Basic Electric Circuits
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Module 8 - Two Port Network
Lecture 40 - Interconnection of networks

Namaskar, so in this week we discussed about few two-port parameters, precisely, z parameters, y parameters and then h parameters, g parameters. Then in the last session we discussed about the T parameters that is transmission parameters. Now, when the system is very large and complex, we cannot analyse simply by putting one two-port network, so in that case it is required that the complete system can be subdivided into multiple two-port networks and those can be connected either in series, parallel or maybe in cascaded formation.

So today we will discuss about how we will interconnect various two-port networks and what would be the best options in a different topological condition such as series, parallel or cascading of the two-port networks.

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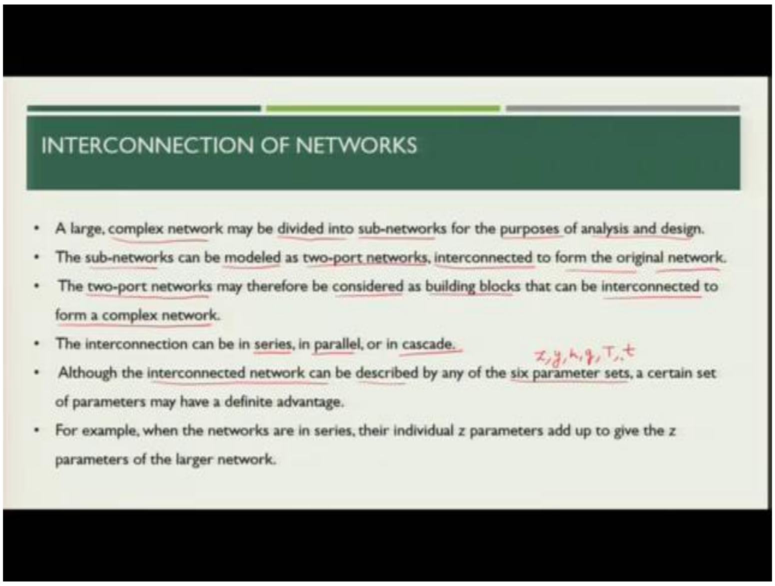

So, let us start the interconnection of the networks, so as we discussed that the large complex network can be divided into sub networks for the purposes of analysis and design. The sub networks can be modelled as two-port networks, so the sub networks which we create out of the large and complex network, we will convert them as a two-port network, and we will interconnect them to perform the original network. So, we can now say that the two-port networks can be considered as a building blocks and that can be interconnected to form a complex network.

As we discussed that interconnection can be either in series, parallel or in cascaded format. Now, although the interconnected network can be described by any of the 6 parameter sets, what were those? We discussed about z parameters, then we discussed about y parameters, then h parameters, then g parameters, then transmission parameters and the inverse transmission parameters, so these were the 6 parameters based on two-port networks we discussed in our previous lectures.

Now, we can describe any interconnected network with the help of these 6 parameter sets, but there are certain sets or parameters which may have a definite advantage, how? Let us take an example that suppose the network is in series means the two-port networks these are whatever the sub networks we have created, if these sub networks are in series then their individual z parameters add up to give the z parameters of the large network. In that case where we see that the networks, sub networks are connected in series, it is better to utilise the z parameters to find out the overall parameter of the larger network.

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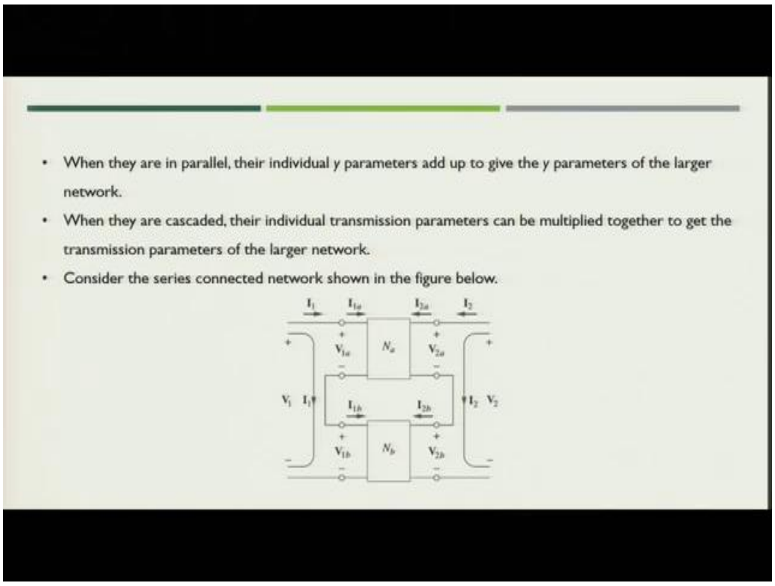

Similarly, in the case when the two-port networks are connected in parallel, in that case y parameters can add up to give the y parameters of the larger network. There may be some cascaded networks also where the output of one network goes as an input to other two-port network, in that case individual transmission parameters can be multiplied together to get the transmission parameters of the larger network. Now these 3 combinations that is series, parallel and cascaded combinations are very important for the circuit analysis, so we will see now how we can utilise these parameters when we interconnect the various networks either in series, parallel or cascaded manner.

So, let us take first case that the network is series connected, so if you see in the figure these, we have the two networks, one is network a and second is network b. Now, these two are in series because the individual voltages that is $\mathbf{V}_{1a}$ and $\mathbf{V}_{1b}$ add up to give the voltage of the complete network that is $\mathbf{V}_1$.

