Basic Electric Circuits

Module 01

Basic Circuit Elements and Waveforms

Lecture-04

Circuit Elements Part-2

By

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Namashkar, yesterday we spoke about the various circuit elements like resistance, inductance and capacitance. We saw their properties and we also saw the governing equations for those elements. We also discussed the Ohm's Law. Now today we will continue our that discussion and try to see what are other basic laws which are important for the analysis of electrical circuit.

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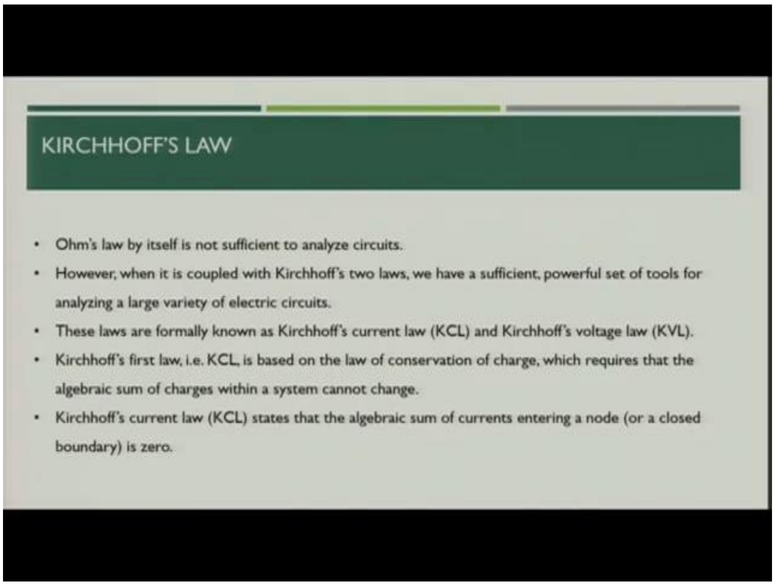

Now, the Ohm's Law itself is not sufficient to analyse the circuit, so the two laws, that is Kirchhoff's two laws were developed which are sufficient and powerful set of tools for analysing large variety of electrical circuit.

What are these two laws? These two laws are Kirchhoff's current law and Kirchhoff's voltage law. The first one that is Kirchhoff's current law is based on law of conservation of charge. That means that the algebraic sum of charges within a particular system cannot change. And this Kirchhoff's current law states that the algebraic sum of currents entering in a particular node or in a closed boundary will always be 0.

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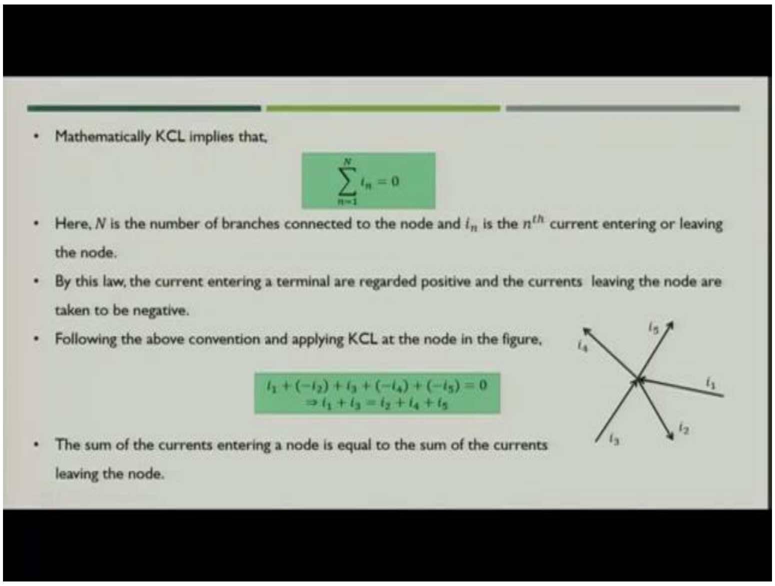

How you will represent it mathematically? You will represent it like summation of i_n, that is current flowing into n^{th} element or the n^{th} branch of the circuit. So, i_n is the current flowing in the n^{th} branch.

So KCL says that the summation of all the currents flowing in the branch, in all the branches is equal to 0 and n is equal to 1 to N. What is N, N is the total number of branches connected to that particular node where we are analysing the circuit or you can say that it is total number of branches coming in or out from a particular closed boundary.

Now in this law current entering the terminals are regarded as positive and the current which are leaving the node are considered to be negative. So, if you see this particular figure and you try to apply the Kirchhoff current law at the node where currents are coming in and going out, the current i_1 is going inside the node so it is positive, current i_2 is coming out so it is negative. Similarly, i_3 is positive and i_4 and i_5 are negative and the sum of all the currents would be equal to 0 which is satisfying the particular equation of KCL or alternatively you can write that i_1 plus i_3 is equal to i_2 plus i_4 plus i_5. So, from this, what you can conclude, that the sum of currents entering the node is equal to sum of currents leaving the node.

