Basic Electric Circuits
Professor Ankush Sharma
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module 8
Two Port Network
Lecture – 36
Impedance Parameters

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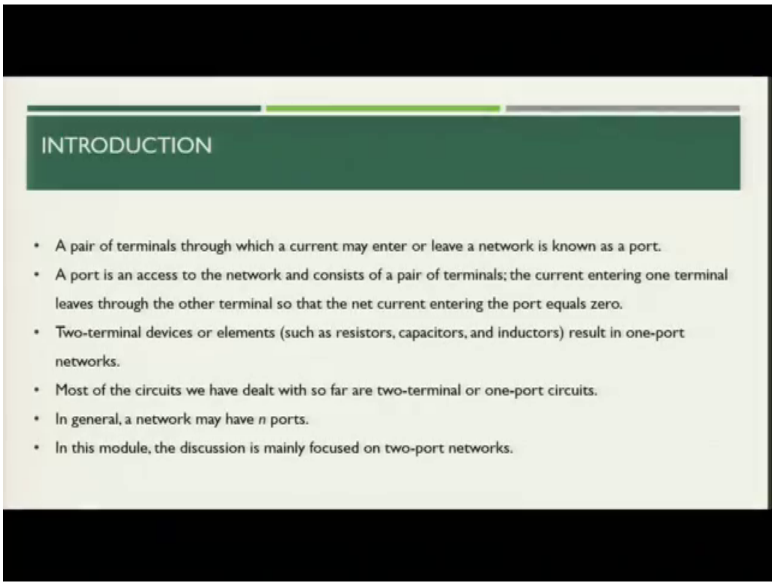

Namaskar. In this week we will start our new module that is on two port networks. Let us understand what two port networks are. Basically, the two-port network is nothing but a pair of terminals through which a current may enter or leave a network. Now, you can also say that the port is nothing but an access to a network and consists of a pair of terminals, the current entering one terminal leaves through the other terminal so that the current entering the port will be equal to zero.

Now, two terminal devices or elements which we discussed till now such as resistors, capacitors, inductors are called as a one port network. So that means that what we have discussed in the circuit like this where we have the resistances, and we try to find out the current flowing through the network. So now if you see when you create the Thevenin equivalent what you do? You generally represent it like Thevenin voltage and the Thevenin resistance. So here you will see that circuit is having two terminals. So, these kinds of circuits are called as a one port network because here we see only two terminals and two terminals will correspond to one single port.

Most of the circuit which we have discussed till now were two terminal or single port network, where we were representing them as a covenant equivalent and we were trying to find out the various parameters of the circuit. But in general, the network can have n number of ports. So, what we will do in this particular module, we will discuss mainly the two port networks.

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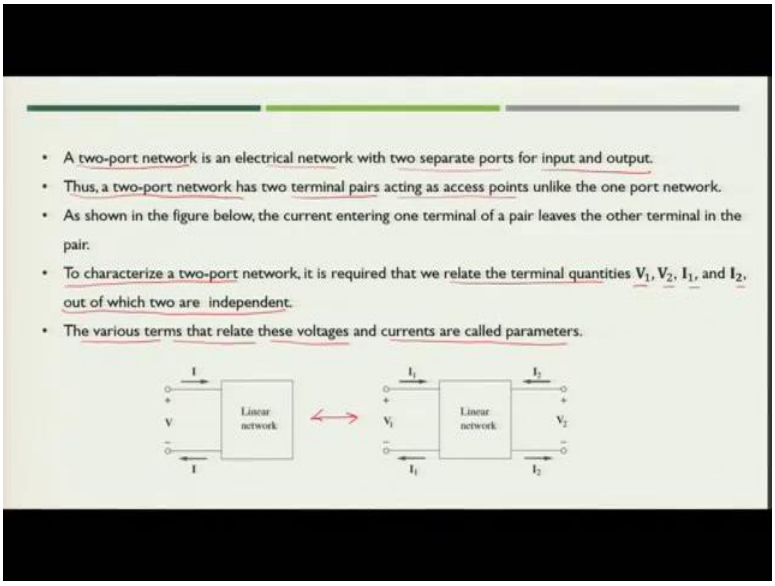

What is a two port network? Two port network is an electrical network with two separate ports for input and output. If you look at this particular figure, this is one port network where the current goes into the linear network and comes out from the other terminal and voltage is applied across these two terminals. This is basically a one port network because it is having only two terminals.

If you have pair of two terminals means you will have two ports, then what will happen that you will give current to the network say $\mathbf{I}_1$ and it will come out from the other terminal and at the other port you will give current $\mathbf{I}_2$ and it will come out from the other terminal, so what you can do you? You can either put two voltage sources on both sides or two current sources on both sides, or you can put the resistor or capacitor or inductor across the terminal and try to find out the values which are required to be calculated related to that network.

Basically, the two-port network has two terminal pairs and acting as access points like we discussed in this figure. This is the major difference between one port network and two port networks. Now, to characterize the two-port network, it is required that we relate the terminal quantities which you see in the two-port network, here $\mathbf{V}_1$ is applied across terminal pair on the left side, and $\mathbf{V}_2$ is applied across the terminal pair on the right side. $\mathbf{I}_1$ flows from port 1, and $\mathbf{I}_2$ flows from port 2.

So, what we must do? If we want to characterize the two port network we have to find out the values of the $\mathbf{V_1}, \mathbf{V_2}, \mathbf{I_1}, \mathbf{I_2}$ or since we have four in variables at least out of that you have to make 2 as an independent quantity so that you can find out the circuit parameters. Now, when you say circuit parameters basically the various terms that relate to these voltages and currents are called circuit or network parameters.

