Basic Electric Circuits
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Module 8 - Two port Network
Lecture 37 - Admittance Parameters

Namaskar. In the last class we were discussing about the Z parameters that is also called as impedance parameters. In this class we will talk about admittance parameters which are also called as Y parameters.

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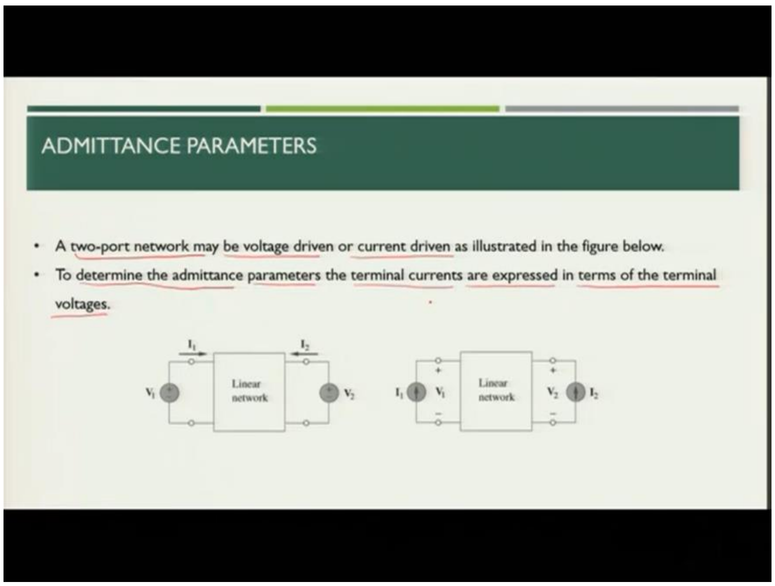

Let us start our discussion about the admittance parameters. Basically, the two port network in this case also can be the voltage driven or current driven, so you can see the figure shown in this slide that you can either apply voltage at both side of the two port network or you can apply current source at both side of the network. Now, in this case what we must do? We must determine the admittance parameter of the terminal currents and when you want to determine the admittance parameter, what you will do?

You will define the terminal currents, or you will express the terminal currents in terms of terminal voltage. So, if you compare with the impedance parameter, we were expressing the terminal voltages in terms of terminal currents. Here, we are expressing the terminal currents in terms of terminal voltages.

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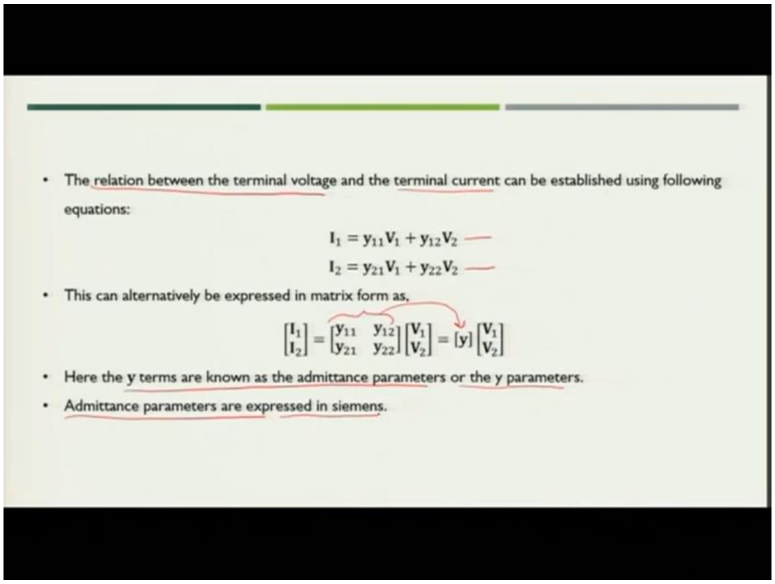

When we express terminal current in terms of terminal voltages, you will get a relationship between terminal voltage and terminal current as follows.

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

Now these values which you have just added in the expression when you compile them in the matrix form it will become the current vector, is nothing but the matrix of these constant terms are

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Now, this term can be represented in the symbolic form called Y and this is known as y parameters or admittance parameters. When you relate the terminal currents by using terminal voltages, the multiplication factors which you will use to express the current symptoms of voltages will give you the admittance parameters. Now, these admittance parameters are expressed in Siemens, you can also say expressed in mhos because these are opposite to impedance.

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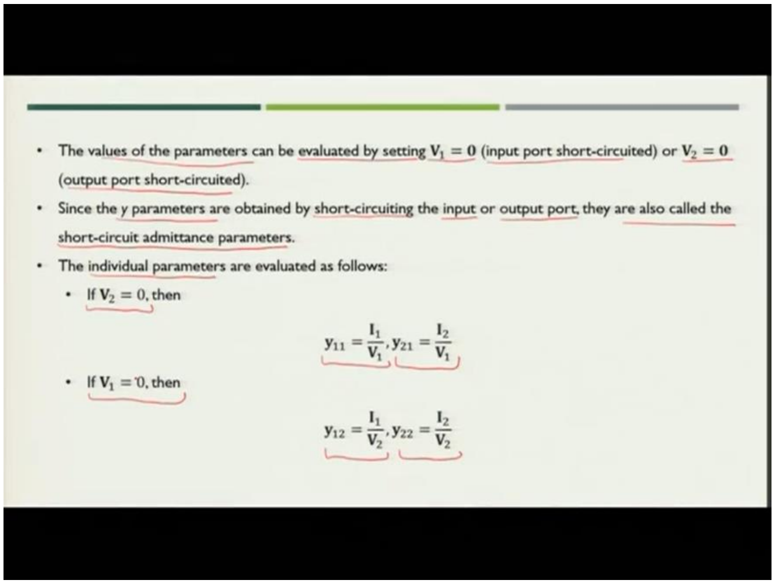

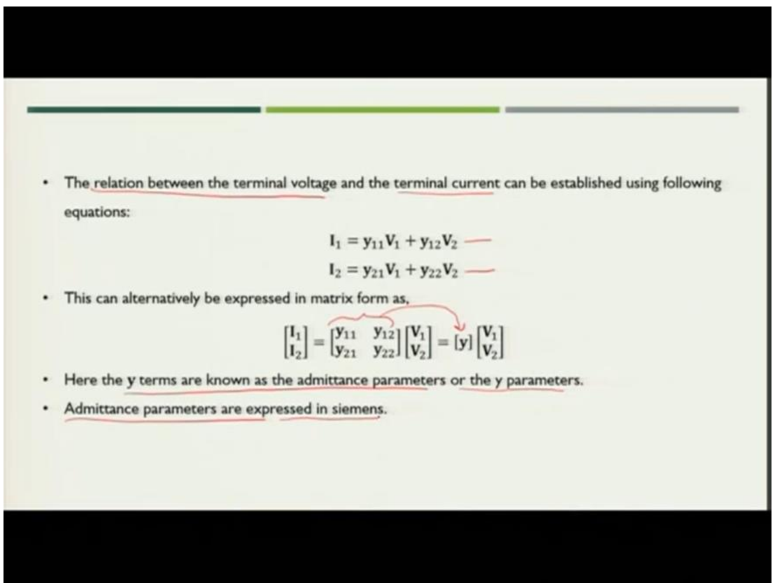

Now, the values of this parameters can be evaluated by setting $\mathbf{V_1 = 0}$ at the input port means you will have the input port short circuited or at the other end you will put the value of $\mathbf{V_2 = 0}$ means output port is short circuited. So, since the y parameters are obtained by short circuiting the input or output ports that is why they are also called as the short circuit admittance parameters. Now, let us try to find out the values of these admittance parameters individually. So, if you first set $\mathbf{V_2 = 0}$ means the output port is short circuited, you will put the value of $\mathbf{V_2 = 0}$ in these two equations, so you will get,

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}, \ \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1}$$

Now, in second case, you now put V 1 is equal to 0 means input port is short circuited so in that case what you will get? You will get y 12.

