile these 2 equations in matrix form as

$$\begin{bmatrix} j50 \\ -j30 \end{bmatrix} = \begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Now as we did in the previous example, in this case also, we will first determine the value of determinant of this matrix as

$$\Delta = \begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix} = 68$$

Now next is we need to find out the value of $\Delta_2$ because we need to find out the value of I 2. So

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.7\angle - 35.22°$$

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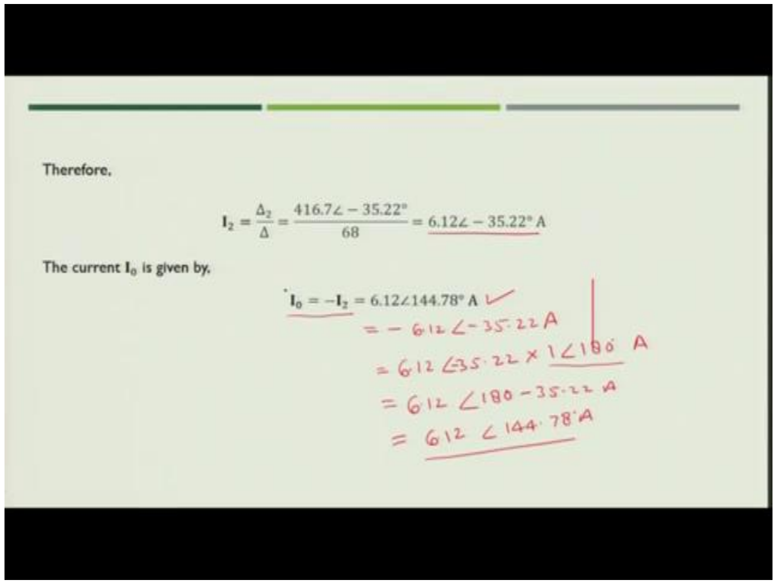


Now let us try to find out the value of current $\mathbf{I}_2$.

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.7\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

Now in the question, we need to find out the value of current $\mathbf{I}_0$. But, $\mathbf{I}_0 = -\mathbf{I}_2$,

$$\mathbf{I}_0 = -\mathbf{I}_2 = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} = 6.12\angle 144.78^\circ \text{ A}$$

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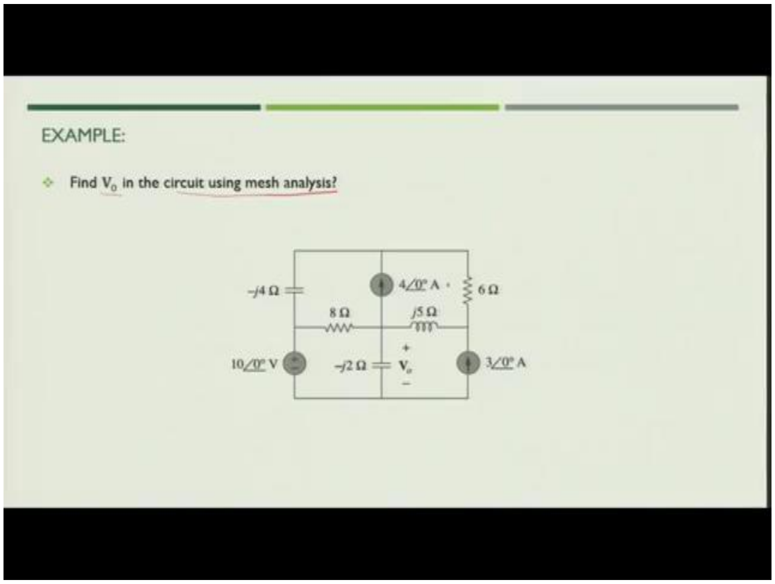


So next, let us try to find out the value of $\mathbf{V}_0$ in this circuit using mesh analysis. We have 1 current source coming between 2 meshes that is between 2 loops. We, therefore, can create a super mesh because if you have current source between 2 meshes, we can create super mesh and solve it.

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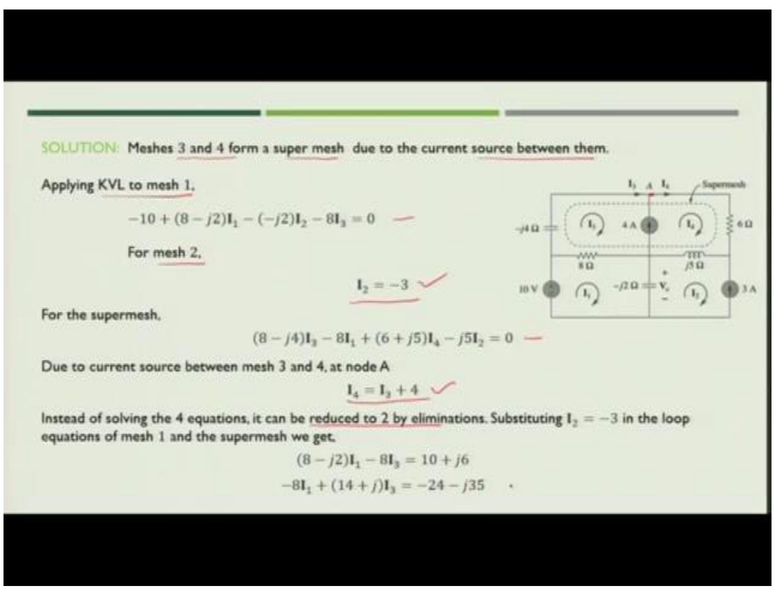