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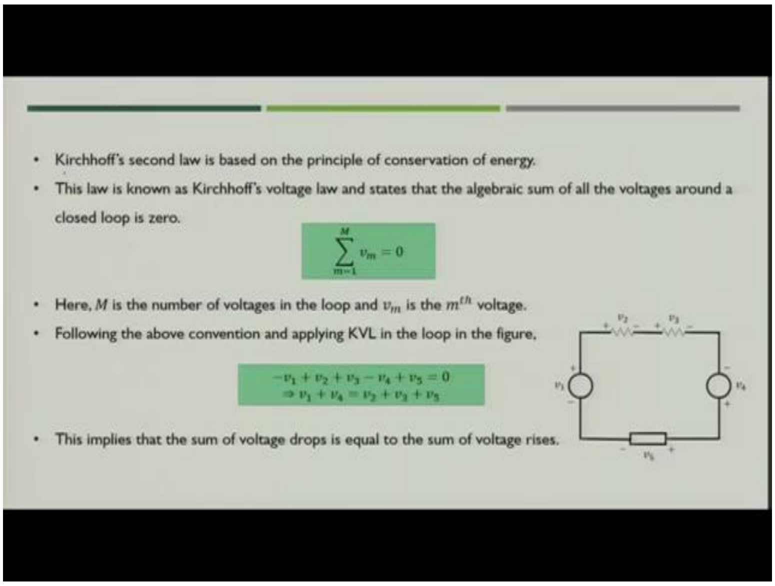

Now, let us talk about the second law of Kirchhoff and this second law is based on principle of conservation of energy. This Kirchhoff's voltage law states that the algebraic sum of all the voltages around a particular closed loop would be 0.

So, in this case, this V_m is the voltage across a particular element connected in a loop and if you sum up all the voltages in a particular loop across all the elements then you will get, the summation of voltage would always be 0 and m is from 1 to M, where M in number of voltages in the loop and V_m that we have considered as the m^{th} voltage. Now, how you will analyze the Kirchhoff voltage law in a particular circuit?

Let us see this particular circuit where all elements are connected in in in a particular loop, so this loop is closed loop. If we apply Kirchhoff voltage law for this particular loop, what we will write? Let us start from v_1. Because we are starting from v_1 that is we are going inside the negative terminal of voltage source and coming out of the positive terminal of voltage source; it means that the voltage is rising up during negative to positive terminal so we represent it like minus of v_1.

And then for the resistance, v_2 is positive because we are entering into the positive side and coming out of negative. So here it is a voltage drop. So, for voltage drop we are writing as positive and voltage rise we are writing as negative. For voltage v_3 we will again write as positive because across the resistance we will see the voltage drop and then again when we go across this voltage source, the voltage is again rising form negative to positive so we have taken voltage as negative of v_4. And then again across resistive element v_5 it is voltage drop so we will say as positive v_5. Now if you rearrange the elements in this equation then we can say that $v_1 + v_4$ is nothing but $v_2 + v_3 + v_5$. Now from this what you can conclude? That sum of the voltage drops in a particular loop is equal to sum of the voltage rises.

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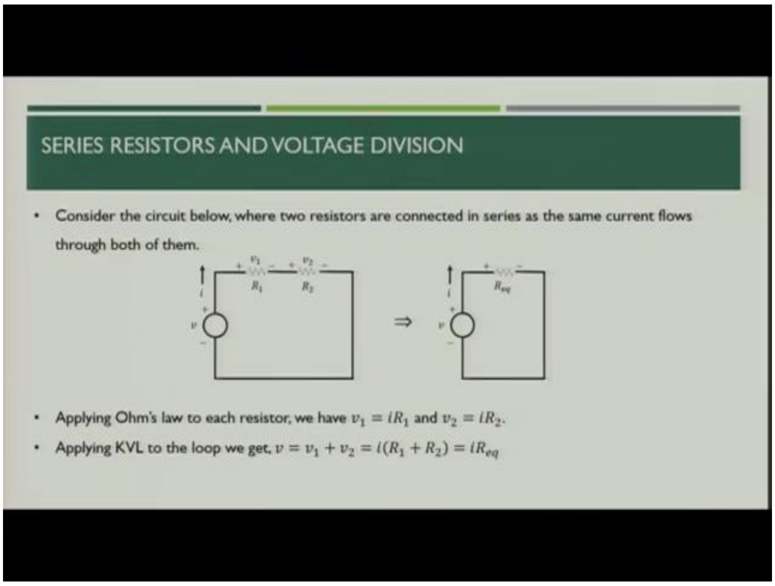

Now, let us talk about the series resistors and the voltage division concepts. Let us see the circuit below where two resistors are connected in series and the same current is flowing through or both of the resistors because those are connected in the series and it is completing the loop. So which law you can apply? You can apply Kirchhoff voltage law. Now the current is same, so, voltage drop across the resistor R_1 can be written as $v_1 = iR_1$ and similarly voltage drop against resistor 2 would be $v_2 = iR_2$.