So, if you see the expression, if you set $\mathbf{V}_1 = 0$, you will get

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2}, \ \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2}$$

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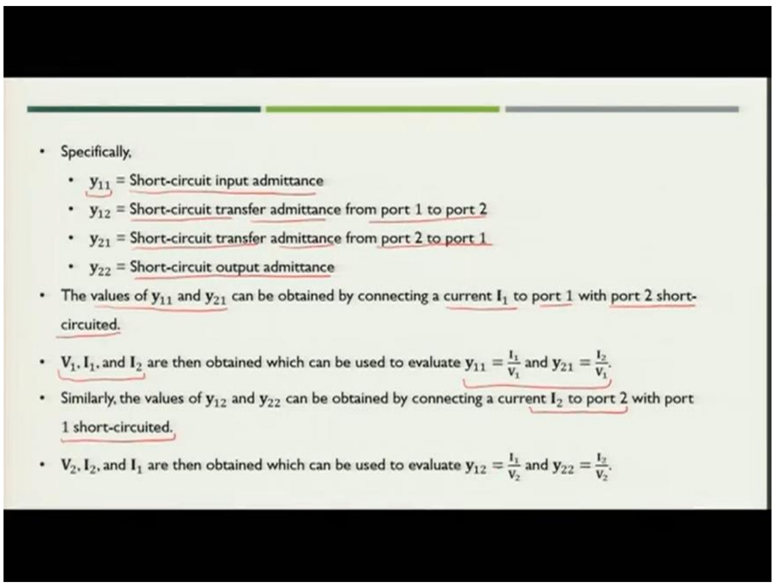

Generally,

- $\mathbf{y}_{11}$ = Short-circuit input admittance
- $\mathbf{y}_{12}$ = Short-circuit transfer admittance from port 2 to port 1
- $\mathbf{y}_{21}$ = Short-circuit transfer admittance from port 1 to port 2
- $\mathbf{y}_{22}$ = Short-circuit output admittance

So, when you connect the current source at port 1 and short circuit the port 2, you will get the values of three quantities that is $\mathbf{V}_1$, $\mathbf{I}_1$ and $\mathbf{I}_2$ and using these you can find out the value of $\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}$ and $\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1}$. Now, in the second case what you will do? You will connect current source $\mathbf{I}_2$ to the second port and you will short circuit the first port. In that case your $\mathbf{V}_1 = \mathbf{0}$, so, rest of the quantities which you obtain are $\mathbf{V}_2$, $\mathbf{I}_2$ and $\mathbf{I}_1$ and using these three quantities you will find out the value of $\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2}$ and $\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2}$.

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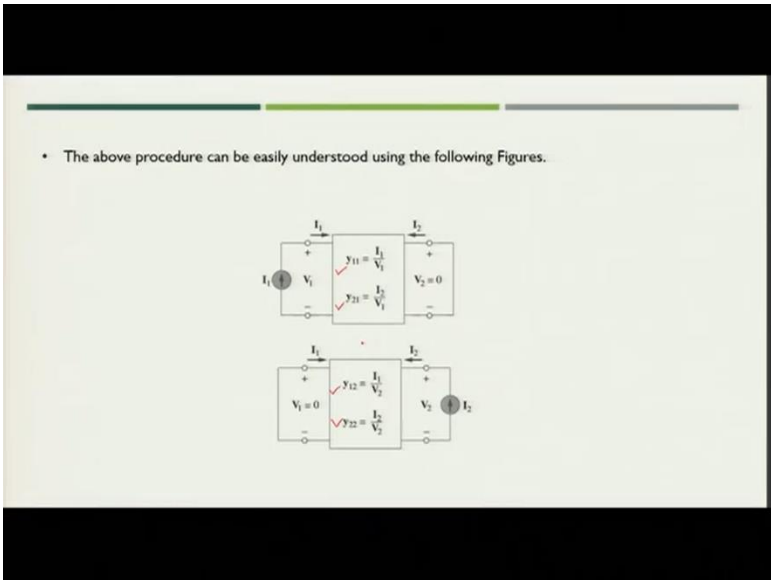

So now, you can summarize our discussion till now related to the calculation of y parameters, so you can summarize our discussion in these two figures. So, if you see this figure, what we are doing? We are applying current $\mathbf{I}_1$ to the input port and output port is short circuited. When you have this scenario, you will get the value of $\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}$ and $\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1}$.

Using current at output port divided by voltage at input port. Similarly, if you connect current source at the output port and the short circuit the input port so $\mathbf{V}_1 = \mathbf{0}$. So, you will get the values $\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2}$, $\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2}$. These two figures will summarize, how you will find out the values of the admittance parameters.

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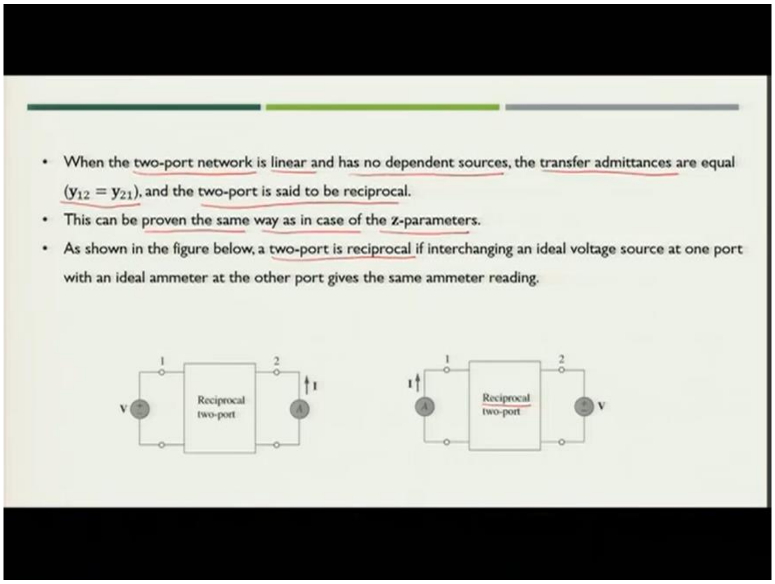

Now, when the two-port network is linear and it has no dependent sources, the transfer admittance will be equal to $\mathbf{y}_{12} = \mathbf{y}_{21}$ and then if this condition holds, you can say that two port is reciprocal. So this is similar to what we discussed in case of Z parameters calculation where $\mathbf{z}_{12} = \mathbf{z}_{21}$ and it was linear and if it was not having any dependent source, the circuit was reciprocal. So, you can correlate the reciprocity of the two port network with respect to z as well as y parameters. Now here also you can prove the scenario in the same way as we explained in case of z parameters, how?