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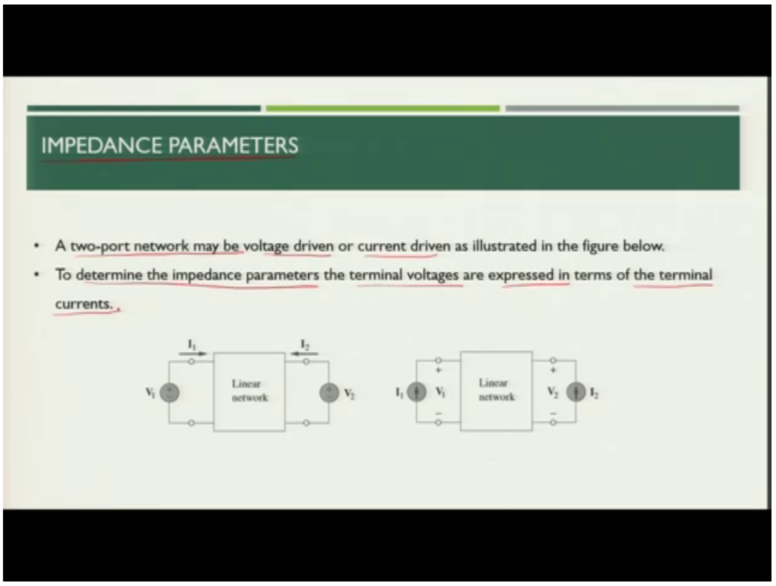

First, we will try to understand the impedance parameters. What are those impedance parameters? Now, the two-port network may be voltage driven or current driven as we can see from the figure. Here at both sides, we are applying the voltage source and the current is flowing to the network or we can apply current source at both sides so that it flows inside the network. Now, to determine the impedance parameters the terminal voltages are expressed in terms of their terminal current.

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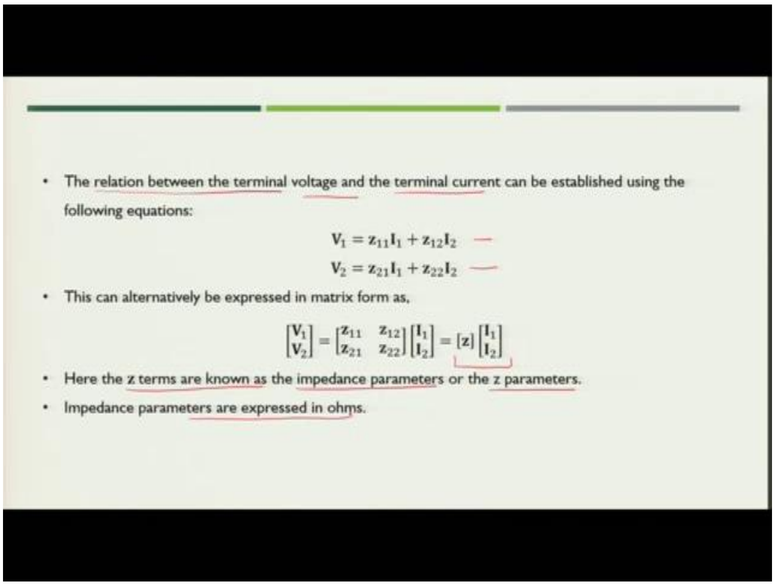

If you see this particular figure, we need to find out the relationship between terminal voltage and terminal current. Suppose there are some parameters $\mathbf{z}_{11}, \mathbf{z}_{12}, \mathbf{z}_{21}, \mathbf{z}_{22}$ so what we can write? We can write,

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

In matrix form you can represent these two equations as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

We can write in short form as $[\mathbf{z}]$ and current vector. So, what you see the matrix in terms of $[\mathbf{z}]$ is called impedance parameters or you can also say the z parameters. So, since the $\mathbf{z}$ is impedance parameter, it will be represented in ohms.

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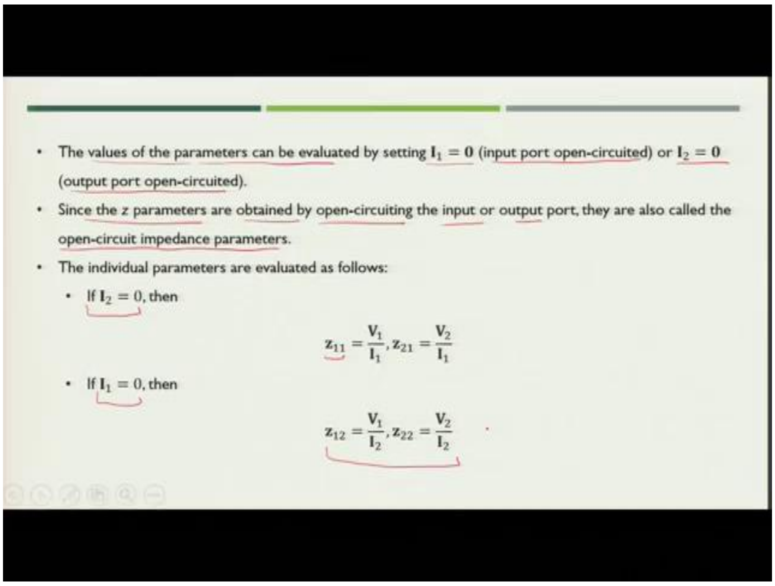

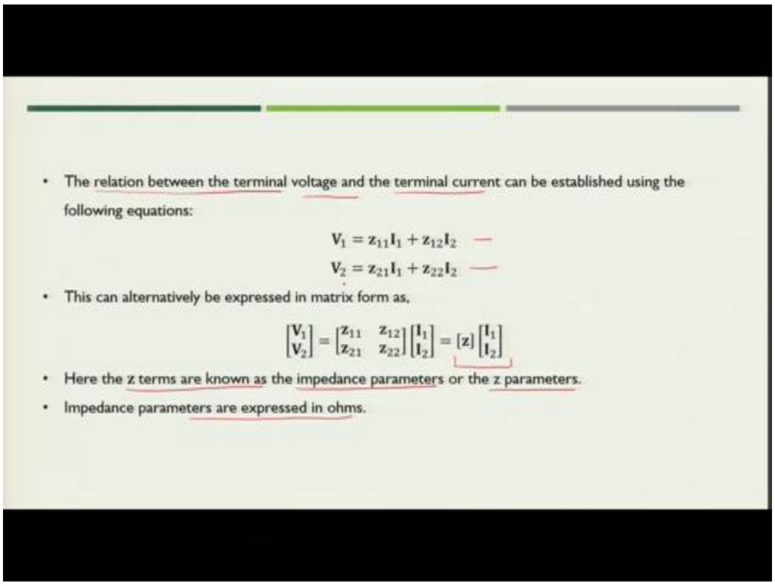

The value of parameters can be calculated by setting up either current $\mathbf{I}_1 = 0$ or current $\mathbf{I}_2 = 0$ that means first you will make input port open circuit means $\mathbf{I}_1 = 0$ or output port open circuited means $\mathbf{I}_2 = 0$. Since these z parameters are obtained by open circuiting either input or output ports, they are also called as open circuit impedance parameters.