Also, the current $\mathbf{I}_1$ which is flowing in the network a will also flow in network b because in case of series network the current flowing through all the networks will remain same. So, here we can say that the network is connected in series because you are adding up the voltages to get the complete network.

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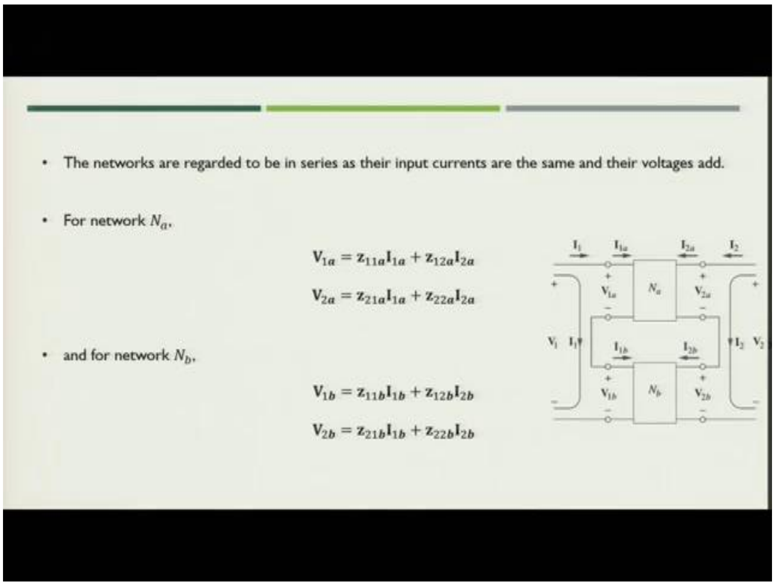

Now, as we know that the total voltage $\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b}$, we will first write the individual z parameter equations for both networks for network a what we can write? We can write $\mathbf{V}_{1a} = \mathbf{z}_{11a}\mathbf{I}_{1a} + \mathbf{z}_{12a}\mathbf{I}_{2a}$. Similarly, $\mathbf{V}_{2a} = \mathbf{z}_{21a}\mathbf{I}_{1a} + \mathbf{z}_{22a}\mathbf{I}_{2a}$, so you can see from this figure this would be the Z parameter equations of this network will be the equation which we just saw. Similarly, for second two-port network that is N b we can write $\mathbf{V}_{1b} = \mathbf{z}_{11b}\mathbf{I}_{1b} + \mathbf{z}_{12b}\mathbf{I}_{2b}$, $\mathbf{V}_{2b} = \mathbf{z}_{21b}\mathbf{I}_{1b} + \mathbf{z}_{22b}\mathbf{I}_{2b}$.

Now, if you see this figure, we can simply say that since the two-port networks are connected in series that means $\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b}$ because the same current will flow in both ports. Similarly, $\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b}$ so these two equations we can use to simplify these four equations which we wrote independently for network a and network b.

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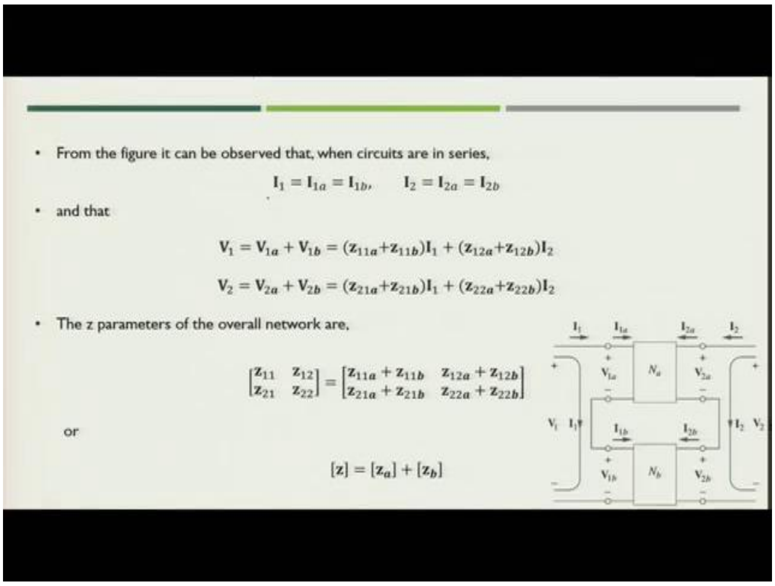

Since we know that $\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b}, \ \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b}$ and $\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b}$, we get

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} = (\mathbf{z}_{11a} + \mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a} + \mathbf{z}_{12b})\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} = (\mathbf{z}_{21a} + \mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a} + \mathbf{z}_{22b})\mathbf{I}_2$$

Now, if you see these 2 equations what you can write,

$$\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11a} + \mathbf{z}_{11b} & \mathbf{z}_{12a} + \mathbf{z}_{12b} \\ \mathbf{z}_{21a} + \mathbf{z}_{21b} & \mathbf{z}_{22a} + \mathbf{z}_{22b} \end{bmatrix}$$

or

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]$$

In short we can write the z parameter matrix of the overall network is the summation of individual z parameter matrix of network a and b.

