Basic Electric Circuits
Professor Ankush Sharma
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module 8 - Two Port Network
Lecture 38 - Hybrid Parameters

Namaskar, in last two sessions we discussed about the z parameters and y parameters. In case of z parameters, we established the relationship for $\mathbf{V}_1$ and $\mathbf{V}_2$ that is the voltages at the two ports in terms of the currents. While in case of y parameters we stabilize the relationship for currents at both sides in terms of voltages. Now, third condition may arise when you have, voltage and current as a dependent component. Then in that case what will happen? That will give the new set of parameters which we will discuss in today's session.

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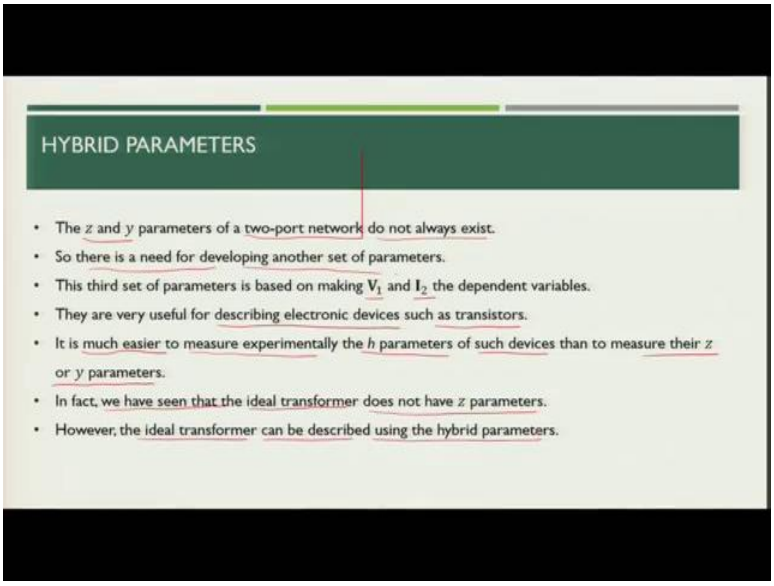

Let us start the discussion of today's lecture, we will discuss about the hybrid parameters in today's session. We have seen z and y parameters, but in case of two port network it is not necessary that you always have a flexibility to find out the z and y parameters. There is a need for developing another set of parameters, wherein you may have the dependent variables as $\mathbf{V}_1$ and $\mathbf{I}_2$, while the independent variables will be to $\mathbf{V}_2$ and $\mathbf{I}_1$.

This kind of relationships you will see mostly in the electronic devices such as transistors. In case of transistors, if you have studied, we developed the equivalent model of transistor in terms of h

parameters. In those cases we can use the h parameter variables which we will discuss in today's session.

Now, it is much easier to measure experimentally the h parameters of such devices, then to measure their z and y parameters. That is why we have more interested into the h parameters calculation. So, if you recollect that we discussed that in case of ideal transformers we cannot create the z parameters for that circuit. But in this case the ideal transformer can be described using hybrid parameters.

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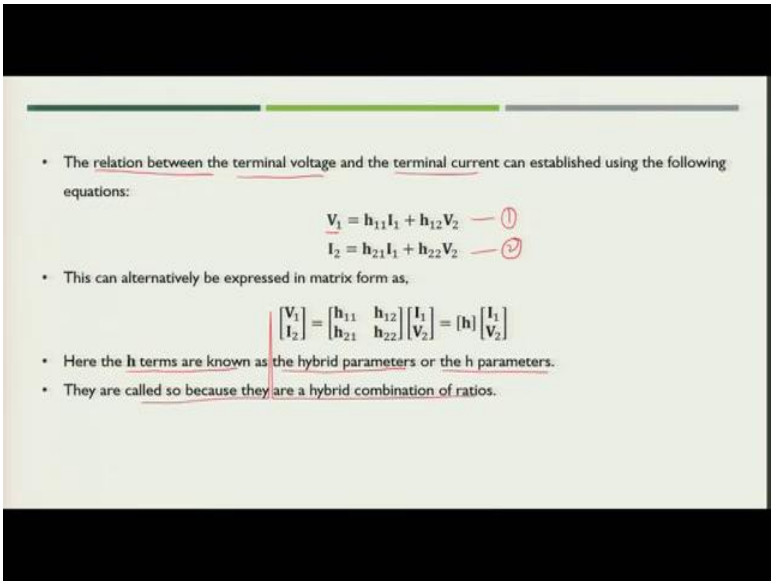

Now, how we will define the relationships between the terminal voltages and the current. In case of h parameters, we will say that $\mathbf{V_1}$ that is the input port voltage will be related to input port current and output port voltages in terms of,

$$\mathbf{V_1 = h_{11}I_1 + h_{12}V_2}$$

Similarly, output port current that is $\mathbf{I_2}$ will have a relationship between input port current and output port voltage and will define it as,

$$\mathbf{I_2 = h_{21}I_1 + h_{22}V_2}$$

These are the two basic equations to find out the h parameters.

Now in matrix form how we will represent? So you will have

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

In summary we can say that we can be present it like a input the dependent vector is equal to h parameter matrix multiplied by independent vector. Now h terms are known as the hybrid parameters or h parameters in this case.

So why they are called is hybrid? Because in this case we have a combination of ratios. So, if you see $\mathbf{h}_{11}$. It will be having the relationship between $\frac{\mathbf{V}_1}{\mathbf{I}_1}$, while $\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2}$. So, when you see these two equations you will come to know that there is no uniformity in the parameters. These are the hybrid kind of parameters which we have in case of h parameters.