Now, let us see how we will evaluate the values of the Z quantities which we have seen in the matrix. Those are $\mathbf{z}_{11}, \mathbf{z}_{12}, \mathbf{z}_{21}, \mathbf{z}_{22}$. So, if you put the first current $\mathbf{I}_2 = 0$ in this equation, so if you set $\mathbf{I}_2 = 0$, you will get $\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$ and $\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$.

Similarly, in this equation now, if you put $\mathbf{I}_1 = 0$ what will happen that you will get the value of $\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}$ and $\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$. So, this you can calculate the z-parameters or impedance parameters of the network.

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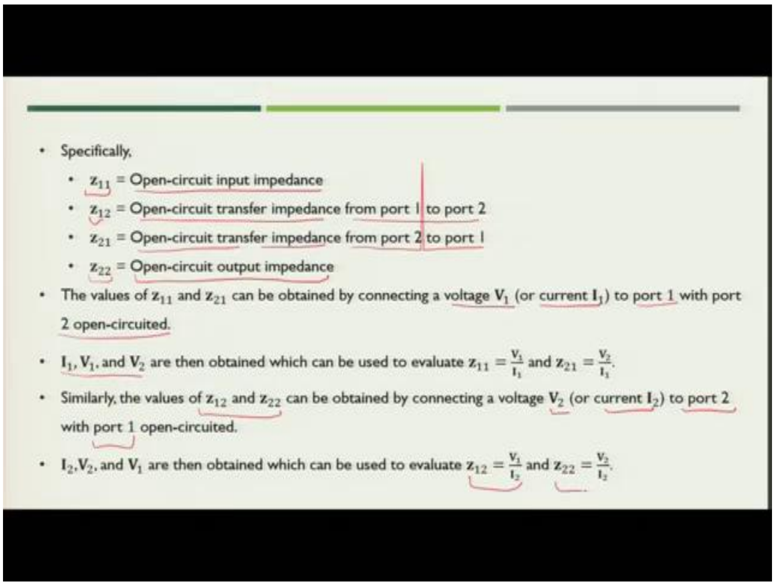

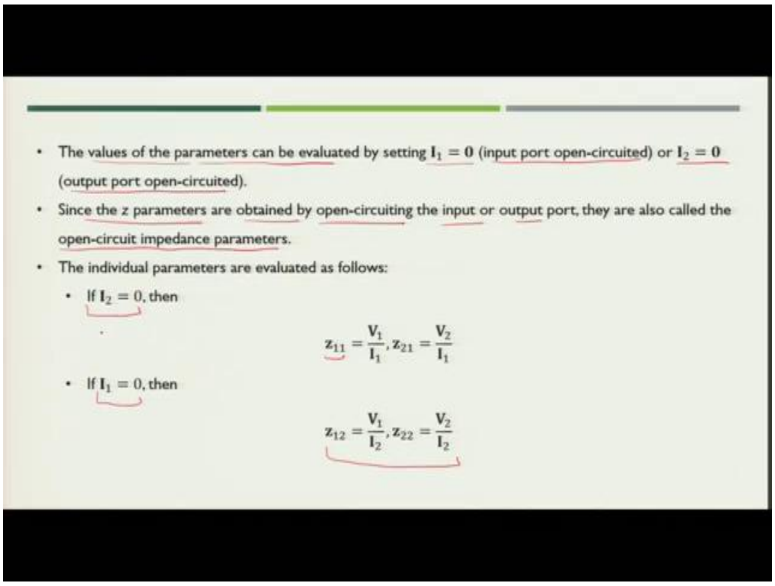

Now, if you see $\mathbf{z}_{11}$ is called as open circuit input impedance because this is calculated when you set $\mathbf{I}_2 = 0$, so you will say that this is open circuit input impedance. Similarly, $\mathbf{z}_{22}$ is open circuit output impedance. Because that is also evaluated when you set $\mathbf{I}_1 = 0$. $\mathbf{z}_{12}$ is open circuit transfer impedance from port 1 to port 2 and $\mathbf{z}_{21}$ is called open circuit transfer impedance from port 2 to port 1.

Now to evaluate $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ you can find the values by connecting a voltage $\mathbf{V}_1$ or current $\mathbf{I}_1$ to port 1 with port 2 open circuited, then what you will do? You will find the value of $\mathbf{I}_1$, $\mathbf{V}_1$, $\mathbf{V}_2$ and then obtain the value that is $\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$ and $\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$. Similarly, if you need to find the value of $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$, what you will do? You will connect voltage $\mathbf{V}_2$ or current $\mathbf{I}_2$ to port 2 and keep port 2 open circuited, then, you will find the value of $\mathbf{I}_2$, $\mathbf{V}_2$, $\mathbf{V}_1$ and based on these values you can calculate the value of $\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}$ and $\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$.

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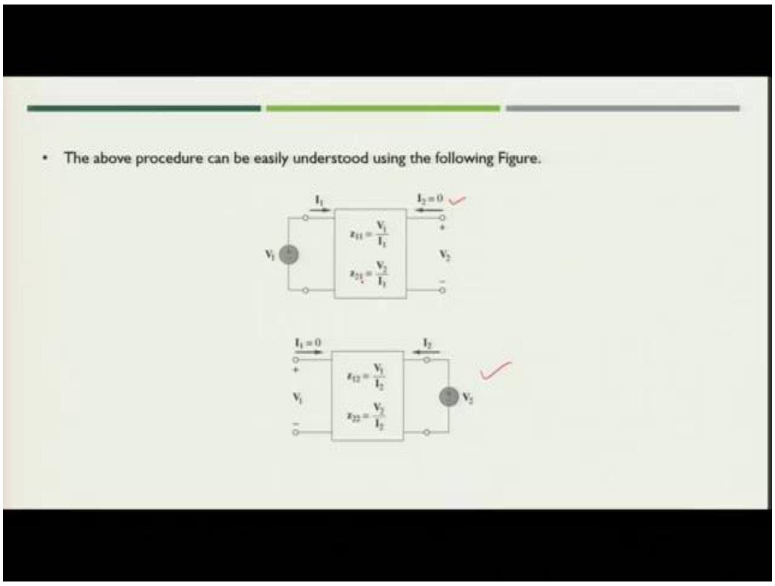

Now you can see that this procedure can be easily understood with the help of the figure. So, in case of 2 port network if you keep output port open circuit means $\mathbf{I}_2 = 0$, then you will get the value of $\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$ and $\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$. Similarly, if you keep input port as open circuited, that means that $\mathbf{I}_1 = 0$, then you will get value of $\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}$ and $\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$. So, these two figures summarise about how we can calculate the values of impedance parameters.

