Basic Electric Circuits
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Module 8 - Two Port Network
Lecture 39 - Transmission parameters

Namaskar, so in last few sessions we discussed about a set of parameters, we started with z parameters, then we spoke about y parameters and then we discussed h parameters as well as g parameters. In today's session we will discuss about transmission parameters.

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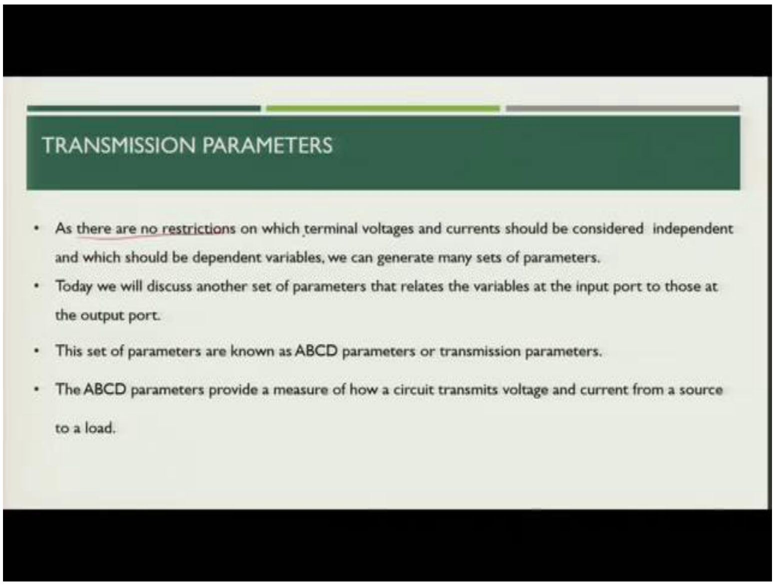

Let us see what the transmission parameters is. Basically, there are no restrictions on which the terminal voltage and current should be considered independent and which should be dependent. We have 4 variables: $\mathbf{V_1}$, $\mathbf{I_1}$ and $\mathbf{V_2}$, $\mathbf{I_2}$. We can choose any one set of voltage or current as independent parameter and any set we can consider as a dependent parameter.

Based on the choice we can generate many sets of such parameters. We discussed few of the parameters in our previous lectures, so today we will discuss about another set of parameters which relates variables at the input port to those at the output port that means the $\mathbf{V_1}, \mathbf{I_1}$ that is the variable we get at the input port will be related with the output port variable that is $\mathbf{V_2}, \mathbf{I_2}$.

And these sets of parameters are known as ABCD parameters or you can also say them as transmission parameters. Now ABCD parameter is very important in powered systems because it provides measure of how circuit transmits voltage and current from source to load. So here the relationship between $\mathbf{V_1}, \mathbf{I_1}$ and $\mathbf{V_2}, \mathbf{I_2}$ is important so we will discuss the ABCD parameters today.

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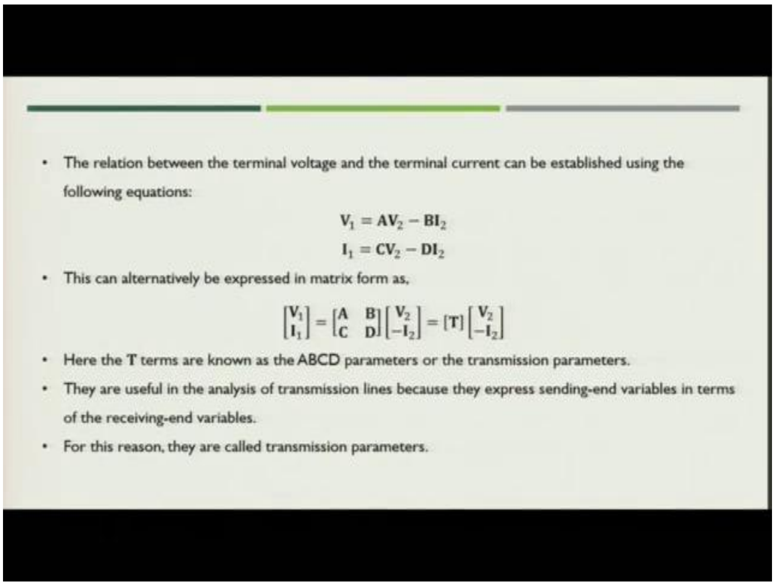

So, how we will relate these ABCD parameters with respect to the circuit variables? The relationship between terminal voltage and terminal current can be stabilised with the help of these two equations. So here the input port voltage that is $\mathbf{V_1}$ can be written in terms of output port voltage and current and it is defined as

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

Similarly, input port current $\mathbf{I}_1$ can be related to $\mathbf{V}_2, \mathbf{I_2}$ and we can write

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

Now in matrix form we can write,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} = [\mathbf{T}] \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

Now in this case if you see ABCD can be written as capital T and this defines the transmission parameters.

So, either you can write ABCD or you can simply write T, both will represent the same set of ABCD parameters. Now this is very important in the analysis of transmission lines because now they are expressing the sending-end voltages and currents in terms of receiving-end voltage and current so because of this particular property we call them as a transmission parameters.