Let us see these two figures. Now in first case, what we are doing? We are applying the voltage $\mathbf{V}$ across input port and finding out the value of current which is flowing inside the output port with the help of emitter. Now, if you see that this particular two-port network is reciprocal, it means that if you exchange the locations of voltage source and the emitter, the quantities that is I and V are not going to change so if this condition holds we will say that the circuit is reciprocal.

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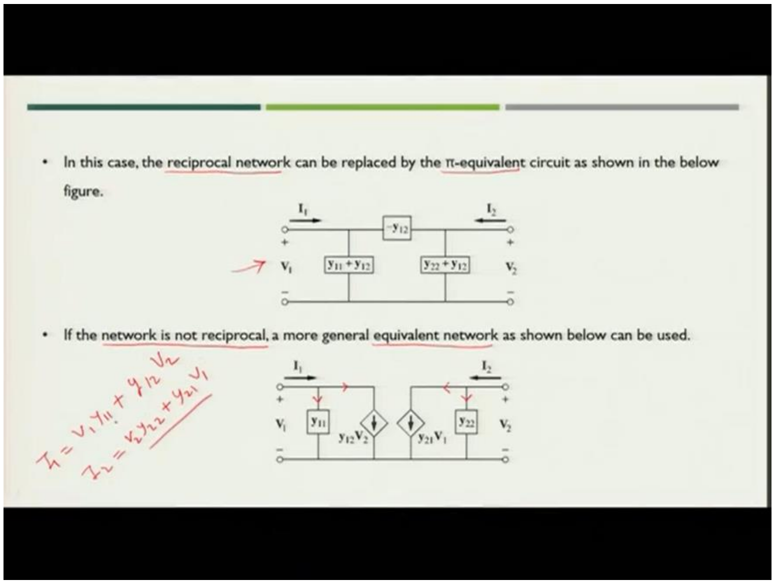

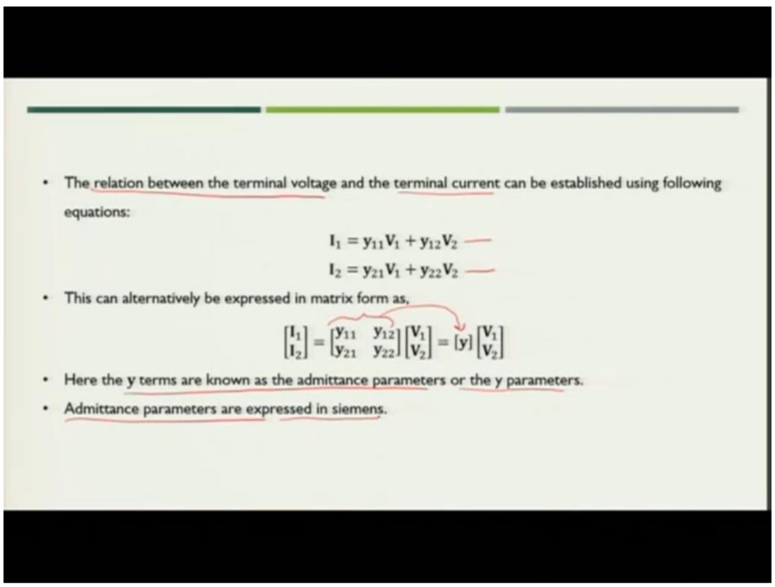

Now, what will happen in case of reciprocal network? If the network is reciprocal, we can represent circuit with its pi equivalent model. If you see the left leg of the pi circuit, the value of left leg admittance would be $\boldsymbol{y}_{11} + \boldsymbol{y}_{12}$, while the value of the right leg admittance is $\boldsymbol{y}_{22} + \boldsymbol{y}_{12}$. Now, here since $\boldsymbol{y}_{12} = \boldsymbol{y}_{21}$ so you can use either $\boldsymbol{y}_{12}$ or $\boldsymbol{y}_{21}$ both are, both will hold true and then the branch which is connecting these two legs will have the admittance equal to $-\boldsymbol{y}_{12}$.

So, if you have a reciprocal network, that network can be represented equivalently with the help of pi equivalent circuit. Now, in general if the network is not reciprocal and you want to represent the circuit in equivalent in the form of equivalent network you can use the following figure. So, if you see the value of current $\mathbf{I}_1$ if you if you use Kirchhoff's current law in the left side of the circuit, so $\mathbf{I}_1$ will be, since $\mathbf{V}_1$ is applied across $\mathbf{y}_{11}$, it will become $\mathbf{V}_1\mathbf{y}_{11} + \mathbf{V}_2\mathbf{y}_{12}$.

Similarly, the value of current $\mathbf{I}_2$, if you see from this figure the value of current flowing through this leg would be $\mathbf{V}_2\mathbf{y}_{22} + \mathbf{V}_1\mathbf{y}_{21}$. So, if you see these two equations these equations are nothing but the equations which we discussed in our previous slide. So, we can say the with the help of this particular equivalent network we can represent any two port network as shown in this particular figure so any two port network can be represented as a equivalent, as an equivalent network which would be represented like shown in the figure.

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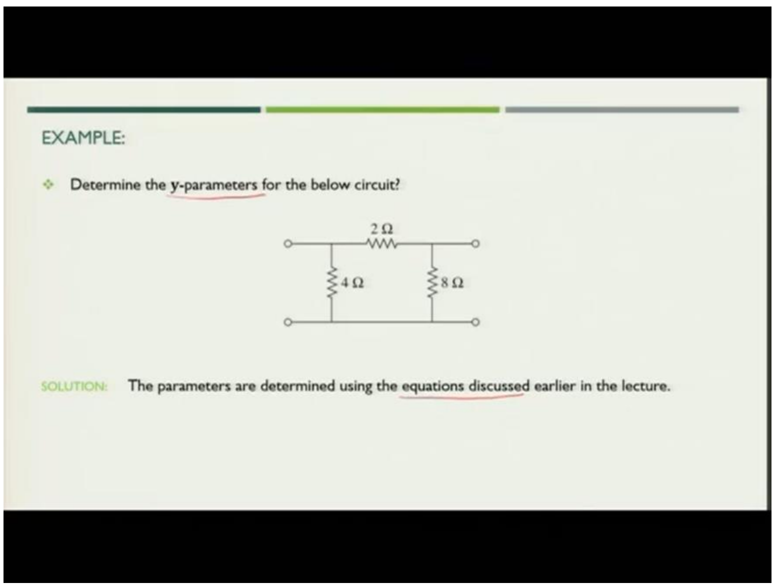