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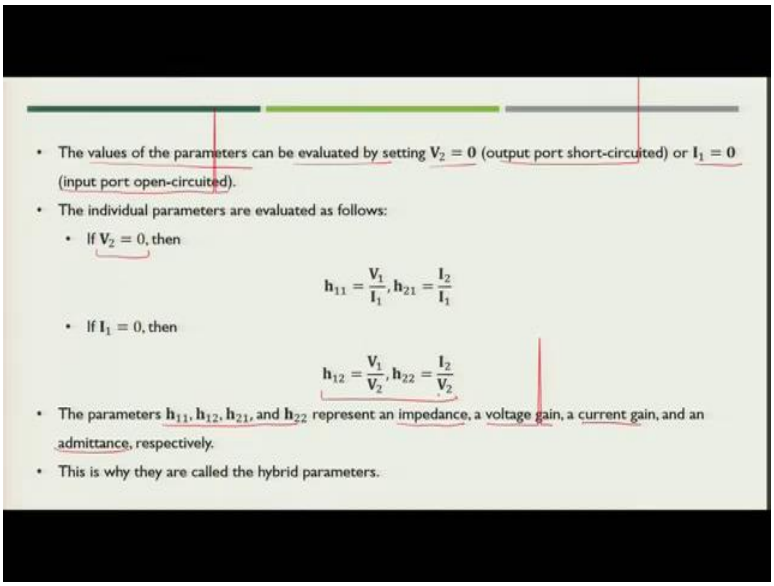

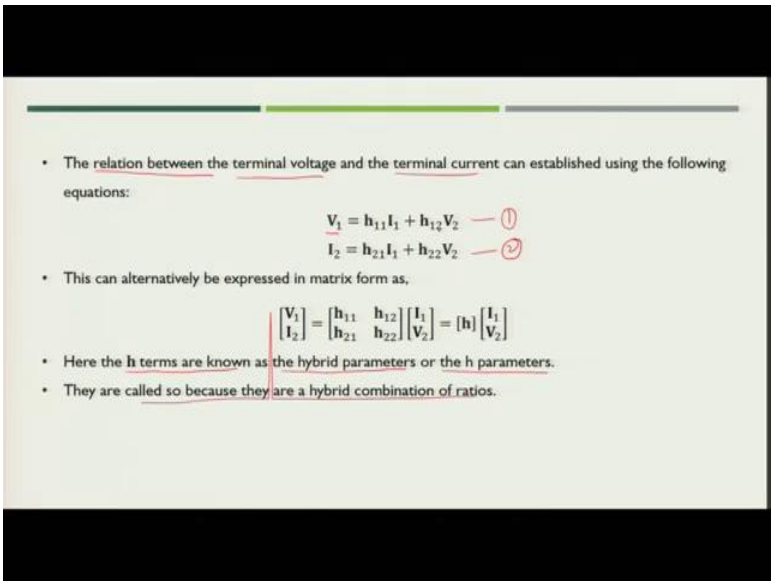

How will you find the h parameters? The value parameters can be evaluated by setting $\mathbf{V_2 = 0}$. That means you set the output port short circuit or $\mathbf{I_1 = 0}$ that is input port open circuit. So, let us state first case in which we set $\mathbf{V_2 = 0}$ that means the output port is short circuited. So, if you set $\mathbf{V_2 = 0}$ in these two equations what will happen,

$$\mathbf{h_{11} = \frac{V_1}{I_1}},\ \mathbf{h_{21} = \frac{I_2}{I_1}}$$

Now, second case is when you set $\mathbf{I_1 = 0}$. So here if you set $\mathbf{I_1 = 0}$, h12 you will get as V1 upon V2. And h22 you will get as I2 upon V2.

$$\mathbf{h_{12} = \frac{V_1}{V_2}},\ \mathbf{h_{22} = \frac{I_2}{V_2}}$$

Now these parameters, what do they represent?

$\mathbf{h_{11}}$ represents the impedance because it is the ratio between $\mathbf{V_1}$ and $\mathbf{I_1}$, $\mathbf{h_{12}}$ is the voltage gain because this is a relationship between $\mathbf{V_1}$ and $\mathbf{V_2}$, $\mathbf{h_{21}}$ is the current gain because it is again the ratio between $\mathbf{I_2}$ and $\mathbf{I_1}$ and $\mathbf{h_{22}}$ is the admittance because it is ratio between $\mathbf{I_2}$ and $\mathbf{V_2}$. So now, since we have impedance, voltage gain, current gain and admittance as a parameter, we call them as hybrid parameters.

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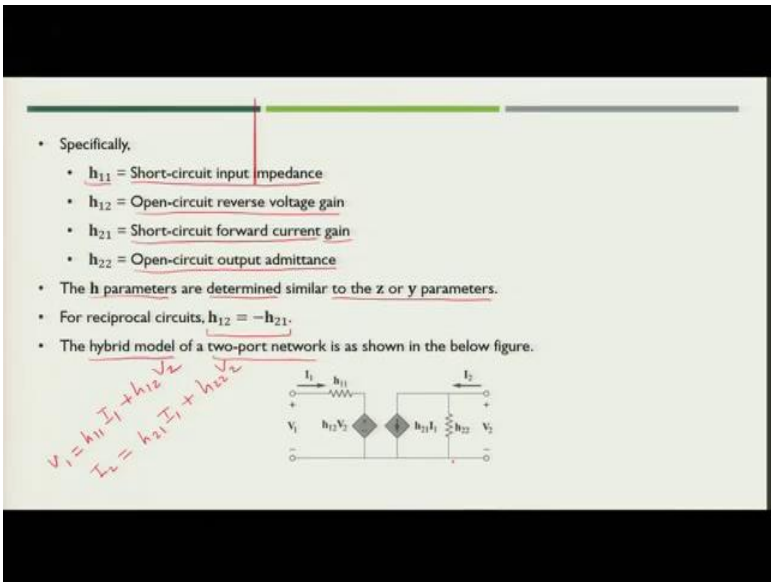