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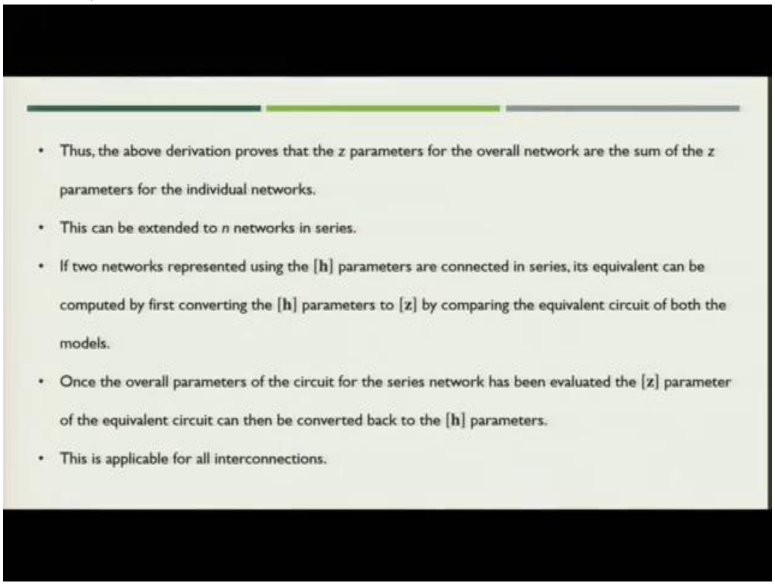

So, with this we can say that the z parameters of the overall network are the sum of the z parameters of the individual networks. So now, whatever we discussed was for two-port networks, we can extend it to n set of sub networks which are part of the overall network of the system. Now, if the two-port networks are represented using any other parameters say h parameter, what we can do?

Since we know that the networks are connected in series, the best thing is that let us first convert the h parameters into corresponding z parameters of individual networks and then use the summation of the z parameters to get the overall z parameters of the network and then when we get all the parameters of the circuit for a series network that is the z parameter of the overall network, we can convert this z parameter again back into h parameter.

This is applicable for any type of interconnection which we may have in the network, so the same is for maybe y or g or any other parameter which you get from the sub networks, so you can first convert them into the z parameters, then add them and the final z parameter matrix can be converted back into h parameters matrix.

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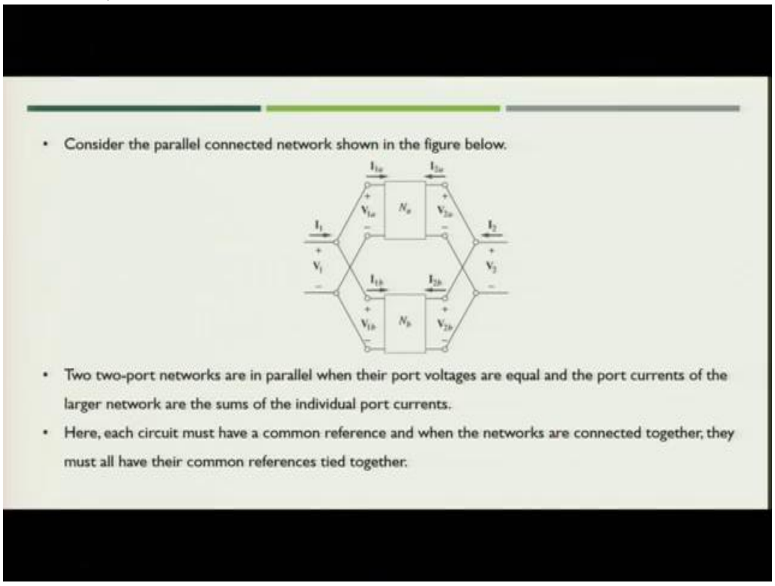

Now, let us consider the second case in which the network is connected in parallel. So here we have network a and network b, so these are connected in parallel when their port voltages are equal and port currents of the larger networks are the sum of individual port currents. So that means that $\mathbf{V}_1$ and $\mathbf{V}_2$ is applied to both sub networks and $\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}$.

Now, in this case the circuit must have one common reference, so in this case this is the common reference for both sub networks, so we will connect them together so that we get the common reference of overall network. So, when you connect all of them in parallel so the common networks you will tie them together so that the networks the sub networks will be in parallel.

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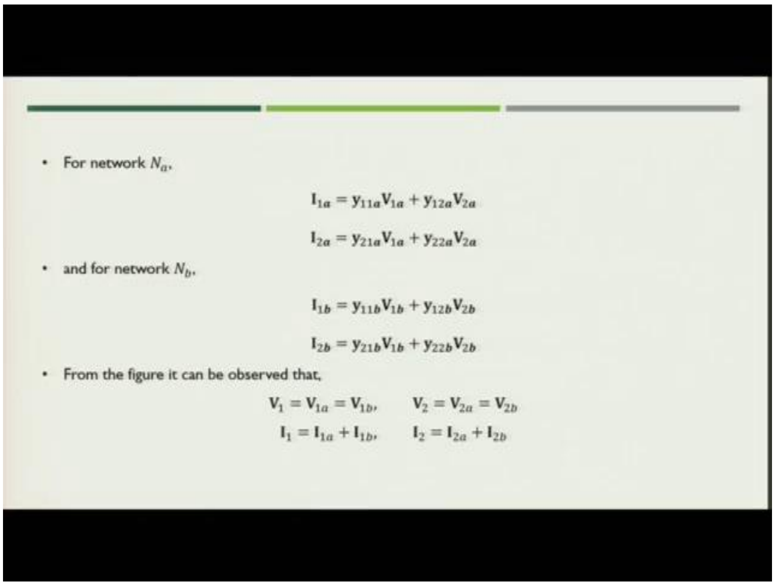