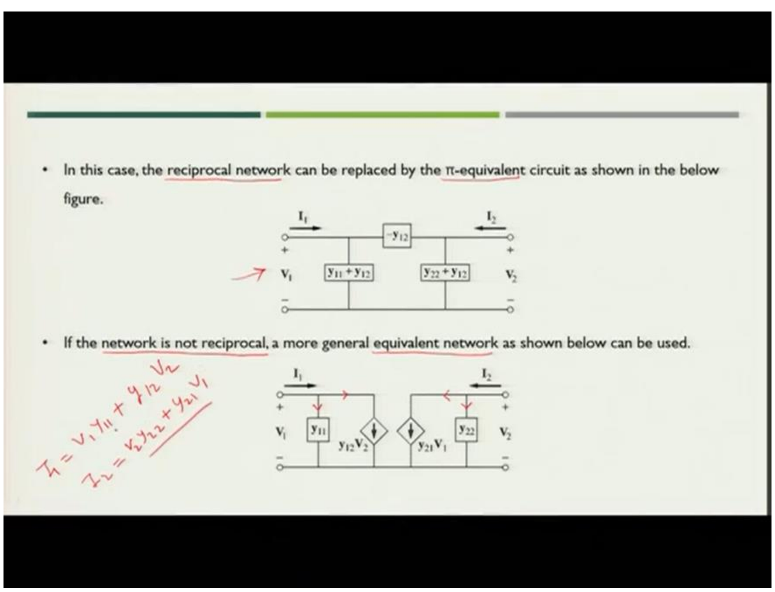

Now, let us understand this understand this concept with the help of few examples, so that you can understand the concept clearly, let us see one example where we have to find out the y parameters of the circuit given below. So, first what we will do? We will use the equations that is y parameters what we just saw means these two these two equations, so we will use these two equations and try to find out the y parameters of this circuit.

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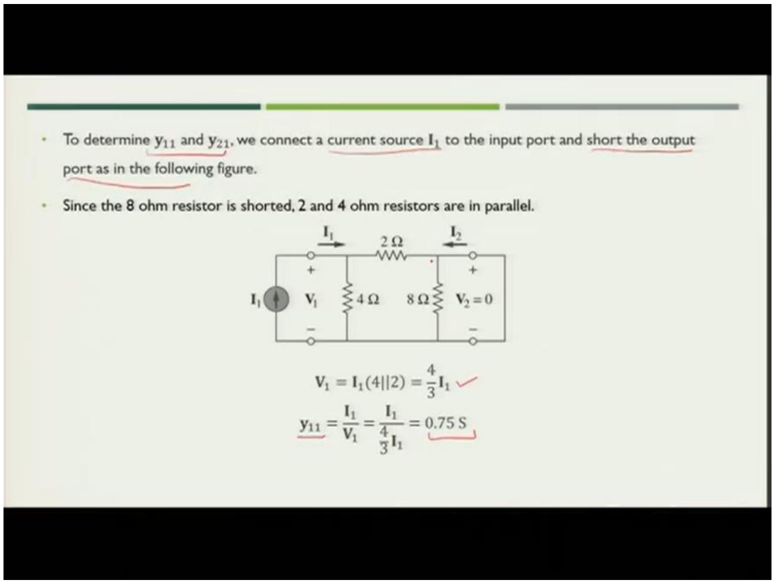

Now, first what we have to do? We will determine the value of $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ so when you are asked to find the value of $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ we will connect one current source $\mathbf{I}_1$ in the input port and we will short circuit the output port so the circuit will be as shown in the figure. Now, if you see this particular figure, the 8 ohm resistor will be now short circuited because it is connect across the terminals of the output port so the current $\mathbf{I}_2$ will be flowing through 2 ohm resistance. Now, since this is short circuited and current $\mathbf{I}_1$ is flowing form the input port we will have, will see these two resistances in parallel.

So, what will be the value of voltage $\mathbf{V}_1$?

$$\mathbf{V}_1 = \mathbf{I}_1(4||2) = \frac{4}{3}\mathbf{I}_1$$

Now, since you have got the value of $\mathbf{V}_1$ in terms of $\mathbf{I}_1$,

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = 0.75\text{S}.$$

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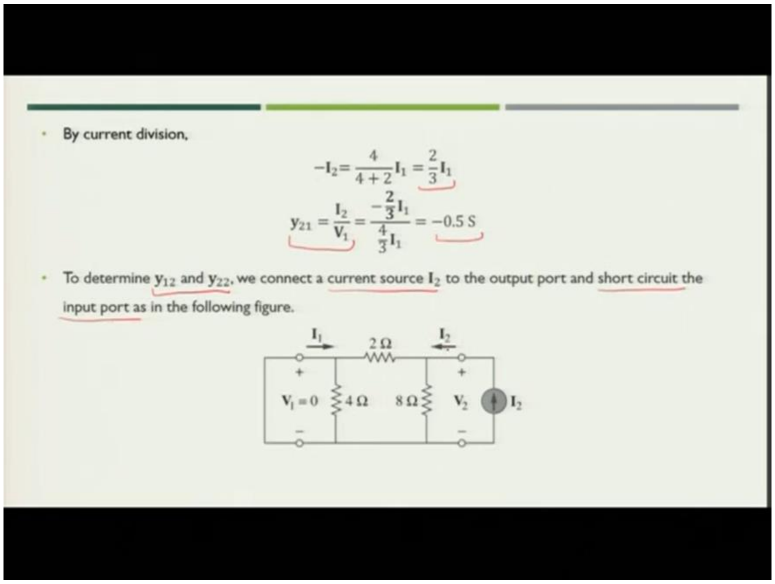

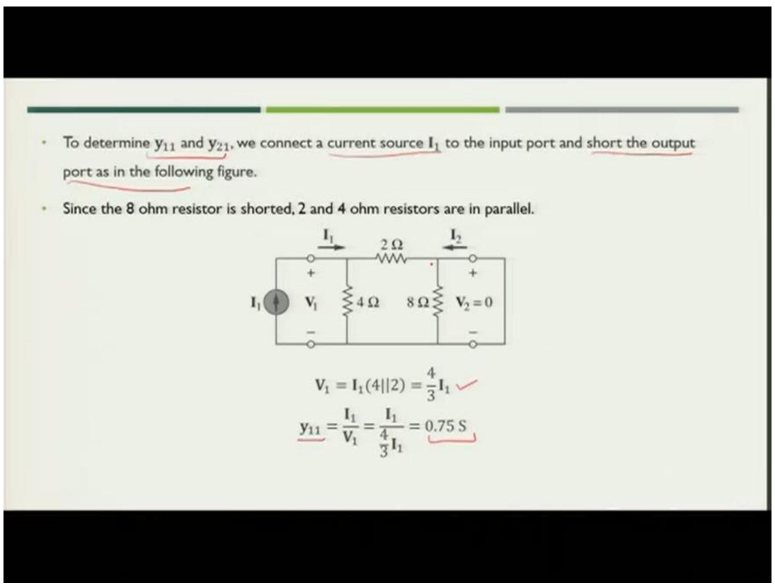

Now, next is you will use the current division and find out the relationship between $\mathbf{I}_2$ and $\mathbf{I}_1$. So, if you see this particular figure you need to find out the value of current flowing through 2 ohm resistance in terms of $\mathbf{I}_1$, with the help of current division. The component of $\mathbf{I}_1$ which is flowing through 2 ohm resistance will be equal to the value of current $-\mathbf{I}_2$ because both would be in opposite directions. So, what you can write?