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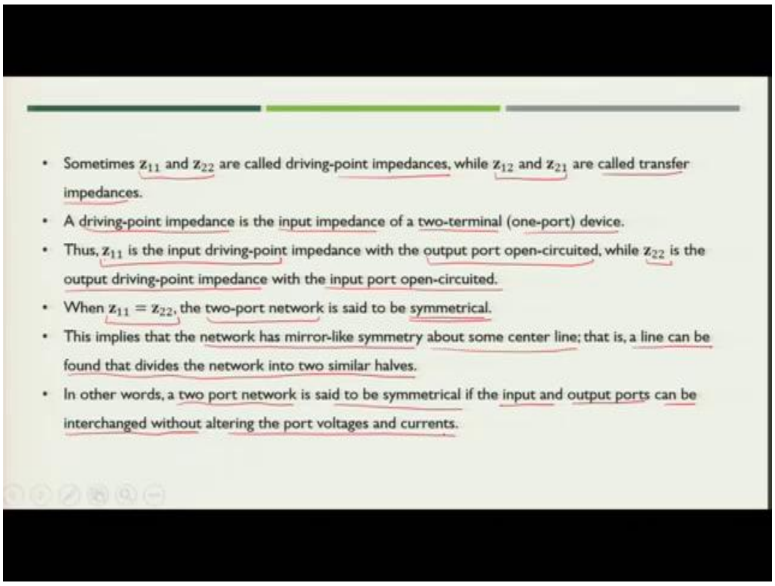

Now, sometimes the $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ are called driving point impedances and $\mathbf{z}_{12}$ and $\mathbf{z}_{21}$ are called transfer impedance. Driving point impedance is the input impedance of two terminal device. So, $\mathbf{z}_{12}$ is the input driving point impedance with the output port open circuited, while you can say that $\mathbf{z}_{22}$ is the output driving point impedance with input port open circuited.

Now in case, when you see that $\mathbf{z}_{11}$= $\mathbf{z}_{22}$, it means that the two-port network is symmetrical. It means that, the network has mirror like symmetry about some center line, so, what would be the center line? Center line would be a line that can be found that divides the network into two similar halves.

So, what you can say that the two-port network is said to be symmetrical if the input and output ports can be interchanged without altering port voltages and currents. Since $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ are equal, so you will say that the network is symmetrical and because of the values that is input driving point impedance for both sides are same means the source or the voltage or current on both sides can be interchanged without any change in the circuit parameters.

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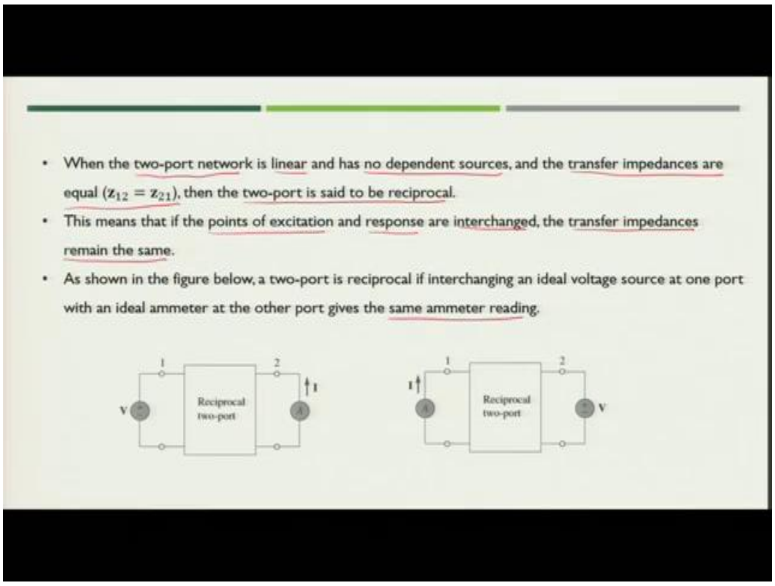

Now, another criterion which we have to keep in mind is that the two port network is linear and has no dependent source. So, if this is the condition and the transfer impedances are equal that is $\mathbf{z}_{12} = \mathbf{z}_{21}$, it means that we can say that two port is reciprocal. What does it mean? It means that the points of excitation and response can be interchanged, but the transfer impedance will remain same.

So, we can understand this with the help of this figure in which at one port that is input port we are applying the voltage and at the output port we are adding the Ammeter to find out the value of current. So, if it is reciprocal two port network, then if you interchange the locations of voltage source and the ammeter will give you same reading. So, in that way you can say that the circuit is a reciprocal circuit. And for that, the condition which we must verify is that first it has to be linear with no dependent source, then transfer impedances should be same; that is $\mathbf{z}_{12} = \mathbf{z}_{21}$.

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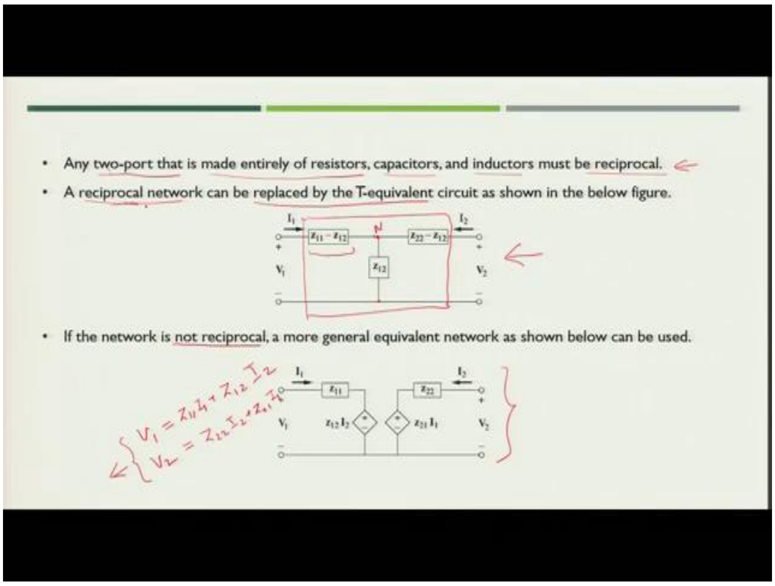