Now, what we have to do? We have to use the y parameters equations to find out the y parameters of overall network. Since these 2 networks are in parallel, we will utilise first y parameter equations. So, what we can write for network a?

$$\mathbf{I}_{1a} = \mathbf{y}_{11a}\mathbf{V}_{1a} + \mathbf{y}_{12a}\mathbf{V}_{2a}$$

$$\mathbf{I}_{2a} = \mathbf{y}_{21a}\mathbf{V}_{1a} + \mathbf{y}_{22a}\mathbf{V}_{2a}$$

Now, for the network b we can again write the equations

$$\mathbf{I}_{1b} = \mathbf{y}_{11b}\mathbf{V}_{1b} + \mathbf{y}_{12b}\mathbf{V}_{2b}$$

$$\mathbf{I}_{2b} = \mathbf{y}_{21b}\mathbf{V}_{1b} + \mathbf{y}_{22b}\mathbf{V}_{2b}$$

Now when we see the figure, we can easily observe that since the both networks are connected in parallel, we can say

$$\mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b}, \ \mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b}$$

$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}, \ \mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b}$$

So now, if we put the values individual currents in these 2 equations, what we get? We

$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b} = (\mathbf{y}_{11a} + \mathbf{y}_{11b})\mathbf{V}_1 + (\mathbf{y}_{12a} + \mathbf{y}_{12b})\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b} = (\mathbf{y}_{21a} + \mathbf{y}_{21b})\mathbf{V}_1 + (\mathbf{y}_{22a} + \mathbf{y}_{22b})\mathbf{V}_2$$

The z parameters of the overall network are,

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11\boldsymbol{a}} + \mathbf{y}_{11\boldsymbol{b}} & \mathbf{y}_{12\boldsymbol{a}} + \mathbf{y}_{12\boldsymbol{b}} \\ \mathbf{y}_{21\boldsymbol{a}} + \mathbf{y}_{21\boldsymbol{b}} & \mathbf{y}_{22\boldsymbol{a}} + \mathbf{y}_{22\boldsymbol{b}} \end{bmatrix}$$

or

$$[\mathbf{y}] = [\mathbf{y}_{\boldsymbol{a}}] + [\mathbf{y}_{\boldsymbol{b}}]$$

So, in short we can write the y parameter equation of overall network is the summation of y parameter matrix of individual sub networks. The y parameters of overall network can be calculated as summing the individual y parameters of the individual networks. Now, again this can be extended to any n number of two-port networks which are connected in parallel.

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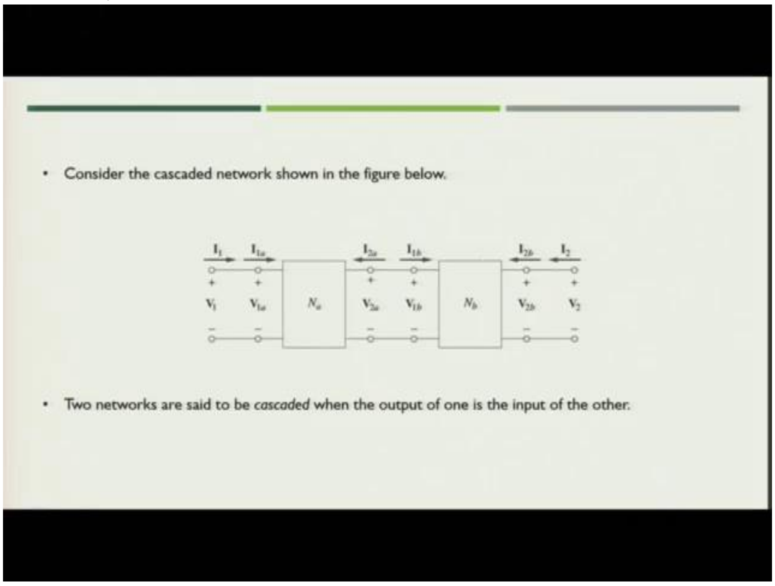

Now, let us see the third case where the network is connected in cascaded fashion. Cascaded fashion means the network a output is given as an input to network b. So, if you see in this figure, we say that current $\mathbf{I}_1$ and $\mathbf{V}_1$ is the overall voltage and current of the overall network, and $\mathbf{I}_2, \mathbf{V}_2$ is the output port current of the overall network. Now $\mathbf{I}_{2\boldsymbol{a}}$ which is output of network a will be given as input which is $\mathbf{I}_{1\boldsymbol{b}}$ to the network b. From this what we can say that,

$$\begin{bmatrix} \mathbf{V}_{2\boldsymbol{a}} \\ -\mathbf{I}_{2\boldsymbol{a}} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1\boldsymbol{b}} \\ \mathbf{I}_{1\boldsymbol{b}} \end{bmatrix}$$

So, let us write first the network equations for these two two-port networks. What we can write?