$$-\mathbf{I}_2 = \frac{4}{4+2}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Now, you know that we have calculated the value of $\mathbf{V}_1$ in terms of $\mathbf{I}_1$ and we have also calculated the value of $\mathbf{I}_2$ in terms of $\mathbf{I}_1$, we can find out the value

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-\frac{\mathbf{2}}{\mathbf{3}}\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = -0.5S$$

Now, next task is to find out the value of $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$. In this case, what we will do? We will connect a current source $\mathbf{I}_2$ at the output port and we will short circuit the input port. So, your figure will look like as shown in the slide where the input port is short circuited in that case your $\mathbf{V}_1$ will become 0 and $\mathbf{I}_2$ is connected to the output port.

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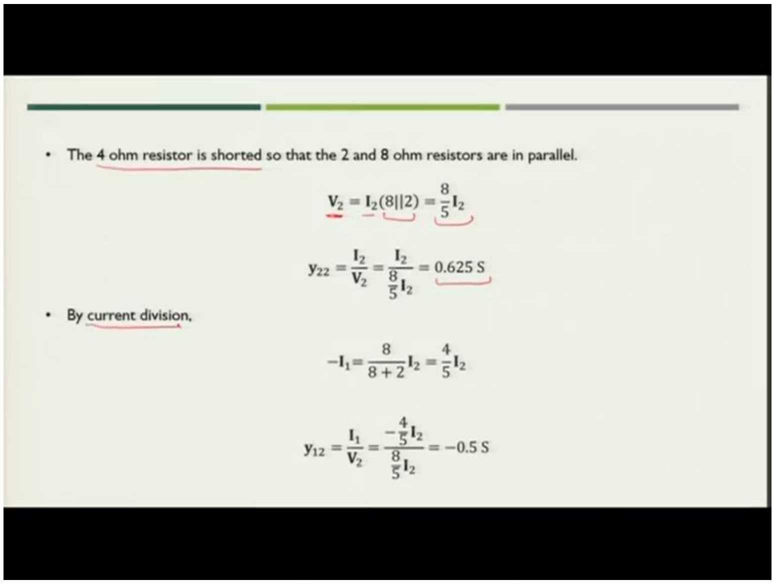

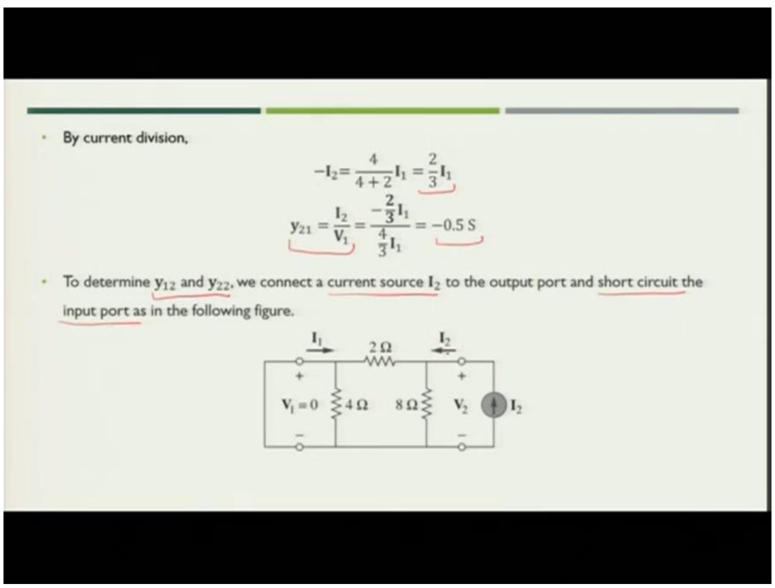

Now, in this case again the 4 ohm resistor will be short circuited because this will be short circuited because of the output port is short circuit. So now,

$$\mathbf{V}_2 = \mathbf{I}_2(8||2) = \frac{8}{5}\mathbf{I}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = 0.625S$$

Now, next again you have to use the current division, so that you can find the value of current $\mathbf{I}_1$ in terms of $\mathbf{I}_2$. So, if you see this figure $\mathbf{I}_1$ will be flowing in this particular resistance that is 2 ohm resistance but one leg of $\mathbf{I}_2$ will also be flowing through this.

You can say that the component of $\mathbf{I}_2$ flowing through 2 ohm resistance is $-\mathbf{I}_1$. Using this you can simply use the current division and find out the value of $\mathbf{I}_1$ in terms of $\mathbf{I}_2$. So, you can say

$$-\mathbf{I}_1 = \frac{8}{8+2}\mathbf{I}_2 = \frac{4}{5}\mathbf{I}_2$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\frac{4}{5}\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = -0.5\text{S}$$

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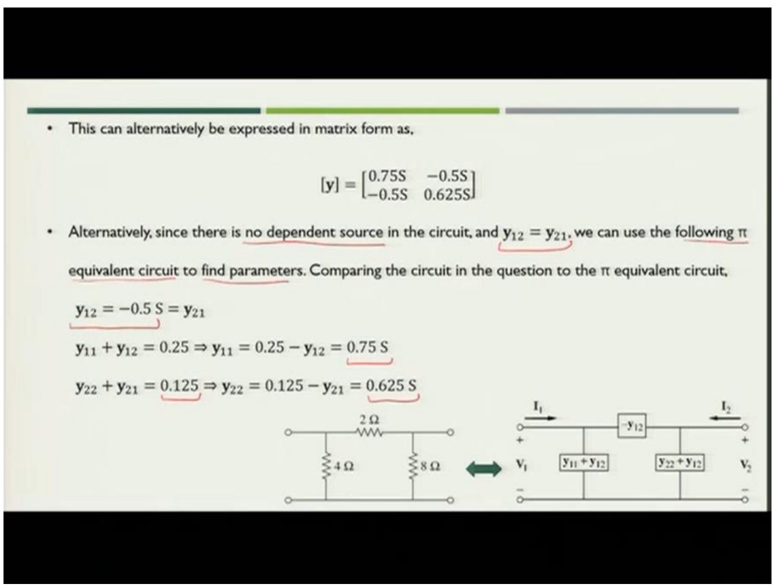

Now, if you compile the y parameters which you have just calculated, the y matrix will be as shown

$$[\mathbf{y}] = \begin{bmatrix} 0.75\text{S} & -0.5\text{S} \\ -0.5\text{S} & 0.625\text{S} \end{bmatrix}$$

Now, if you see this matrix you can easily say y 12 is equal to y 21. Now since we do not have any dependent source in the circuit, we can say that the circuit is reciprocal so in that case, what we can do? We can use the pi equivalent circuit to find the parameters directly.