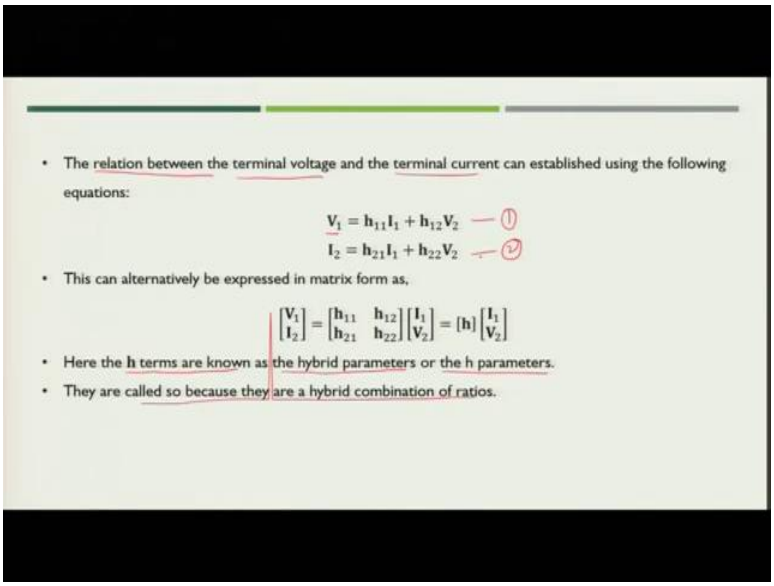

So now, you can say that,

- $\mathbf{h}_{11}$ = Short-circuit input impedance
- $\mathbf{h}_{12}$ = Open-circuit reverse voltage gain
- $\mathbf{h}_{21}$ = Short-circuit forward current gain
- $\mathbf{h}_{22}$ = Open-circuit output admittance

Now, the h parameters you can determine in a similar way as we determine in case of z or y parameters means you can just write the relationships h parameter equations. And then you can easily find out the h parameters. In case of reciprocal circuits $\mathbf{h_{12} = -h_{21}}$. So, hybrid model of a two Port network you can define like this. How you will define, how you will draw this kind of circuit?

If you see the first equation which we have $\mathbf{V_1 = h_{11}I_1 + h_{12}V_2}$, that is $\mathbf{h}_{11}$ multiplied by current flowing through this and then you have $\mathbf{h_{12}V_2}$ that is dependent voltage source. So, it will become $\mathbf{h_{12}V_2}$. This is simply a series combination of the resistor and the dependent voltage source that is why you define it in terms of voltage $\mathbf{V_1}$.

Now here, it is a parallel combination so you can straightaway utilize the nodal analysis. So, if you use Kirchhoff's current law, tou will get $\mathbf{I_2 = h_{21}I_1 + h_{22}V_2}$. These two are the same equations which we represented initially to define the h parameters. The same equations can be utilized to draw the hybrid model of a two port network, so this will be your equivalent h parameter circuit for a particular network.

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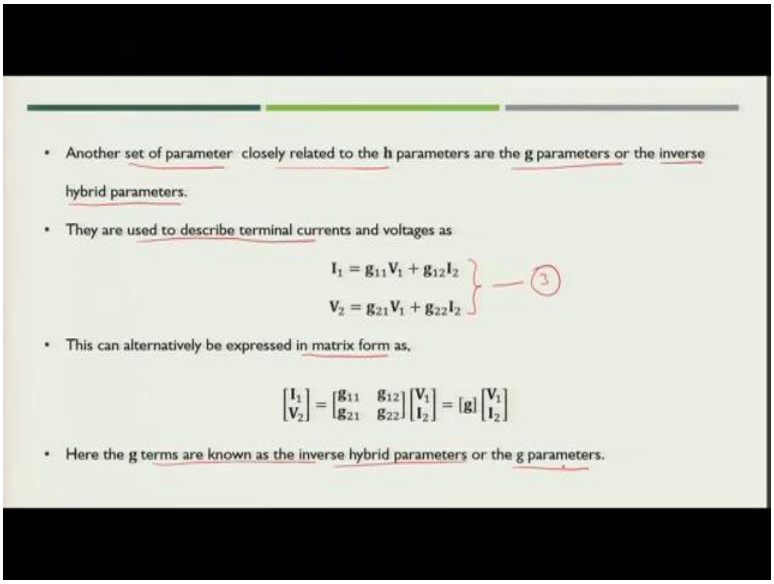

Now, there is another set of parameters which are closely related to h parameters. And what they are called? They are called as a g parameter or inverse hybrid parameters. Now in case of h parameters we stabilize the relationship between $\mathbf{V}_1$ and $\mathbf{I}_2$. So $\mathbf{V}_1$ and $\mathbf{I}_2$ were the dependent

variables in case of h parameters. Now, in this case when you are defining the g parameters or you can say the inverse of h parameters, we will have the dependent variables as $\mathbf{I}_1$ and $\mathbf{V}_2$, independent variable will be $\mathbf{V}_1$ and $\mathbf{I}_2$.

So now, how you will describe the terminal currents and voltages?

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

This can be another set of equations which will define the g parameters. In matrix from how you can write? You can write as a dependent variable vector,

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{g}_{11} & \mathbf{g}_{12}\\ \mathbf{g}_{21} & \mathbf{g}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_2\end{bmatrix} = [\mathbf{g}]\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_2\end{bmatrix}$$

So, since if you see these are inverse of h parameters, because in case of h parameters we were having $\mathbf{V}_1$ and $\mathbf{I}_2$ as dependent variable, here we are having the opposite of that, that is $\mathbf{I}_1$ and $\mathbf{V}_2$ as dependent variable. So, we will say g as inverse of hybrid parameters or we say in short as g parameters.