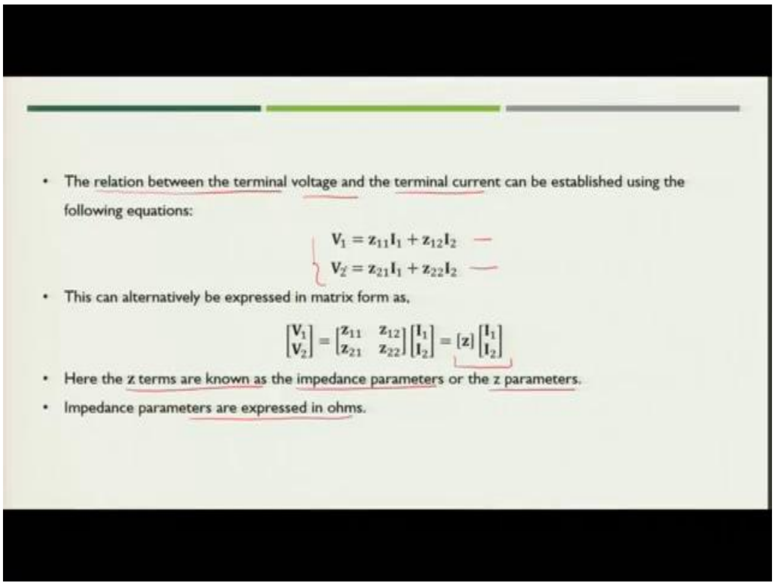

Now any two network, two port network that is made entirely of resistors, capacitors and inductor must be reciprocal. So, this is another important property which you must keep in mind and when you have a reciprocal network, you can replace that network by an equivalent T-circuit. So, if you see this T-circuit, so the unknown that is basically the network which is there inside the box can be equivalently represented by a T-equivalent circuit where VI I1 are the input voltage and input currents and V2 I2 are the output voltage and output current at the other port.

Now, what would be the value of T equivalent circuit parameters? The values of impedances for T equivalent would be like in the left side you will see there will be the value of Z11 minus Z12 that is the first leg of T, then on the right you will see that is Z22 minus Z12 that is the right leg of the T and then the branch that is Z12, which is connecting the node which is at the centre and the reference node. So, with this you will see that you can represent a network which is reciprocal into a T-equivalent circuit or you can also say this is Y-equivalent circuit because Y or T are used interchangeably in the literature, but most of the time you will say that it is considered as a T-equivalent.

Now, if the network is not reciprocal is still with the help of the equations which we saw in the previous slides that is $\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$ and $\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$. So, these two equations can be represented with the help of a equivalent circuit. So, here, if you see you have one impedance that is called $\mathbf{z}_{11}$ and in series with you will have one dependent voltage source that is having the value of $\mathbf{z}_{12}\ \mathbf{I}_2$, so what would be the value of $\mathbf{V}_1$?

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

From this side, if you check that V2 is applied across the port, current I2 is flowing across the through the impedance Z22 and there is a dependent voltage source having value Z21 I1. So now, if you want to write the loop equation for this particular port so you can write V2 is nothing but Z22 I2 plus Z21 I1. So, these are the same equations which we just saw in the previous slide, so that means that any two-port network can be equivalently represented with the help of the circuit shown in the slide.

So, here it need not to be reciprocal because this is a generic equivalent network having Z parameters, while the figure which we saw above the generic circuit is T-equivalent circuit which is only applicable when your network is reciprocal. So, these are the two commonly used circuits based on the specific conditions in the analysis of circuit.

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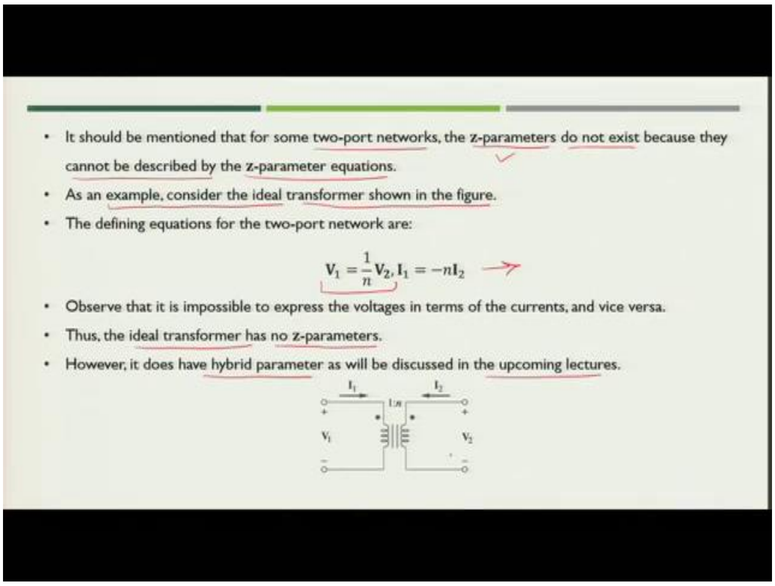

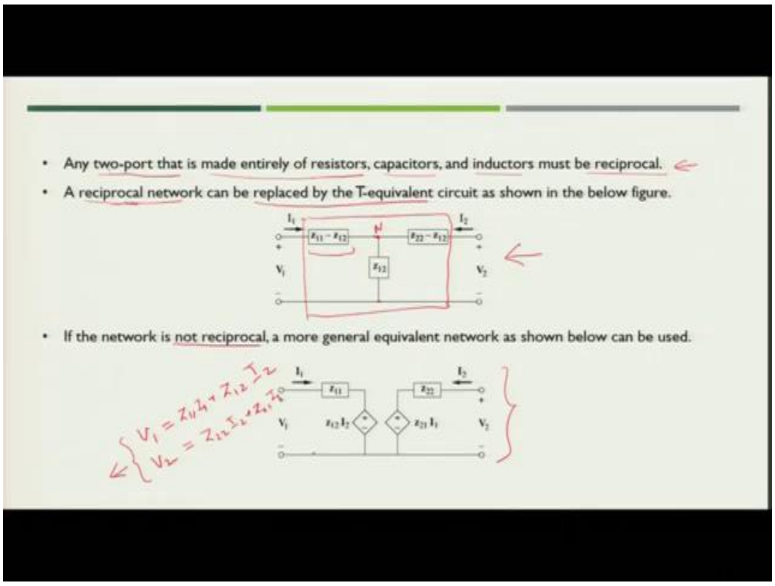

Now, few more things which we must understand in for two-port network. The Z parameters does not exist for some of the two-port networks because they cannot be described by the Z parameter equations which we saw. So, if you cannot represent the two-port network with the help of these two Z parameter equations it means that those two port networks can be represented in the form of, cannot be represented in the form of Z parameters.