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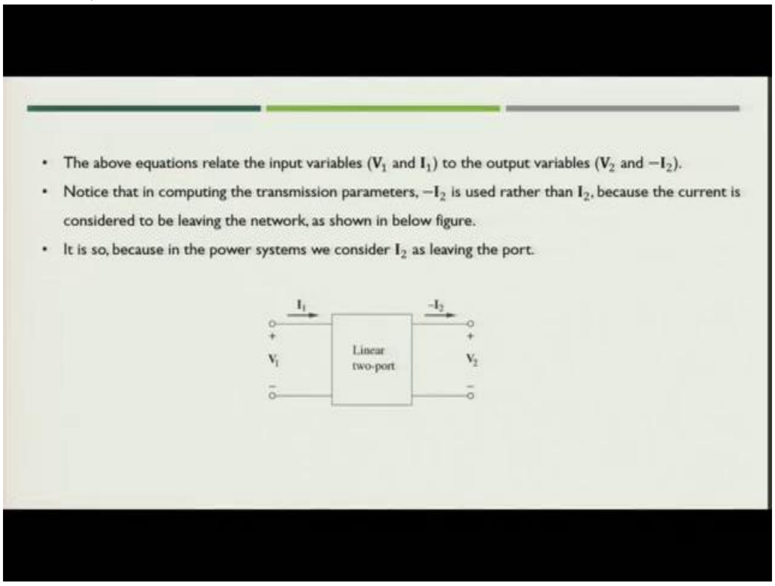

Now, the above equation which we discussed relate the input variables $\mathbf{V}_1$ and $\mathbf{I}_1$ to output variable $\mathbf{V}_2$ and $-\mathbf{I}_2$. So here the important thing is that till now we shown the relationship between $\mathbf{V}_1$, $\mathbf{I}_1$ and $\mathbf{V}_2$, $\mathbf{I}_2$, here we are considering the opposite sign of $\mathbf{I}_2$ because when you see the transmission network you take the simple power system network where the $\mathbf{V}_1$, $\mathbf{I}_1$ is inside the network and $\mathbf{V}_2$ is the output port voltage and current generally goes out of the circuit.

Because of this specific condition, related to power system network, rather than establishing the relationship between $\mathbf{V}_1$, $\mathbf{I}_1$ and $\mathbf{V}_2$, $\mathbf{I}_2$ establishes the relationship between $\mathbf{V}_1$, $\mathbf{I}_1$ and $\mathbf{V}_2$, $-\mathbf{I}_2$. So, $\mathbf{I}_2$ we generally consider in this direction, but since the load if you connect consumes current rather than providing current so that is why the transmission parameters can best be described by reversing the sign of current $\mathbf{I}_2$ so that is why we use here $-\mathbf{I}_2$ rather than $\mathbf{I}_2$.

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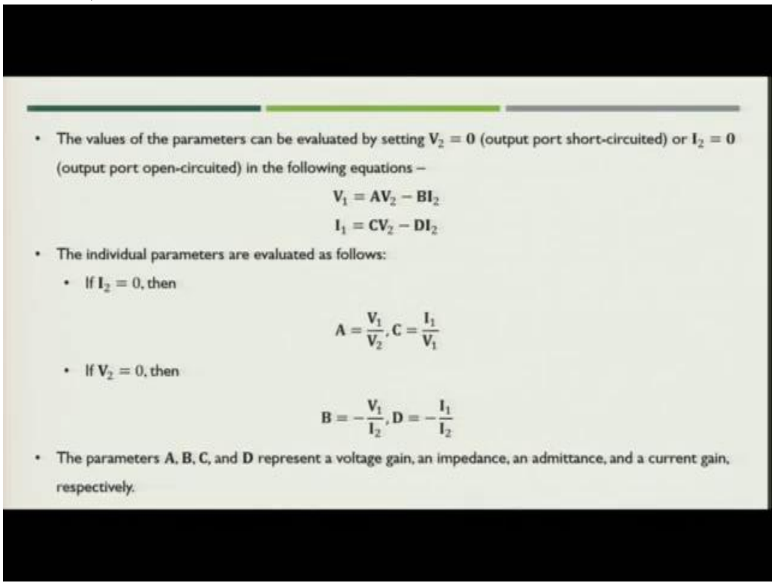

Now, let us see how we will calculate these parameters. The parameters we can calculate by setting $\mathbf{V_2} = \mathbf{0}$ that means you set the output port as short-circuited or $\mathbf{I_2} = \mathbf{0}$ that means you set output port open circuit. Now, the equations which we saw in terms of ABCD parameters which relates $\mathbf{V_1}$, $\mathbf{I_1}$ and $\mathbf{V_2}$, $\mathbf{I_2}$ is like this shown in the slide,

$$\mathbf{V_1} = \mathbf{AV_2} - \mathbf{BI_2}$$

$$\mathbf{I_1} = \mathbf{CV_2} - \mathbf{DI_2}$$

Now in these equations, if you set $\mathbf{I_2} = 0$ that means you are setting output port as open circuit. In that scenario,

$$\mathbf{A} = \frac{\mathbf{V_1}}{\mathbf{V_2}}, \mathbf{C} = \frac{\mathbf{I_1}}{\mathbf{V_1}}$$

Now, next is the condition when you set $\mathbf{V_2} = 0$ that means you are keeping output port short-circuited, you get,

$$\mathbf{B} = -\frac{\mathbf{V_1}}{\mathbf{I_2}},\ \mathbf{D} = -\frac{\mathbf{I_1}}{\mathbf{I_2}}$$

Now, what these parameters are representing? If you see A, A is nothing but a reverse voltage gain, B is nothing but the impedance, C is the admittance and D is current gain.