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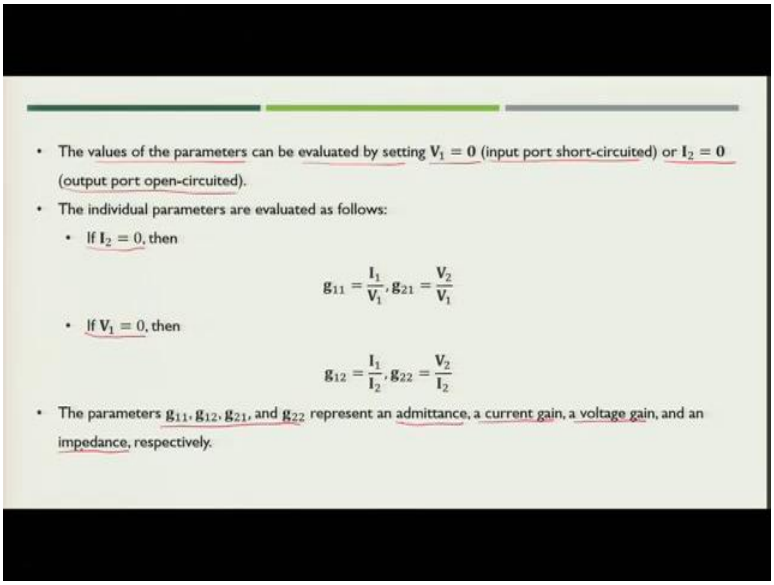

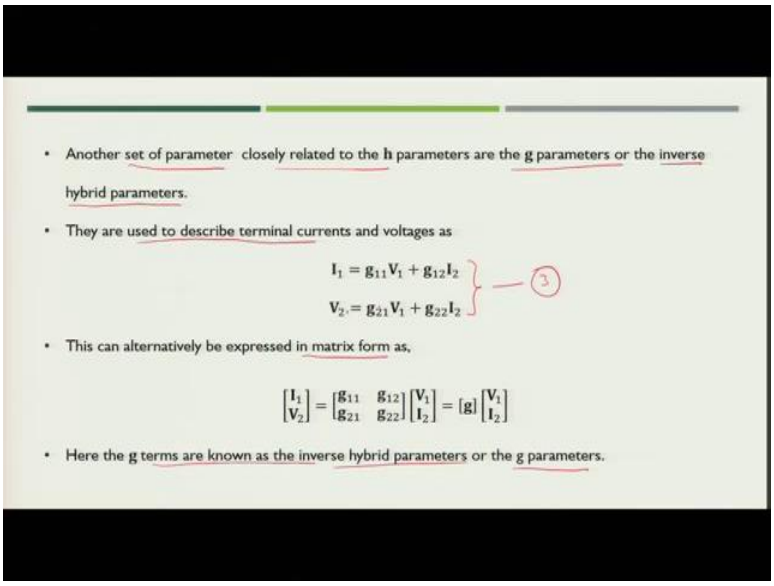

Now, how you will find out the values of g parameters? The values of these parameters can be evaluated by now setting $\mathbf{V_1 = 0}$. That is input ports short circuited. So in case of h parameters we were having $\mathbf{V_2 = 0}$ that is output port was short circuited. Here $\mathbf{V_1 = 0}$ means input port is short circuited and $\mathbf{I_2 = 0}$ that means output port open circuit. In case of h parameters, we set $\mathbf{I_1 = 0}$ that is input port open circuited.

Now, if you set $\mathbf{I_2 = 0}$, we will get,

$$\mathbf{g_{11} = \frac{I_1}{V_1},\ g_{21} = \frac{V_2}{V_1}}$$

If you set $\mathbf{V_1 = 0}$, then,

$$\mathbf{g_{12} = \frac{I_1}{I_2},\ g_{22} = \frac{V_2}{I_2}}$$

The parameters $\mathbf{g_{11}, g_{12}, g_{21},}$ and $\mathbf{g_{22}}$ represent an admittance, a current gain, a voltage gain, and an impedance, respectively. You can compare them with the h parameters where h11 was impedance, h12 was voltage gain, h21 was current gain and h22 was admittance. Here you will see the parameters are opposite to what we had in case of h parameters.

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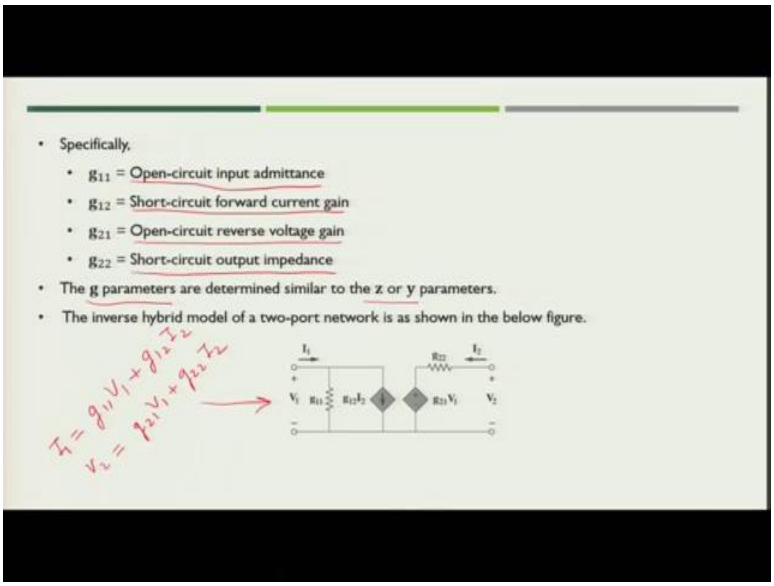