So total voltage from Kirchhoff voltage law, you can write $v = v_1 + v_2$. Now since current is common you can take current out and you will get $v = v_1 + v_2 = i(R_1 + R_2) = iR_{eq}$. So, this equation, this particular segment of this particular equation will be represented with the help of this figure, where we have a v voltage source and the equivalent resistor of both of the resistors both R_1 and R_2. Hence,

$$R_{eq} = R_1 + R_2$$

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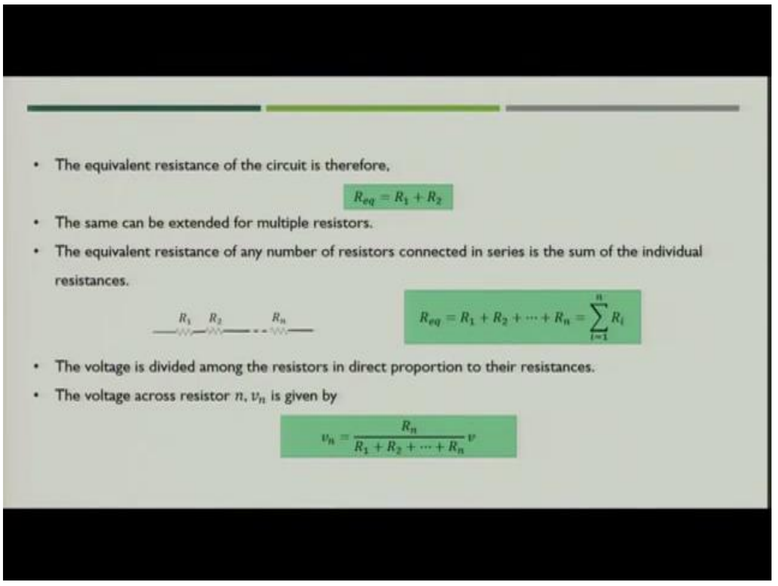

Now, if you generalize this particular equation, you can extend it for multiple resistors. So, equivalent resistance for any number of resistors connected in series would be the summation of individual resistances. So, suppose if you have n resistances connected in series like R_1, R_2 upto R_n what would be the equivalent resistance? Equivalent resistance would be summation of all the resistances connected in the series.

$$R_{eq} = R_1 + R_2 + \cdots + R_n = \sum_{i=1}^{n} R_i$$

Now, the voltage is divided among the resistors is directly proportional to their resistances. Suppose there is a resistor R_n and voltage across that resistor is v_n. Then,

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_n} v$$

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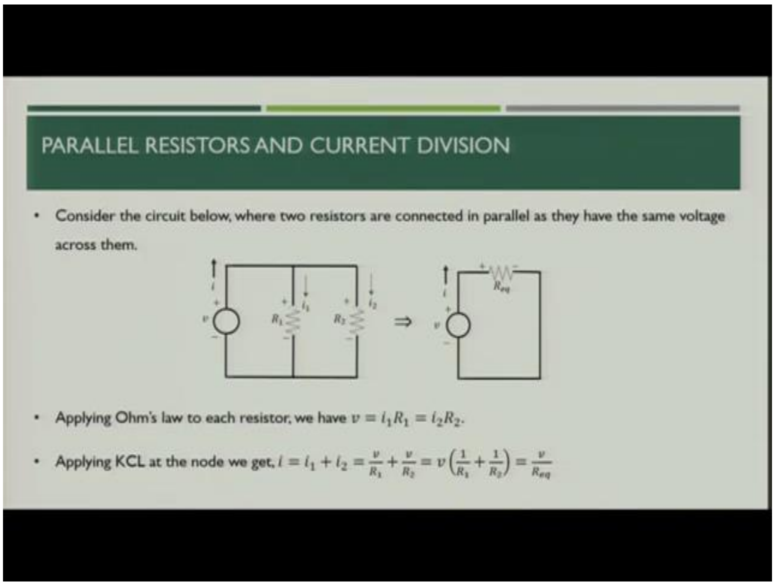

Now, let us talk about the case where the resistors are connected in parallel, there we will say that the current division will be followed. So, let us take now this figure where voltage is connected to R1 and R2 both which are in parallel, so current flowing through R1 would be i1 and current flowing through R2 would be i2. Since these two are in parallel, what you can apply? You can apply Kirchhoff's current law.

So basically, one important thing which you can always take as a thumb rule is that whenever you see elements connected in series you can simply apply Kirchhoff's voltage law and when you see the elements connected in parallel fashion, you can use Kirchhoff's current law. So, these are the 2 important laws and based on the circuit you can apply them.

So now let us see this particular circuit where current is being divided into i1 and i2, i1 is flowing through R1 and i2 is flowing through R2. Since voltage is same across R1 and R2 both, you can simply find out the value of i1 is nothing but v/R_1 using Ohm's law. Now, similarly for i2 also you can write i2 is nothing but v/R_2. So finally,

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

This you can represent using the equivalent figure where voltage is connected in series with resistance R equivalent.