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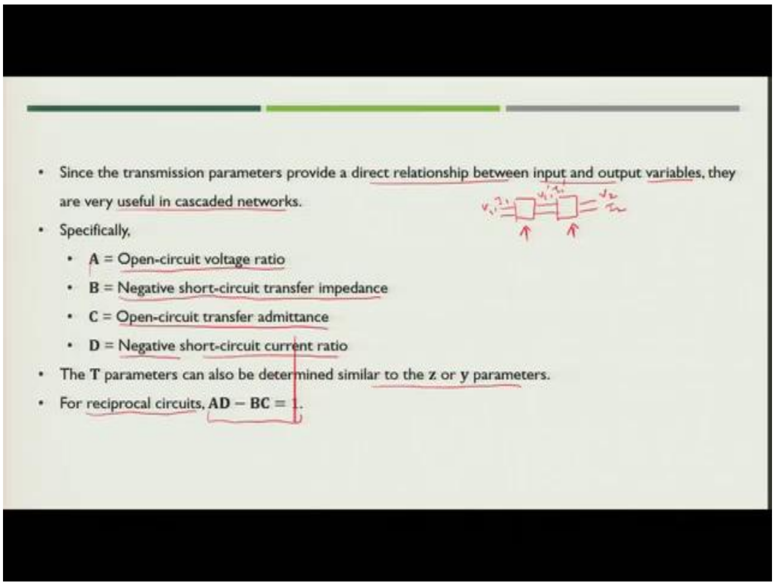

So, what we can say that since the transmission parameters provides a direct relationship between input and output variables, they are very useful in cascaded networks that means if you have two networks connected in cascaded fashion so you can say that here the parameters are $\mathbf{V}_1$, $\mathbf{I}_1$, intermittent parameters you can say $\mathbf{V}_1{}'$, $\mathbf{I}_1{}'$ and output you may have $\mathbf{V}_2$, $\mathbf{I}_2$ as the variables. The transmission parameters will help you to stabilise the relationship in those cases when you have cascaded networks. Analysis of these kind of networks we will see the next lecture when we discuss about the interconnection of various networks, we will see that particular aspect in detail.

For today's lecture we will first discuss the key properties of the transmission parameters, so we will see what are those the properties. First let us see what ABCD defines; A will be your open circuit voltage ratio because this you calculate when you keep output port as open circuit, B is negative short-circuit transfer impedance, C is open circuit transfer admittance and D is again negative short-circuit current ratio.

Now, one of the important properties in case of ABCD parameters is that if you want to see whether the two port network is reciprocal or not, you need to find out with the help of this condition. So, if $\mathbf{AD} - \mathbf{BC} = 1$, we will say that the circuit is reciprocal. Now, how we will define, determine the values of ABCD? We will define the values of ABCD similar to what we did in case of z, y or in case of h and g parameters, means we will first open circuit the output port and then we will short-circuit the output port and find out the value of ABCD parameters.

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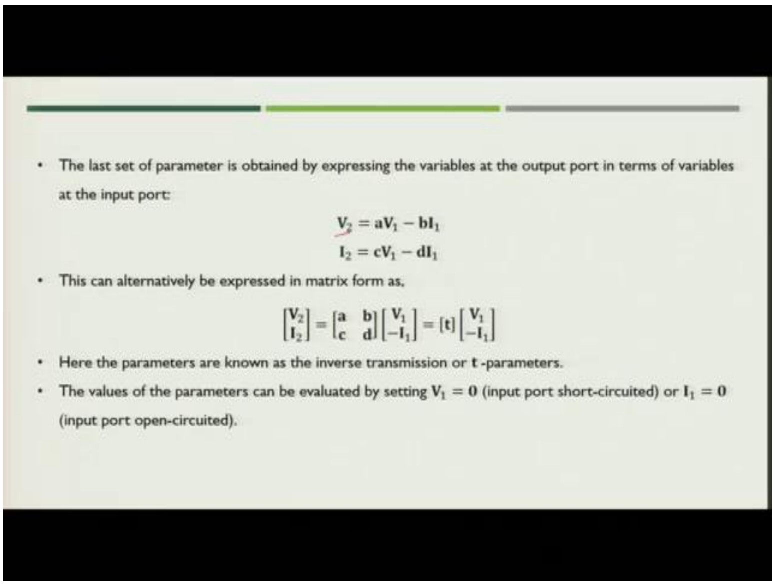

Now, next is the opposite of what we discussed till now means the relationship will now be $\mathbf{V}_2$, $\mathbf{I}_2$ will be related with respect to $\mathbf{V}_1$, $\mathbf{I}_1$ that means we can also set $\mathbf{V}_1$, $\mathbf{I}_1$ as independent variables and $\mathbf{V}_2$, $\mathbf{I}_2$ as dependent variables, in that case the relationship which we get is called as a inverse transmission or inverse T parameters.