So, if you compare the given circuit with equivalent, pi equivalent of the circuit what you can now figure out is that, the value of $\mathbf{y}_{12} = -0.5\text{S} = \mathbf{y}_{21}$. Now, next is we need to find $\mathbf{y}_{11} + \mathbf{y}_{12} =$

$0.25 \Rightarrow \mathbf{y}_{11} = 0.25 - \mathbf{y}_{12} = 0.75S$. Now, if you put the value of y 12 here, you will get simply the value of y 11 as 0.75 Siemens. Now, next is if you see the right left, the value of $\mathbf{y}_{22} + \mathbf{y}_{21} = 0.125 \Rightarrow \mathbf{y}_{22} = 0.125 - \mathbf{y}_{21} = 0.625S$.

So, using the circuit pi equivalent also you can find the value of y parameters as we saw in case of finding out the y parameters using equations. In this way if you know that the circuit is reciprocal without going into the details of y parameter equations and then finding out the value of y parameters. You can directly use the pi equivalent circuit and find out the value of y parameters.

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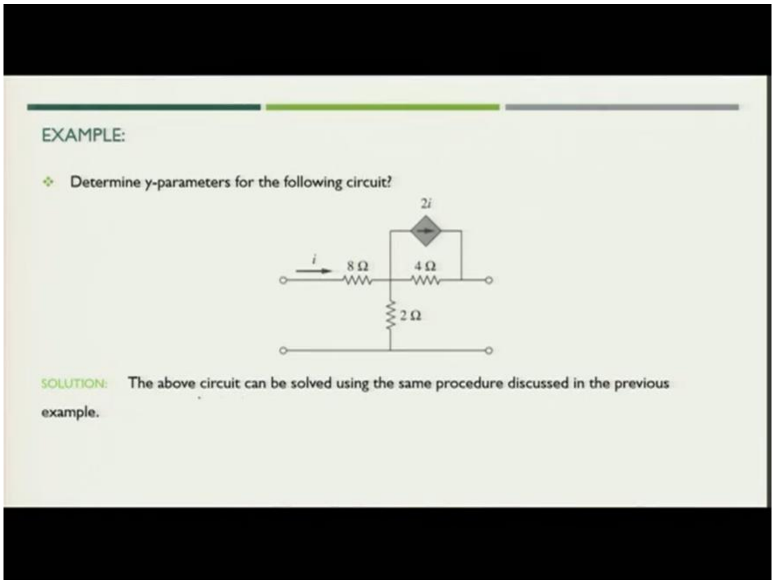

Now, next let us see another example, here the circuit is having one dependent current source, so we will use the parameter equations to find out the y parameters and we will justify that when you have the dependent source the circuit will not be reciprocal.

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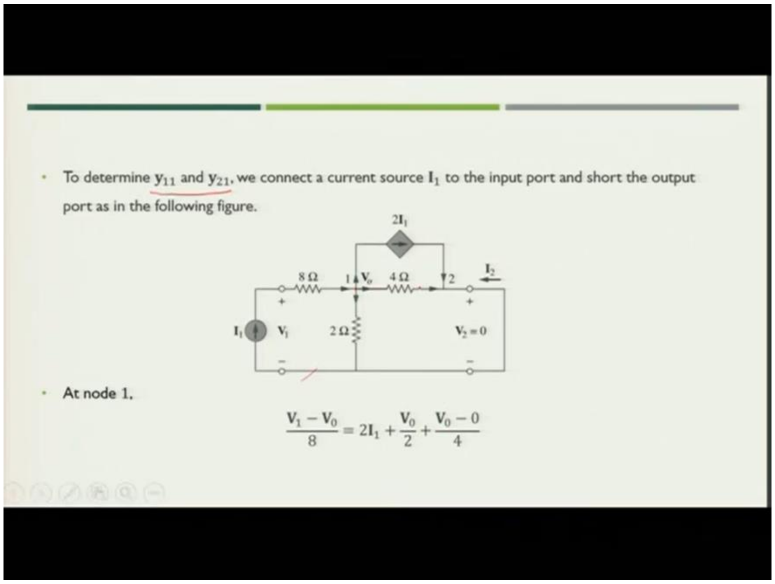

We will again apply the same procedure which we did in the previous example, we will first apply current source that is $\mathbf{I}_1$ at the input port and we will short circuit the output port that is the right side port of the two-port network and we will find out the values of admittance parameters. So, when you have this kind of circuit arrangement you will be able to get the values of $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$. Now, if you see this circuit, let us assume that the voltage at this node is $\mathbf{V}_0$, so what we will do? We will apply KCL at this node and try to solve the equation.

So, if you apply KCL at this node you will see that current flowing in 8 ohm resistance will be $\frac{\mathbf{V}_1-\mathbf{V}_0}{8}$. So this is going, this is the current going inside the node, others are going outside, coming outside of the node so the summation of all 4 currents will be 0 at this node. So using that KCL theorem we will the particular law we can utilize the equation and find out the values of I 1.

So, what we can write? We can write,

$$\frac{\mathbf{V}_1 - \mathbf{V}_0}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_0}{2} + \frac{\mathbf{V}_0 - 0}{4}$$

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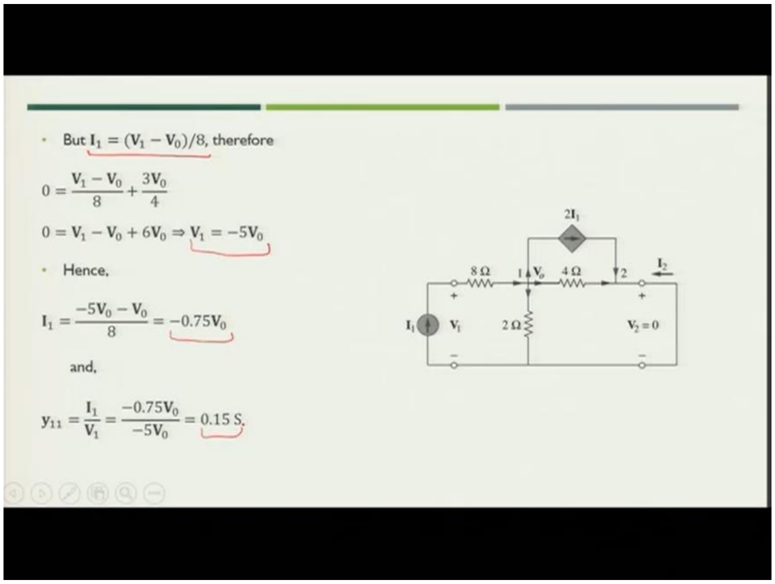