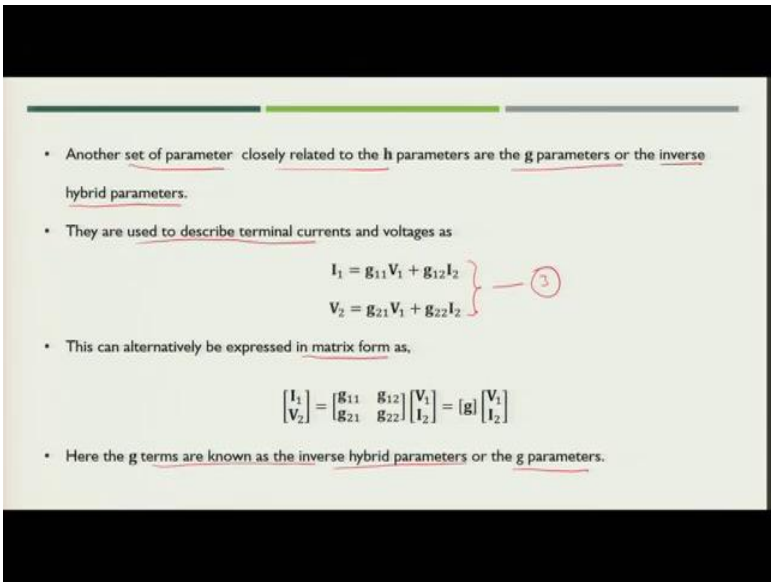

Now, you can say that

- $\mathbf{g}_{11}$ = Open-circuit input admittance
- $\mathbf{g}_{12}$ = Short-circuit forward current gain
- $\mathbf{g}_{21}$ = Open-circuit reverse voltage gain
- $\mathbf{g}_{22}$ = Short-circuit output impedance

Now, here also you can find out the g parameters similar to what we did in case of z or y parameters. Here the inverse hybrid model that is the g parameter model. You can draw like this. If you see here the current source and the resistances are in parallel so you can use nodal analysis in this case.

If you apply Kirchhoff's current law here, then $\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$ that is the dependent current source you have got. This you will get in case of this part of the circuit. In case of second part you will see that the resistances in series with dependent voltage source so you will use Kirchhoff's voltage law. When you use Kirchhoff's voltage law you will write $\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$. So now, you will see that these two are the same equations which we initially mentioned in case of g parameters. So, these equations can be used to draw the equivalent circuit for g parameters.

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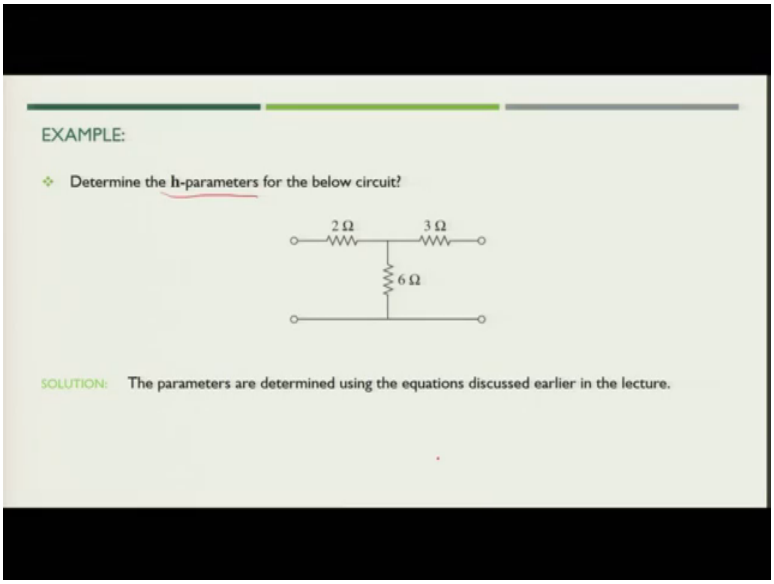

Now, let us understand this concept with the help of few examples. Let us take first the example in case of h parameters. So here if you see you have the T circuit which has one 2 ohm resistance, 3 ohm resistance and 6 ohm resistance. Now we need to find out the h parameters, so we will use the basic definition for in case of h parameters.

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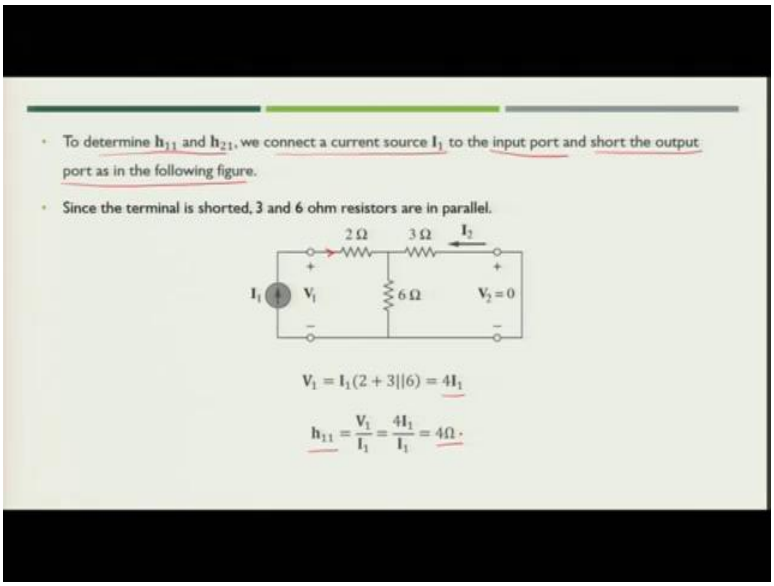

So we will first determine the h11, h21. How? We will first connect the current source $\mathbf{I}_1$ to the input port and short circuit the output port. So, if you see the updated circuit here you have the T circuit of the, T circuit given in the question. Now, you are making output port short circuit means $\mathbf{V}_2 = 0$ in this case and you are applying the current source $\mathbf{I}_1$ to the input port.