We establish this relationship with the help of small abcd because it is opposite to the capital ABCD parameter which we have discussed till now where we establish the relationship of $\mathbf{V}_1$, $\mathbf{I}_1$ with respect to $\mathbf{V}_2$, $\mathbf{I}_2$. In this case the relationship will be $\mathbf{V}_2$, $\mathbf{I}_2$ with respect to $\mathbf{V}_1$, $\mathbf{I}_1$, so here $\mathbf{V}_1$, $\mathbf{I}_1$ is independent, $\mathbf{V}_2$, $\mathbf{I}_2$ is dependent.

So, how we will write?

$$\mathbf{V}_2 = \mathbf{a}\mathbf{V}_1 - \mathbf{b}\mathbf{I}_1$$

$$\mathbf{I}_2 = \mathbf{c}\mathbf{V}_1 - \mathbf{d}\mathbf{I}_1$$

Alternatively, in the matrix form you can write it like,

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} = [\mathbf{t}] \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix}$$

So, we represent this abcd matrix with small t, so we say it as a inverse transmission or a small t parameters. Now, how you will find out the values? You will apply the same criteria which we did in the case of transmission parameters. Here the value of parameters can be evaluated by now setting $\mathbf{V_1} = \mathbf{0}$ that means (open port is) input port is short-circuited and $\mathbf{I_1} = \mathbf{0}$ that means input port is open circuited.

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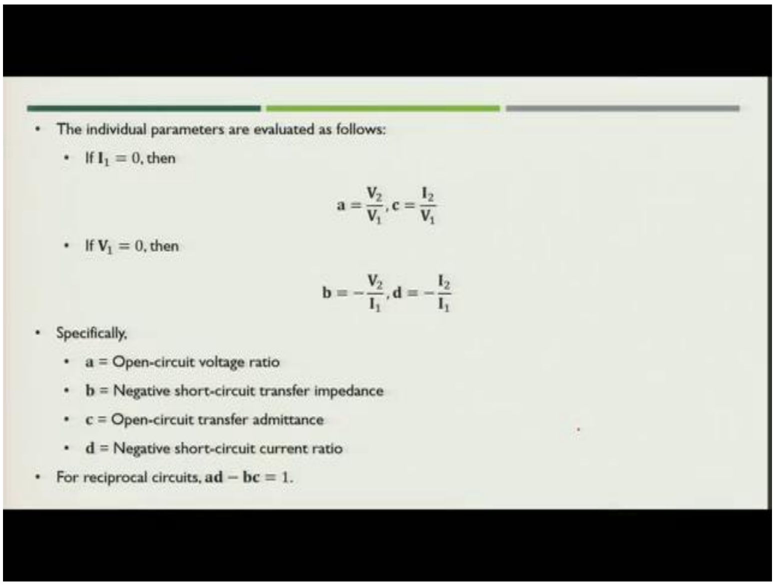

How you will find the values of parameters? So, when you set $\mathbf{I_1} = 0$, you will get,

$$\mathbf{a} = \frac{\mathbf{V_2}}{\mathbf{V_1}}, \mathbf{c} = \frac{\mathbf{I_2}}{\mathbf{V_1}}$$

Then we will set $\mathbf{V_1} = 0$, to get,

$$\mathbf{b} = -\frac{\mathbf{V_2}}{\mathbf{I_1}}, \ \mathbf{d} = -\frac{\mathbf{I_2}}{\mathbf{I_1}}$$

So, what you can write? a is nothing but open circuit voltage ratio, what is b? b is negative short-circuit transfer impedance, because it is basically the relationship ratio between V and I, so it is negative short-circuit transfer impedance, c is open circuit transfer admittance because it is $\frac{\mathbf{I_2}}{\mathbf{V_1}}$ ratio and then d is negative short-circuit current ratio that is relationship between the ratio between $\frac{\mathbf{I_2}}{\mathbf{I_1}}$.

So, these small abcd parameters can also be utilised in case of the transmission lines particularly the power systems because here also we get the impact of output variables that is $\mathbf{V_2}$, $\mathbf{I_2}$ on the input variables that is $\mathbf{V_1}$, $\mathbf{I_1}$. So here we will say that the circuit is reciprocal again if $\mathbf{ad} - \mathbf{bc} = 1$.