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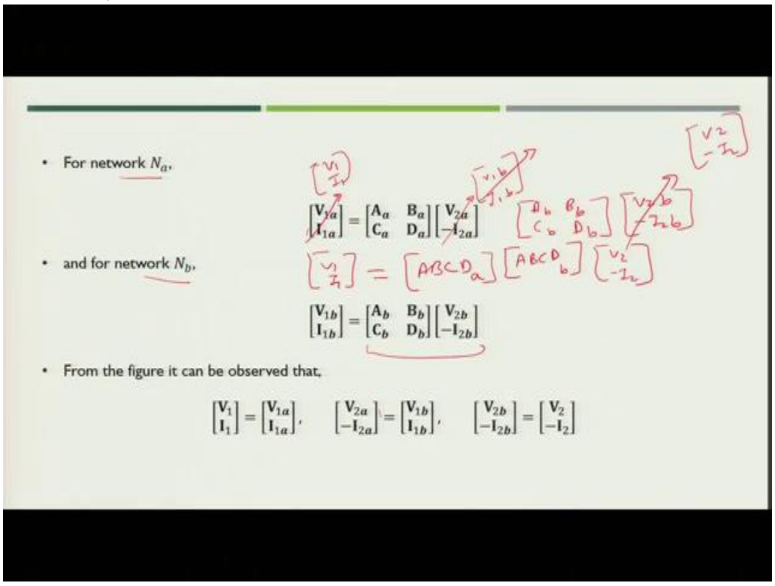

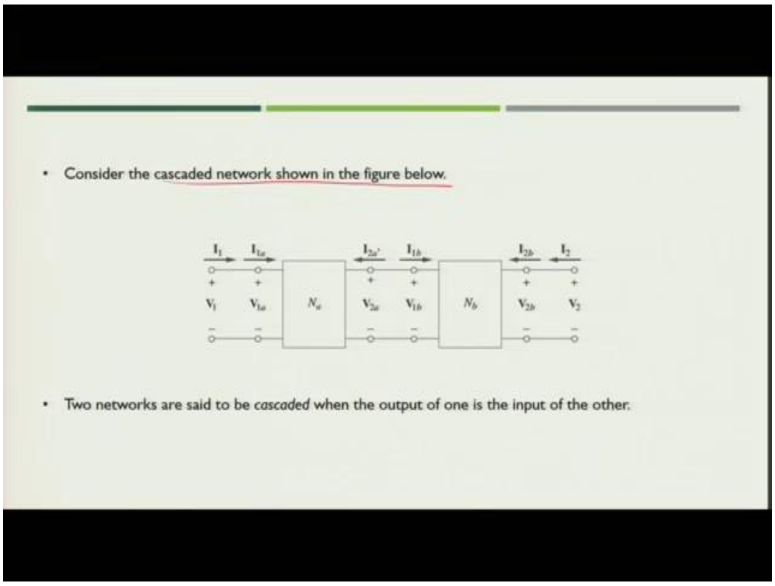

We can write for network a as $\begin{bmatrix}\boldsymbol{V_{1a}}\\\boldsymbol{I_{1a}}\end{bmatrix}$ are the input side vector equal to ABCD of network a into $\begin{bmatrix}\mathbf{V}_{2a}\\-\mathbf{I}_{2a}\end{bmatrix}$ that is the output vector for the network a. Similarly, for network b also we can write $\begin{bmatrix}\mathbf{V}_{1b}\\\mathbf{I}_{1b}\end{bmatrix}$ as an input vector equal to ABCD parameters for network b multiplied by vector output vector of $\begin{bmatrix}\mathbf{V}_{2b}\\-\mathbf{I}_{2b}\end{bmatrix}$. Now from the figure what we can observe? We can say that,

$$\begin{bmatrix}\mathbf{V}_1\\\mathbf{I}_1\end{bmatrix}=\begin{bmatrix}\mathbf{V}_{1a}\\\mathbf{I}_{1a}\end{bmatrix}$$

Similarly,

$$\begin{bmatrix} \mathbf{V}_{2\boldsymbol{a}} \\ -\mathbf{I}_{2\boldsymbol{a}} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1\boldsymbol{b}} \\ \mathbf{I}_{1\boldsymbol{b}} \end{bmatrix}$$

Similarly, at the output side what we can write?

$$\begin{bmatrix} \mathbf{V}_{2\boldsymbol{b}} \\ -\mathbf{I}_{2\boldsymbol{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{2} \\ -\mathbf{I}_{2} \end{bmatrix}$$

Now we have this information from the figure, if we see these two equations what we can write, we can write that,

$$\begin{bmatrix} \mathbf{V}_{1} \\ \mathbf{I}_{1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\boldsymbol{a}} & \mathbf{B}_{\boldsymbol{a}} \\ \mathbf{C}_{\boldsymbol{a}} & \mathbf{D}_{\boldsymbol{a}} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2\boldsymbol{a}} \\ -\mathbf{I}_{2\boldsymbol{a}} \end{bmatrix}$$

Since we know $\begin{bmatrix} \mathbf{V}_{2\boldsymbol{a}} \\ -\mathbf{I}_{2\boldsymbol{a}} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1\boldsymbol{b}} \\ \mathbf{I}_{1\boldsymbol{b}} \end{bmatrix}$,

$$\begin{bmatrix} \mathbf{V}_{1} \\ \mathbf{I}_{1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\boldsymbol{a}} & \mathbf{B}_{\boldsymbol{a}} \\ \mathbf{C}_{\boldsymbol{a}} & \mathbf{D}_{\boldsymbol{a}} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{1\boldsymbol{b}} \\ \mathbf{I}_{1\boldsymbol{b}} \end{bmatrix}$$

But,

$$\begin{bmatrix} \mathbf{V}_{1\boldsymbol{b}} \\ \mathbf{I}_{1\boldsymbol{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\boldsymbol{b}} & \mathbf{B}_{\boldsymbol{b}} \\ \mathbf{C}_{\boldsymbol{b}} & \mathbf{D}_{\boldsymbol{b}} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2\boldsymbol{b}} \\ -\mathbf{I}_{2\boldsymbol{b}} \end{bmatrix}$$