Meshes 3 and 4 will now form this upper mesh due to current source between them. We then apply KVL first to mesh 1. When you applied KVL to mesh 1, it will become,

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

For mesh 2,

$$\mathbf{I}_2 = -3$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0$$

Due to current source between mesh 3 and 4, at node A

$$\mathbf{I}_4 = \mathbf{I}_3 + 4$$

So now we have 4 equations.

Using $\mathbf{I}_2$ and the value of $\mathbf{I}_4$, we can put these 2 values in these 2 equations and simplify. So when we simplify we reduce it to only 2 equations as,

$$(8 - j2)\mathbf{I}_1 - 8\mathbf{I}_3 = 10 + j6$$

$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35$$

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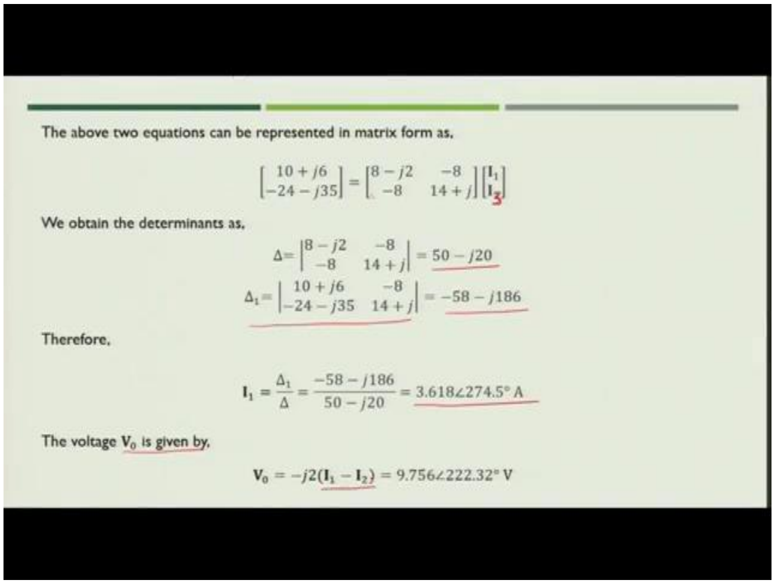


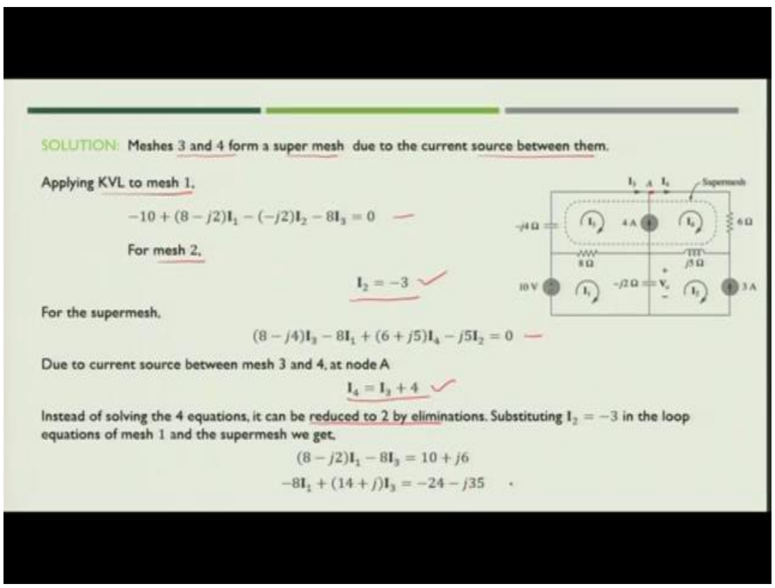


So we have now these 2 equation which we can compile in the form of matrix as,

$$\begin{bmatrix}10+j6\\-24-j35\end{bmatrix}=\begin{bmatrix}8-j2 & -8\\-8 & 14+j\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\\mathbf{I}_2\end{bmatrix}$$

We obtain the determinants as,

$$\Delta=\begin{vmatrix}8-j2 & -8\\-8 & 14+j\end{vmatrix}=50-j20$$

$$\Delta_1 = \begin{vmatrix} 10+j6 & -8 \\ -24-j35 & 14+j \end{vmatrix} = -58 - j186$$

Therefore,

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\angle 274.5° \text{ A}$$

The voltage $\mathbf{V}_0$ is given by,

$$\mathbf{V}_0 = -j2(\mathbf{I}_1 - \mathbf{I}_2) = 9.756\angle 222.32° \text{ V}$$

Using these techniques which we discuss in case of a dc circuit analysis, same we applied in the case of ac circuit analysis and solved the circuit so with this week close over today's session thank you.