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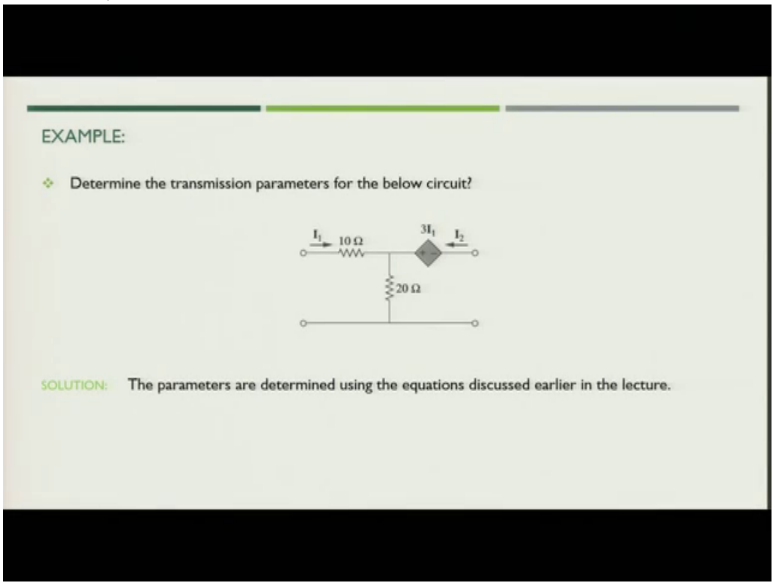

Now, let us a couple of examples to understand these parameters. Let us see there is one circuit as given in the figure, we have two resistances and one dependent voltage source and we need to find out the transmission parameters for this particular circuit so we will use the equations and we will set first voltage the output port as open circuit and then short-circuit to find out the circuit parameters.

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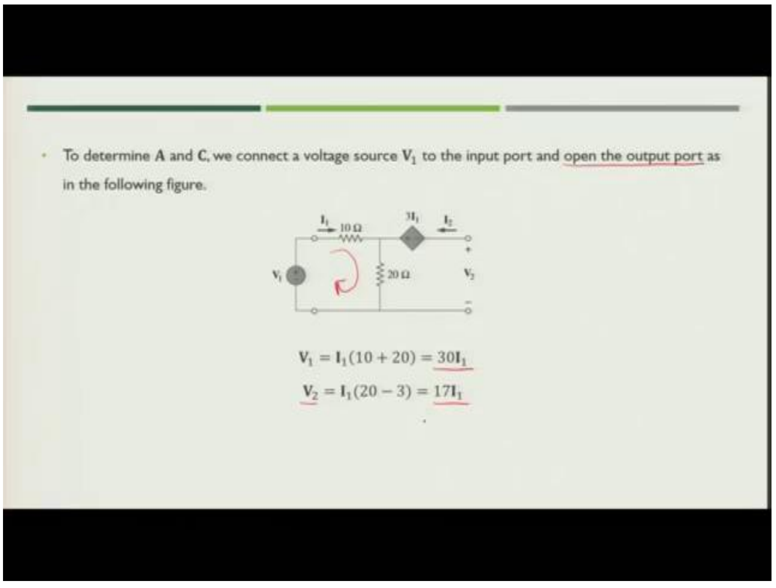

So now, in first case what we are doing? We are opening the output port and the input port we have a voltage connected that is $\mathbf{V}_1$. And when we have this particular condition, we put these values in the ABCD equation that is $\mathbf{V}_1, \mathbf{I}_1$ equations.

From this figure,

$$\mathbf{V}_1 = \mathbf{I}_1(10 + 20) = 30\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_1(20 - 3) = 17\mathbf{I}_1$$

So, $\mathbf{V}_1$ and $\mathbf{V}_2$ we have got with respect to input current $\mathbf{I}_1$.

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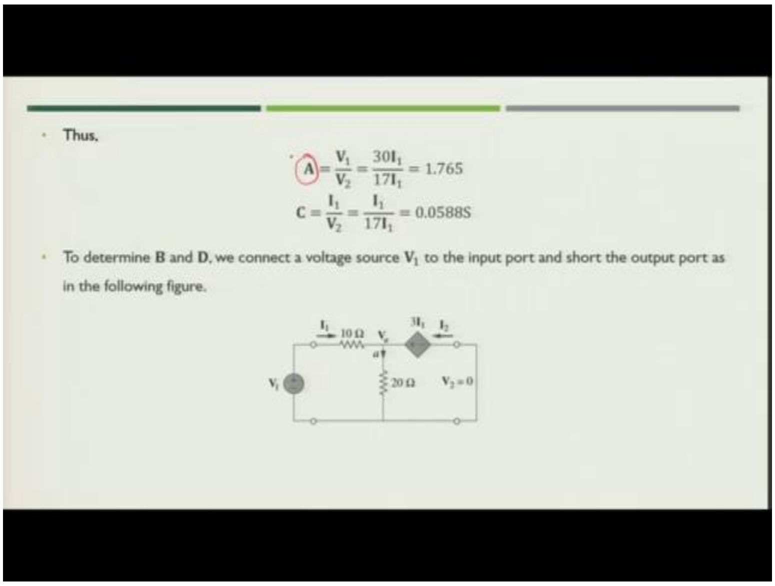

So, what you can write now with the help of equations for ABCD parameters, we know that,

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{30\mathbf{I}_1}{17\mathbf{I}_1} = 1.765$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_1}{17\mathbf{I}_1} = 0.0588\text{S}$$

Now, next we have found the value of A and C, next we need to find out the value of B and D. So for that what we have to do? We have to short-circuit the output port and apply a voltage $\mathbf{V}_1$ in the input port and solve the equations.