Now when you short circuit the output port these 3 ohm and 6 ohm resistance will now be in parallel. So what you can write

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3||6) = 4\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{4\mathbf{I}_1}{\mathbf{I}_1} = 4\Omega$$

Now, in the second case what you have to do? Now you will utilize the current division at this point. So, if you see the current flowing in this particular segment that is 3 ohm resistance will be the current which is flowing here will be divided in 3 ohm and 6 ohm resistance so using current division you can find out what is the value of current flowing in the through this particular 3 ohm the resistance. So, what you get using current division? You will get the value of current flowing in this. But here the value of current is also $-\mathbf{I}_2$ because the direction of current is opposite.

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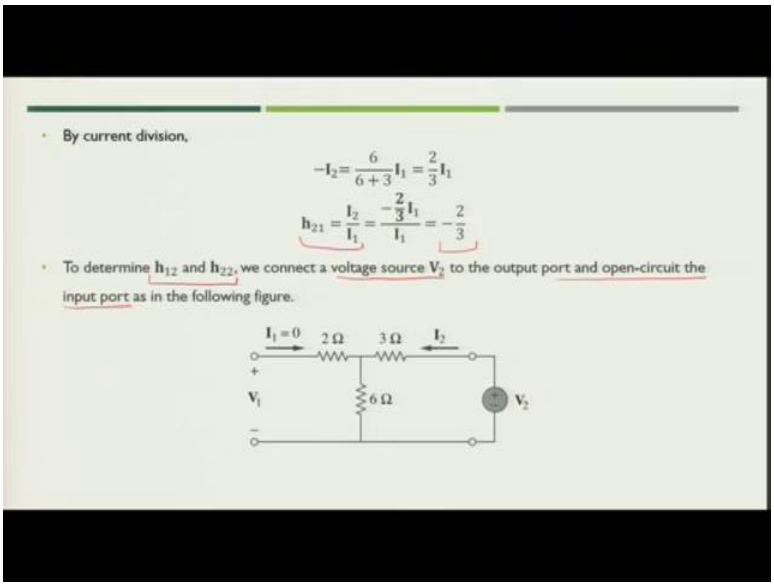

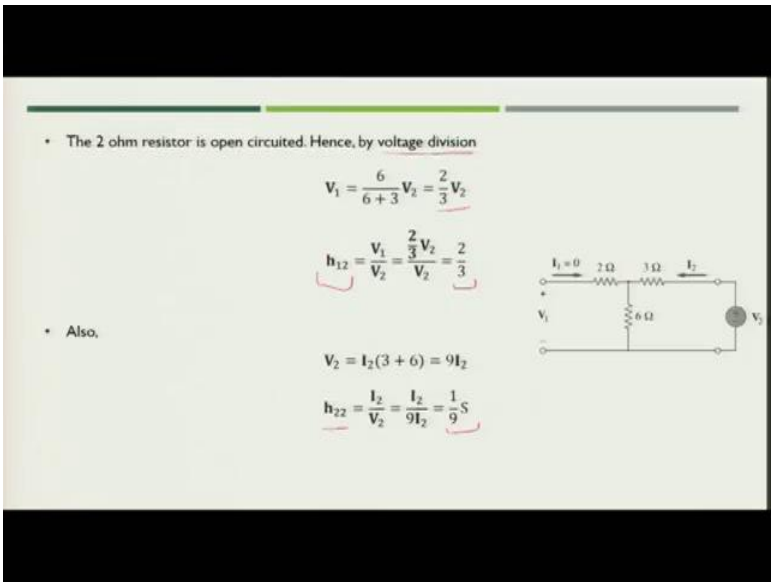

You can write

$$-\mathbf{I}_2 = \frac{6}{6+3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\frac{\mathbf{2}}{\mathbf{3}}\mathbf{I}_1}{\mathbf{I}_1} = -\frac{2}{3}$$

Now next, what we have to do? We need to find out the values of $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$. So, what we will do? In this case will connect a voltage source $\mathbf{V}_2$ to the output port and we will keep the input port as open circuit. In this case $\mathbf{I}_1$ will become 0 and we will apply $\mathbf{V}_2$ to the output port. So what will happen now?

So, if you use the Voltage division at this particular point so you will see that the value

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2$$

$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{\frac{\mathbf{2}}{\mathbf{3}}\mathbf{V}_2}{\mathbf{V}_2} = \frac{2}{3}$$

Now, also you can say that,

$$\mathbf{V}_2 = \mathbf{I}_2(3+6) = 9\mathbf{I}_2$$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{9\mathbf{I}_2} = \frac{1}{9}\mathrm{S}$$

Now, if you see $\mathbf{h}_{12} = 2/3$ and $\mathbf{h}_{21} = -2/3$. So, you can simply say that this circuit is reciprocal circuit.

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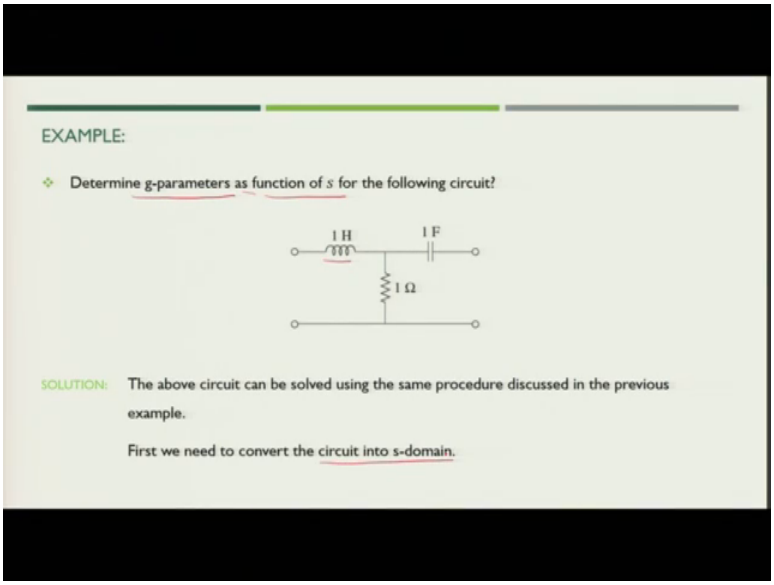

Now next, let us try to find the g parameters of the circuit which is given in this figure, here you will see that the circuit is having inductor of one henry capacitor of 1 farad and one resistance of 1 ohm. We have to find the g parameters as a function of s? So, means that we have to convert first this circuit into Laplace domain that is S domain.