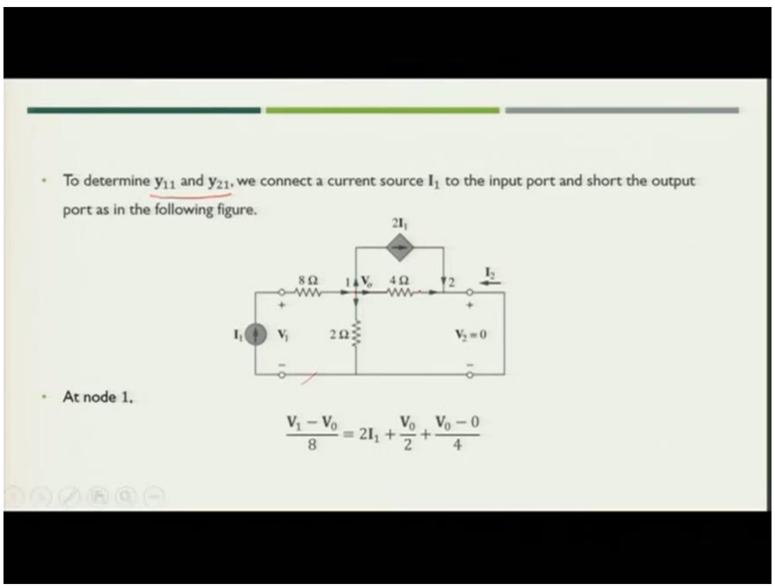

Now if you simplify, you will get the value of the now, let us see another thing when you are saying that voltage across the current flowing through 8 ohm resistance is $\mathbf{I}_1$ because you are connecting the current source $\mathbf{I}_1$ at the input port. So when you connect $\mathbf{I}_1$ it means that the value of current flowing through 8 ohm resistance will be $\mathbf{I}_1$. But at the same time you can also say that the value of current $\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_0)/8$.

You can put the value of $\mathbf{I}_1$ in the previous equation and simplify when you simply you will get the value of $\mathbf{V}_1$ in terms of $\mathbf{V}_0$. You can say $\mathbf{V}_1 = -5\mathbf{V}_0$ after simplification. So, this you have got, now you can put the value of V 1 in the above equation so you can also get the

$$\mathbf{I}_1 = \frac{-5\mathbf{V}_0 - \mathbf{V}_0}{8} = -0.75\mathbf{V}_0$$

Now you know that

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{-0.75\mathbf{V}_0}{-5\mathbf{V}_0} = 0.15\text{S}$$

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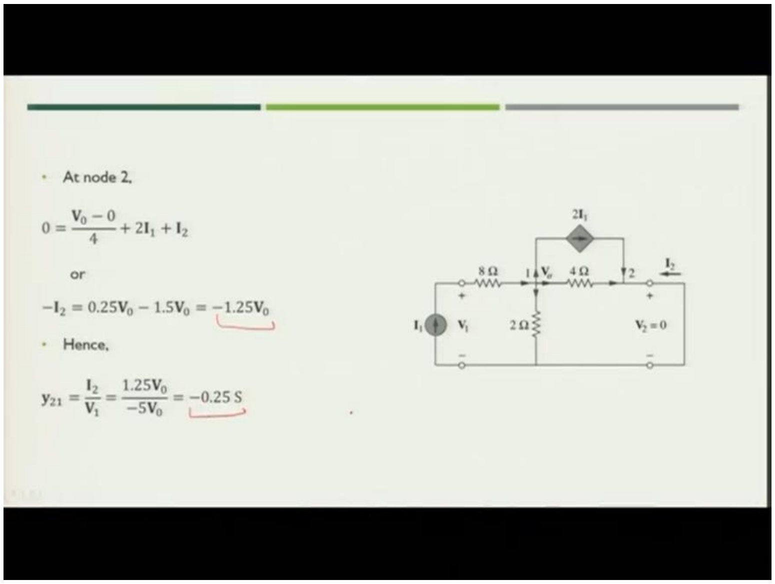

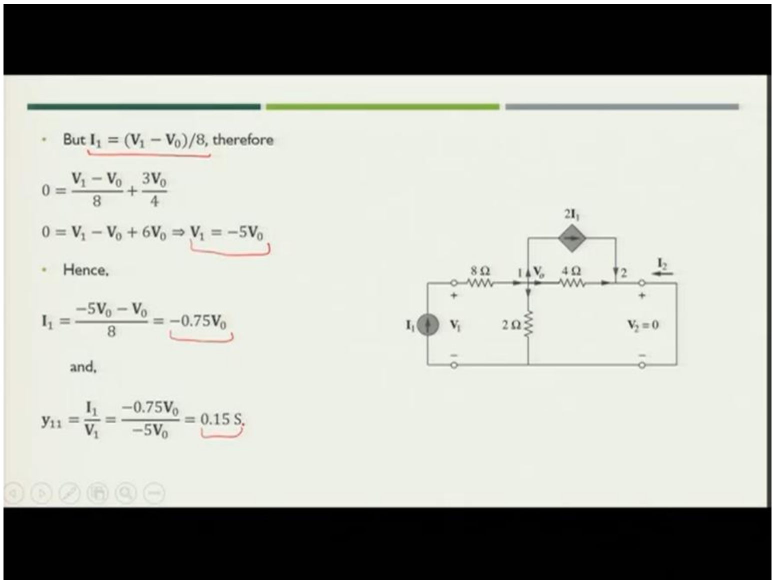

Now, next task is that you have to find the value of $\mathbf{y}_{21}$, so if you see the second node you apply KCL at this node also, what you will get?

$$0 = \frac{\mathbf{V}_0 - 0}{4} + 2\mathbf{I}_1 + \mathbf{I}_2$$

or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_0 - 1.5\mathbf{V}_0 = -1.25\mathbf{V}_0$$

- Hence,

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1.25\mathbf{V}_0}{-5\mathbf{V}_0} = -0.25\text{S}$$

So, you got y 11 and y 21 from this circuit.