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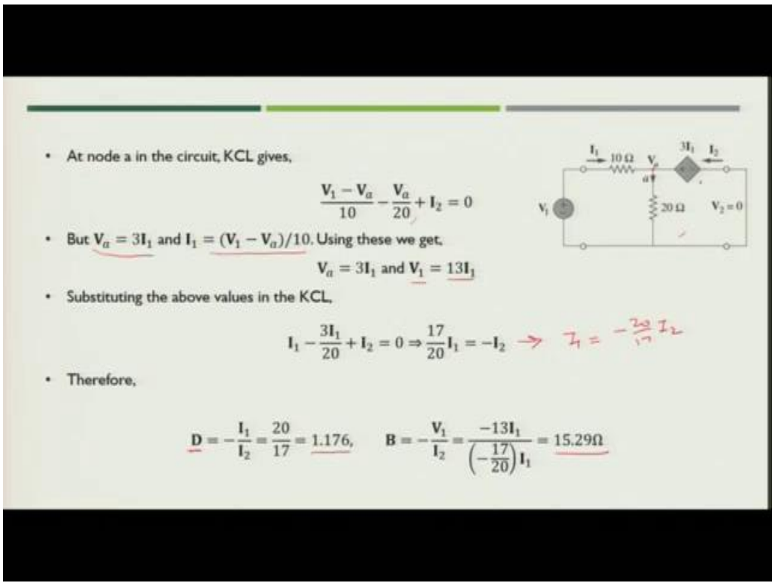

You will, if you see the circuit you can apply Kirchhoff current law at this node, let say the voltage at this node is $\mathbf{V}_a$ and we apply Kirchhoff current law at this node, so what we can write? We can write the value of $\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_a)/10$ and the current is going inside, so we are taking current going inside as positive or current going outside as negative. So applying KCL,

$$\frac{\mathbf{V}_1 - \mathbf{V}_a}{10} - \frac{\mathbf{V}_a}{20} + \mathbf{I}_2 = 0$$

Now, if you see this circuit you can say that $\mathbf{V}_a$ is nothing but equal to voltage the of dependent voltage source because this is the voltage which is applied across the 20 ohms resistance and the voltage which is applied across 20 ohms resistance is nothing but $\mathbf{V}_a$. So, what you can write? You can write $\mathbf{V}_a = 3\mathbf{I}_1$ and from this section we have already seen that $\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_a)/10$. So we have these two information in our hand, so we can now say the value of $\mathbf{V}_a = 3\mathbf{I}_1$ and $\mathbf{V}_1 = 13\mathbf{I}_1$.

Substituting the above values in the KCL,

$$\mathbf{I}_1 - \frac{3\mathbf{I}_1}{20} + \mathbf{I}_2 = 0 \Rightarrow \frac{17}{20}\mathbf{I}_1 = -\mathbf{I}_2$$

Therefore,

$$\mathbf{D} = -\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{20}{17} = 1.176, \;\; \mathbf{B} = -\frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{-13\mathbf{I}_1}{\left(-\frac{17}{20}\right)\mathbf{I}_1} = 15.29\Omega$$

So, using the technique of putting output port open circuit and short-circuit one by one you can easily find out the values of ABCD parameter.

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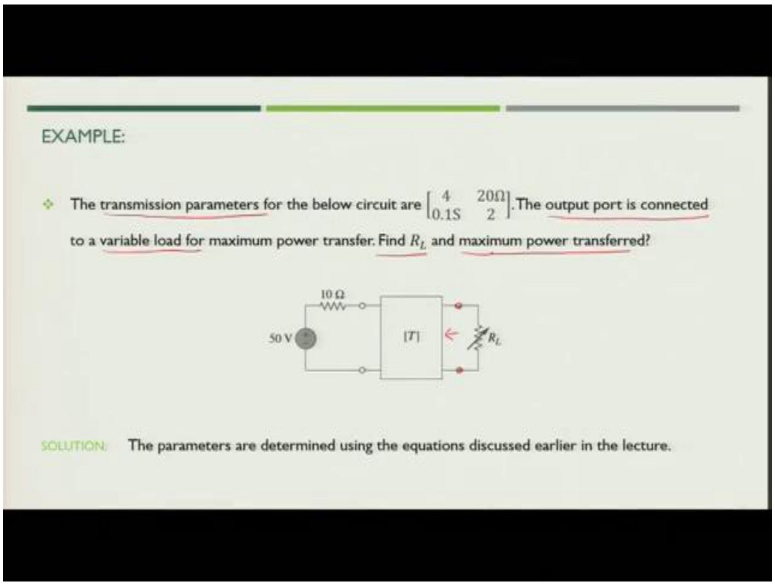

Now, let us see another example where the transmission parameters are given, so here you see the transition parameters in the matrix form, the output port is connected to a variable load for maximum power transfer. Now, what we must do? We need to find out the value of R_L at which maximum power transfer will take place and we need to find out how much maximum power will be transferred.

So, what we will do? We will utilise the same process which we discussed, we will first put output port open circuit and then short-circuit to find out the relationships and when you are required to find out the value of R_L for maximum power transfer, the objective will be to find out the Thevenin equivalent of the circuit which is left to the R_L. This segment needs to be represented as Thevenin equivalent. We will use ABCD parameters and find out the value of V_{Th} and R_{Th} so that we can find out the value of maximum power to be transferred.