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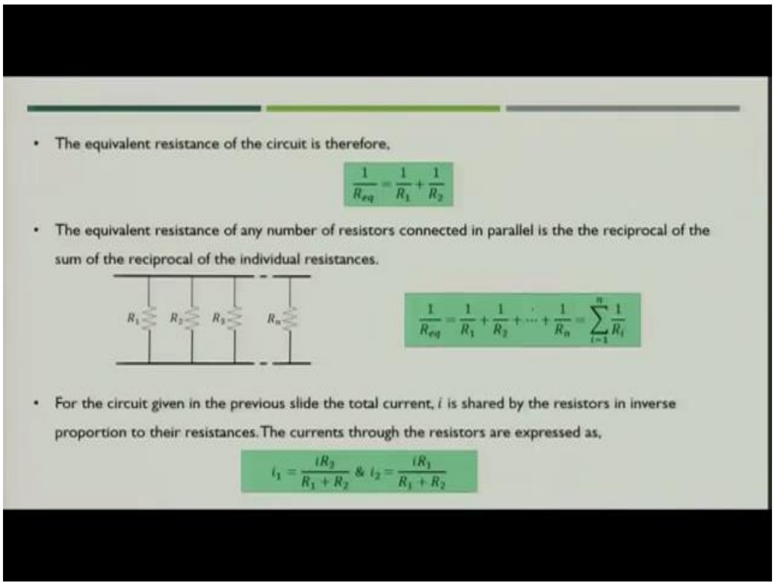

So,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Now if you want to generalise it what will you write? Suppose if there are n resistances in parallel then 1 upon R equivalent is nothing but 1 by R1 plus 1 by R2 and so on upto 1 by Rn or in mathematical terms you can write like summation of i is equal to 1 to N of 1 by Ri, i is ranging between 1 to N

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$$

where N is number of elements that is number of resistors connected in parallel fashion. So how can you summarize this particular aspect? This says that equivalent resistance of any number of resistors connected in parallel is the reciprocal of the reciprocal of individual resistances.

So reciprocal of individual resistances is $1/R_i$ and then you will sum up and whatever you will get finally you will take again the reciprocal of same and you will get the value of equivalent resistance. It means that you can simply write R equivalent is nothing but reciprocal of the summation of the reciprocal of individual resistances, right.

Now when you talk about the current division, the circuit which you saw in the previous slide, you will get the current flowing through the individual resistors as

$$i_1 = \frac{iR_2}{R_1 + R_2} \ \& \ i_2 = \frac{iR_1}{R_1 + R_2}$$

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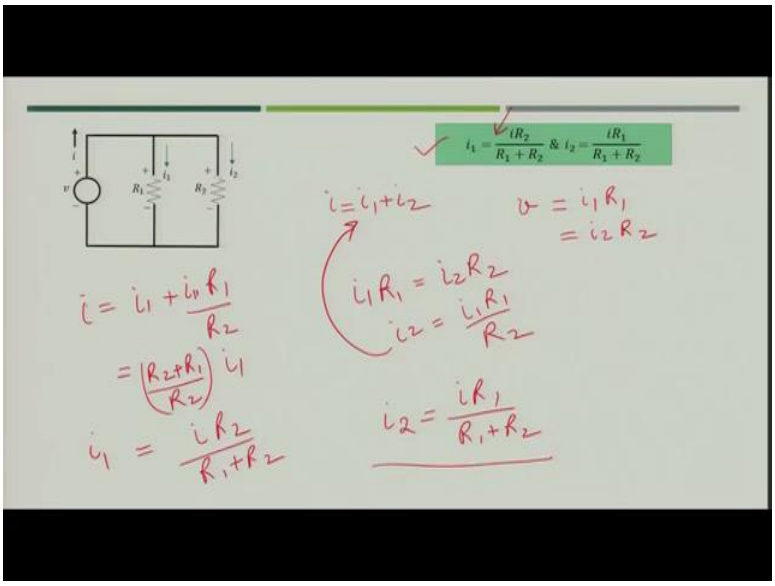

Now you will ask how can we determine these values, so let us see how you can find out the value of i1 and i2 by seeing this particular figure. So now you know that i is nothing but i1 plus i2 and voltage v is anyway applied across both of the resistors, so $v = i_1R_1 = i_2R_2$, so from here you can find out the value of i_2 that is nothing but i_1R_1/R_2. Now

$$\mathrm{i} = i_1 + \frac{i_1R_1}{R_2}$$

Now if you simplify,

$$i = i_1\frac{R_1 + R_2}{R_2}$$

Therefore,

$$i_1 = i\frac{R_2}{R_1 + R_2}, \quad i_2 = i\frac{R_1}{R_1 + R_2}$$

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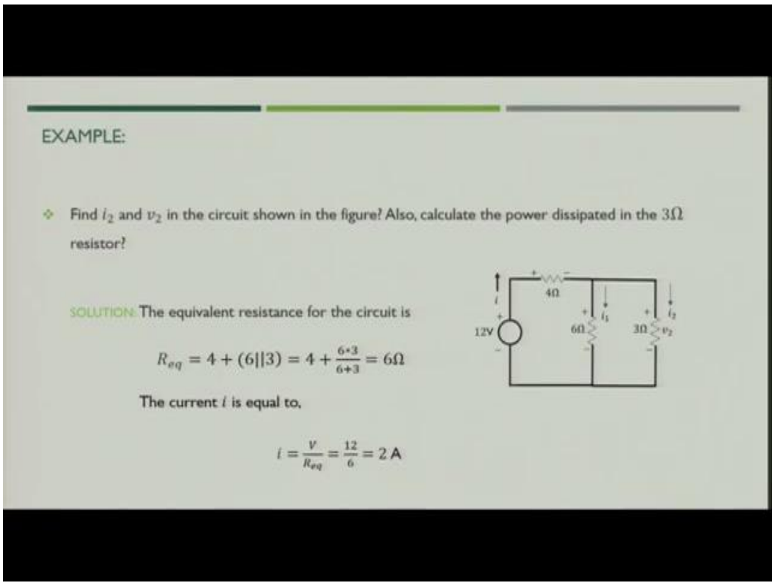