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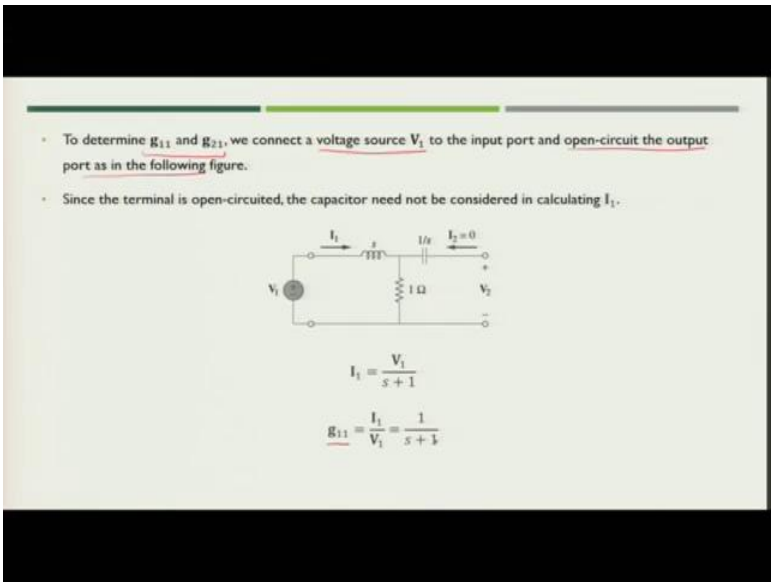

So first we will convert it to Laplace domain that is we will say that 1 henry in s domain we will be s. Capacitor will be represented as 1 by s. And the resistance will remain the same. Now, to find out the value of $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$. What we will do? We will connect the voltage source $\mathbf{V}_1$ to the input port and we will open circuit the output port. As we see in this figure. So here now what will happen? $\mathbf{I}_2$ will be zero and we will have current $\mathbf{I}_1$ flowing from the input port.

Now what is the value of $\mathbf{I}_1$? I1 you can simply represent as,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{s+1}$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{s+1}$$

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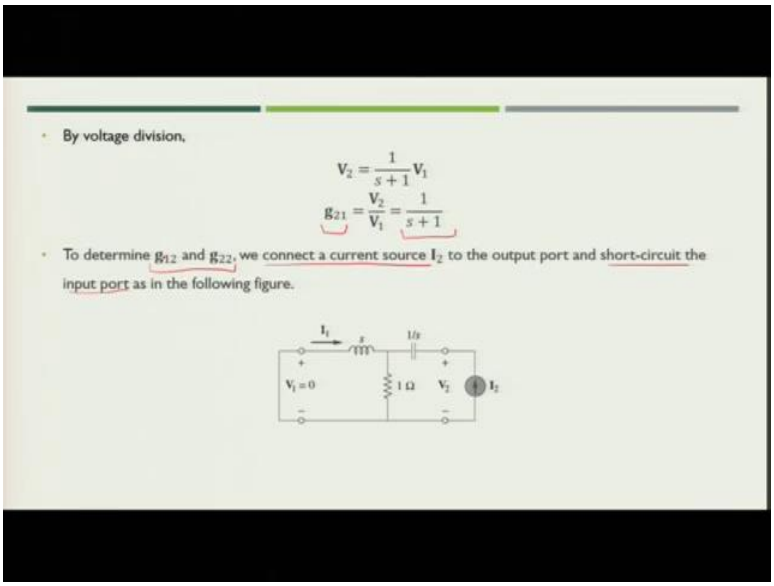

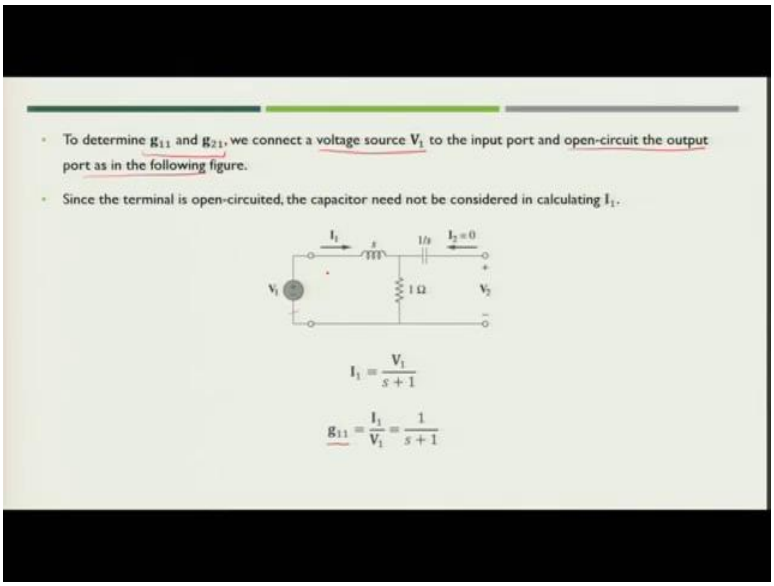

Now next, what do you have to do? You must find out the value of voltage $\mathbf{V_2}$ in terms of $\mathbf{V_1}$. So you can simply use voltage division here. Because $\mathbf{V_2}$ is applied across 1 ohm resistance, it can be represented in terms of $\mathbf{V_1}$ with the help of voltage division.