Now, what can be the example? If you consider an ideal transformer, if you see in the figure below it is an ideal transformer having some trans ratio 1 is to n, so what you can write? You can write V1 is nothing but 1 by n of V2 and I1 is minus n of I2. Now, if you see these two equations you cannot represent these two equations in the form of Z parameter equations. So, that is why in case of ideal transformers we cannot calculate the Z parameters. But we can have the hybrid parameters which we will discuss in the next lectures. So, we have to keep in mind that not all circuits can be converted into the equivalent Z parameters to a few of them are like we discussed with the help of ideal transformer example.

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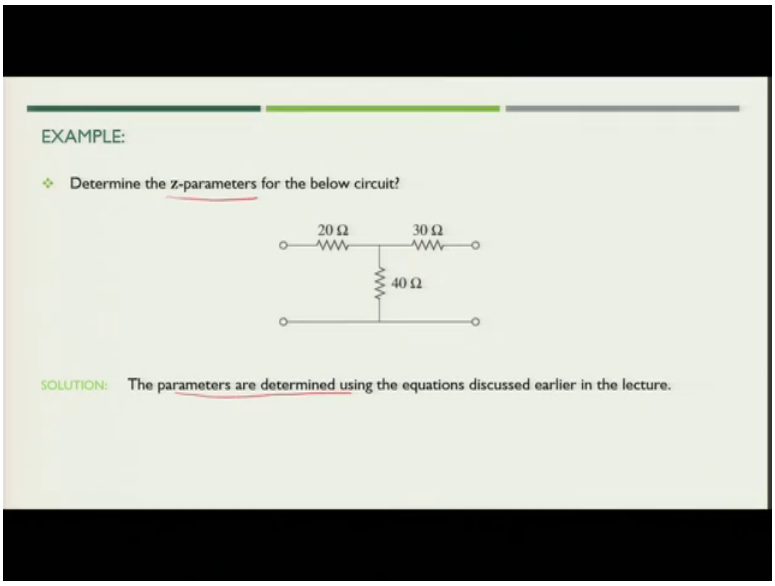

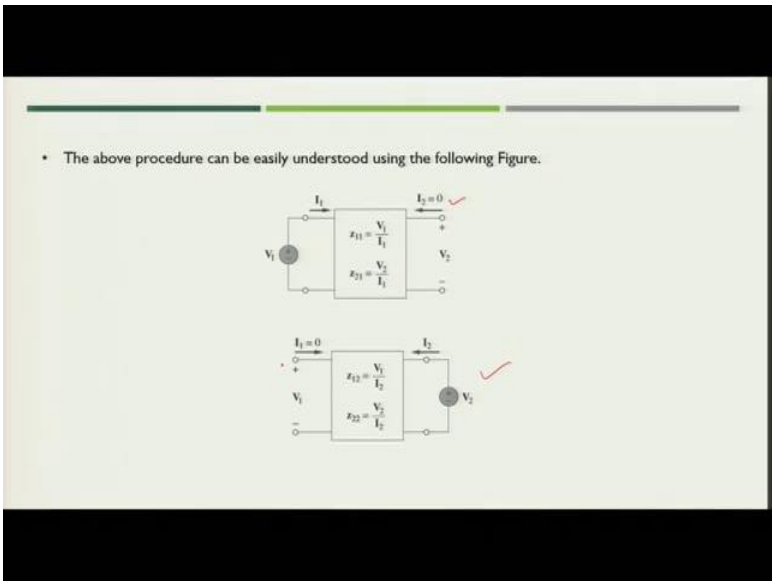

Now, let us understand the concept which we discussed related to Z parameters with the help of few examples so that you can understand the concept more clearly. Let us see the circuit which is given in the figure we need to find out the Z parameters for the circuit shown in the figure. Now what we have to do? We have to first use the circuit which we discussed previously, so what are those two circuits? First is you create the circuit when you put I2 is equal to 0, in second you put I1 equal to 0 and you will find out the value of Z parameters.

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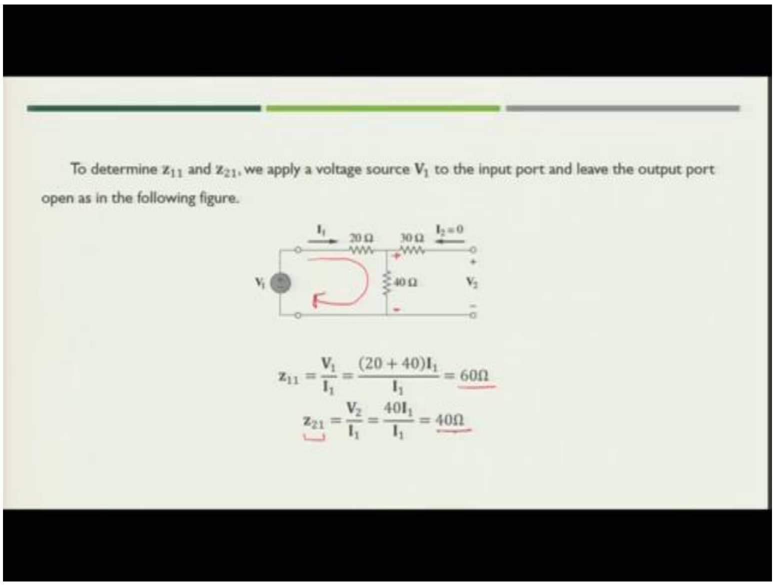

So, what we are doing here? So, in the input port we are applying V1 voltage and we will find out the current I1, while at the other end V2 is there but current I2 is 0.