Hence,

$$\begin{bmatrix} \mathbf{V}_{1} \\ \mathbf{I}_{1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\boldsymbol{a}} & \mathbf{B}_{\boldsymbol{a}} \\ \mathbf{C}_{\boldsymbol{a}} & \mathbf{D}_{\boldsymbol{a}} \end{bmatrix} \begin{bmatrix} \mathbf{A}_{\boldsymbol{b}} & \mathbf{B}_{\boldsymbol{b}} \\ \mathbf{C}_{\boldsymbol{b}} & \mathbf{D}_{\boldsymbol{b}} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2\boldsymbol{b}} \\ -\mathbf{I}_{2\boldsymbol{b}} \end{bmatrix}$$

Now,

$$\begin{bmatrix} \mathbf{V}_{2\boldsymbol{b}} \\ -\mathbf{I}_{2\boldsymbol{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{2} \\ -\mathbf{I}_{2} \end{bmatrix}$$

So finally, what we get? We get,

$$\begin{bmatrix} \mathbf{V}_{1} \\ \mathbf{I}_{1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\boldsymbol{a}} & \mathbf{B}_{\boldsymbol{a}} \\ \mathbf{C}_{\boldsymbol{a}} & \mathbf{D}_{\boldsymbol{a}} \end{bmatrix} \begin{bmatrix} \mathbf{A}_{\boldsymbol{b}} & \mathbf{B}_{\boldsymbol{b}} \\ \mathbf{C}_{\boldsymbol{b}} & \mathbf{D}_{\boldsymbol{b}} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2} \\ -\mathbf{I}_{2} \end{bmatrix}$$

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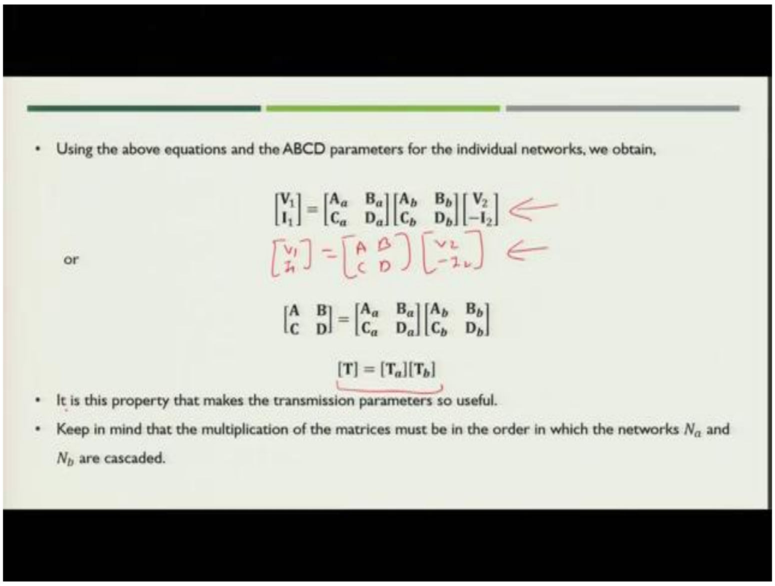

So, that means that if the network is connected in cascaded fashion, the ABCD parameter of overall network can be V multiplication of individual ABCD parameters of the sub networks. So when you compare

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\boldsymbol{a}} & \mathbf{B}_{\boldsymbol{a}} \\ \mathbf{C}_{\boldsymbol{a}} & \mathbf{D}_{\boldsymbol{a}} \end{bmatrix} \begin{bmatrix} \mathbf{A}_{\boldsymbol{b}} & \mathbf{B}_{\boldsymbol{b}} \\ \mathbf{C}_{\boldsymbol{b}} & \mathbf{D}_{\boldsymbol{b}} \end{bmatrix}$$

In short you can write that

$$[\mathbf{T}] = [\mathbf{T}_{\boldsymbol{a}}][\mathbf{T}_{\boldsymbol{b}}]$$

T is nothing but transmission parameter right, which is also called as ABCD parameter. This is basically very important property for the transmission the cascaded network that is why makes the transition parameters very useful in case of cascaded network.

One thing which we must always keep in mind that multiplication of matrices that is we are seeing here must be in the order in which networks N a and N b are cascaded. So, if b is following the network a then matrix multiplication would be $[\mathbf{T}_{\boldsymbol{a}}][\mathbf{T}_{\boldsymbol{b}}]$, not $[\mathbf{T}_{\boldsymbol{b}}][\mathbf{T}_{\boldsymbol{a}}]$.

Whenever we have very large network and it is connected as a set of cascaded two-port networks, the T parameter of overall network would be the multiplication of all the T parameters matrices of individual sub networks. So, if we say there are n two-port networks which are part of the overall bigger network, all n T parameter matrices would be multiplied in that order in which the cascaded networks are connected.