Now, let us take the example, if you are asked to find the value of i2 and v2 in the circuit and also power dissipated in the 3 ohm resistor, what you will do? Since these two are connected in parallel, you can use the Kirchhoff's current law at this node and try to find out what is the equivalent resistance. So you will get equal resistance

$$R_{eq} = 4 + (6||3) = 4 + \frac{6*3}{6+3} = 6\Omega$$

So total R equivalent of the circuit would be 6 ohm and accordingly you can find out the value of current as,

$$i = \frac{V}{R_{eq}} = \frac{12}{6} = 2 \text{ A}$$

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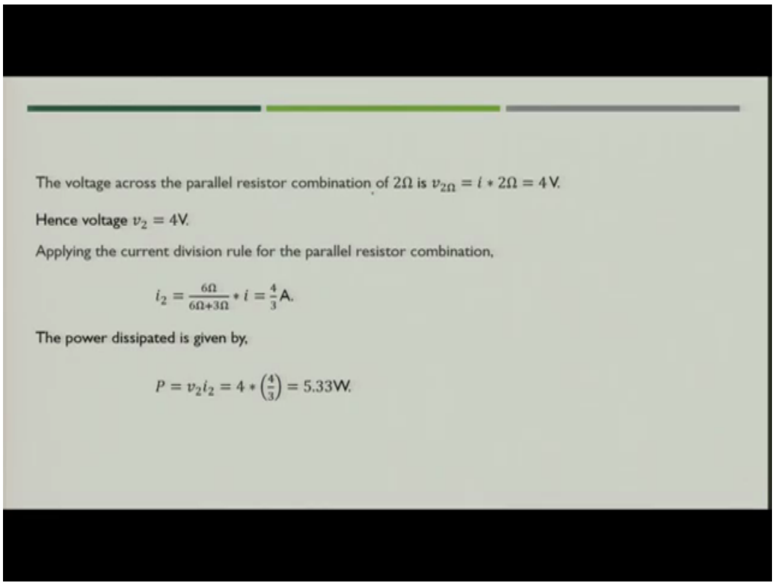

Now, the voltage across the parallel resistance of 2 Ohm that is basically the equivalent resistance of this particular segment would be is $v_{2\Omega} = i * 2\Omega = 4$ V. Since this voltage v_2 is common for both resistors, you can find out the value v_2, for this combination and then finally using the current division you can find out the value of i_2. So, you get the value as 4 Volt for the equivalent combination of 6 Ohm and 3 Ohm parallel resistors, which is v_2, then using current division rule you can find the value of current i_2 as

$$P = v_2 i_2 = 4 * \left(\frac{4}{3}\right) = 5.33\text{W}$$

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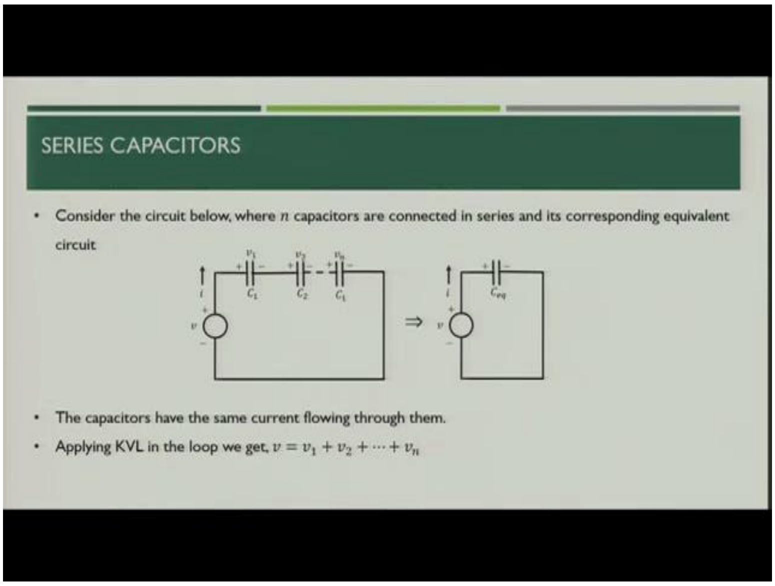

Now, let us talk about the series capacitors. Now consider the circuit below, where there are n capacitors connected in series. You can see that Kirchhoff's Voltage law can be used because these are connected in series. So, when applying Kirchhoff's Voltage law, you get

$$v = v_1 + v_2 + \cdots + v_n$$

because these voltages are the voltage across the individual capacitors.