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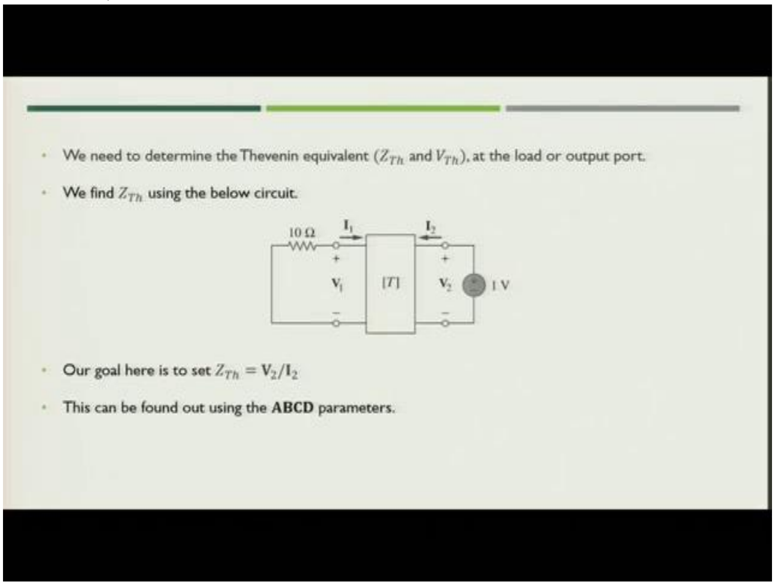

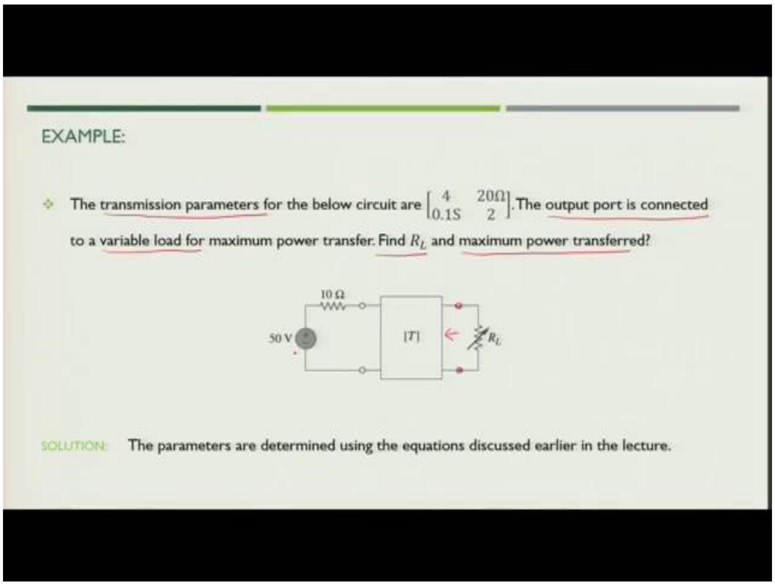

Now, first what we need to do? We must determine the Thevenin impedance and Thevenin voltage, first let us find out the value of Thevenin impedance. So for that what we have to do? To find out the value of Thevenin impedance we need to short-circuit all the independent voltage source, so you here this independent voltage source we will short-circuit and then to find out the value of Z_{Th} we will apply one voltage at the output port so that we can find the value of $\mathbf{V}_2$, so when you get the value of $\mathbf{V}_2$ and $\mathbf{I}_2$ you can simply find out the value of Z_{Th}. So this relationship $\mathbf{V}_2$ and $\mathbf{I}_2$ we will get with the help of ABCD parameters.

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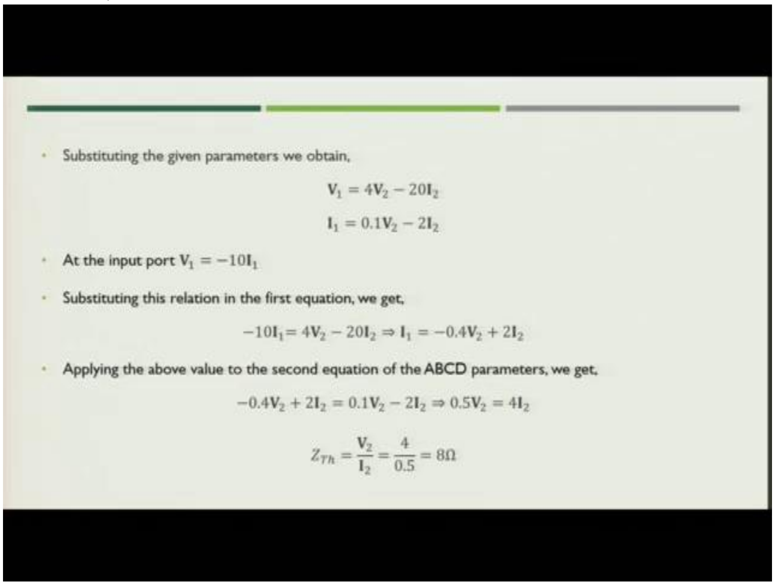