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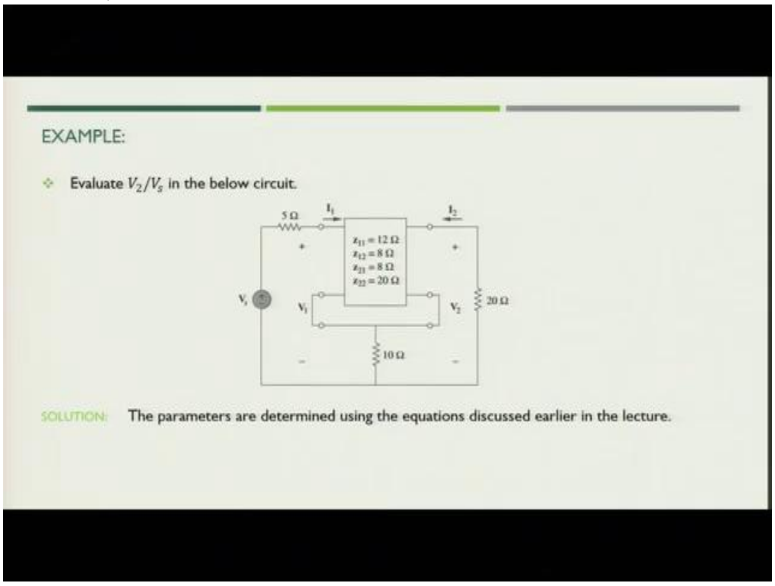

Now, let us understand the concepts, the interconnection of the network which we discussed till now with the help of few examples. Now, in this case the figure the Z parameters of one two-port network is given, the Z parameter of second network we need to find out, since these two are connected in series we can simply take the Z parameters of the second two-port network because this is the second two-port network you will see and we can sum up these two Z parameter matrices to get the Z parameter of the overall network.

So first the task is that we know the Z parameters of the upper network, next we have to find out the Z parameters of the lower network. Now, if you compare this particular two-port network, you can say it is a form of T network.

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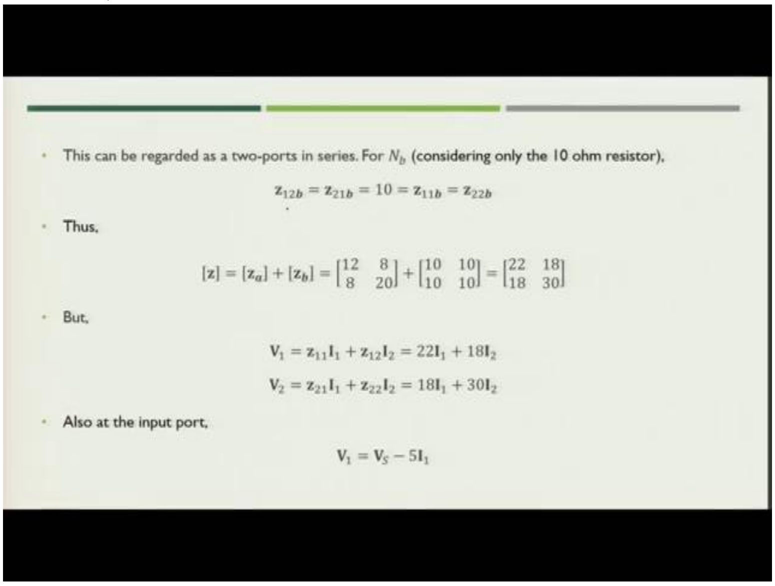

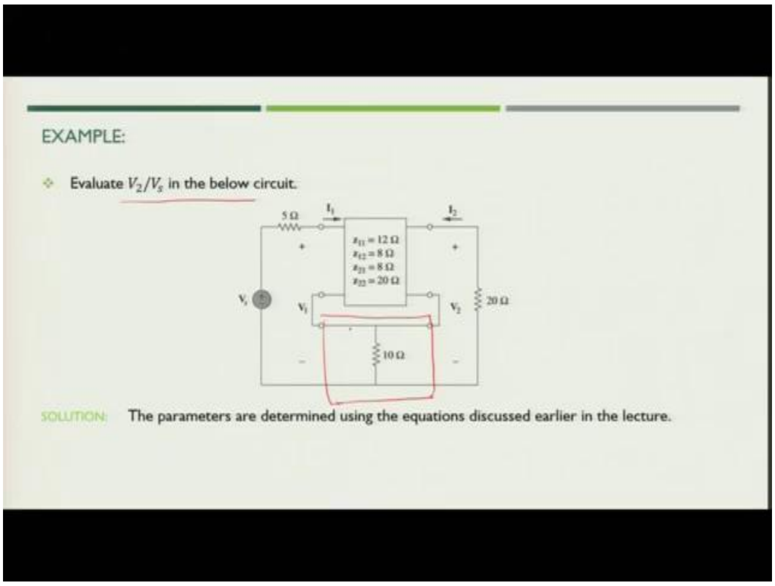

So, we can simply write the Z parameters of this type of network as $\mathbf{z}_{12\boldsymbol{b}} = \mathbf{z}_{21\boldsymbol{b}} = 10 = \mathbf{z}_{11\boldsymbol{b}} = \mathbf{z}_{22\boldsymbol{b}}$

In case of second two-port network let us say it is Nb, we can compile the Z parameter matrix as having all the elements as equal to 10 and for the first two-port network we already have the Z parameters in hand so we can simply get the Z parameter of the overall network.