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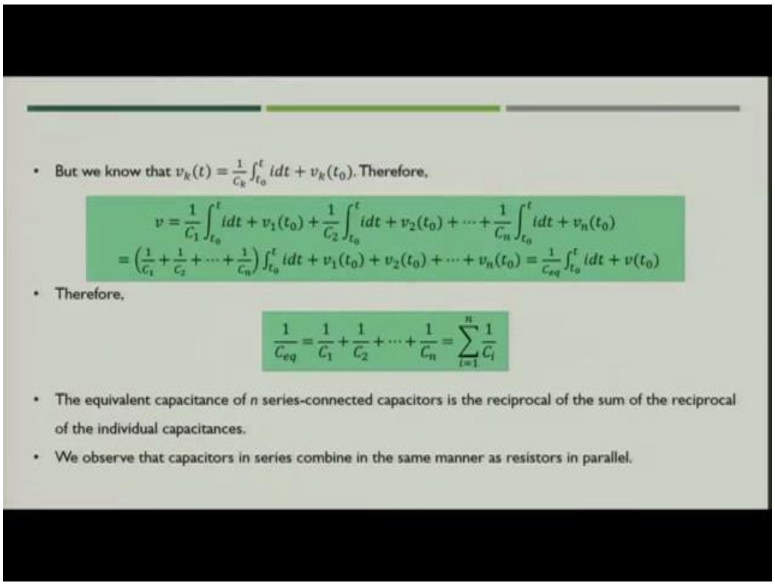

Now, what is voltage v? Voltage v for the kth capacitor can defined as

$$v_k(t) = \frac{1}{C_k}\int_{t_0}^{t} idt + v_k(t_0)$$

So, this is something which we discussed in the previous class, where we analysed the various resistor, inductor and capacitor elements. Now this is for kth capacitor if you add up the voltages for all the capacitors which are connected in series, you will get the value of total voltage v as,

$$v = \frac{1}{C_1}\int_{t_0}^{t} idt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} idt + v_2(t_0) + \cdots + \frac{1}{C_n}\int_{t_0}^{t} idt + v_n(t_0)$$

So, when you re-arrange this particular equation you can club all of the capacitances outside the integral because these are the constant quantities and then the initial voltages to obtain

$$\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right)\int_{t_0}^{t} idt + v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0)$$

So, what can you can write?

$$\frac{1}{C_{eq}}\int_{t_0}^{t} idt + v(t_0)$$

Now if you compare these two, you will easily conclude that $\frac{1}{C_{eq}}$ is nothing but the summation of the reciprocal of individual capacitances.

So, you can simply write the

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}$$

that is number of capacitors connected in series. So now what you can say, equivalent capacitance of n series connected capacitors is reciprocal of the sum of the reciprocal of individual capacitances, that is c equivalent is nothing but reciprocal of the sum of the reciprocal of the individual capacitances, right. Now if you compare with the, the previous discussion related to resistance you can observe that capacitors in series combine in the same manner as you see resistance in the parallel.

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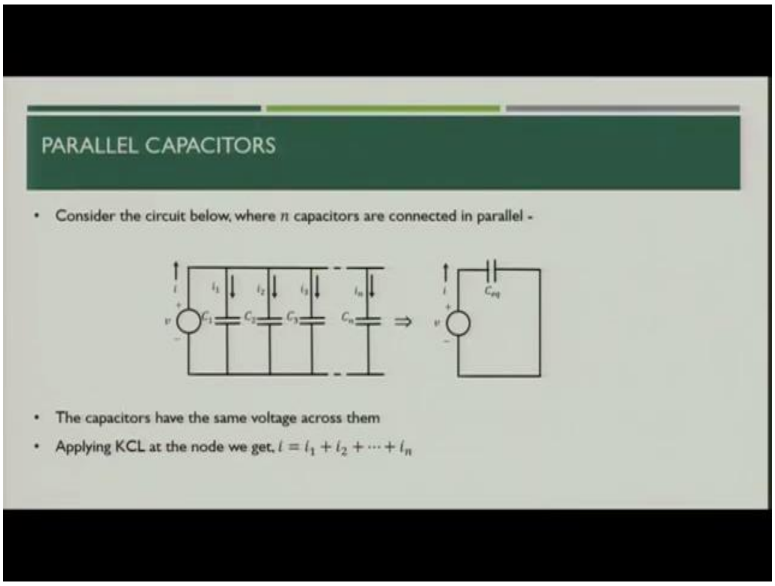

Now, let us talk about the capacitors which are connected in parallel. So here what you can do? You can simply apply Kirchhoff current law and you can (identify) find out the value of equivalent capacitance.

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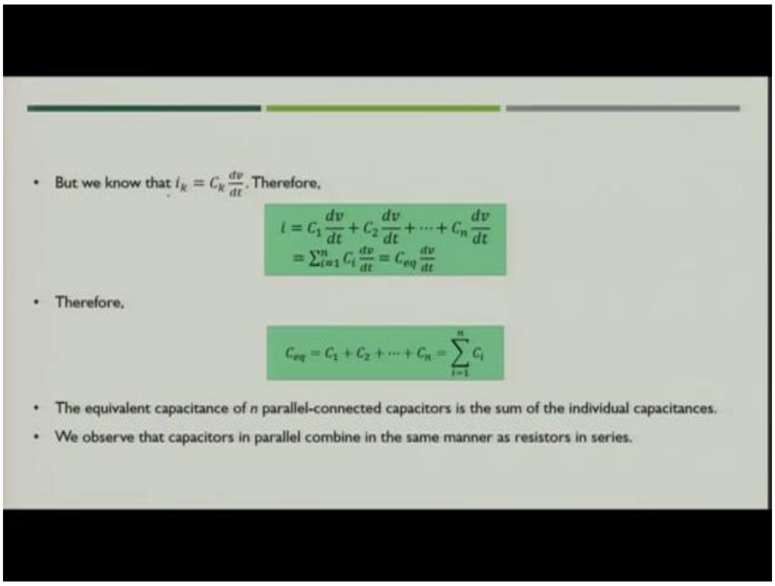

So, when you apply Kirchhoff current law, you know that the current flowing in the i^{th} capacitor can be written as

$$i_k = C_k \frac{dv}{dt}$$

This is what we saw in the previous lecture.