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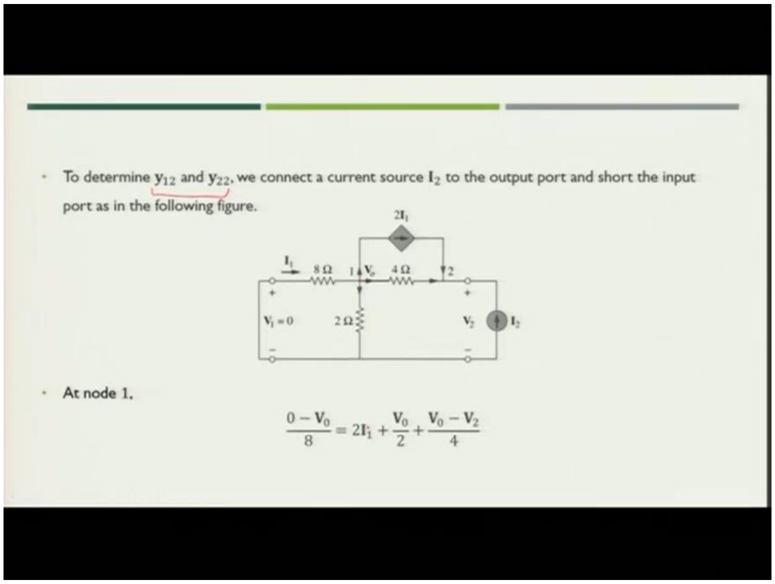

Now, next task is that we will add $\mathbf{I}_2$ as a current source at output port and short circuit the input port, so when we will do that we will get again the values of remaining Y parameters that is $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$. Now, if you see, if you apply KCL at node 1, you get,

$$\frac{0 - \mathbf{V}_0}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_0}{2} + \frac{\mathbf{V}_0 - \mathbf{V}_2}{4}$$

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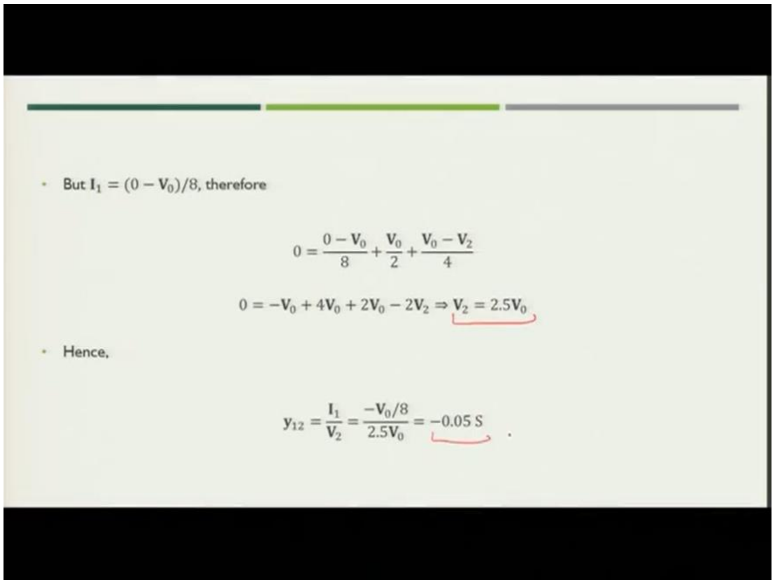

You can put the value of $\mathbf{I}_1 = (0 - \mathbf{V}_0)/8$ and when you will simplify,

$$0 = \frac{0 - \mathbf{V}_0}{8} + \frac{\mathbf{V}_0}{2} + \frac{\mathbf{V}_0 - \mathbf{V}_2}{4}$$

$$0 = -\mathbf{V}_0 + 4\mathbf{V}_0 + 2\mathbf{V}_0 - 2\mathbf{V}_2 \Rightarrow \mathbf{V}_2 = 2.5\mathbf{V}_0$$

Hence,

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\mathbf{V}_0/8}{2.5\mathbf{V}_0} = -0.05\mathrm{S}$$

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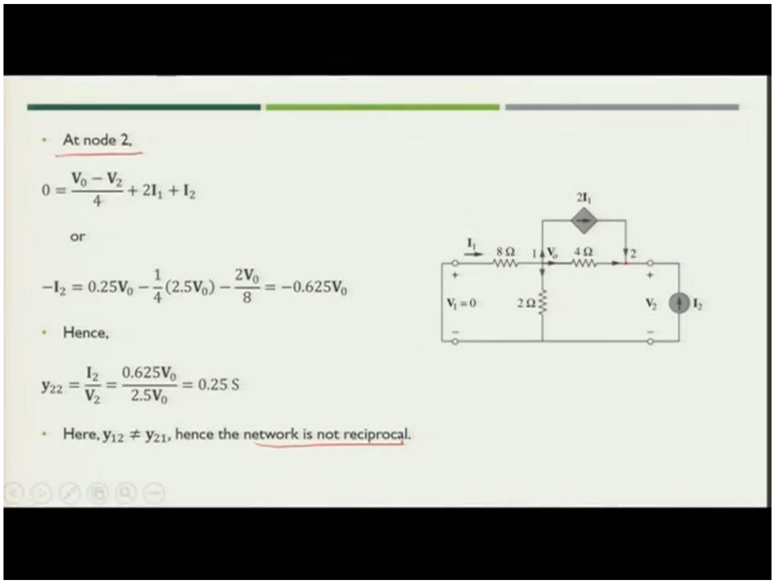

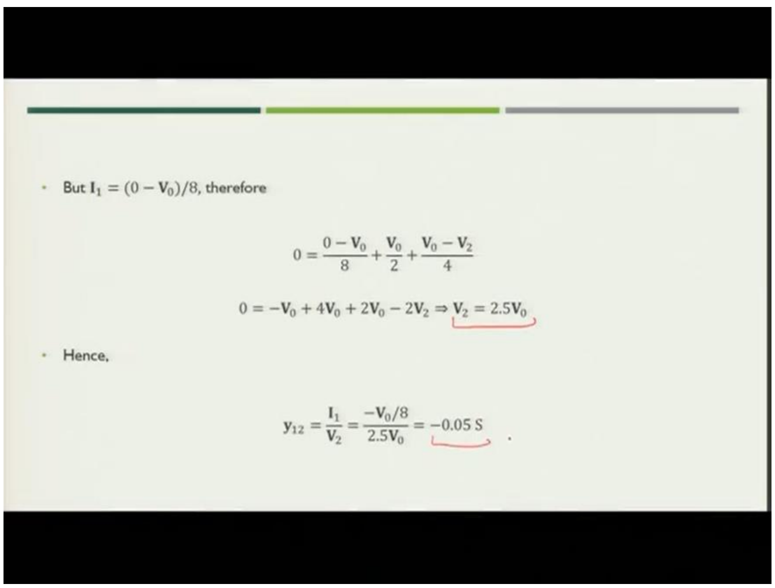

Now, at node 2, you will apply KCL so the summation of all currents is 0

$$0 = \frac{\mathbf{V}_0 - \mathbf{V}_2}{4} + 2\mathbf{I}_1 + \mathbf{I}_2$$

or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_0 - \frac{1}{4}(2.5\mathbf{V}_0) - \frac{2\mathbf{V}_0}{8} = -0.625\mathbf{V}_0$$

Hence,

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{0.625\mathbf{V}_0}{2.5\mathbf{V}_0} = 0.25\text{S}$$

Now in this case if you see y 12 is not equal to y 21 because y 12 you have calculated as (point) minus of 0.05 Siemens, while y 21 is -0.25 Siemens, so you can say that this particular circuit is not reciprocal. So with this we can close our today's session in which we discussed about the admittance parameters. Thank you.