So, if you apply voltage division,

$$\mathbf{V_2} = \frac{1}{s+1}\mathbf{V_1}$$

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s+1}$$

Now next, we need to find out $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$. We have to connect a current source $\mathbf{I}_2$ to the output port and short circuit the input port. We will short circuit the input port that means $\mathbf{V}_1 = \mathbf{0}$ and we will apply $\mathbf{I}_2$ as a current source at the output terminal. Now we need to find out the value of $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$.

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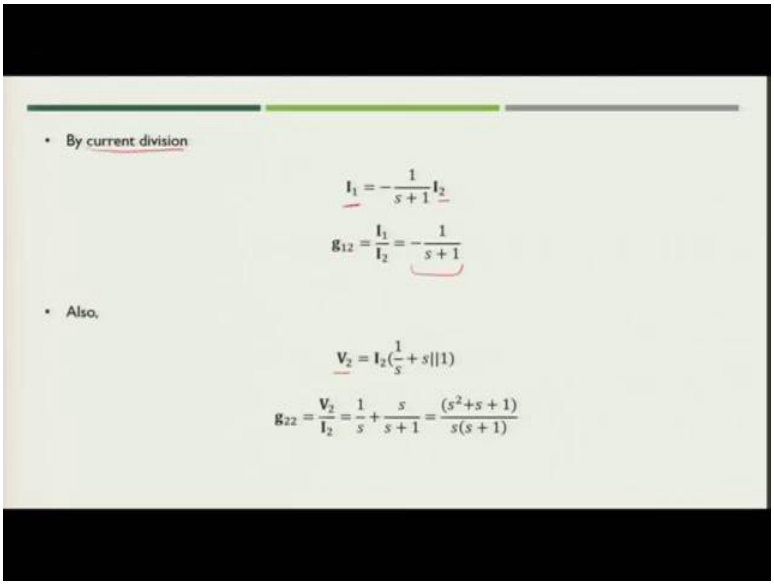

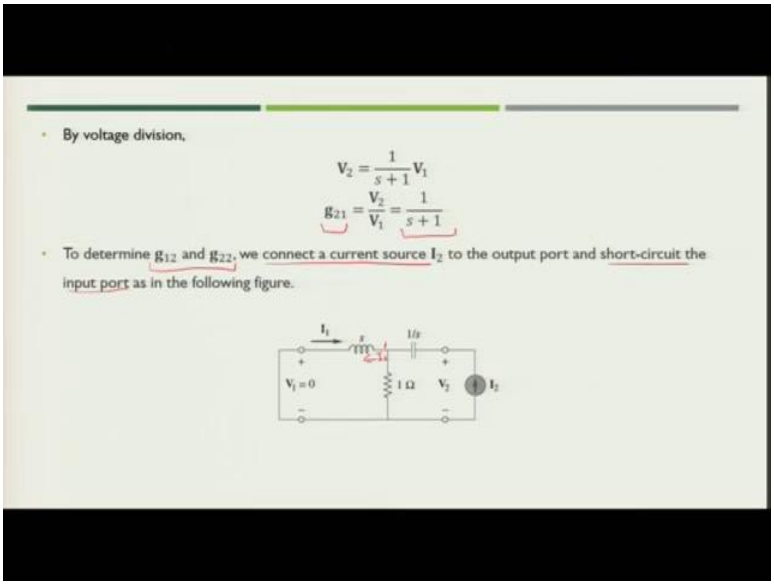

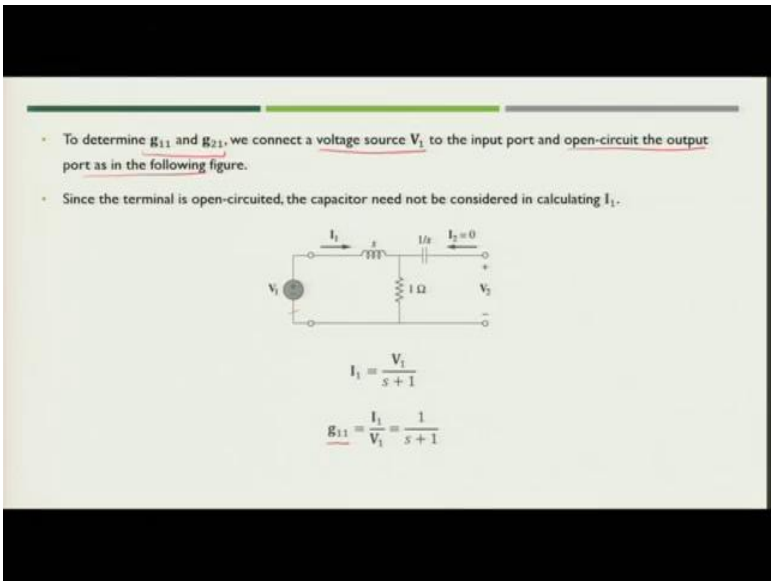

So what we will do? We will use the current division. Here if you see the current $\mathbf{I}_2$ is flowing 1 ohm resistance as well as the inductor. So, if you see that the component of current flowing through the inductor the component of I2 will be nothing but equal to minus of I1. So what we have to do? We have to find out the portion of I2 flowing through the Inductor. So for that we can simply utilize the current division.

So, if you utilize current division,

$$\mathbf{I}_1 = -\frac{1}{s+1}\mathbf{I}_2$$

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -\frac{1}{s+1}$$

Now, we need to find out the relationship between V2 and I2. Value of V2 is,

$$\mathbf{V}_2 = \mathbf{I}_2\left(\frac{1}{s} + s||1\right) = 9\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{s} + \frac{s}{s+1} = \frac{(s^2+s+1)}{s(s+1)}$$

So now, whether it is a simple network or if you get the network having that resistance, inductance and capacitance. You can easily find out the parameters that maybe z parameters, y parameters, h

parameters or g parameters. So with this we can close our today's session. In this session we discussed about two important parameters which were h parameters and g parameters. Thank you.