So, if you sum up all the currents, it will be equal to current shown in the circuit. So that is the total current nothing but i_1 plus i_2 to i_n, so when you sum up you will get

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \cdots + C_n \frac{dv}{dt}$$

If you sum up then you can write

$$\sum_{i=1}^{n} C_i \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

eventually you will say that c equivalent is nothing but summation of individual capacitances which are connected in the circuit in parallel.

Hence, equivalent capacitance of n parallel connected capacitor is nothing but the sum of the individual capacitances. So now if you observe the, the correlation of the circuits related to resistors and the capacitors, you can say that resistors connected in parallel are combined in the same manner as the resistors in series.

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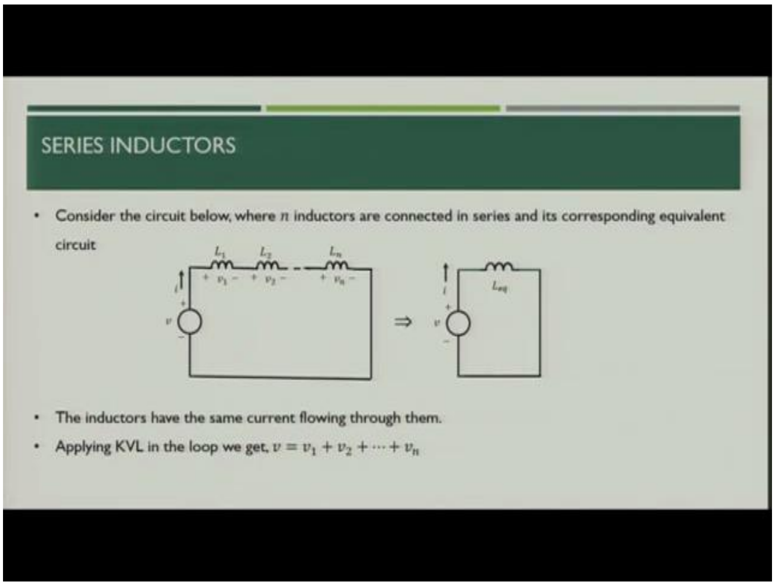

Now, let us talk about inductors, suppose the inductors, there are L_n inductors which are connected in series. So, what can you apply? You can apply Kirchhoff's voltage law, because it is, these are connected in loop. v is nothing but the summation of individual voltages across the individual inductors

$$v = v_1 + v_2 + \cdots + v_n$$

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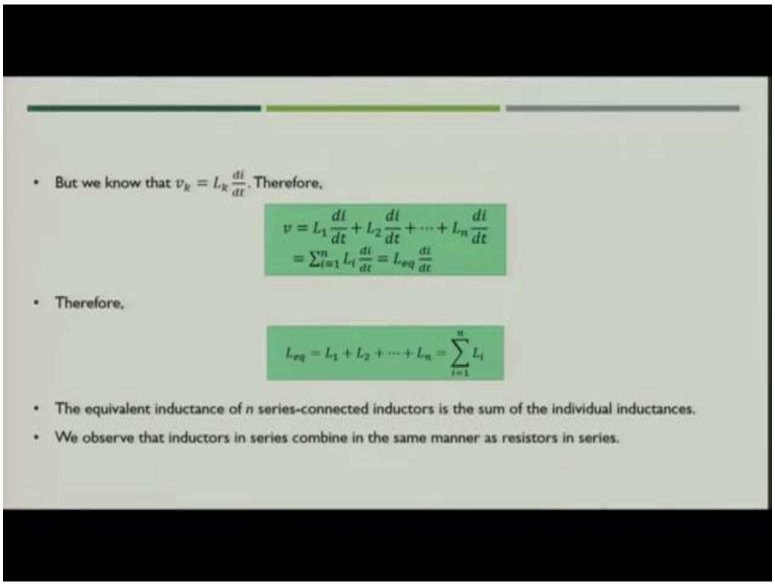

So, as we know that $v_k = L_k \frac{di}{dt}$, i is same in all the inductors, so what can you write?

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + \cdots + L_n\frac{di}{dt}$$

So when you sum up the all the inductors you can simply

$$\sum_{i=1}^{n} L_i \frac{di}{dt} = L_{eq}\frac{di}{dt}$$

You can conclude that L equivalent is nothing but summation of all the individual inductances connected in series in that particular circuit. So, equivalent inductance of n series connected inductors is some of the individual inductances.

$$L_{eq} = L_1 + L_2 + \cdots + L_n = \sum_{i=1}^{n} L_i$$

So, if you correlate with the series combined resistors you will say that the inductors in the series combined will have equivalent inductance in the same manner as the resistors which are connected in series.