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{(20 + 40)\mathbf{I}_1}{\mathbf{I}_1} = 60\Omega$$

Now for $\mathbf{z}_{21}$,

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40\mathbf{I}_1}{\mathbf{I}_1} = 40\Omega$$

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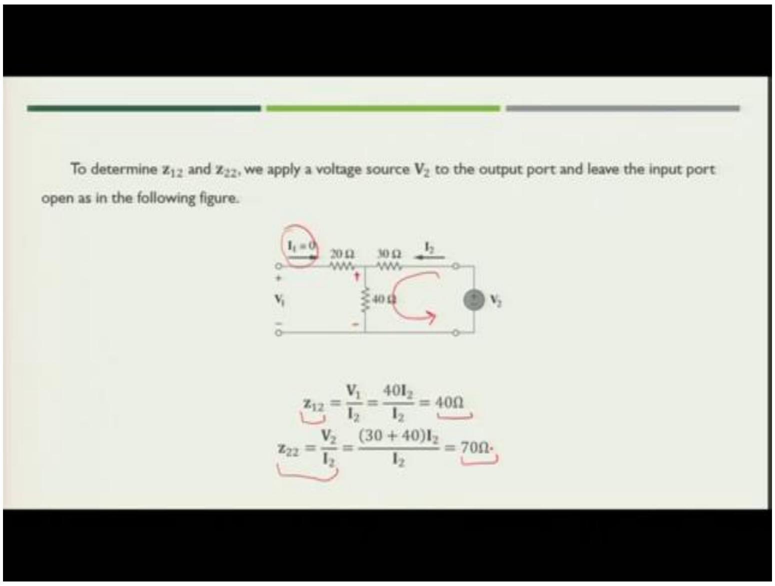

Now, next what you have to do? You must keep input port as open circuit and apply voltage V2 across the output port and we measure the value of I2. So, what would be the value of Z12? Since, I1 is equal to 0, the value of V1 would be across again the 40 ohm resistance.

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{40\mathbf{I}_2}{\mathbf{I}_2} = 40\Omega$$

Now, next is you need to find out the value of $\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$.

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{(30+40)\mathbf{I}_2}{\mathbf{I}_2} = 70\Omega$$

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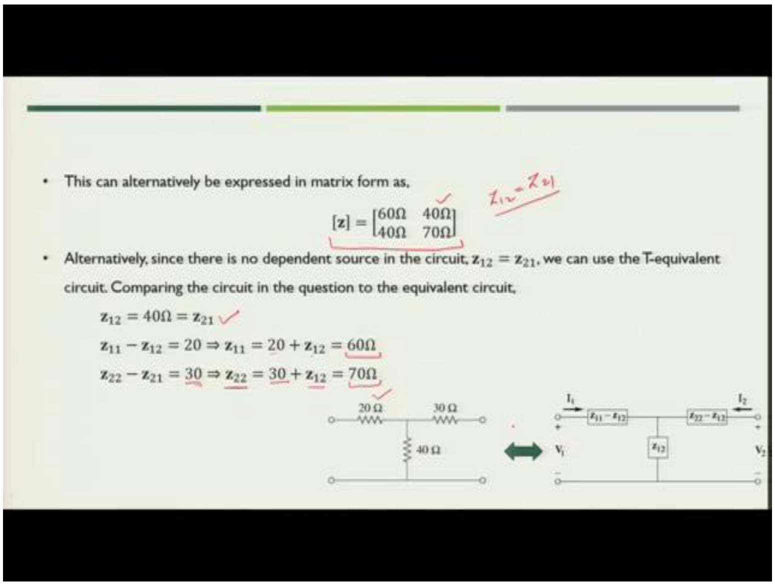

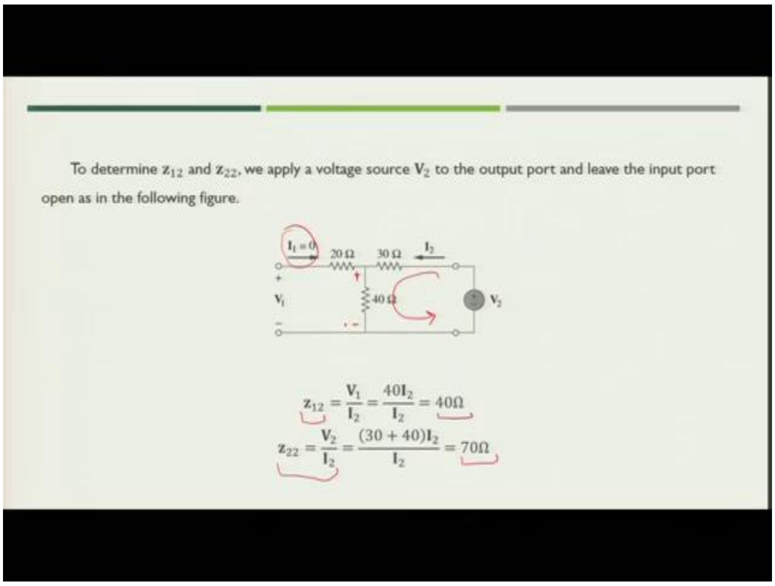

So, now you can compile the Z matrix which is,

$$[\mathbf{z}] = \begin{bmatrix} 60\Omega & 40\Omega \\ 40\Omega & 70\Omega \end{bmatrix}$$

Now, if you see this particular matrix, you will see that $\mathbf{z}_{12} = 40\Omega = \mathbf{z}_{21}$, what does it mean? It means that you can represent this circuit as T-equivalent. Anyway, this circuit itself is represented as T-equivalent, so this is reciprocal circuit and when we compare with the T-equivalent circuit with the circuit given in the example you can easily find out the value of the Z parameters.

How? Now, if you compare these two circuits you can say that $\mathbf{z}_{11} - \mathbf{z}_{12} = 20$ and $\mathbf{z}_{12} = 40\Omega$. So $\mathbf{z}_{11} = 20 + \mathbf{z}_{12} = 60\Omega$.

Finally, the value of $\mathbf{z}_{22} - \mathbf{z}_{21} = 30$. So, what you will get? You will get,

$$\mathbf{z}_{22} = 30 + \mathbf{z}_{12} = 70\Omega$$

So, you can use either the Z parameter equations to find out the Z parameters, or you, if you know that the circuit is, a circuit can be represented as a T-equivalent, so you can compare the circuit with T-equivalent circuit and straight away you can find out the value of Z parameters.