So, what we can write?

$$[\mathbf{z}] = [\mathbf{z}_{\boldsymbol{a}}] + [\mathbf{z}_{\boldsymbol{b}}] = \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

But,

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 = 22\mathbf{I}_1 + 18\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 = 18\mathbf{I}_1 + 30\mathbf{I}_2$$

Also at the input port,

$$\mathbf{V}_1 = \mathbf{V}_S - 5\mathbf{I}_1$$

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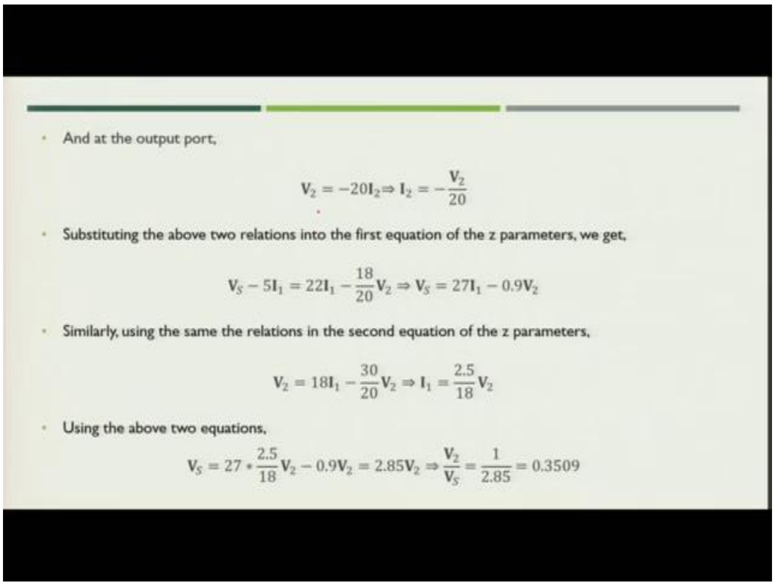

And at the output port,

$$\mathbf{V}_2 = -20\mathbf{I}_2 \Rightarrow \mathbf{I}_2 = -\frac{\mathbf{V}_2}{20}$$

Substituting the above two relations into the first equation of the z parameters, we get,

$$\mathbf{V}_S - 5\mathbf{I}_1 = 22\mathbf{I}_1 - \frac{18}{20}\mathbf{V}_2 \Rightarrow \mathbf{V}_S = 27\mathbf{I}_1 - 0.9\mathbf{V}_2$$

Similarly, using the same the relations in the second equation of the z parameters,

$$\mathbf{V}_2 = 18\mathbf{I}_1 - \frac{30}{20}\mathbf{V}_2 \Rightarrow \mathbf{I}_1 = \frac{2.5}{18}\mathbf{V}_2$$

Using the above two equations,

$$\mathbf{V}_S = 27 * \frac{2.5}{18}\mathbf{V}_2 - 0.9\mathbf{V}_2 = 2.85\mathbf{V}_2 \Rightarrow \frac{\mathbf{V}_2}{\mathbf{V}_S} = \frac{1}{2.85} = 0.3509$$

So you can see, with the given circuit you can convert them into two-port sub networks and since in this case the network is connected in series, we can utilise the Z parameters to find out the answer that is V 2 by VS given in this particular example.

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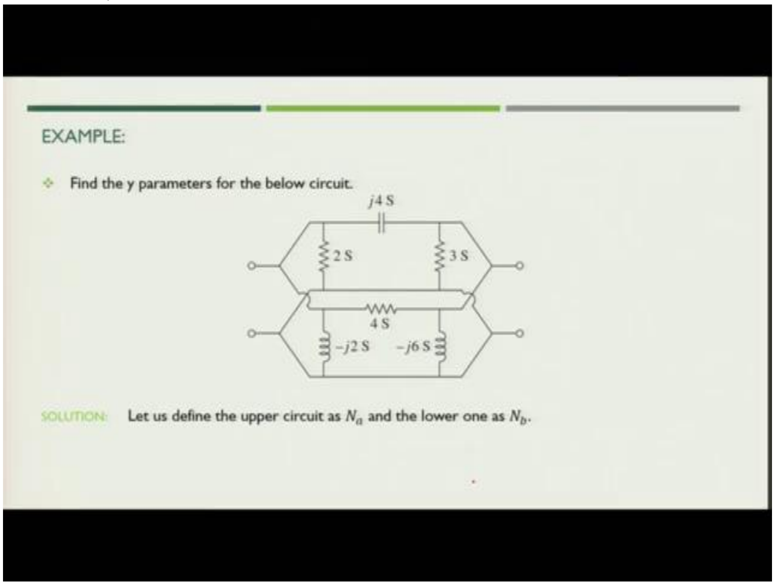

Now, let us take another example where we need to find out the Y parameters of the circuit, so if you see these two-port sub networks are connected in parallel so we can easily find out the Y parameters. What we can do? We can find the Y parameters of individual sub networks, so individual sub networks means one is this sub networks and another is this sub network.

So, if you recollect in case of Y parameters how we compile the individual elements of Y parameters? We can write

$$\mathbf{y}_{12\boldsymbol{a}} = \mathbf{y}_{21\boldsymbol{a}} = -j4,\ \ \mathbf{y}_{11\boldsymbol{a}} = 2 + j4,\ \ \mathbf{y}_{22\boldsymbol{a}} = 3 + j4$$

$$\mathbf{y}_{12\boldsymbol{b}} = \mathbf{y}_{21\boldsymbol{b}} = -4,\ \ \mathbf{y}_{11\boldsymbol{b}} = 4 - j2,\ \ \mathbf{y}_{22\boldsymbol{b}} = 4 - j6$$

Thus,

$$[\mathbf{y}] = [\mathbf{y}_{\boldsymbol{a}}] + [\mathbf{y}_{\boldsymbol{b}}] = \begin{bmatrix} 6 + j2 & -4 - j4 \\ -4 - j4 & 7 - j2 \end{bmatrix} \text{S}$$

So now, with this we can close our today's session in which we discussed about the interconnection of various two-port networks, so in this session we can say that various two-port parameters which we have discussed in previous lectures can be interconnected in any fashion to analyse the larger networks, thank you.