So, if you put the values of A, B, C and D you can simply write

$$\mathbf{V}_1 = 4\mathbf{V}_2 - 20\mathbf{I}_2$$

$$\mathbf{I}_1 = 0.1\mathbf{V}_2 - 2\mathbf{I}_2$$

At the input port $\mathbf{V}_1 = -10\mathbf{I}_1$

Substituting this relation in the first equation, we get,

$$-10\mathbf{I}_1 = 4\mathbf{V}_2 - 20\mathbf{I}_2 \Rightarrow \mathbf{I}_1 = -0.4\mathbf{V}_2 + 2\mathbf{I}_2$$

Applying the above value to the second equation of the ABCD parameters, we get,

$$-0.4\mathbf{V}_2 + 2\mathbf{I}_2 = 0.1\mathbf{V}_2 - 2\mathbf{I}_2 \Rightarrow 0.5\mathbf{V}_2 = 4\mathbf{I}_2$$

$$Z_{Th} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{4}{0.5} = 8\Omega$$

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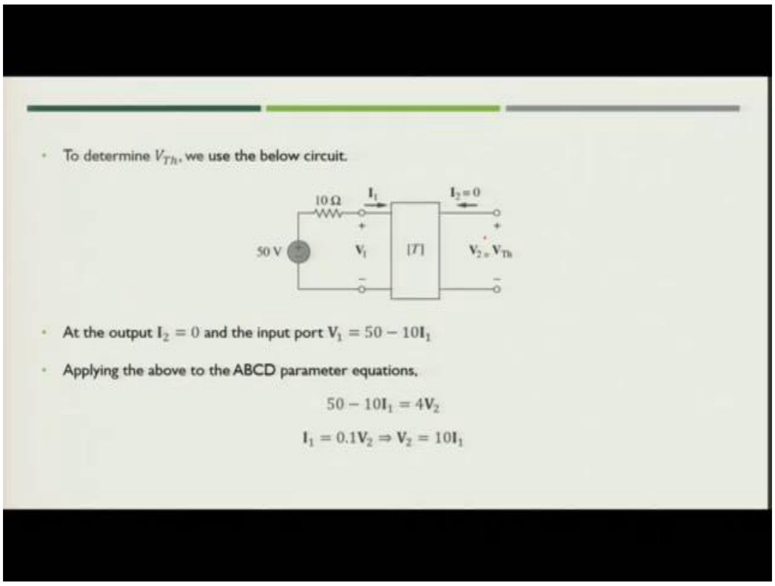

Now, next task is we need to find out the value of V_{Th}, so for finding out the value of V_{Th} the value of current $\mathbf{I}_2 = 0$ because this port will be open circuited and the input port the value of $\mathbf{V}_1 = 50 - 10\mathbf{I}_1$.

Now we apply this to ABCD parameter equations so what we can write? We can write

$$50 - 10\mathbf{I}_1 = 4\mathbf{V}_2$$

$$\mathbf{I}_1 = 0.1\mathbf{V}_2 \Rightarrow \mathbf{V}_2 = 10\mathbf{I}_1$$

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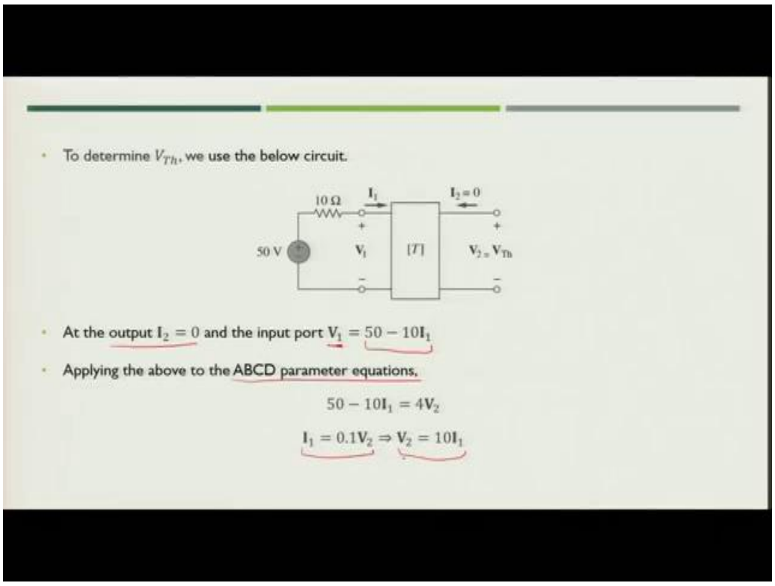

$$50 - 10\mathbf{I}_1 = 4\mathbf{V}_2 \Rightarrow 50 - \mathbf{V}_2 = 4\mathbf{V}_2$$

$$5\mathbf{V}_2 = 50 \Rightarrow \mathbf{V}_2 = 10$$

Thus,

$$V_{Th} = \mathbf{V}_2 = 10\text{V}$$

For maximum power transfer,

$$R_L = Z_{Th} = 8\Omega$$

The maximum power transfer is therefore,

$$P = I^2 R_L = \left(\frac{V_{Th}}{2R_L}\right)^2 R_L = 3.125\text{W}$$

So, with this we can close our today's session in which we can discussed about the transmission parameter and inverse transmission parameter. In next lecture we will discuss about the interconnection of various two-port networks which we have discussed in last 4 sessions, thank you.