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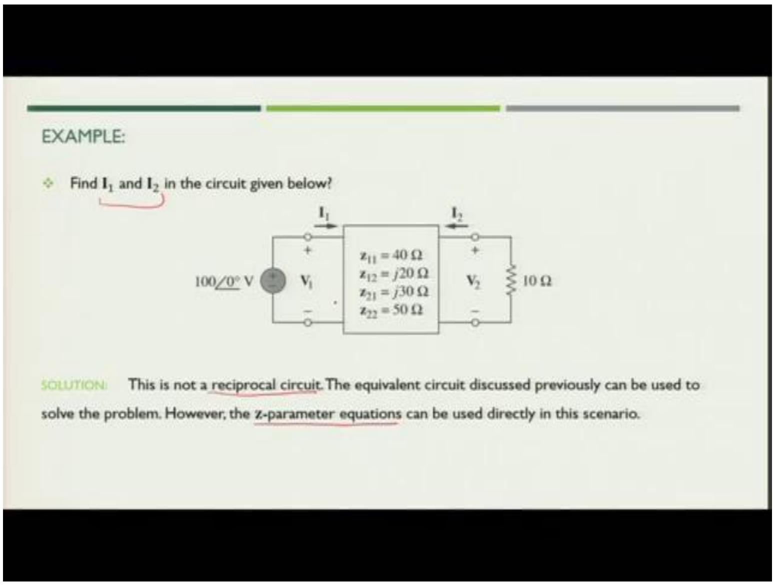

Now, let us take another example. Suppose if we are asked to find out the value of $\mathbf{I}_1$ and $\mathbf{I}_2$, the circuit, the Z parameters are given and if you see in this case $\mathbf{z}_{12}$ is not equal to $\mathbf{z}_{21}$, so that means the circuit is not reciprocal. So, what we have to do? We have to use the Z parameter equations to find out the values of $\mathbf{I}_1$ and $\mathbf{I}_2$.

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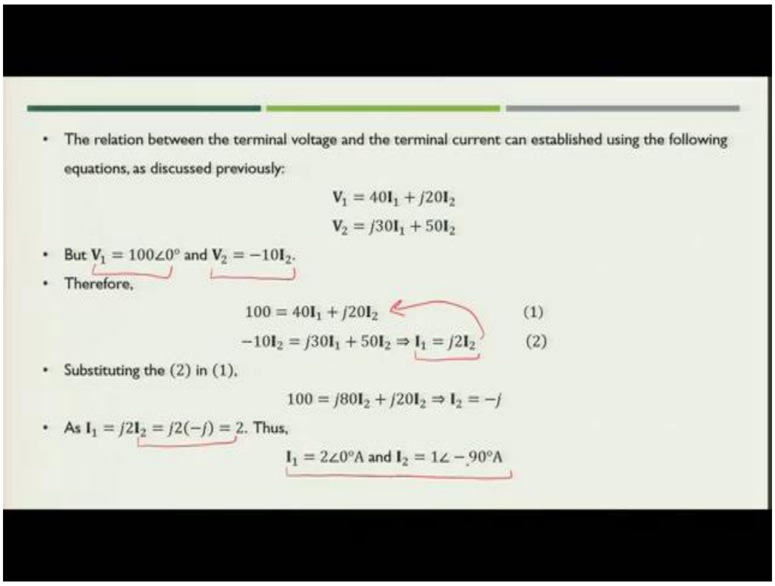

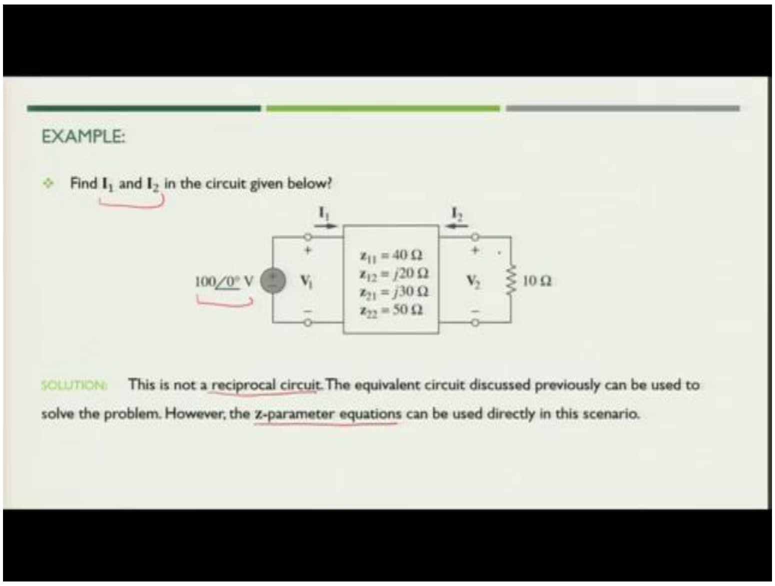

Now, if we write the circuit parameter equations that is Z parameter equations. So,

$$\mathbf{V}_1 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$

$$\mathbf{V}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2$$

Now, if you see the circuit having voltage $\mathbf{V}_1 = 100\angle 0°$ and $\mathbf{V}_2 = -10\mathbf{I}_2$ because $\mathbf{I}_2$ is in this direction. So, these two values we can get from the circuit, so we can put these values in the above equation, so we get,

$$100 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$

$$-10\mathbf{I}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \Rightarrow \mathbf{I}_1 = j2\mathbf{I}_2$$

Now if you put the value of $\mathbf{I}_1$ in this equation that is equation 1, you can say that

$$100 = j80\mathbf{I}_2 + j20\mathbf{I}_2 \Rightarrow \mathbf{I}_2 = -j$$

and when you put the value of $\mathbf{I}_2$ in the condition which we just found

$$\mathbf{I}_1 = j2\mathbf{I}_2 = j2(-j) = 2.$$

So, what we can write? $\mathbf{I}_1 = 2\angle 0°$A and $\mathbf{I}_2 = 1\angle -90°$A. This way you can calculate the value of $\mathbf{I}_1$ and $\mathbf{I}_2$. So now, we close the today's session here. We will discuss few more parameters in the next few lectures. Thank you.