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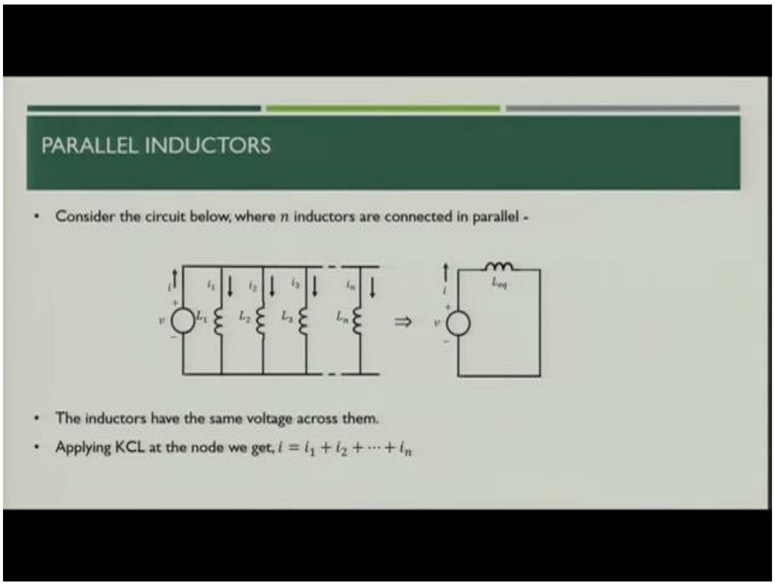

Now, let us talk about the inductors connected in parallel. In this figure you will see that these inductors are connected in parallel. When these inductors are connected in parallel it means you can apply Kirchhoff’s current law. If you if you apply Kirchhoff’s current law, you will get i that is the current coming from the voltage source is nothing but the summation of individual currents flowing in the individual inductors and finally the objective is to find out the value of equivalent inductance of the circuit.

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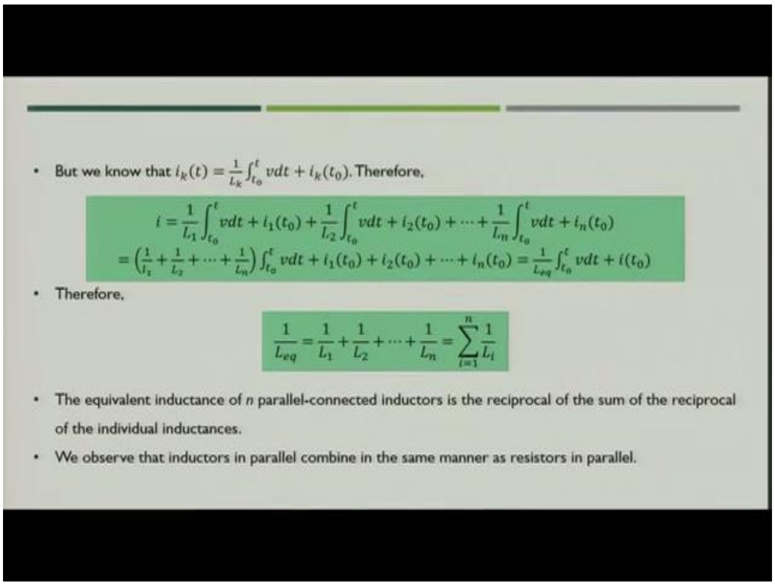

So what do you have to do? You have to find out the value of the current flowing individually in the in each inductor so you write

$$i_k(t) = \frac{1}{L_k}\int_{t_0}^{t} vdt + i_k(t_0)$$

i can be written in integral form and the same has been utilised here to find out the value of total current. So, what do you have to do? You have to just sum up the individual currents flowing in each inductor. The voltage is same across each inductor, for first inductor we will write

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} vdt + i_1(t_0)$$

this is something which we say that the, initially we are assuming that at time t_0, there is some current in the inductor. So that is considered to be as the initial condition for that inductor 1. Similarly, for inductor 2 also we will write $\frac{1}{L_2}\int_{t_0}^{t} vdt$ plus the initial current flowing in the inductor and when we will sum up all the elements, we can write

$$i = \frac{1}{L_1}\int_{t_0}^{t} vdt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} vdt + i_2(t_0) + \cdots + \frac{1}{L_n}\int_{t_0}^{t} vdt + i_n(t_0)$$

$$= \left(\frac{1}{l_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}\right)\int_{t_0}^{t} vdt + i_1(t_0) + i_2(t_0) + \cdots + i_n(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} vdt + i(t_0)$$

Now if you compare these two, you can write,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} = \sum_{i=1}^{n}\frac{1}{L_i}$$

Hence, equivalent inductance of N parallel connected inductors is reciprocal of the sum of the reciprocal of individual inductances. That means that L equivalent can be simply written as reciprocal of the sum of the reciprocal of individual inductances, right.

So, we can observe that inductors which are connected in parallel are combined in the same manner as the resistors in parallel. We saw the parallel and series combination of all the three resistors, capacitors and inductors. In next class we will try to find out the phasor relationship between these elements, that is the voltage and current phasor relationship for resistor, capacitor and inductor and then we will stabilize the relationship of impedance for the sinusoidal varying voltage and current. We close this session today. In next class we will carry forward with the phasor representation of these quantities, Thank you.